

# THE ELECTROMAGNETIC MASS DIFFERENCE OF PIONS AT LOW TEMPERATURE

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## Abstract

We compute low temperature corrections to the electromagnetic mass difference of pions in the chiral limit. The computation is done in a model independent way in the framework of chiral perturbation theory, using the background field method and the hard thermal loop approximation. We also generalize at low temperature the sum rule of Das et al. We find that the mass difference between the charged and neutral pions decreases at low temperature  $T$  with respect to the  $T = 0$  value. This is so in spite of the fact that charged particles always get a thermal correction to their masses of order  $\sim eT$ , where  $e$  is the gauge coupling constant. Our result can be understood as a consequence of the tendency towards chiral symmetry restoration at finite temperature.

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## I. INTRODUCTION

At low energies the strong interactions are successfully described in the framework of chiral perturbation theory ( $\chi$ PT) [1,2]. This theory only involves the low energy modes of the QCD spectrum, such as the (pseudo) Goldstone bosons of the spontaneously broken chiral symmetry. There are eight (pseudo) Goldstone bosons,  $(\pi's, K's, \eta)$ , whose interactions can be understood in terms of symmetry considerations. A chiral Lagrangian is expanded in derivatives of the Goldstone fields, and in the masses of the three light quarks, which break explicitly chiral symmetry. Electromagnetic interactions can also be included in a chiral Lagrangian, although they break the chiral symmetry explicitly.

The mass difference between the charged and neutral pions in the chiral limit (i.e. massless quarks) was computed more than thirty years ago using a sum rule approach [3]. With very general hypothesis, namely, soft pion theorems, current algebra, Weinberg's sum rules [4], and vector meson dominance, Das et al. reached to the theoretical prediction  $(M_{\pi^\pm} - M_{\pi^0})_{th} \sim 5.0$  MeV. That value was in a remarkable good agreement with the experimental one  $(M_{\pi^\pm} - M_{\pi^0})_{exp} \sim 4.6$  MeV. This computation allowed one to understand this mass difference as being due, essentially, to electromagnetic effects. This result was also reproduced in the framework of  $\chi$ PT, by considering resonance exchange within a photon loop [5]. Better theoretical estimates of the electromagnetic mass difference of pions have since then been achieved [6].

In the chiral limit Dashen's theorem [7] states that the mass difference between charged and neutral kaons should be the same as the one of pions. Electromagnetic effects can be parametrized in a chiral Lagrangian in such a way that Dashen's theorem is fulfilled in the chiral limit [5]. However, there is no good agreement between the predicted electromagnetic mass difference and the experimental value  $(M_{K^\pm} - M_{K^0})_{exp} \sim -4.0$  MeV. This is understood from the fact that the kaon's mass difference receives important QCD contributions of the order of the quark masses. On the other hand, corrections to Dashen's theorem due to the effect of the quark masses have been studied thoroughly in the literature (see, *e.g.* Refs. [8] and [9] and references therein).

In this article we will compute the electromagnetic mass difference of pions and kaons in the chiral limit at low  $T$ . Exactly as it happens at  $T = 0$ , we expect that our results for those masses should be trusted for pions. To get a better estimate of  $(M_{K^\pm} - M_{K^0})(T)$  further corrections should be taken into account, but we will not compute them here. We will only consider low temperatures, that is,  $T \ll f_\pi$ , where  $f_\pi = 92.4$  MeV is the pion decay constant. This is the temperature regime where the contribution of the lightest particles of the QCD spectrum is the relevant one, since those of heavier states are exponentially suppressed. Thus, the thermal contribution of vector resonances will not be taken into account in this article.

The computation will be first done in a model independent way in the framework of  $\chi$ PT, using the imaginary time formalism. We will consider the lowest order chiral Lagrangian extended to include electromagnetism, and obtain the one-loop thermal effective action for soft modes with the help of the background field method. We will work in the hard thermal loop (HTL) approximation [10,11]. This methodic approximation allows us to identify the leading thermal corrections to any Feynman diagram, while ensuring the gauge independence

of the results.

Finally, we will generalize to low  $T$  the sum rule computation of Das et al. [3]. Using the same kind of soft pion techniques as in the  $T = 0$  case, we will check our previous results for pions. In this approach we will make use of the evaluation of thermal correlators of vector and axial vector currents derived in Ref. [12].

We will make a big emphasis on the assumptions and approximations involved in any of the two above mentioned approaches. It will then be clearly established that the standard soft pion techniques at low  $T$  in the chiral limit have their diagrammatic counterpart in the HTL approximation.

This paper is structured as follows. In Subsect. II A we review the  $\chi$ PT framework used in our computation. In Subsect. II B we obtain the one-loop thermal effective action for the soft Goldstone bosons. The sum rule computation of Das et al [3] is reviewed in Subsect. III A. This computation is then generalized at low  $T$  in Subsec. III B. In Subsect. III C we present the numerical results, and we end with the conclusions in Sect. IV. A general discussion on HTL's is presented in the Appendix.

## II. CHIRAL PERTURBATION THEORY COMPUTATION

### A. Lowest Order Chiral Lagrangian with Electromagnetic Interactions

In this subsection we present a brief summary of  $\chi$ PT in the lowest order extended to include electromagnetic interactions. We also explain how the background field method (BFM) is used in this specific case. We use natural units, so that  $\hbar = c = k_B = 1$ .

