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Neutrino masses from new generations

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Abstract: We reconsider the possibility that Majorana masses for the three known neutrinos are generated radiatively by the presence of a fourth generation and one right-handed neutrino with Yukawa couplings and a Majorana mass term. We find that the observed light neutrino mass hierarchy is not compatible with low energy universality bounds in this minimal scenario, but all present data can be accommodated with five generations and two right-handed neutrinos. Within this framework, we explore the parameter space regions which are currently allowed and could lead to observable effects in neutrinoless double beta decay, $\mu - e$ conversion in nuclei and $\mu \to e\gamma$ experiments. We also discuss the detection prospects at LHC.

Keywords: LHC, Neutrino Physics, Lepton Flavour Violation, Beyond Standard Model.

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1. Introduction

Neutrino oscillation data [1–14] require at least two massive neutrinos with large mixing, providing one of the strongest evidences of physics beyond the Standard Model (SM). However, the new physics scale responsible for neutrino masses is largely unknown. With the starting of the LHC, new physics scales of order TeV will become testable through direct production of new particles, so it is very interesting to explore low-energy scenarios for neutrino masses. Moreover, typically, these scenarios also lead to observable signatures in precision experiments, such as violations of universality, charged lepton flavour violating (LFV) rare decays such as $\ell_i \to \ell_j \gamma$ or $\mu - e$ conversion in nuclei, which, being complementary to the LHC measurements, may help to discriminate among different models. Regarding the fundamental question of the neutrino mass nature, Dirac or Majorana, lepton-number-violating low-scale models may give additional contributions to neutrinoless double beta ($0\nu2\beta$) decay process, shedding new light on this issue.

On the other hand, one of the most natural extensions of the SM that has been extensively explored in the last years is the addition of one (or more) sequential generations of quarks and leptons [15]. This extension is very natural and has a rich phenomenology both at LHC as well as in LFV processes. Moreover, new generations address some of the open questions in the SM and can accommodate emerging hints on new physics (see for instance [16] for a recent review).

Theoretically, apart from simplicity, there are no compelling arguments in favour of only three families. In theories with extra dimensions one can relate the number of families to the topology of the compact extra dimensions or set constraints on the number of chiral families and allowed gauge groups by requiring anomaly cancellation. Then, one can build models to justify only three generations at low energies. However, one could also build other models in order to justify four or more generations. In the SM in four dimensions anomalies cancel within each generation and, therefore, the number of families is in principle free.

From the phenomenological point of view it seems that the most striking argument against new generations is the measurement of the invisible Z-boson decay width, $\Gamma_{\rm inv}$, which effectively counts the number of light degrees of freedom coupled to the Z-boson (lighter than $m_Z/2$) which is very close to 3 [17]. However, if neutrinos from new families are heavy they do not contribute to $\Gamma_{\rm inv}$ and, then, additional generations are allowed. Still, pairs of virtual heavy fermions from new generations contribute to the electroweak parameters and spoil the agreement of the SM with experiment. Global fits of models with additional generations to the electroweak data have been performed [18, 19] and the conclusion is that they favour no more that five generations with appropriate masses for the new particles. Although some controversy exists on the interpretation of the data (see for instance [20]) most of the fits make some simplifying assumptions on the mass spectrum of the new generations and do not consider Majorana neutrino masses for the new generations or the possibility of breaking dynamically the gauge symmetry via the condensation of the new generations' fermions; all these will give additional contributions to the oblique parameters and will modify the fits. Therefore, in view that soon we will see or exclude new generations thanks to the LHC, it is wise to approach this possibility with an open mind.

From the discussion above, it seems that neutrinos from new generations are very different from the ones discovered up to now, since they should have a mass 10^{11} times larger. However, this apparent difference is naturally explained within the framework that we are going to explore. In the SM neutrinos are massless because there are no right-handed neutrinos and because, with the minimal Higgs sector, lepton number is automatically conserved. We now know that neutrinos have masses, therefore the SM has to be modified to accommodate them; the simplest possibility is to add three right-handed neutrinos with Dirac mass terms, like for the rest of the fermions in the SM. If one then considers the SM with four generations (and four right-handed neutrinos), it is very difficult to justify why the neutrino from the fourth generation is 10^{11} times heavier than the three observed ones. This difficulty is alleviated if right-handed neutrinos have Majorana masses at the electroweak scale and the Dirac masses of the neutrinos are of the order of magnitude of their corresponding charged leptons [21]. Then, the see-saw mechanism is operative and gives neutrino masses $m_1 \sim m_e^2/M$, $m_2 \sim m_\mu^2/M$, $m_3 \sim m_\tau^2/M$, $m_4 \sim m_E^2/M \sim m_E$ (we denote by E the fourth generation charged lepton). Although with a common Majorana mass M at the electroweak scale it is not possible to obtain m_3 light enough to fit the observed neutrino masses, this could be solved by allowing different Majorana masses for the different generations; but then one should explain why $M_2, M_3 \gg M_4$.

