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Double parton correlations and constituent quark models: a Light Front approach to the valence sector

M. Rinaldi[a] **S. Scopetta**[a] **M. Traini**[b] **V. Vento**[c]

[a] *Dipartimento di Fisica e Geologia, Università degli Studi di Perugia, and INFN, sezione di Perugia,*
via A. Pascoli 06100 Perugia, Italy

[b] *Dipartimento di Fisica, Università degli studi di Trento, and INFN - TIFPA,*
Via Sommarive 14, I - 38123 Povo (Trento), Italy

[c] *Departament de Fisica Teòrica, Universitat de València and Institut de Fisica Corpuscular, Consejo Superior de Investigaciones Científicas,*
46100 Burjassot (València), Spain

E-mail: matteo.rinaldi@pg.infn.it, sergio.scopetta@pg.infn.it, traini@science.unitn.it, vicente.vento@uv.es

Abstract: An explicit evaluation of the double parton distribution functions (dPDFs), within a relativistic Light-Front approach to constituent quark models, is presented. dPDFs encode information on the correlations between two partons inside a target and represent the non-perturbative QCD ingredient for the description of double parton scattering in proton-proton collisions, a crucial issue in the search of new Physics at the LHC. Valence dPDFs are evaluated at the low scale of the model and the perturbative scale of the experiments is reached by means of QCD evolution. The present results show that the strong correlation effects present at the scale of the model are still sizable, in the valence region, at the experimental scale. At the low values of x presently studied at the LHC the correlations become less relevant, although they are still important for the spin-dependent contributions to unpolarized proton scattering.

Contents

1 Introduction

Multiple hard partonic collisions occurring in a single hadronic scattering, the so called multiple parton interactions (MPI), have been studied since a long time ago [1]. MPI are suppressed by a power of $\Lambda^2_{\rm QCD}/Q^2$ with respect to single parton interactions, Q being the partonic center-of-mass energy in the collision. Technically, they are therefore higher twist distributions. Despite of this, experimental evidence of these processes has been obtained already several years ago [2]. At the LHC, MPI, representing a background for the search of new Physics, are of great relevance. In recent years therefore a strong debate around MPI has arised (see Refs. [3–5] for comprehensive papers on the subject). Several dedicated workshops have been organized, starting from that illustrated in Ref. [6].

The subject of this work is related to double parton scattering (DPS). It is now understood that DPS contributes to same-sign WW and same-sign dilepton productions [7–10]. New signatures of DPS, i.e., double Drell-Yan processes, have been also identified [11]. DPS represents besides a background for Higgs studies in the channel $pp \to WH \to \ell\nu b\bar{b}$ [12–15]. Evidence of DPS at the LHC has been established [16].

In their seminal work, the authors of Ref. [1] wrote the DPS cross section in terms of double parton distribution functions (dPDFs), $F_{ij}(x_1, x_2, \vec{z}_\perp)$, describing the joint probability of having two partons with flavors $i, j = q, \bar{q}, g$, longitudinal momentum fractions x_1, x_2 and transverse separation $\vec{z}_\perp$ inside a hadron:

$$\begin{aligned} \mathrm{d}\sigma = \frac{1}{S} \sum_{i,j,k,l} \int \mathrm{d}\vec{z}_\perp\, F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) F_{kl}(x_3, x_4, \vec{z}_\perp, \mu) \\ \times \hat{\sigma}_{ik}(x_1 x_3 \sqrt{s}, \mu) \hat{\sigma}_{jl}(x_2 x_4 \sqrt{s}, \mu)\,. \end{aligned} \tag{1.1}$$

The partonic cross sections $\hat{\sigma}$ refer to the hard, short-distance processes, S is a symmetry factor, present if identical particles appear in the final state and μ is the renormalization scale. For clarity of presentation, such a scale has been taken to be the same for both partons, which is not the case in actual processes, in general.

In Eq. (1.1), contributions due to flavor, spin and color correlations between the two partons, present in QCD, are neglected, as well as parton-exchange interference contributions [4, 5, 17, 18]. Two main assumptions are usually made, for the dPDFs, in DPS analyses:
i) the dependences upon the transverse separation and the momentum fractions or parton flavors are not correlated:

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)\,; \tag{1.2}$$

ii) a factorized form is chosen also for the dependence upon x_1, x_2:

$$\begin{aligned} &F_{ij}(x_1, x_2, \mu) \\ &= q_i(x_1, \mu) q_j(x_2, \mu)\, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n\,, \end{aligned} \tag{1.3}$$

where q is the usual parton distribution function (PDF). The expression $\theta(1-x_1-x_2)(1-x_1-x_2)^n$, where $n > 0$ is a parameter to be fixed phenomenologically, introduces the kinematic constraint $x_1 + x_2 \leq 1$.

dPDFs, describing soft Physics, are nonperturbative objects. The dynamical origin of double parton correlations, having potential effects in the dPDFs and, in turn, in DPS, has been discussed in semi-inclusive deep inelastic scattering and in hard exclusive processes [19]. Positivity bounds have been obtained for polarized dPDFs [20]. Being non perturbative, dPDFs have not been evaluated in QCD. As it happens for the usual PDFs, they can be at least estimated at a low scale, $Q_0 \sim \Lambda_{\rm QCD}$, the so called hadronic scale, using quark models. The results of these calculations should be then evolved using perturbative QCD (pQCD) in order to compare them with data taken at a momentum scale $Q > Q_0$, according to a well established procedure, proposed already in Refs. [21, 22]. The evolution of dPDFs, namely the way they change from Q_0 to $Q > Q_0$, known since a long time [23, 24], is currently systematically studied [4, 5, 18, 25–33]. The result of these analyses is important to relate not only data from different experiments with each other, but also model calculations at the hadronic scale to data taken at high energy. In this way, the analysis of data involving DPS can be guided.

The first model calculation of dPDFs in the valence region has been presented in Ref. [34], in a bag model framework for the proton, at the hadronic scale Q_0, without evolution to $Q > Q_0$. In a model where the valence quarks carry all the momentum, such as the bag model, the scale Q_0 has to be taken quite low. If the bag were assumed to be rigid, as in the so-called cavity approximation [35], the quarks would be independent and none of the relevant correlations described by dPDFs would be found. In Ref. [34], therefore, a prescription is used to recover momentum conservation, already applied in model calculations of PDFs (see, e.g. [36]). In this way, quark correlations in the bag are found. The analysis of Ref. [34] has been retaken in a constituent quark model (CQM) framework in Ref. [37]. CQM calculations of parton distributions have been proven to be able to predict the gross features of PDFs [38–40], generalized parton distributions (GPDs)

[41] and transverse momentum dependent parton distributions (TMDs) [42–44]. Similar expectations motivated the analysis of Ref. [37]. With respect to the approach of Ref. [34], the non relativistic (NR) dynamics includes from the very beginning correlations into the scheme. The main result of Refs. [34] and [37] was that Eq. (1.3) holds reasonably well, but Eq. (1.2) is strongly violated. Actually, problems with Eq. (1.2) had already been pointed out in Refs. [3, 45, 46] on a general ground. In the CQM picture of Ref. [37], the fact that correlations are naturally included helped to understand their dynamical origin, for example that of the breaking of the approximation Eq. (1.2). To have predictions in different models is important of course to understand which, among the features of the results, are the model dependent ones.

Both the analysis of Refs. [34] and [37] are somehow incomplete. The missing items are mainly two. First of all, for different reasons, they both lead to the so called "bad support" problem. This means that dPDFs are not vanishing in the forbidden kinematic region, $x_1 + x_2 > 1$. In the bag model this is due to the lack of momentum conservation, i.e., proton states are not momentum eigenstates. In the CQM calculation this is due to the impossibility to treat correctly the off-shellness of the interacting partons. Secondly, as already stressed, the results of Refs. [34] and [37] are valid at the hadronic scale and only in the valence quark region. For these studies to be directly used in the LHC data analysis, having for the moment high statistics only for small values of x, far from the valence region, corresponding to high momentum transfer, additional studies, in particular the pQCD evolution of the model results, are necessary.