At leading order the effective low energy Lagrangian describing the physics of the (pseudo) Goldstone bosons of the spontaneously broken chiral symmetry is, in Minkowski space-time [5]

$$\begin{aligned} \mathcal{L}_2 = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2a}(\partial_\mu A^\mu)^2 \\ & + \frac{f_\pi^2}{4} \text{Tr} (\nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma) + \frac{f_\pi^2}{4} \text{Tr} (\chi^\dagger \Sigma + \chi \Sigma^\dagger) + C \text{Tr} (Q \Sigma Q \Sigma^\dagger) , \end{aligned} \quad (2.1)$$

where  $A_\mu$  is the photon field,  $F_{\mu\nu}$  is the photon field strength,  $a$  is the gauge-fixing parameter, and  $\Sigma$  is a  $SU(N)$  unitary matrix written in terms of the (pseudo) Goldstone bosons  $\Phi$  as

$$\Sigma = \exp (i\Phi/f_\pi) . \quad (2.2)$$

For  $N = 3$ ,  $\Phi = \Phi^a \lambda^a$ , where  $\lambda^a$  are the Gell-Mann matrices, and in terms of the physical particles, one can write

$$\Phi = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix} . \quad (2.3)$$

The matrix  $Q$  is the quark charge matrix, which for the  $N = 3$  case reads

$$Q = \frac{e}{3} \begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}, \quad (2.4)$$

and  $e$  is the electromagnetic coupling constant.

The covariant derivative is defined as

$$\nabla_\mu \Sigma = \partial_\mu \Sigma - i(v_\mu + a_\mu + QA_\mu)\Sigma + i\Sigma(v_\mu - a_\mu + QA_\mu), \quad (2.5)$$

$v_\mu$  and  $a_\mu$  being external vector and axial vector hermitian and traceless sources, respectively. One also defines

$$\chi = 2B(s + ip), \quad (2.6)$$

where  $B$  is a constant related to the quark condensate  $\langle \bar{q}q \rangle = -f_\pi^2 B [1 + O(m_q)]$ , and  $s$  and  $p$  are scalar and pseudoscalar external sources, respectively. The external scalar source can incorporate the mass matrix of quarks, which we put to zero since we will work in the chiral limit. In order to maintain the usual chiral counting, it is convenient to count the photon field as a quantity of order  $O(1)$  and the electromagnetic coupling  $e$  of  $O(p)$ , so the Lagrangian  $\mathcal{L}_2$  is of order  $O(p^2)$  [8].

The parameter  $C$  gives the purely electromagnetic part of the masses of the charged pions and kaons in the chiral limit

$$M_{\pi^\pm}^2 = M_{K^\pm}^2 = \frac{2e^2 C}{f_\pi^2} + O(m_q). \quad (2.7)$$

The constant  $C$  obeys a sum rule which relates its value to the correlators of vector and axial vector currents (see Subsec. III A). The low energy constant  $C$  is a model independent quantity, which enters naturally in the chiral Lagrangian.

To ensure the  $SU(N)_R \times SU(N)_L$  symmetry of the Lagrangian (2.1) one introduces local spurions  $Q^R(x)$  and  $Q^L(x)$  instead of the constant charge matrix, Eq. (2.4). The Lagrangian (2.1) is then modified so that the covariant derivative becomes

$$\nabla_\mu \Sigma = \partial_\mu \Sigma - i(F_\mu^R + Q^R A_\mu)\Sigma + i\Sigma(F_\mu^L + Q^L A_\mu), \quad (2.8)$$

where, for convenience, we have defined

$$F_\mu^R = v_\mu + a_\mu, \quad F_\mu^L = v_\mu - a_\mu. \quad (2.9)$$

The last term in Eq. (2.1) also becomes

$$C \text{Tr} (Q_R \Sigma Q_L \Sigma^\dagger). \quad (2.10)$$

Under a  $SU(N)_R \times SU(N)_L$  symmetry all fields transform as

$$\Sigma'(x) = U_R(x)\Sigma(x)U_L^\dagger(x) , \quad (2.11a)$$

$$Q^l(x) = U_l(x)Q^l(x)U_l^\dagger(x) , \quad l = R, L , \quad (2.11b)$$

$$\left(F_\mu^R(x) + Q^R(x)A_\mu(x)\right)' = U_R(x) \left(F_\mu^R(x) + Q^R(x)A_\mu(x)\right) U_R^\dagger(x) + iU_R(x)\partial_\mu U_R^\dagger(x) \quad (2.11c)$$

$$\left(F_\mu^L(x) + Q^L(x)A_\mu(x)\right)' = U_L(x) \left(F_\mu^L(x) + Q^L(x)A_\mu(x)\right) U_L^\dagger(x) + iU_L(x)\partial_\mu U_L^\dagger(x) , \quad (2.11d)$$

$$(s(x) + ip(x))' = U_R(x) (s(x) + ip(x)) U_L^\dagger(x) , \quad (2.11e)$$

$$U_l(x) \in SU(N)_l , \quad l = R, L . \quad (2.11f)$$

At this point one is ready to compute one-loop corrections to the lowest order chiral Lagrangian with the background field method. Let us recall that at the end of the computation one has to put the spurions  $Q_R(x) = Q_L(x) = Q$ .

In the BFM all fields are split into classical and quantum pieces. The photon gauge field is split additively

$$A_\mu(x) = \bar{A}_\mu(x) + \alpha_\mu(x) , \quad (2.12)$$

where  $\bar{A}_\mu$  is the background field, and  $\alpha_\mu$  is the quantum one. The matrix containing the Goldstone fields is split multiplicatively

$$\Sigma(x) = \xi(x) \exp(i\phi/f_\pi)\xi(x) , \quad (2.13)$$

where

$$\bar{\Sigma}(x) = \xi(x)\xi(x) , \quad (2.14)$$

is the background or classical field, and  $\phi$  is the quantum fluctuation.