Right-handed neutrinos, however, do not have gauge charges and are not needed to cancel anomalies, therefore their number is not linked to the number of generations. In fact, an extension of the SM with four generations and just one right-handed neutrino with both Dirac and a Majorana masses at the electroweak scale leads, at tree level, to three massless and two heavy Majorana neutrinos. Since lepton number is broken in the model, the three massless neutrinos acquire Majorana masses at two loops therefore providing a natural explanation for the tiny masses of the three known neutrinos [22] [1]. More generally, it has been shown that in the SM with n_L lepton doublets, n_H Higgs doublets and $n_R < n_L$ right-handed neutrino singlets with Yukawa and Majorana mass terms there are $n_L - n_R$ massless Majorana neutrinos at tree level, of which $n_L - n_R - \max(0, n_L - n_H n_R)$ states acquire mass by neutral Higgs exchange at one loop [24–26]. The remaining $\max(0, n_L - n_H n_R)$ states get masses at two loops. Similar extensions could be built with additional hyperchargeless fermion triplets, like in type III see-saw.

In this work we reconsider the model of ref. [22], without enlarging the scalar sector of the SM but allowing for extra generations. The paper is organised as follows. In section 2 we summarize current neutrino data and searches for new generations. In section 3 we review the radiative neutrino mass generation at two loops, and show that the observed light neutrino mass hierarchy can not be accommodated in the minimal scenario with four generations. In section 4 we present a five generation example which leads to the observed neutrino masses and (close to tribimaximal) mixing. We introduce a simple parametrization of the model and explore the parameter space allowed by current neutrino data, universality, charged lepton flavour violating rare decays $\ell_i \to \ell_j \gamma$ and $0\nu 2\beta$ decay, as well as the regions that will be probed in near future experiments (MEG, $\mu - e$ conversion in nuclei). Section 5 is devoted to collider phenomenology and we summarize our results in section 6.

2. Framework and review

It has been well established in the last decade that neutrinos are massive, thanks to the results obtained with solar [1–4,11], and atmospheric [6,7,10] neutrinos, confirmed in experiments using man-made beams: neutrinos from nuclear reactors [5] and accelerators [8,12].

The minimum description of all neutrino data requires mixing among the three neutrino states with definite flavour $(\nu_e, \nu_\mu, \nu_\tau)$, which can be expressed as quantum superpositions of three massive states ν_i (i=1,2,3) with masses m_i. The standard parametrization of the

[1] Two-loop quantum corrections within the SM with only two massive Majorana neutrinos also lead to a (tiny) mass for the third one [23].

leptonic mixing matrix, $U_{\rm PMNS}$, is:

$$U_{\rm PMNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & 1 \end{pmatrix} \tag{2.1}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. In addition to the Dirac-type phase δ, analogous to that of the quark sector, there are two physical phases ϕ_i if neutrinos are Majorana particles. The measurement of these parameters is by now restricted to oscillation experiments which are only sensitive to mass-squared splittings ($\Delta m^2_{ij} \equiv m_i^2 - m_j^2$). Moreover, oscillations in vacuum cannot determine the sign of the splittings. As a consequence, an uncertainty in the ordering of the masses remains; the two possibilities are:

$$m_1 < m_2 < m_3, \tag{2.2}$$

$$m_3 < m_1 < m_2. \tag{2.3}$$

The first option is the so-called normal hierarchy spectrum while the second one is the inverted hierarchy scheme; in this form they correspond to the two possible choices of the sign of $\Delta m^2_{31} \equiv \Delta m^2_{atm}$, which is still undetermined, while $\Delta m^2_{21} \equiv \Delta m^2_{sol}$ is known to be positive. Within this minimal context, two mixing angles and two mass-squared splittings are relatively well determined from oscillation experiments (see table 1), there is a slight hint of $\theta_{13} > 0$ and nothing is known about the phases.