The present work is a step towards an improvement in both the items listed above. This is obtained within a relativistic, fully Poincaré covariant Light-Front approach (for comprehensive reports see, e.g., [47, 48]). In this framework, successfully applied in Hadronic Physics in general (see, e.g., [49]) and for the calculation of parton distributions in particular (see, e.g., [50–56]), the active particles are on-shell and the "bad-support" problem does not arise. As it will be shown, this fact helps to recover the symmetries of the results, expected on general grounds, and to properly evaluate Mellin moments of the obtained distributions, for a formally correct pQCD evolution of the results. This last procedure is also performed, for the valence sector only, for the moment being. This represents an important step towards a proper treatment of the second issue listed here above.

The paper is structured as follows. In the next section, the formalism to evaluate the dPDFs in a LF CQM is illustrated and the main equations presented. In the third one, results at the hadronic scale, obtained within the relativistic hypercentral CQM [50], already used for the calculation of parton distributions in Refs. [50–56], are presented. In the fourth section, the pQCD evolution of the calculated dPDFs is illustrated and discussed. Eventually, conclusions are drawn in the last section.

2 Double parton distributions in a Light-Front Constituent quark model

In this section, the main features of the calculation of the dPDFs within a LF CQM are described. A quantitative analysis of the validity of the approximations Eqs. (1.2) and (1.3) in this framework will be therefore possible. Among the forms of relativistic dynamics, the LF is the one with maximum number of kinematical, interaction independent generators [57]. In particular, LF boosts are kinematical and boost-invariant states can be defined, which makes the LF form very convenient when high momentum transfers are present, such as in DIS processes, where light-cone Physics naturally arises. The Poincaré covariance obtained in a LF approach allows to preserve the symmetries of the problem. For a comprehensive introduction to LF methods see, for example, Refs [47, 48].

Let us introduce now formally the dPDFs, the subject of our study. As in the model calculations of Refs. [34, 37], the quantity of interest will be the Fourier- transformed dPDF in momentum space, $F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{k}_\perp)$:

$$F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{k}_\perp) = \int d\vec{z}_\perp \, e^{\mathrm{i}\vec{z}_\perp \cdot \vec{k}_\perp} F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{z}_\perp) \ , \tag{2.1}$$

where the coordinate-space expression $F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{z}_\perp)$ reads, in terms of Light-Cone (LC) quantized fields q_i for a quark of flavor i, helicity λ_i and LC normalized proton states, for an unpolarized proton, as follows (see, e.g., [5])

$$\begin{aligned} F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{k}_\perp) = & \,(-8\pi P^+)\frac{1}{2}\sum_\lambda \int d\vec{z}_\perp \, e^{\mathrm{i}\vec{z}_\perp \cdot \vec{k}_\perp} \\ & \times \int \left[\prod_l^3 \frac{dz_l^-}{4\pi}\right] e^{ix_1 P^+ z_1^-/2} e^{ix_2 P^+ z_2^-/2} e^{-ix_1 P^+ z_3^-/2} \\ & \times \langle \lambda, \vec{P} = \vec{0} | \hat{\mathcal{O}}_i^1 \left(z_1^- \frac{\bar{n}}{2}, z_3^- \frac{\bar{n}}{2} + \vec{z}_\perp\right) \hat{\mathcal{O}}_j^2 \left(z_2^- \frac{\bar{n}}{2} + \vec{z}_\perp, 0\right) | \vec{P} = \vec{0}, \lambda \rangle \ , \end{aligned} \tag{2.2}$$

where, for generic 4-vectors z and z', the operator

$$\hat{\mathcal{O}}_i^k(z, z') = \bar{q}_i(z)\hat{O}(\lambda_k)q_i(z') \tag{2.3}$$

has been defined, with

$$\hat{O}(\lambda_k) = \frac{\not{\bar{n}}}{2}\frac{1 + \lambda_k \gamma_5}{2} \ . \tag{2.4}$$

Besides, the light-like four vector, $\bar{n} = (1, 0, 0, -1)$, and the rest frame state of the nucleon with helicity λ, $|\vec{P} = \vec{0}, \lambda\rangle$, have been introduced. In Eq. (2.2) color-correlated and interference dPDFs, Sudakov suppressed at high energies, have not been considered [5, 58].

Here and in the following, the "$\pm$" components of a four-vector b are defined according to $b^\pm = b_0 \pm b_z$ and $x_i = \dfrac{k_i^+}{P^+}$ is the fraction of the system momentum carried by the parton "i". Moreover, the notation $\tilde{b} = (b^+, \vec{b}_\perp)$ is used for LC vectors.

In order to evaluate the above quantities, use has been made of the LC free quark fields [59, 60]:

$$q_i(\xi) = \sum_r \int \frac{d\tilde{k}}{2(2\pi)^3\sqrt{k^+}} \theta(k^+) e^{-i\xi^- k^+} a^i_{\tilde{k},r} \; u_{LF}(\tilde{k}, r) \tag{2.5}$$

where the operator $a^i_{\tilde{k},r}$ destroys a quark of flavor i, helicity r and LC momentum $\tilde{k}$. The $u_{LF}(\tilde{k}, r)$ spinors read [48]:

$$u_{LF}(\tilde{k}, +) = \frac{m}{\sqrt{k^+}} \begin{pmatrix} \dfrac{k^+}{m} \\ \\ \dfrac{k_x + ik_y}{m} \\ \\ 1 \\ \\ 0 \end{pmatrix} ; \quad u_{LF}(\tilde{k}, -) = \frac{m}{\sqrt{k^+}} \begin{pmatrix} 0 \\ \\ 1 \\ \\ -\dfrac{k_x - ik_y}{m} \\ \\ \dfrac{k^+}{m} \end{pmatrix} . \tag{2.6}$$

Now one has to establish a direct link between the above expressions for the dPDF and the proton wave function corresponding to a given LF CQM. To this aim, the procedure proposed in Ref. [51] for the model evaluation of GPDs has been extended to the present case.

Starting from the general expression Eq. (2.2), by expanding the proton state $|\vec{0}, \lambda\rangle$ in its Fock components and retaining only the first, valence contribution, $|\vec{0}, \lambda, val\rangle$, one obtains, in terms of the LF one-quark states of isospin τ_i, $|\tilde{k}_i, \lambda^f_i, \tau_i\rangle$ [48]:

$$\begin{aligned} |\vec{0}, \lambda\rangle \simeq |\vec{0}, \lambda^f, val\rangle &= \sum_{\lambda^f_i \tau_i} \int \left[\prod_{i=1}^{3} \frac{dx_i}{\sqrt{x_i}}\right] \delta\left(1 - \sum_{i=1}^{3} x_i\right) \left[\prod_{i=1}^{3} \frac{d\vec{k}_{i\perp}}{2(2\pi)^3}\right] \delta\left(\sum_{i=1}^{3} \vec{k}_{i\perp}\right) \\ &\times \; 2(2\pi)^3 \Psi^{[f]}_\lambda(\{x_i, \vec{k}_{i\perp}, \lambda^f_i, \tau_i\}) \prod_{i=1}^{3} |\tilde{k}_i, \lambda^f_i, \tau_i\rangle \; . \end{aligned} \tag{2.7}$$