In the spirit of the BFM, one expands the Lagrangian keeping only terms which are quadratic in the fluctuations

$$\mathcal{L}_2 = \mathcal{L}_2^{(0)} + \mathcal{L}_2^{(2)} + \dots \quad (2.15)$$

We will work in Euclidean space-time, so that we rotate the Minkowski Lagrangian  $\mathcal{L}_2$  to the Euclidean one  $\mathcal{L}_{2,E}$ . In Euclidean space-time one has [8]

$$\begin{aligned} \mathcal{L}_{2,E}^{(0)} = & -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - \frac{1}{2a}(\partial_\mu\bar{A}^\mu)^2 \\ & - f_\pi^2 \text{Tr}(\bar{\Delta}_\mu)^2 - \frac{f_\pi^2}{4} \text{Tr}(M^+) - \frac{C}{4} \text{Tr}(H_R^2 - H_L^2) , \end{aligned} \quad (2.16)$$

and

$$\begin{aligned} \mathcal{L}_{2,E}^{(2)} = & \frac{1}{4} \text{Tr}(\bar{d}_\mu\phi)^2 - \frac{1}{4} \text{Tr}([\bar{\Delta}_\mu, \phi])^2 + \frac{1}{8} \text{Tr}(M^+\phi^2) \\ & - \frac{1}{4} (\partial_\mu\alpha_\nu - \partial_\nu\alpha_\mu)^2 - \frac{1}{2a} (\partial_\mu\alpha^\mu)^2 + \frac{f_\pi}{2} \text{Tr}([\bar{\Delta}_\mu, \phi] H_R) \alpha^\mu \\ & - \frac{C}{8f_\pi^2} \text{Tr}([H_R + H_L, \phi][H_R - H_L, \phi]) \\ & - \frac{f_\pi}{2} \text{Tr}(\bar{d}_\mu\phi H_L) \alpha^\mu + \frac{f_\pi^2}{4} \text{Tr}(H_L^2)\alpha^\mu\alpha_\mu \end{aligned} \quad (2.17)$$

where

$$\bar{d}_\mu \phi = \partial_\mu \phi + [\bar{\Gamma}_\mu, \phi] , \quad (2.18a)$$

$$\bar{\Gamma}_\mu = \frac{1}{2} \left( \xi^\dagger \bar{\nabla}_\mu^R \xi + \xi \bar{\nabla}_\mu^L \xi^\dagger \right) , \quad (2.18b)$$

$$\bar{\Delta}_\mu = \frac{1}{2} \left( \xi^\dagger \bar{\nabla}_\mu^R \xi - \xi \bar{\nabla}_\mu^L \xi^\dagger \right) , \quad (2.18c)$$

$$\bar{\nabla}_\mu^l = \partial_\mu - i(F_\mu^l + Q^l \bar{A}_\mu) , \quad l = R, L , \quad (2.18d)$$

$$M^+ = \xi^\dagger \chi \xi^\dagger + \xi \chi^\dagger \xi , \quad (2.18e)$$

$$H_R = \xi^\dagger Q_R \xi + \xi Q_L \xi^\dagger , \quad (2.18f)$$

$$H_L = \xi^\dagger Q_R \xi - \xi Q_L \xi^\dagger , \quad (2.18g)$$

We will work in the massless quark limit. In this situation all particles at tree level are massless except for the charged pions and kaons.

To obtain the one-loop effective action one has to integrate out the quantum fluctuations  $\phi$  and  $\alpha_\mu$ , while treating the background fields as external sources. The functional integral can be done since  $\mathcal{L}_{2,E}^{(2)}$  is only quadratic in the quantum fields. In a diagrammatic approach, one should only consider diagrams with quantum fields running inside the loop, while the background or classical fields only generate external vertices. At  $T = 0$  the one-loop functional generated by  $\mathcal{L}_{2,E}^{(2)}$  has been evaluated in Ref. [8], using dimensional regularization and renormalization to deal with the ultraviolet (UV) divergencies. In Ref. [8] the next-to-leading order chiral Lagrangian  $\mathcal{L}_4$  has also been constructed. We will not need it at the order of the computation we are working.

## B. One-Loop Thermal Effective Action for Soft Modes

In this subsection we use the BFM to compute one-loop thermal corrections to the low energy constant  $C$  for soft modes. We will only consider thermal corrections and refer to [8] for a more general  $T = 0$  analysis. Let us mention that the UV divergencies which appear at  $T = 0$  and at finite  $T$  are the same, so the renormalization procedure does not change at all in our analysis with respect to the  $T = 0$  one.

With respect to UV finite corrections in the chiral limit, and if we restrict our computation to soft modes, that is momenta  $P \ll T$ , then one can ensure that the leading thermal corrections, which are typically of order  $T^2/f_\pi^2$ , are dominant with respect to the  $T = 0$  ones, which are of order  $P^2/f_\pi^2$ . Therefore, if one neglects corrections of order  $P^2/f_\pi^2$ , there are only two expansion parameters in the chiral limit. One of them is the gauge coupling constant squared,  $e^2$ , and the other is the dimensionless quantity  $T^2/f_\pi^2$ . We will assume that the two expansion parameters are equally important.

The computations will be done using the imaginary time formalism (ITF). Feynman rules for propagators and vertices are straightforwardly derived from Eq. (2.17). We will denote Euclidean momentum with capital letters, so  $K^2 = k_0^2 + \mathbf{k}^2$ . In ITF and for bosonic fields  $k_0 = 2\pi nT$  for integral  $n$ . The standard notation for the thermal momentum measure

$$\int \frac{d^4 K}{(2\pi)^4} \equiv T \sum_{n=-\infty}^{n=\infty} \int \frac{d^3 k}{(2\pi)^3}, \quad (2.19)$$

will be used throughout this article.

The leading thermal corrections to any Feynman diagram when the external momenta is *soft*, that is  $\ll T$ , arises when the momenta running inside the loop is *hard*, or  $\sim T$ . Those diagrams are called *hard thermal loops* (HTL's) [10,11]. There are systematic set of rules to extract from each Feynman diagram the corresponding HTL. Those will be taken into account in the computation.