Regarding the absolute neutrino mass scale, it is constrained by laboratory experiments searching for its kinematic effects in Tritium β-decay, which are sensitive to the so-called effective electron neutrino mass,

$$m^2_{\nu_e} \equiv \sum_i m_i^2 |U_{ei}|^2. \tag{2.4}$$

The present upper limit is $m_{\nu_e} < 2.2$ eV at 95% confidence level (CL) [28,29], while a new experimental project, KATRIN [30], is underway, with an estimated sensitivity limit $m_{\nu_e} \sim 0.2$ eV. However, cosmological observations provide the tightest constraints on the absolute scale of neutrino masses, via their contribution to the energy density of the Universe and the growth of structure. In general these bounds depend on the assumptions made about the expansion history as well as on the cosmological data included in the analysis [31]. Combining CMB and large scale structure data quite robust bounds have been obtained: $\sum_i m_i < 0.4$ eV at 95% CL within the ΛCDM model [32] and $\sum_i m_i < 1.5\,$eV at 95% CL when allowing for several departures from ΛCDM [33].

Light neutrino best fit values
$\Delta m^2_{21} = (7.64^{+0.19}_{-0.18}) \times 10^{-5}\ {\rm eV}^2$
$\Delta m^2_{31} = \begin{cases} (2.45 \pm 0.09) \times 10^{-3}\ {\rm eV}^2 & {\rm NH} \\ -(2.34^{+0.10}_{-0.09}) \times 10^{-3}\ {\rm eV}^2 & {\rm IH} \end{cases}$
$\sin^2\theta_{12} = 0.316 \pm 0.016$
$\sin^2\theta_{23} = \begin{cases} 0.51 \pm 0.06 & {\rm NH} \\ 0.52 \pm 0.06 & {\rm IH} \end{cases}$
$\sin^2\theta_{13} = \begin{cases} 0.017^{+0.007}_{-0.009} & {\rm NH} \\ 0.020^{+0.008}_{-0.009} & {\rm IH} \end{cases}$

Table 1: The best fit values of the light neutrino parameters and their 1σ errors from [27].

Finally, if neutrinos are Majorana particles complementary information on neutrino masses can be obtained from $0\nu 2\beta$ decay. The contribution of the known light neutrinos to the $0\nu 2\beta$ decay amplitude is proportional to the effective Majorana mass of ν_e, $m_{ee} = |\sum_i m_i U_{ei}^2|$, which depends not only on the masses and mixing angles of the U_{PMNS} matrix but also on the phases. The present bound from the Heidelberg-Moscow group is $m_{ee} < 0.34$ eV at 90% CL [34], but future experiments can reach sensitivities of up to $m_{ee} \sim 0.01$ eV [35].

We now briefly review the current status of searches for new sequential generations. Direct production of the 4^{th} generation quarks t' and b', assuming $t' \to Wq$ and $b' \to Wt$ has been searched in CDF, leading to the lower mass bounds $m_{t'} > 335$ GeV and $m_{b'} > 385$ GeV [36, 37]. Limits on new generation leptons, from LEP II, are weaker: $m_{\ell'} > 100.8$ GeV and $m_{\nu'} > 80.5\,(90.3)$ GeV for pure Majorana (Dirac) particles, assuming that the 4^{th} generation leptons are unstable, i.e., their mixing with the known leptons is large enough so that they decay inside the detector [17]. When neutrinos have both Dirac and Majorana masses, their coupling to the Z boson may be reduced by the neutrino mixing angle and the bound on the lightest neutrino mass may be relaxed to 63 GeV [38]. While the bound on a charged lepton stable on collider lifetimes is still about 100 GeV, in the case of stable neutrinos the only limit comes from the LEP I measurement of the invisible Z width, $m_{\nu'} > 39.5\,(45)$ GeV for pure Majorana (Dirac) particles [17].