In terms of the canonical, Instant-Form (IF) one-quark states $|\vec{k}_i, \lambda^c_i, \tau_i\rangle$, the same proton state reads instead:

$$|\vec{0}, \lambda\rangle \simeq |\vec{0}, \lambda^c, val\rangle = \sum_{\lambda^c_i \tau_i} \int \left[\prod_{i=1}^{3} d\vec{k}_i\right] \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \Psi^{[c]}_\lambda(\{\vec{k}_i, \lambda^c_i, \tau_i\}) \prod_{i=1}^{3} |\vec{k}_i, \lambda^c_i, \tau_i\rangle \; . \tag{2.8}$$

Here and in the following the short-hand notation $\{\alpha_i\}$ is adopted for $\alpha_1, \alpha_2, \alpha_3$. The orthogonality conditions for the LF and IF states read, respectively:

$$\begin{aligned} \langle \tau, \lambda^f, \tilde{k} | \tilde{k}', \lambda'^f, \tau' \rangle &= 2(2\pi)^3 k^+ \delta(k^+ - k'^+) \; \delta^2(\vec{k}_\perp - \vec{k}'_\perp) \delta_{\tau,\tau'} \delta_{\lambda^f, \lambda'^f} \; ; \\ \langle \tau, \lambda^c, \vec{k} | \vec{k}', \lambda'^c, \tau' \rangle &= \delta^3(\vec{k} - \vec{k}') \delta_{\tau,\tau'} \delta_{\lambda^c, \lambda'^c} \; . \end{aligned} \tag{2.9}$$

The above descriptions can be related to each other through the following fundamental relation [47]

$$|\tilde{k}, \lambda^f, \tau\rangle = \sqrt{\omega}(2\pi)^{3/2} \sum_{\lambda^c} D^{1/2}_{\lambda^f \lambda^c}(R_{cf}(\vec{k}))|\vec{k}, \lambda^c, \tau\rangle \ , \tag{2.10}$$

expressing the LF spin state of a particle from its canonical IF one, where $\omega = \sqrt{m^2 + \vec{k}^2}$ and the Melosh rotations $D^{1/2}_{\mu\lambda}(R_{cf}(\vec{k}))$ naturally arise.

The above equation relates *free* canonical and light-front states. Actually we are interested in interacting quarks in a proton and the connection of instant form and light front states for composite systems is, in this case, much more complicated. Nevertheless, if one chooses a suitable representation of the Poincaré operators, such as the Bakamjian-Thomas construction [61], Eq. (2.10) can be generalized to interacting states in composite systems [50–56]. The dynamical framework used in the following consists in a relativistic mass equation built in accord with the Bakamjian-Thomas construction. For the physical situations relevant to the subject of this paper, therefore, one can relate the valence contributions to the nucleon state in its rest frame, in the LF and IF, given by Eqs. (2.7) and (2.8) respectively, as follows (see, e.g., Ref. [51]):

$$|\vec{0}, \lambda^f, val\rangle = \sqrt{M_0}(2\pi)^{3/2}|\vec{0}, \lambda^c, val\rangle \ , \tag{2.11}$$

where $M_0 = \sum_i \omega_i$ is the free quarks energy, in terms of which the free mass, invariant for LF boosts, is defined as follows:

$$M_0^2 = \sum_i \frac{m_i^2 + \vec{k}_{i\perp}^2}{x_i} \ . \tag{2.12}$$

The Melosh operators read, in our notation:

$$D^{1/2}_{\mu\lambda}(R_{cf}(\vec{k}_i)) = \langle\mu| \ \hat{D}_i \ |\lambda\rangle \ , \tag{2.13}$$

with

$$\hat{D}_i = \frac{m + x_i M_0 - i\vec{\sigma}_i \cdot (\hat{z} \times \vec{k}_\perp)}{\sqrt{(m + x_i M_0)^2 + \vec{k}_\perp^2}} \ . \tag{2.14}$$

Substituting the LF quark states, Eq. (2.10), into the proton LF state, Eq. (2.7), one gets the latter in terms of IF quark states and Melosh operators. Then, using Eq. (2.11) for the obtained expression, one gets a direct link between the LF proton wave function (LFWF), $\psi^{[f]}_\lambda$, appearing in Eq. (2.7), and the IF one, $\psi^{[c]}_\lambda$, appearing in Eq. (2.8) [51]:

$$
\begin{aligned}
\psi_\lambda^{[f]}(\{x_i, \vec{k}_{i\perp}, \lambda_i^f, \tau_i, \}) &= 2(2\pi)^3 \left[\frac{\omega_1\omega_2\omega_3}{M_0 x_1 x_2 x_3}\right]^{1/2} \prod_{i=1}^{3} \left[\sum_{\lambda_i^c} D^{*1/2}_{\lambda_i^c \lambda_i^f}(R_{cf}(\vec{k}_i))\right] \\
&\quad \times \psi_\lambda^{[c]}(\{\vec{k}_i, \lambda_i^c, \tau_i\}) \\
&= 2(2\pi)^3 \left[\frac{\omega_1\omega_2\omega_3}{M_0 x_1 x_2 x_3}\right]^{1/2} \Psi(\vec{k}_1, \vec{k}_2, \vec{k}_3; \{\lambda_i^f, \tau_i\}) \; .
\end{aligned}
\tag{2.15}
$$

One should notice that, in principle, LFWFs are eigensolutions of the LF hamiltonian which, in turn, is derived from the QCD Lagrangian. Our simplified approach, as it is apparent from the above equations and as it has been already stated, consists in using LFWFs obtained properly boosting eigenstates of a relativistic effective mass equation, consistent with the Bakamjian-Thomas construction and reproducing some of the QCD symmetries.

In Eq. (2.15), for the sake of convenience, the function Ψ has been introduced. For the model under scrutiny in the following, it can be split into a momentum space wave function, ψ, and a spin-orbital-flavor part, as follows:

$$
\begin{aligned}
\Psi(\vec{k}_1, \vec{k}_2, \vec{k}_2; \{\lambda_i^f, \tau_i\}) &= \psi(\vec{k}_1, \vec{k}_2, \vec{k}_3) \prod_{i=1}^{3} \left[\sum_{\lambda_i^c} D^{*1/2}_{\lambda_i^c \lambda_i^f}(R_{cf}(\vec{k}_i))\right] \\
&\quad \times \Phi(\lambda_1^c, \lambda_2^c, \lambda_3^c, \tau_1, \tau_2, \tau_3) \; .
\end{aligned}
\tag{2.16}
$$

Now, the wave function Eq. (2.15), with the expression (2.16) for Ψ, can be used to evaluate the valence component of the LF proton state, Eq. (2.7). This state is in turn inserted, together with the quark field expression, Eq. (2.5), in the general definition of the dPDFs, Eq. (2.2). Then, using ordinary operator algebra and properly treating the δ function on the x variables appearing in Eq. (2.7), a straightforward procedure leads to the general expression for the evaluation of the valence contribution to the dPDF Eq. (2.1) within a given LF CQM. For quarks of flavor q_1 and q_2, longitudinal momentum fractions x_1 and x_2 and transverse distance $\vec{k}_\perp$ in momentum space, it reads as follows:

$$
\begin{aligned}
F_{q_1 q_2}^{\lambda_1,\lambda_2}(x_1, x_2, \vec{k}_\perp) &= 3(\sqrt{3})^3 \int \left[\prod_{i=1}^{3} d\vec{k}_i \sum_{\lambda_i^f \tau_i}\right] \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \\
&\quad \times \Psi^*\left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2}, \vec{k}_3; \{\lambda_i^f, \tau_i\}\right) \\
&\quad \times \widehat{P}_{q_1}(1)\widehat{P}_{q_2}(2)\widehat{P}_{\lambda_1}(1)\widehat{P}_{\lambda_2}(2)\, \Psi\left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2}, \vec{k}_3; \{\lambda_i^f, \tau_i\}\right) \\
&\quad \times \delta\left(x_1 - \frac{k_1^+}{P^+}\right) \delta\left(x_2 - \frac{k_2^+}{P^+}\right) \; .
\end{aligned}
\tag{2.17}
$$