In this paper we are interested in finding the thermal corrections to the masses of the soft charged pions and kaons. So we are looking for one-loop corrections to the term in  $\mathcal{L}_{2,E}^{(0)}$

$$- \frac{C}{4} \text{Tr} \left( H_R^2 - H_L^2 \right). \quad (2.20)$$

Since we will only compute the leading thermal correction to the effective action, and the masses of the charged quantum fields are soft, we will neglect the masses of the quantum fields in the propagators. This means that we are neglecting corrections of order  $M_{\pi^\pm}/T$  in the final answers. On the other hand, considering those masses as soft quantities means than we are assuming  $T^2 \gtrsim C/f_\pi^2$ .

There are only three Feynman diagrams to be evaluated, namely those generated by the last three terms of Eq. (2.17), all of which give, in the HTL approximation, a tadpole type contribution at one-loop order. We use dimensional regularization to deal with the UV divergencies. In dimensional regularization and renormalization the  $T = 0$  contribution to the tadpole vanishes, so we only need to consider the thermal contributions to each diagram.

The first diagram we consider is a tadpole of quantum fields  $\phi$ . The generators of  $SU(N)$   $\lambda^a$  are normalized as  $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ , and  $[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$ , where  $f^{abc}$  are the structure constants of  $SU(N)$ . Then this tadpole gives the correction to Eq. (2.20)

$$\frac{C}{f_\pi^2} f^{abc} f^{a'bc} (H_R + H_L)^a (H_R - H_L)^{a'} \int \frac{d^4 K}{(2\pi)^4} \frac{1}{K^2} = C \frac{N T^2}{24 f_\pi^2} \text{Tr} \left( H_R^2 - H_L^2 \right). \quad (2.21)$$

In Eq. (2.21) we have used the  $SU(N)$  relation  $f^{abc} f^{a'bc} = N\delta^{aa'}$ .

Taking into account the relation

$$\text{Tr} H_L^2 = -\frac{1}{2} \text{Tr} \left( H_R^2 - H_L^2 \right) + \text{Tr} \left( Q_R^2 + Q_L^2 \right), \quad (2.22)$$

one sees that there are two other diagrams which give corrections to Eq. (2.20). One of them is a tadpole of the quantum gauge field, and the other is a diagram with a quantum boson  $\phi$  and a quantum gauge field  $\alpha_\mu$  circulating inside the loop. We evaluate the two above diagrams in the Feynman gauge  $a = 1$ . However, since we only retain the HTL of the diagram, one can ensure that our results are gauge independent [10,11]. We thus have

$$\frac{f_\pi^2}{4} \text{Tr} H_L^2 \left( \int \frac{d^4 K}{(2\pi)^4} \frac{\delta^{\mu\nu} \delta_{\mu\nu}}{K^2} - \int \frac{d^4 K}{(2\pi)^4} \frac{(K+P)_\mu (K+P)_\nu \delta^{\mu\nu}}{K^2 (K+P)^2} \right). \quad (2.23)$$

Using the relation (2.22) we end up with

$$-\frac{f_\pi^2}{8} \text{Tr} \left( H_R^2 - H_L^2 \right) \int \frac{d^4 K}{(2\pi)^4} \frac{3}{K^2} = -\frac{f_\pi^2 T^2}{32} \text{Tr} \left( H_R^2 - H_L^2 \right) , \quad (2.24)$$

plus an irrelevant constant term proportional to  $\text{Tr} (Q_R^2 + Q_L^2)$ .

The one-loop thermal effects for soft background fields involving the  $\bar{\Gamma}_\mu$  and  $\bar{\Delta}_\mu$  fields arising from the two first pieces of Eq. (2.17) have already been obtained in Refs. [13,14]. One can also easily compute the thermal correction to the term  $\text{Tr} M^+$ , just by evaluating a tadpole of  $\phi$  fields, taking into account the  $SU(N)$  relation  $\{\lambda^a, \lambda^b\} = \frac{4}{N} \delta^{ab} + 2d^{abc} \lambda^c$ . Finally, one can write the complete one-loop thermal effective action for soft modes as

$$\begin{aligned} \mathcal{Z}_2 = S_2 + \delta S_{2,T} = & \int d^4 x \left( -\frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} - \frac{1}{2a} (\partial_\mu \bar{A}^\mu)^2 \right) \\ & - \frac{NT^2}{12} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \int d^4 x d^4 y \text{Tr} \left( \bar{\Gamma}_{\mu\lambda}(x) \langle x | \frac{Q^\mu Q_\nu}{-(Q \cdot \bar{d})^2} | y \rangle \bar{\Gamma}^{\nu\lambda}(y) \right) \\ & - \int d^4 x \left( f_\pi^2(T) \text{Tr}(\bar{\Delta}_\mu^2(x)) + \frac{C(T)}{4} \text{Tr} (H_R^2(x) - H_L^2(x)) \right. \\ & \left. - \frac{f_\pi^2}{4} \left( 1 - \frac{N^2 - 1 T^2}{12N f_\pi^2} \right) \text{Tr}(M^+(x)) \right) , \end{aligned} \quad (2.25)$$

where  $Q^\mu$  is a light-like four vector (see Ref. [13]), and [15–17]

$$f_\pi(T) = f_\pi \left( 1 - \frac{N T^2}{24 f_\pi^2} \right) , \quad (2.26)$$

and

$$\bar{\Gamma}_{\mu\nu} = \partial_\mu \bar{\Gamma}_\nu - \partial_\nu \bar{\Gamma}_\mu + [\bar{\Gamma}_\mu, \bar{\Gamma}_\nu] , \quad (2.27)$$

$$C(T) = C \left( 1 - \frac{N T^2}{6 f_\pi^2} \right) + \frac{T^2 f_\pi^2}{8} . \quad (2.28)$$

If we put the external sources  $v_\mu = a_\mu = 0$ , and  $Q_R = Q_L = Q$ , the third term in Eq. (2.25) is just the HTL effective action [18], which includes the Debye mass for the electric field.

With the above effective action one can read the thermal correction to the quark condensate, just by performing a functional derivative of  $\mathcal{Z}_2$  with respect to the external source  $s(x)$ . It is given by

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_{T=0} \left( 1 - \frac{N^2 - 1 T^2}{12N f_\pi^2} \right) , \quad (2.29)$$

in agreement with Ref. [15].

The value of the masses of the charged Goldstone bosons are obtained by finding the poles of the two-point functions of the axial current (see Ref. [2] for details about the computation). So one gets



$$M_{\pi^\pm}^2(T) = \frac{2e^2 C(T)}{f_\pi^2(T)} . \quad (2.30)$$

In the low  $T$  limit, and using the values of  $C(T)$  and  $f_\pi(T)$  above one then finds, at leading order,

$$M_{\pi^\pm}^2(T) = M_{K^\pm}^2(T) \approx M_{\pi^\pm}^2 \left( 1 - \frac{N T^2}{12 f_\pi^2} \right) + \frac{e^2 T^2}{4} . \quad (2.31)$$

If one puts  $C = 0$ , then the thermal masses of the charged bosons agree with those computed in Ref. [19] for scalar QED. Temperature corrections to the electromagnetic mass difference of pions have also been considered in [20]. We agree with the results of [20], except for the fact that thermal corrections of order  $T^2/f_\pi^2$  were not considered there.

Finally, let us stress that in the massless quark limit Dashen's theorem remains valid at finite  $T$ , as expected.

### III. SUM RULE APPROACH TO THE ELECTROMAGNETIC MASS DIFFERENCE OF PIONS

#### A. Computation at Zero Temperature

In this subsection we review the sum rule computation of the electromagnetic mass difference of pions. We will follow closely the notation and conventions of Ref. [6], working in this subsection in Minkowski space-time. In the following subsection we will generalize the computation at low  $T$ .

At  $T = 0$  one works with the assumption that  $SU(2)_R \times SU(2)_L$  is a symmetry of the QCD Hamiltonian which is spontaneously broken to  $SU(2)_{L+R}$ . The pions are then the associated Goldstone bosons.

In the chiral limit the neutral pion is massless,  $M_{\pi^0} = 0$ , while the charged pions get a mass from one-photon exchange contributions. The mass of the charged pions is given by

$$M_{\pi^\pm}^2 = \frac{ie^2}{2} \int d^4x \Delta^{\mu\nu}(x) \langle \pi(p) | \mathcal{T} (J_\mu(x) J_\nu(0)) | \pi(p) \rangle , \quad (3.1)$$

where  $\Delta^{\mu\nu}$  is the photon propagator, and  $J_\mu$  is the electromagnetic current. In momentum space and in Feynman gauge, Eq. (3.1) becomes

$$M_{\pi^\pm}^2 = \frac{ie^2}{2} \int \frac{d^4q}{(2\pi)^4} \frac{g^{\mu\nu}}{q^2} T_{\mu\nu}(q^2, p \cdot q) , \quad (3.2)$$

where

$$T_{\mu\nu}(q^2, p \cdot q) = i \int d^4x e^{iq \cdot x} \langle \pi(p) | \mathcal{T} (J_\mu(x) J_\nu(0)) | \pi(p) \rangle . \quad (3.3)$$

Eq. (3.3) can be separated into non-contact (NC) and contact (C) contributions [6]. The last one refers to one which has both photons interacting at the same vertex.

One can reduce the non-contact contribution to the Compton scattering amplitude to vacuum polarization functions [3,6]. This is done using the soft pion theorem

$$\lim_{p_\mu \rightarrow 0} \langle \pi^a(p) \beta | \mathcal{O} | \alpha \rangle = -\frac{i}{f_\pi} \langle \beta | [Q_5^a, \mathcal{O}] | \alpha \rangle , \quad (3.4)$$

where  $\alpha$  and  $\beta$  are arbitrary states, and  $Q_5^a$  is the axial charge, and also the current algebra

$$[Q_5^a, \mathcal{V}_\mu^b] = i f^{abc} \mathcal{A}_\mu^c , \quad [Q_5^a, \mathcal{A}_\mu^b] = i f^{abc} \mathcal{V}_\mu^c . \quad (3.5)$$

In Eq. (3.5)  $\mathcal{V}_\mu^a$  and  $\mathcal{A}_\mu^a$  are the vector current and axial vector current, respectively. Thus, in the soft pion limit,

$$\begin{aligned} \lim_{p_\mu \rightarrow 0} T_{\mu\nu}^{(NC)}(q^2, p \cdot q) &\equiv \frac{2i}{f_\pi^2} \left( \Pi_{\mu\nu}^{V,3}(q) - \Pi_{\mu\nu}^{A,3}(q) \right) \\ &= \frac{2i}{f_\pi^2} \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{T} \left( \mathcal{V}_\mu^3(x) \mathcal{V}_\nu^3(0) - \mathcal{A}_\mu^3(x) \mathcal{A}_\nu^3(0) \right) | 0 \rangle . \end{aligned} \quad (3.6)$$

The above correlators are written in terms of spectral functions as

$$\Pi_{\mu\nu}^{V,3}(q) = i q^2 \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \int_0^\infty ds \frac{\rho_V(s)}{q^2 - s} , \quad (3.7)$$

$$\Pi_{\mu\nu}^{A,3}(q) = i q^2 \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \int_0^\infty ds \frac{\rho_A(s)}{q^2 - s} - i f_\pi^2 \frac{q_\mu q_\nu}{q^2} . \quad (3.8)$$

The contact term contribution to the Compton scattering amplitude is [6]

$$T_{\mu\nu}^{(C)}(q^2, p \cdot q) = 2g_{\mu\nu} . \quad (3.9)$$

Adding the non-contact and contact contributions one gets

$$\lim_{p_\mu \rightarrow 0} T_{\mu\nu}(q^2, p \cdot q) = - \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left( -2 + \frac{2}{f_\pi^2} \int_0^\infty ds (\rho_V(s) - \rho_A(s)) \frac{q^2}{q^2 - s} \right) . \quad (3.10)$$

Therefore, in the mass formula one has

$$M_{\pi^\pm}^2 = 3ie^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} - \frac{3ie^2}{f_\pi^2} \int \frac{d^4q}{(2\pi)^4} \int_0^\infty ds \frac{\rho_V(s) - \rho_A(s)}{q^2 - s} . \quad (3.11)$$

The above integrals are evaluated using dimensional regularization. Thus, the first term in Eq. (3.11) vanishes. The second one is logarithmically divergent, but the coefficient of the divergent piece vanishes due to the second Weinberg sum rule. If one chooses a different UV regulator, there are also quadratic divergences which cancel due to the first Weinberg sum rule [5].