In order to estimate the dynamical correlations between two unpolarized or longitudinally polarized quarks, the flavor and spin projectors, acting on the state of the i particle, have been introduced in the above equation. They read:

$$\begin{aligned}
\hat{P}_{u(d)}(i) &= \frac{1 \pm \tau_3(i)}{2} \,, \\
\hat{P}_{\lambda_k}(i) &= \frac{1 + \lambda_k \sigma_3(i)}{2} \,.
\end{aligned} \tag{2.18}$$

Eq. (2.17) is the general expression needed to evaluate the (spin-dependent) dPDFs in a Light-Front CQM. In particular, if $q_1 = q_2 = u$, assuming an SU(6) symmetry for the canonical proton wave function, as it will be done in this paper, one gets:

$$\begin{aligned}
u_{\uparrow(\downarrow)} u_{\uparrow(\downarrow)}(x_1, x_2, k_\perp) &= 2(\sqrt{3})^3 \int d\vec{k}_{1\perp} d\vec{k}_{2\perp} \, \frac{E_1 E_2 E_3}{k_1^+ x_1 x_2 (1 - x_1 - x_2) j} \\
&\times \langle \tilde{P}_1^{\uparrow(\downarrow)} \rangle \langle \tilde{P}_2^{\uparrow(\downarrow)} \rangle \, \psi^* \left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2}, -\vec{k}_1 - \vec{k}_2 \right) \\
&\times \psi \left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2}, -\vec{k}_1 - \vec{k}_2 \right) .
\end{aligned} \tag{2.19}$$

The following relations hold among the quantities appearing in this equation and the integration variables:

$$\begin{aligned}
k_1^+ &= \sqrt{x_1 \left\{ m^2 \left[1 + \frac{x_1}{x_2} + \frac{x_1}{1 - x_1 - x_2} \right] + k_{1\perp}^2 + \frac{x_1}{x_2} k_{2\perp}^2 + \frac{x_1}{1 - x_1 - x_2} k_{3\perp}^2 \right\}} \, ; \\
k_2^+ &= \frac{x_2}{x_1} k_1^+, \quad k_3^+ = \frac{1 - x_1 - x_2}{x_1} k_1^+, \quad k_{iz} = -\frac{m^2 + k_{i\perp}^2}{2k_i^+} + \frac{k_i^+}{2} \, ; \\
E_i &= \sqrt{m^2 + k_{iz}^2 + \vec{k}_{i\perp}^2} \, ; \\
j &= \left| \frac{m^2 + k_{1\perp}^2}{2k_1^{+2}} + \frac{m^2 + k_{2\perp}^2}{2 \frac{x_2}{x_1} k_1^{+2}} + \frac{m^2 + k_{3\perp}^2}{2 \frac{1 - x_1 - x_2}{x_1} k_1^{+2}} + \frac{1}{2x_1} \right| .
\end{aligned} \tag{2.20}$$

Besides, the spin structure of Eq. (2.19) is described by the coupling of the Melosh rotation with the spin projection operators defined in Eq. (2.18), as follows:

$$\tilde{P}_i^{\uparrow(\downarrow)} = \hat{D}_i \hat{P}_{\uparrow(\downarrow)}(i) \hat{D}_i^\dagger \,, \tag{2.21}$$

so that, using the canonical spin-isospin state corresponding to the SU(6) symmetry, the matrix elements appearing in Eq. (2.19) read:

$$\langle \tilde{P}_i^{\uparrow(\downarrow)} \rangle = \left\langle \frac{\chi_{MS} \phi_{MS} + \chi_{MA} \phi_{MA}}{\sqrt{2}} \middle| \tilde{P}_i^{\uparrow(\downarrow)} \middle| \frac{\chi_{MS} \phi_{MS} + \chi_{MA} \phi_{MA}}{\sqrt{2}} \right\rangle . \tag{2.22}$$

As one can see in Eq. (2.17), the dPDFs evaluated by using the Light-Front treatment depend on delta functions, defining the longitudinal momentum fractions carried by the quarks inside the proton, which are function of P^+, the plus component of the proton momentum, which, in the LF intrinsic frame, reads:

$$P^+ = \sum_i^3 k_i^+ = \sum_i^3 \omega_i = M_0 \ , \tag{2.23}$$

where M_0 is the free quarks total energy, already defined in Eq. (2.12). Thanks to this relation, the correct support is obtained, i.e., dPDFs are vanishing in the forbidden kinematic region, $x_1 + x_2 > 1$. This result is natural in the LF approach (see, e.g., Refs. [51–56]).

In particular, in the present investigation, the following combinations of the spin components of the expression Eq. (2.19) will be described:

$$uu(x_1, x_2, k_\perp) = \sum_{i,j=\uparrow,\downarrow} u_i u_j(x_1, x_2, k_\perp), \tag{2.24}$$

i.e., the dPDF describing two unpolarized quarks, and

$$\Delta u \Delta u(x_1, x_2, k_\perp) = \sum_{i=\uparrow,\downarrow} u_i u_i(x_1, x_2, k_\perp) - \sum_{i\neq j=\uparrow,\downarrow} u_i u_j(x_1, x_2, k_\perp) \ , \tag{2.25}$$

i.e., the one in the case of two polarized quarks. These are the only distributions contributing to the total cross section in events involving unpolarized protons (see, e.g., Ref. [4]).

3 Results at the hadronic scale

In order to calculate the dPDFs, a wave function within a proper CQM is needed. In this paper, as an example, use will be made of the relativistic hyper-central CQM, firstly introduced in Ref. [50], providing a reasonable description of the light baryon spectrum. This model has been systematically applied to the evaluation of parton distributions in Refs. [50–56]. The present analysis is exploratory since there are no data available for the observables under scrutiny. This model gives us sufficient guarantee to be able to grasp the most relevant features of dPDFs. Together with the LF treatment, it provides a scheme that reproduces the symmetries which are relevant in the problem. Details on the construction of the model and on the fixing of its parameters to low energy properties of the proton can be found in Refs. [50, 55, 56]. For the discussion here it is enough to know that the total proton state is given by a momentum space wave function and a spin-isospin part, dictated by the SU(6) symmetry. In the model under scrutiny here the momentum space wave function, ψ_{00}, does not depend on the angular variables but only on the hyper-radius k_ξ, being a solution of the Mass equation

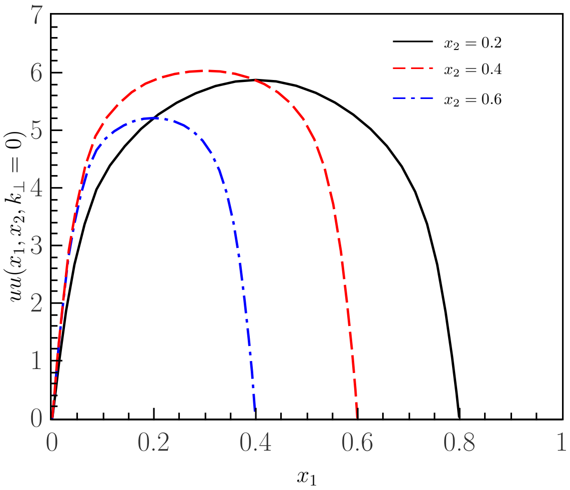

Figure 1. The unpolarized dPDF, Eq. (2.24), as a function of x_1 and for three values of x_2, at $k_\perp = 0$.