The second term in Eq. (3.11) can be easily evaluated if at this point one assumes that the spectral functions  $\rho_V$  and  $\rho_A$  are dominated by the vector and axial vector mesons [3]

$$\rho_V(s) = F_\rho^2 \delta(s - M_\rho^2) , \quad (3.12)$$

$$\rho_A(s) = F_A^2 \delta(s - M_A^2) . \quad (3.13)$$

Using the relations

$$F_\rho^2 M_\rho^2 = F_A^2 M_A^2 , \quad F_A^2 = F_\rho^2 - f_\pi^2 , \quad (3.14)$$

which are derived from Weinberg's sum rules [4]

$$\int_0^\infty ds (\rho_V(s) - \rho_A(s)) = f_\pi^2 , \quad (3.15)$$

$$\int_0^\infty ds s (\rho_V(s) - \rho_A(s)) = 0 , \quad (3.16)$$

one finds

$$M_{\pi^\pm}^2 = -\frac{3e^2}{16\pi^2} M_\rho^2 \frac{F_\rho^2}{f_\pi^2} \ln \frac{M_\rho^2}{M_A^2} , \quad (3.17)$$

which is the classical result of Das et al. [3].

Better estimates of the correlator of vector and axial vector currents, not necessarily assuming a narrow-width spectral function as in Eqs. (3.12)-(3.13), have been considered in the literature [6], [21].

## B. Computation at Low Temperature

In this subsection we generalize the previous sum rule computation at low  $T$ , working in Euclidean space-time and in the ITF. At low  $T$  one can assume that the main hypothesis that hold at  $T = 0$  are still valid, and that one only needs to consider small thermal corrections to the previous computation.

At low  $T$  chiral symmetry is still spontaneously broken, although at high enough  $T$  it is supposed to be restored. We make the assumption that at low  $T$  one can use the same kind of soft pion theorems than at  $T = 0$ . In the order of the computation we are working knowledge of the  $T = 0$  Weinberg's sum rules will be sufficient for us. A generalization of Weinberg's sum rules at finite  $T$  can be found in Ref. [22].

We start from Eq. (3.1), but now evaluated in a thermal bath at equilibrium. We will follow the same steps as in Sect. III A to reduce the thermal amplitude

$$\langle \pi(p) | \mathcal{T} (J_\mu(x) J_\nu(0)) | \pi(p) \rangle_T \quad (3.18)$$

to a thermal polarization function. Thus, we use the low  $T$  generalization of the soft pion theorem Eq. (3.4) at leading order in the thermal correction, i.e.,

$$\lim_{p_\mu \rightarrow 0} \langle \pi^a(p) \beta | \mathcal{O} | \alpha \rangle_T = -\frac{i}{f_\pi(T)} \langle \beta | [Q_5^a, \mathcal{O}] | \alpha \rangle_T , \quad (3.19)$$

and the current algebra. It is important to realize that Eq. (3.19) only holds at low  $T$  and at leading order in  $T^2/f_\pi^2$ . At order  $T^4/f_\pi^4$  there are two distinct pion decay constants [17], due to the loss of Lorentz invariance, and therefore at that order Eq. (3.19) would be modified.

The thermal amplitude Eq. (3.18) is then written in terms of the thermal correlators of vector and axial vector currents. In Euclidean space-time, these correlators are given by

$$\Pi_{\mu\nu}^{V,a}(K, T) = \int d^4x e^{iK \cdot x} \sum_n \langle n | \mathcal{T} \mathcal{V}_\mu^a(x) \mathcal{V}_\nu^a(0) e^{(\Omega-H)/T} | n \rangle , \quad (3.20)$$

$$\Pi_{\mu\nu}^{A,a}(K, T) = \int d^4x e^{iK \cdot x} \sum_n \langle n | \mathcal{T} \mathcal{A}_\mu^a(x) \mathcal{A}_\nu^a(0) e^{(\Omega-H)/T} | n \rangle , \quad (3.21)$$

where the sum is over the full set of eigenstates of the Hamiltonian  $H$ , and  $e^{-\Omega} = \sum_n \langle n | e^{-H/T} | n \rangle$ , and  $K$  is the Euclidean momentum. These thermal correlators have been computed in Ref. [12] assuming that the lightest particles of the spectrum of  $H$ , that is, the pions, give the main contribution to the sum. Then, just by using the same soft pion theorems that hold at  $T = 0$  and the current algebra, and after integrating over the pionic thermal space, one gets for massless pions

$$\Pi_{\mu\nu}^V(K, T) = \left( 1 - \frac{T^2}{6f_\pi^2} \right) \Pi_{\mu\nu}^V(K, T = 0) + \frac{T^2}{6f_\pi^2} \Pi_{\mu\nu}^A(K, T = 0) , \quad (3.22)$$

$$\Pi_{\mu\nu}^A(K, T) = \left( 1 - \frac{T^2}{6f_\pi^2} \right) \Pi_{\mu\nu}^A(K, T = 0) + \frac{T^2}{6f_\pi^2} \Pi_{\mu\nu}^V(K, T = 0) . \quad (3.23)$$

The two above low  $T$  correlators can thus be expressed in terms of their  $T = 0$  values, although there is a mixing between the vector and axial vector ones. Let us stress that at finite  $T$ , and due to the loss of Lorentz invariance in the thermal bath, the general form of the thermal correlator of two currents should depend on more unknown functions than the ones at  $T = 0$ . However, at low  $T$  one can assume that these correlators retain, approximately, their covariant form [12].

Once we know the value of the above thermal correlators, we are ready to compute the thermal corrections to the mass of the charged pions. We work in the ITF, where the thermal momentum measure is the one written in Eq. (2.19). From Eqs. (3.22)-(3.23) we know that

$$\Pi_{\mu\nu}^V(K, T) - \Pi_{\mu\nu}^A(K, T) = \left( 1 - \frac{T^2}{3f_\pi^2} \right) \left( \Pi_{\mu\nu}^V(K, T = 0) - \Pi_{\mu\nu}^A(K, T = 0) \right) . \quad (3.24)$$

In Feynman gauge, one then has

$$M_{\pi^\pm}^2(T) = 3e^2 \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^2} - \frac{3e^2}{f_\pi^2(T)} \left( 1 - \frac{T^2}{3f_\pi^2} \right) \int \frac{d^4K}{(2\pi)^4} \int_0^\infty ds \frac{\rho_V(s) - \rho_A(s)}{K^2 + s} . \quad (3.25)$$

The thermal contribution of the first term in Eq. (3.25) does not vanish, as it happens for the  $T = 0$  contribution in dimensional regularization. Notice that we have neglected the thermal factors in front of the first term of Eq. (3.25). Those terms would yield thermal corrections of order  $e^2 T^4/f_\pi^4$ , which are subleading at the order of the computation that we are working. If one uses dimensional regularization to cure the UV divergencies of the above

integral, then the quadratic divergencies of Eq. (3.25) vanish. If one uses a different UV regulator then to check that the quadratic divergencies which multiply the  $T^2$  corrections cancel one would need the thermal factors that we have neglected in Eq. (3.25). To simplify our computation, we will work with dimensional regularization, where Eq. (3.25) holds exactly.

At this stage one can apply the hypothesis of vector meson dominance, which is valid at  $T = 0$ . We should then compute the integrals

$$\int \frac{d^4 K}{(2\pi)^4} \left( \frac{F_\rho^2}{K^2 + M_\rho^2} - \frac{F_A^2}{K^2 + M_A^2} \right). \quad (3.26)$$

After performing the sum over Matsubara frequencies one then gets the  $T = 0$  contribution (already evaluated in the previous subsection), plus the pure thermal part. However, since the masses of the vector resonances are much bigger than the temperatures we are considering,  $M_\rho, M_A \gg T$ , the thermal corrections associated to those particles are suppressed as  $\sim \exp(-M_{\rho,A}/T)$ .

Thus, collecting all the thermal corrections, we get

$$M_{\pi^\pm}^2(T) = \left( 1 - \frac{T^2}{6f_\pi^2} \right) M_{\pi^\pm}^2 + \frac{e^2 T^2}{4}, \quad (3.27)$$

plus subleading corrections. Therefore, we have checked our formula (2.30) for the  $N = 2$  case.

It is worthwhile emphasizing that since the thermal corrections to the second term of Eq. (3.25) factorize, our results are also valid if we use a better evaluation of the spectral functions than the ones given in Eqs. (3.12)-(3.13).

### C. Results

At zero temperature, the difference of the squared pion masses in the chiral limit is given by the result of Das et al., Eq. (3.17). Using the relations (3.14) to eliminate the parameters of the axial vector meson,  $A$ , it can be written as

$$M_{\pi^\pm}^2 - M_{\pi^0}^2 = \frac{3\alpha}{4\pi} M_\rho^2 \frac{F_\rho^2}{f_\pi^2} \ln \frac{F_\rho^2}{F_\rho^2 - f_\pi^2}, \quad (3.28)$$

where  $\alpha$  is the electromagnetic coupling constant. Although this result was obtained in the chiral limit, it can be used to give an estimate of the electromagnetic mass difference of pions. The difference of the squared pion masses is

$$M_{\pi^\pm}^2 - M_{\pi^0}^2 \sim 2M_\pi \Delta M_\pi. \quad (3.29)$$

Taking the physical value of the pion mass  $M_\pi = 135$  MeV, and the following values for the other parameters involved:  $F_\pi = 92.4$  MeV,  $M_\rho = 770$  MeV,  $F_\rho = 153$  MeV and  $\alpha = 1/137$ , one gets  $\Delta M_\pi = 4.8$  MeV, which is in very good agreement with the experimental value  $(\Delta M_\pi)_{exp} = 4.6$  MeV.

We thus expect that  $M_{\pi^\pm}^2(T) - M_{\pi^0}^2(T)$  at low temperature is also well approximated by the chiral limit calculation Eq. (3.27), i.e.,

$$\Delta M_\pi^2(T) \equiv M_{\pi^\pm}^2(T) - M_{\pi^0}^2(T) = \left(1 - \frac{T^2}{6f_\pi^2}\right) \Delta M_\pi^2 + \pi\alpha T^2, \quad (3.30)$$

where  $\Delta M_\pi^2 = M_{\pi^\pm}^2 - M_{\pi^0}^2$ , given in Eq. (3.28). The result is plotted in Fig. 1, as a function of the temperature. Although our computation is only strictly valid for low  $T$ , we have extrapolated it up to  $T \sim \sqrt{6}f_\pi \sim 220$  MeV. The dashed-dotted line in Fig. 1 represents the part proportional to the electromagnetic mass difference at  $T = 0$ ,  $\Delta M_\pi^2$ , in Eq. (3.30), and we see that it decreases with  $T$ . The dashed line corresponds to the typical thermal mass of charged particles, which grows with  $T$ , while the solid line is the full result.