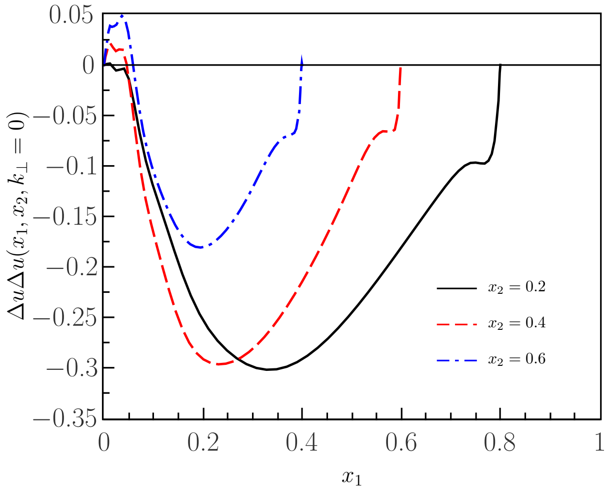

Figure 2. The same as in Fig. 1, but for the spin-dependent dPDF Eq. (2.25).

$$(M_0 + V)\psi_{00}(k_\xi) = \left(\sum_i^3 \sqrt{\vec{k}_i^2 + m^2} - \frac{\tau}{\xi} + \kappa\xi \right) \psi_{00}(k_\xi) = M\psi_{00}(k_\xi) \ . \tag{3.1}$$

ξ is the variable conjugated to k_ξ and the parameters of the potential are fixed in order to reproduce the essential features of the Nucleonic spectrum. Their values are for the relativistic (for the NR reduction of the mass operator Eq. (3.1), see Ref. [62]) case: $\tau = 3.30\,(4.59)$ and $\kappa = 1.80\,(1.61)$ fm^{-2}. The solution of this Mass equation is described, e.g., in Ref. [50]. It is instructive to notice that the intrinsic momentum space wave function is simply given by $\psi(\vec{k}_1, \vec{k}_2) = \psi(k_\xi)$, where here k_ξ is the hyperradius in momentum space,

$$k_\xi = \sqrt{2(\vec{k}_1^2 + \vec{k}_2^2 + \vec{k}_1 \cdot \vec{k}_2)} \ , \tag{3.2}$$

so that the wave function depends on the product $\vec{k}_1 \cdot \vec{k}_2$. This fact, as it happens in the model calculations of Ref. [37], is responsible for the presence of spin-independent correlations between the two quarks in the CQM calculation. In the spin-dependent case, the presence of the Melosh rotations represents an additional source of (spin) correlations.

The numerical results for the calculation of the unpolarized and longitudinally polarized dPDFs at the hadronic scale of the model, performed using this momentum space wave function in Eq. (2.19), for example in the case of two quarks of flavor $q_1 = u$, $q_2 = u$, are now described and discussed. First of all, let us remark that, in proper limits, the calculated unpolarized dPDFs reproduce the single particle PDFs evaluated in the same LF hypercentral scheme and shown in Ref. [51].

In Figs. 1 and 2 the distributions $uu(x_1, x_2, k_\perp = 0)$ and $\Delta u \Delta u(x_1, x_2, k_\perp = 0)$ are shown for three different values of x_2. At variance with the results obtained in previous

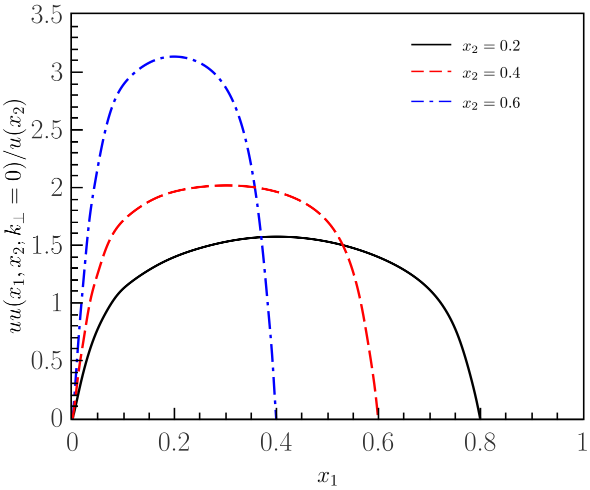

Figure 3. The ratio r_1, Eq. (3.4), at $k_\perp = 0$, evaluated for three values of x_2, as a function of x_1.

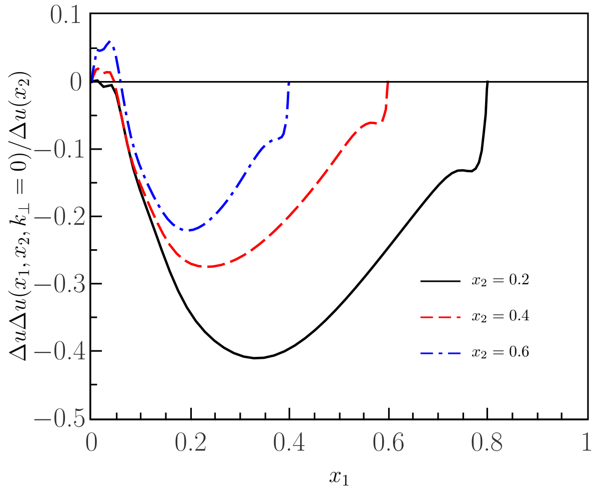

Figure 4. The ratio r_2, Eq. (3.5), at $k_\perp = 0$, evaluated for three values of x_2, as a function of x_1.

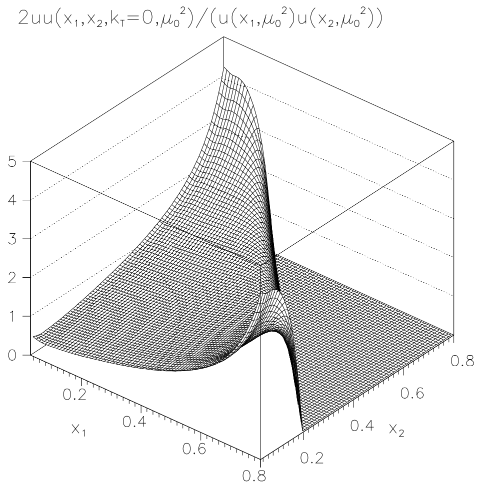

Figure 5. The ratio Eq. (3.6) at the hadronic scale.

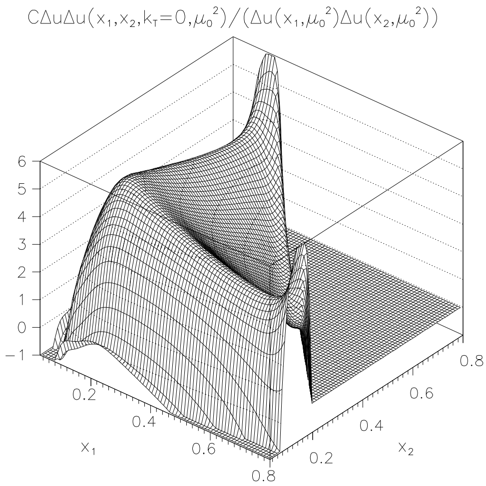

Figure 6. The ratio Eq. (3.7) at the hadronic scale.

works, Refs. [30, 37], a correct support is obtained, i.e., the dPDFs are different from zero only in the physical region, $x_1 + x_2 < 1$. Thanks to the same property, the symmetry of the dPDF in exchanging x_1 and x_2, due to the indistinguishability of the two quarks when $k_\perp = 0$, is restored and the probabilistic interpretation of the distribution is recovered. In particular, for the unpolarized dPDF, where the SU(6) spin-isospin symmetry breaking is not apparent as it is in the polarized case due to the presence of the Melosh rotation, one finds the following condition:

$$uu(x_1, x_2, k_\perp = 0) = uu(1 - x_1 - x_2, x_2, k_\perp = 0) \; . \tag{3.3}$$

This equation reflects the fact that, once the isospin factor coming from the SU(6) symmetry has been taken into account in the calculation, the dPDF is completely symmetric under the exchange of any pair of the three quarks.

As already pointed out, the main interest in this work is to understand the dynamical origin of the correlations, and in particular to verify if the factorized *ansätze* of Eqs. (1.2,1.3) are valid in the valence quark region, where the used CQM could give predictions on the distributions of interest here. To this aim, in order to estimate how much this factorization could be violated, it is instructive to analyze and show the ratios

$$r_1(x_1, x_2) = \frac{uu(x_1, x_2, k_\perp = 0)}{u(x_2)} \, , \tag{3.4}$$

$$r_2(x_1, x_2) = \frac{\Delta u \Delta u(x_1, x_2, k_\perp = 0)}{\Delta u(x_2)} \, , \tag{3.5}$$

where $u(x_2)$ and $\Delta u(x_2)$ are the standard one-body unpolarized and polarized PDFs, respectively. The above ratios are shown in Figs. 3 and 4. If the approximation Eq. (1.3) were valid, the above ratios should not depend on x_2. On the contrary, it is clearly seen that in both the polarized and unpolarized case the factorization is strongly violated, due to the correlations between the two active quarks, present in the hypercentral CQM under scrutiny here. These results are qualitatively in agreement with those found in Refs. [30, 37]; of course, the amount of violation of the factorization and the shape of the ratio are model dependent features.

A useful pictorial representation of this analysis can be obtained by drawing two other ratios,

$$r_3(x_1, x_2) = \frac{2uu(x_1, x_2, k_\perp = 0)}{u(x_1)u(x_2)} \tag{3.6}$$

and

$$r_4(x_1, x_2) = \frac{C \Delta u \Delta u(x_1, x_2, k_\perp = 0)}{\Delta u(x_1) \Delta u(x_2)} \, , \tag{3.7}$$

where C is the constant

$$C = \frac{[\int dx_1 \Delta u(x_1)]^2}{\int dx_1 \, dx_2 \, \Delta u \Delta u(x_1, x_2, k_\perp = 0)} \, , \tag{3.8}$$

in a three-dimensional plot. The factors 2 and C in the numerator of Eqs. (3.6) and (3.7), respectively, have been added in order to have these ratios equal to 1 in the kinematical regions where the factorization *ansatz* Eq. (1.3) is valid. Clearly, the constant C is a model dependent quantity and its value is 8/3 in a pure NR SU(6) scheme and -6.17 in the present LF hypercentral approach. The ratios Eqs. (3.6) and (3.7) are shown in Figs. 5 and 6, respectively. It is clear that in our approach the factorization property, Eq. (1.3), is badly violated, in particular in the polarized case where, due to the contribution

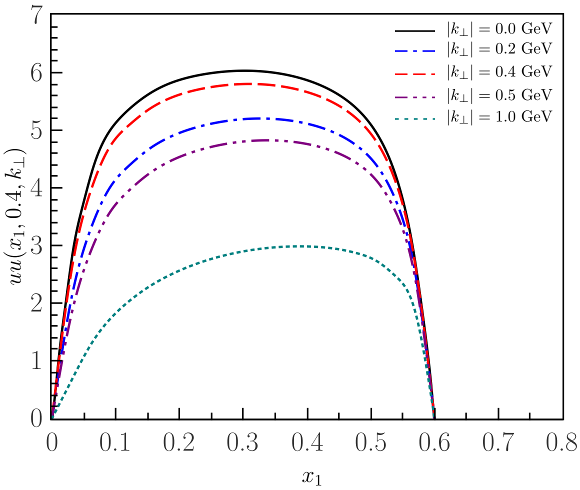

Figure 7. The unpolarized dPDF, Eq. (2.24), evaluated at $x_2 = 0.4$ and at five values of $k_\perp$.

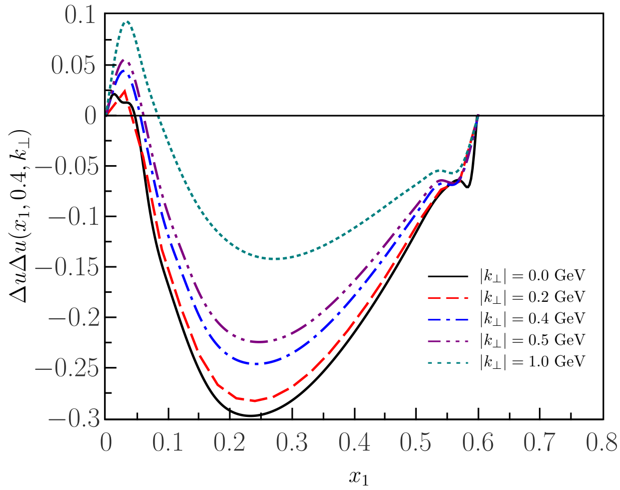

Figure 8. The polarized dPDF, Eq. (2.25), evaluated at $x_2 = 0.4$ and at five values of $k_\perp$.

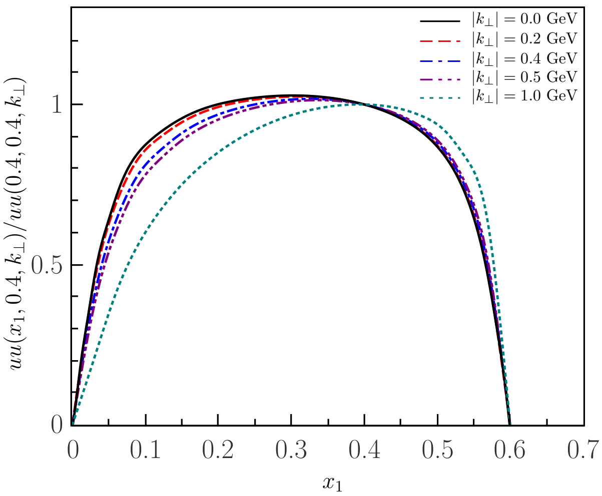

Figure 9. The ratio r_5, Eq. (3.9), for five values of $k_\perp$.

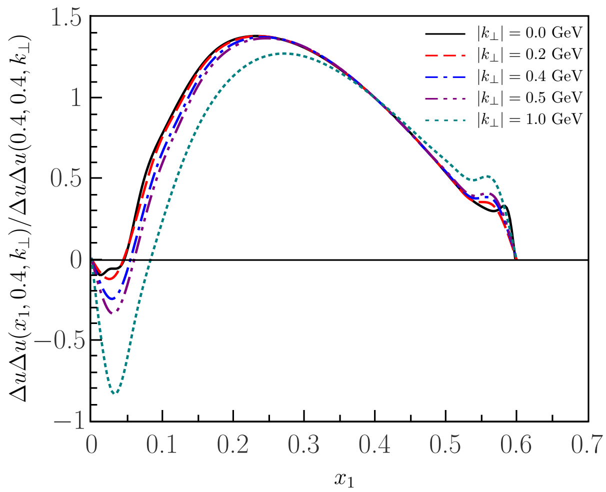

Figure 10. The ratio r_6, Eq. (3.10), for five values of $k_\perp$.

of the Melosh rotation, the correlations between the two quarks are so strong to lead to a change of the sign of the dPDF with respect to the single particle PDF, which determines a severe violation of the approximation, for any value of x. It is interesting to notice that, if one had used a NR SU(6) scheme, Fig. 5 and 6 would be exactly the same. The shown difference is a model dependent, relativistic effect. In the unpolarized case, the shape of the ratio Eq. (3.6) is indeed not too different from that obtained in [37] in a conventional CQM approach. In the polarized case, relativity produces a big difference. This seems to indicate that the factorization ansatz should be used with great care in the spin-dependent case.