#### IV. CONCLUSIONS

The difference of correlators of vector and axial vector currents can be taken as an order parameter of chiral symmetry breaking [23,24]. The sum rule

$$f_\pi^2 \delta^{ab} = \frac{i}{3} \int d^4x \langle 0 | \mathcal{T} \left( \mathcal{V}_\mu^a(x) \mathcal{V}^{\mu b}(0) - \mathcal{A}_\mu^a(x) \mathcal{A}^{\mu b}(0) \right) | 0 \rangle$$

implies the asymmetry of the vacuum, since the operator  $\mathcal{V}_\mu^a(x) \mathcal{V}^{\mu a}(0) - \mathcal{A}_\mu^a(x) \mathcal{A}^{\mu a}(0)$  transforms as the irreducible representation (3,3) of the symmetry group  $SU(2)_L \times SU(2)_R$ , and therefore if the vacuum were invariant, the above correlator should be zero. If chiral symmetry is restored at high  $T$ , this order parameter should then vanish.

Low  $T$  computations of the vector and axial vector correlators show a tendency towards chiral symmetry restoration [12]. Although those computations cannot be trusted at higher  $T$ , they give a good estimate of the critical temperature of the phase transition,  $T_c \sim 160$  MeV, which agrees with the values found in lattice computations [25].

The electromagnetic mass difference of pions depends on the correlators of vector and axial vector currents [3]. In concordance with the signs of chiral symmetry restoration at finite  $T$ , one would naturally expect that this mass difference should decrease with  $T$ . This is what we have obtained for low  $T$ , although it is not *a priori* obvious from our Eq. (3.30). We have found a slow decrease of this mass difference with  $T$  but only after a numerical analysis involving the  $T = 0$  values of the different parameters. It should be stressed that even if chiral symmetry is restored at a certain  $T_c$ , typical thermal effects occurring in the plasma of pions and photons will always generate a thermal mass for the charged particles going as  $\sim eT$ . Therefore, one can never expect that the electromagnetic mass difference of pions should vanish at any high  $T$ .

Our computation was done in the limit of massless quarks. In order to get a more trustworthy value of the mass difference  $(M_{K^\pm}^2 - M_{K^0}^2)(T)$ , one would need to take into account also QCD corrections proportional to the quark masses, apart from the pure electromagnetic correction computed here. This was, however, beyond the scope of this article.

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## APPENDIX A: HARD THERMAL LOOPS

In a gauge field theory at finite  $T$  hard thermal loops are one-loop diagrams which are as important as the tree amplitudes for soft momenta. Soft denotes a energy scale  $\ll T$ . If  $g$  is the gauge coupling constant, and it is assumed that  $g \ll 1$ , then a soft scale is typically of the order  $\sim gT$ . For soft fields, then HTL's have to be resummed in order to take into account those one-loop effects consistently [10].

Braaten and Pisarski realized that HTL's only arise when all the external momenta of a diagram are soft and the internal one is hard  $\sim T$ . They were then able to establish some power counting rules to extract from each Feynman diagram the corresponding HTL.

In the non-linear sigma model HTL's also appear for soft external momenta [14,13] In  $\chi$ PT in the chiral limit without gauge interactions the expansion parameter for thermal corrections is  $T^2/f_\pi^2$  and it is this scale the one which allows to define the scales soft  $\sim \sqrt{T^2/f_\pi^2} T$  or hard  $T$ . Resummation is not required in this case.

In the chiral Lagrangian with electromagnetic interactions included there are then essentially two expansion parameters,  $e$  and  $\sqrt{T^2/f_\pi^2}$ . We have assumed in this article that the two of them are of the same order.

In our case, one can then apply the same power counting rules derived in Ref. [10]. Following Braaten and Pisarski's analysis for the Lagrangian (2.17), it is then easy to realize that vertices which do not carry a momentum dependence do not produce a HTL, except for tadpole-like diagrams. So apart from the tadpoles, the only vertices which produce HTL's are

$$\frac{1}{4} \text{Tr} \left( \partial_\mu \phi [\bar{\Gamma}^\mu, \phi] \right) , \tag{A1}$$

and

$$- \frac{f_\pi}{2} \text{Tr} (\partial_\mu \phi H_L) \alpha^\mu . \tag{A2}$$

## FIGURES

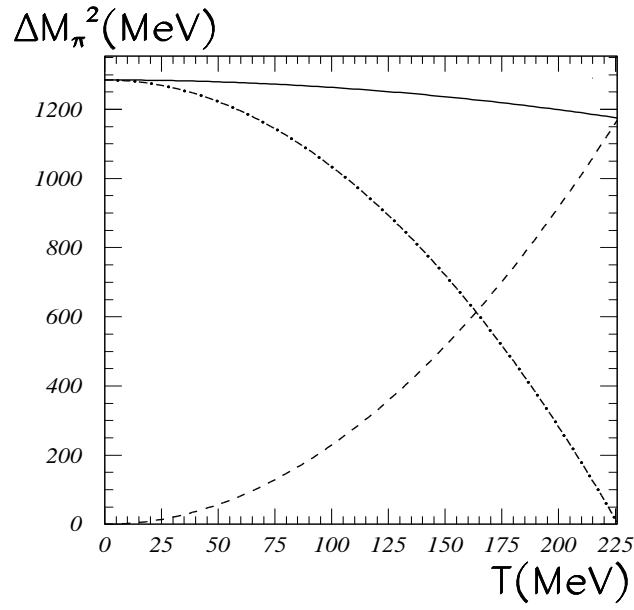


FIG. 1. Electromagnetic mass difference of pions  $\Delta M_\pi^2(T)$  as a function of the temperature in the chiral limit. The dashed-dotted line corresponds to the piece proportional to the electromagnetic mass difference at  $T = 0$ ,  $\Delta M_\pi^2$ , the dashed line to the term proportional to  $\alpha$  which grows with  $T$ , and the solid line is the full result



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