In Figs. 7 and 8, the polarized and unpolarized dPDFs are respectively shown, for five values of the difference in transverse momentum, $k_\perp$. The behavior of the dPDFs, decreasing with increasing $k_\perp$, is similar to that described in Refs. [30, 37]. A better insight in the $k_\perp$ dependence is seen in Figs. 9 and 10, where the ratios

$$r_5(x_1, x_2, k_\perp) = \frac{uu(x_1, 0.4, k_\perp)}{uu(0.4, 0.4, k_\perp)} \tag{3.9}$$

and

$$r_6(x_1, x_2, k_\perp) = \frac{\Delta u \Delta u(x_1, 0.4, k_\perp)}{\Delta u \Delta u(0.4, 0.4, k_\perp)}\ , \tag{3.10}$$

are shown for five values of $k_\perp$. This quantity, already analyzed in Refs. [30, 37], has been chosen in order to check the validity of the approximation Eq. (1.3), i.e., the possibility to factorize the dependences on x_1, x_2 and $k_\perp$ in the dPDFs. The violation of this *ansatz* grows smoothly with increasing $k_\perp$. In the present LF approach, this is somehow expected. First of all, since the invariant mass Eq. (2.12) is defined in terms of both x and $k_\perp$, the dependence of a LFWF on these variables cannot be separated. This is relevant, for example, in studies of leading-twist lensing effects (see, e.g., Ref. [63]). The breaking of the x and $k_\perp$ factorization is correctly found in our model LFWFs. Separable forms (often invoked) are rejected by the structure of our relativistically invariant approach. Moreover, in our scheme, the relativistic effect provided by the Melosh rotations, which are present also in the unpolarized case, when $k_\perp \neq 0$, is another source of unfactorized x and $k_\perp$ dependencies. It could be worth to remind instead that, in a pure SU(6) NR scenario, the factorization holds [37].

In closing this section we notice that, to have a flavor of the behavior of the dPDFs at high momentum transfer and, possibly, in the low-x kinematical region, $x \lesssim 10^{-2}$, presently accessible at the LHC, the pQCD evolution of the model results is necessary. An analysis of this kind, i.e. the check of the validity of the approximations Eqs. (1.2,1.3) at high momentum scales, is reported in the next section.

4 pQCD evolution of the model results

In this section, the procedure and the results of the pQCD evolution of the model calculation will be described. For the moment being, evolved unpolarized and polarized dPDFs are presented at Leading-Order (LO), only when the transverse distance between the two partons in momentum space is zero, i.e., $k_\perp = 0$. Besides, as everywhere in the paper, the scale has been taken to be the same for both partons, which is not the case in actual processes, in general.

The evolution equations for the dPDFs, introduced in Refs. [23, 24] and later used and discussed in Refs. [4, 5, 8, 18, 25–33], are a generalization of the DGLAP equations so that, also in this case, the solution can be found by using the Mellin transformations of the calculated functions. Since we analyse only the valence quark contribution, the inhomogeneous part of the evolution equations is not involved (see also Refs. [31–33]). Thanks to this fact, the evolved dPDFs can be found as follows.

We use the Mellin moments of the weighted distributions:

$$
\begin{aligned}
&x_1 x_2 F_{i_1,i_2}(x_1, x_2, Q^2)\,, \\
&x F_i(x, Q^2)\,,
\end{aligned}
\tag{4.1}
$$

i.e., explicitly

$$
\begin{aligned}
\langle x F_i(Q^2)\rangle_n &= \int_0^1 dx\, x^{n-2}\, x F_i(x, Q^2)\,, \\
\langle x_1 x_2 F_{i_1,i_2}(Q^2)\rangle_{n_1,n_2} &= \int_0^1 dx_1 \int_0^1 dx_2\, x_1^{n_1-2}\, x_2^{n_2-2}\, x_1 x_2 F_{i_1,i_2}(x_1, x_2, Q^2)\,.
\end{aligned}
\tag{4.2}
$$

Here a somehow shorthand notation is adopted, where the PDF for a quark of flavour i is named $F_i(x)$ and the dPDFs for two polarized or unpolarized quarks of flavours i_1 and i_2, is $F_{i_1 i_2}(x_1, x_2, k_\perp = 0) = F_{i_1 i_2}(x_1, x_2)$. Their LO evolution reads

$$
\begin{aligned}
\langle x F_i(Q^2)\rangle_n &= \left(\frac{a_s}{a_{s0}}\right)^{-\frac{P_{\rm NS}^{(0)}(n)}{\beta_0}} \langle x F_i(\mu_0^2)\rangle\,, \\
\langle x_1 x_2 F_{i_1,i_2}(Q^2)\rangle_{n_1,n_2} &= \left(\frac{a_s}{a_{s0}}\right)^{-\frac{P_{\rm NS}^{(0)}(n_1)}{\beta_0}} \cdot \left(\frac{a_s}{a_{s0}}\right)^{-\frac{P_{\rm NS}^{(0)}(n_2)}{\beta_0}} \langle x_1 x_2 F_{i_1,i_2}(\mu_0^2)\rangle_{n_1,n_2}
\end{aligned}
\tag{4.3}
$$

where: $a_{s0} = \frac{\alpha_s(\mu_0^2)}{4\pi}$ and $a_s = \frac{\alpha_s(Q^2)}{4\pi}$ and, at LO, $a_s \equiv a_{s,\rm LO} = 1/(\beta_0 \ln(Q^2/\Lambda^2_{\rm LO}))\,,$ $\beta_0 = 11 - 2n_f/3$. $P_{\rm NS}^{(0)}(n)$ is n^{th} moment of the Non-Singlet splitting function at LO. Taking the n, n_1 and n_2 moments complex, the evolved distributions in x-space are obtained in terms of the inverse Mellin transformations. Namely

$$
\begin{aligned}
x F_i(x, Q^2) &= \frac{1}{2\pi i} \oint_{\mathcal{C}} dn\, x^{(1-n)}\, \langle x F_i(Q^2)\rangle_n = \\
x_1 x_2 F_{i_1,i_2}(x_1, x_2, Q^2) &= \frac{1}{2\pi i} \oint_{\mathcal{C}} dn_1\, \frac{1}{2\pi i} \oint_{\mathcal{C}} dn_2\, x_1^{(1-n_1)}\, x_2^{(1-n_2)}\, \langle x_1 x_2 F_{i_1,i_2}(Q^2)\rangle_{n_1,n_2}
\end{aligned}
\tag{4.4}
$$

We have implemented the numerical evaluation of the dPDFs evolution. Our code reproduces correctly the results shown in Ref. [31], if we use as an input the simple factorized *ansatz* used in that paper. The good support property of the present LF calculation helps to obtain a proper evaluation of the Mellin moments for a precise evolution procedure.

The present calculation scheme does not fix the hadronic scale μ_o^2. To this aim, we have followed here the proposal of, e.g., Ref. [38]: μ_o^2 is the scale at which the valence quarks of the model carry all the proton momentum, i.e., the second moment of the unpolarized valence parton distribution is 1. Knowing the experimental value of this quantity at a high momentum scale, one can evolve it using LO pQCD to lower scales. μ_o^2 is the scale at which the second moment of the unpolarized valence parton distribution turns out to be one. In this way, a very low value for μ_o^2 is found. In agreement with Ref. [38], we fixed the hadronic scale to the value of 0.08 GeV^2, consistent with that used in the analyses shown,

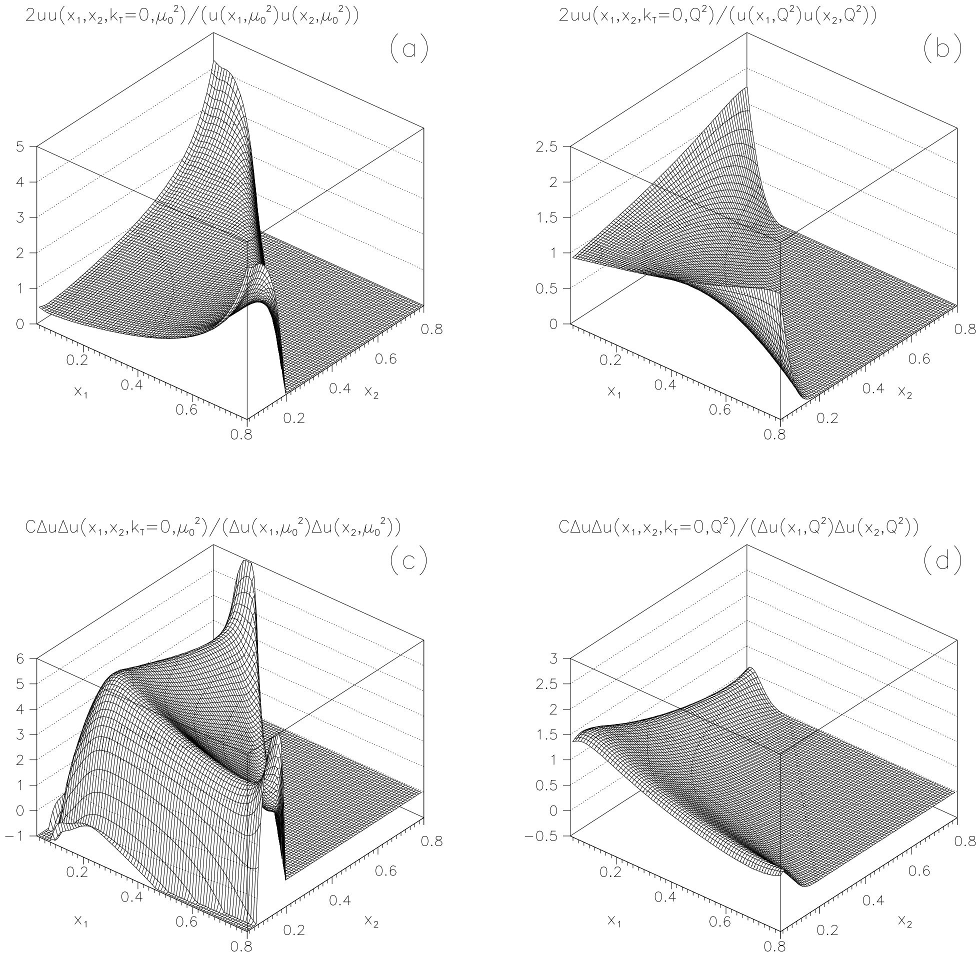

Figure 11. a) The ratio Eq. (3.6) at the hadronic scale; b) the same quantity at a scale $Q^2 = 10$ GeV^2; c) the ratio Eq. (3.7) at the hadronic scale; d) this last quantity at a scale $Q^2 = 10$ GeV^2. The vertical scale of panels (b) and (d) is reduced by a factor of 2 with respect to panels (a) and (c), respectively.

e.g., in Refs. [39, 41, 42, 50, 53, 55, 56] for PDFs and GPDs. A very low starting scale has been tested also for dPDFs in Ref. [32]. Actually, recent studies have proposed, using different arguments, that models can be associated to a little higher scales, which would make a difference in the numerical results (see, e.g., [64]). Nevertheless we are performing here an exploratory calculation since there are no available data for these observables. We think that we should not care of this aspect for the moment being. For the same reason we have taken a standard final scale, $Q^2 = 10$ GeV^2, although the one at the LHC is certainly larger. We think in fact that an evolution from 0.08 to 10 GeV^2 is extreme enough to simulate qualitatively the change in reaching, through pQCD, the experimental situation from a model calculation.

In Figs. 11 (b) and 11 (d) the results of this procedure are shown for the ratios Eqs. (3.6) and (3.7), starting from the model distributions at the hadronic scale giving the ratios in Figs. 11 (a) and 11 (c), respectively, already discussed in the previous section and presented here again for readers convenience. Now, Fig. 11 provides a complete summary of the results of the present analysis. All the shown ratios are constructed to yield one if correlations are not active. It is clearly seen that, in the unpolarized case, the effect of correlations keeps being sizable after evolution in the valence x region. At the same time, it gets less important at small x, where it is investigated presently at the LHC, as already noticed in Refs. [31–33] using simple test functions as inputs in the evolution equations. However, in the model under scrutiny, it is found that spin-dependent correlations, present also in the case of scattering of unpolarized protons, are important, after evolution, even at low x.

In closing, we note that a realistic evaluation of DPS at low x would require a complete study of the evolution of dPDFs, considering therefore not only the valence, non-singlet sector but also the produced gluons and sea quarks, which are mixing in the evolution of the singlet sector. For the moment being, the results reported here are intended to give a first useful glance on the general effects of the evolution.

5 Conclusions

The double parton distribution functions (dPDFs) have been evaluated, in the valence sector, in both the spin-independent and spin dependent case, accessible, in principle, in the scattering of unpolarized protons occurring at, e.g., the LHC. The chosen framework has been a Light-Front constituent quark model. The main aim was to check to what extent constituent quark model estimates support the factorization *ansatz* often used in phenomenological applications. According to this procedure parton correlations are neglected, e.g., dPDFs are factorized in two terms, one depending on the longitudinal momentum fractions of the two quarks, x_1 and x_2, and another on the transverse momentum separation, $\vec{k}_\perp$, e.g. a sort of $x - \vec{k}_\perp$ factorization is assumed. In addition to that, also an $x_1 - x_2$ factorization assumption is made, e.g., the distribution on the longitudinal momenta x_1, x_2 is taken to be uncorrelated.

Previous quark model analyses have been performed in a properly modified version of the standard uncorrelated bag model and in a Non-Relativistic constituent quark model, where correlations are present from the very beginning. The outcome was that, in both models, the $x_1 - x_2$ factorization is strongly violated, while the $x - k_\perp$ factorization is mildly violated. These results have been tested here within a relativistic, fully Poincaré covariant Light-Front approach, already used for the calculation of single particle parton distributions. As a consequence of the Poincaré covariance, some symmetries expected on general grounds, lost in previous analyses, are clearly recovered and the so called "bad support" problem, i.e., the fact that dPDFs are not vanishing in the forbidden kinematic region, $x_1 + x_2 > 1$, does not arise. The main results are as follows. At the low momentum scale of the model, the strong violation of the $x_1 - x_2$ factorization and the mild one of the $x - k_\perp$ factorization are basically confirmed. The obtained valence dPDFs have been then pQCD evolved to a high momentum scale, to give an idea on what could happen in the experimentally accessible region. After pQCD evolution the situation is somehow more subtle. While the effect of correlations keeps being sizable after evolution in the valence x region, for the unpolarized distribution it gets less important at small x. However, spin-dependent correlations, present also in the case of scattering of unpolarized protons, are not washed out by the evolution and are sizable even at low x values.

Further studies, including the evolution of the singlet sector, important for the description of the low x region presently observed at the LHC, and the evaluation of the contributions of DPS to the cross sections of specific relevant channels, are in progress. Besides, from the formal point of view, our LF approach to double parton correlations could be tested by studying the cluster decomposition properties at weak binding of the LFWFs. This is a relevant feature that correctly defined LFWFs should show [65]. Another way to access parton correlations in the free proton is the analysis of interference effects in proton nucleus collisions, firstly addressed in [66]. Calculations of interference effects generalizing the approach recently used in Ref. [67] are going on.

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