Even if new generation fermions are very heavy and cannot de directly produced, they affect electroweak observables through radiative corrections. Recent works have shown that a fourth generation is consistent with electroweak precision observables [20,39,40], provided there is a heavy Higgs and the mass splittings of the new SU(2) doublets satisfy [40] [2]

$$|m_{t'} - m_{b'}| < 80 \text{ GeV}, \tag{2.5}$$

$$|m_{\ell'} - m_{\nu'}| < 140 \text{ GeV} . \tag{2.6}$$

Notice, however, that a long-lived fourth generation can reopen a large portion of the parameter space [41].

In addition to these phenomenological bounds one can place some upper limits by using perturbative unitarity, triviality and by imposing the stability of the Higgs potential at one loop. Typically one obtains limits of the order of the TeV [42] for degenerate lepton doublets and about 600 GeV for degenerate quark doublets.

A very striking effect of new generations is the enhancement of the Higgs-gluon-gluon vertex which arises from a triangle diagram with all quarks running in the loop. This vertex is enhanced approximately by a factor 3 (5) in the presence of a heavy fourth (fifth) generation [39, 43]. Therefore, the Higgs production cross section through gluon fusion at the Tevatron and the LHC is enhanced by a factor of 9 (25) in the presence of a fourth (fifth) generation. Thus, a combined analysis from CDF and D0 for four generations has excluded a SM-like Higgs boson with mass between 131 GeV and 204 GeV at 95% CL [44], while LHC data already excludes 144 GeV $< m_H <$ 207 GeV at 95% CL [45]. From these results, we estimate roughly that $m_H > 300$ GeV in the case of five generations. However, these

[2] The allowed quark mass splittings depend on the Higgs mass, according to the approximate formula $m_{t'} - m_{b'} \simeq \left(1 + \frac{1}{5}\log(\frac{m_H}{115 \text{ GeV}})\right) \times 50$ GeV from ref. [39].

limits may be softened if the fourth generation neutrinos are long-lived and the branching ratio of the decay channel $H \to \nu_4 \bar{\nu}_4$ is significant [46].

Putting all together, a general analysis seems to suggest that at most only two extra generations are allowed [19] unless new additional physics is invoked. If extra generations exist, the Higgs should be heavy. Extra generation quarks should also be quite heavy and be almost degenerate within a generation. The constraints on new generation leptons are milder; charged lepton and Dirac neutrino masses should be in the range 100–1000 GeV and, as we will see in section 4.3, this range will increase if neutrinos have both Dirac and Majorana mass terms.

3. Four generations

If we add one right-handed neutrino ν_R to the SM with three generations and we do not impose lepton number conservation, so that there is a Majorana mass term for the right-handed neutrino, a particular linear combination of ν_e, ν_μ, ν_τ, call it ν'_3, will couple to ν_R and get a Majorana mass at tree level. The other two linear combinations are massless at tree level but, since lepton number is broken, no symmetry protects them from acquiring a Majorana mass at the quantum level. In fact, they obtain a mass at two loops by the exchange of two W bosons (same diagram as in figure 1, but with $\nu_3, \nu_{\bar{3}}$ running in the loop). This leads to two extremely small neutrino masses, as desired, but there is a huge hierarchy between the tree-level mass, for ν_3, and the two-loop-level masses, for ν_1 and ν_2, therefore this possibility cannot accommodate the observed neutrino masses.[3]

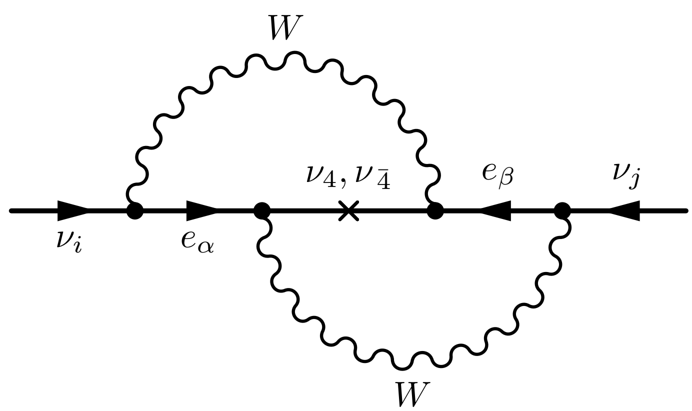

Figure 1: Two-loop diagram contributing to neutrino masses in the four-generation model.

Analogously, we can extend the SM by adding a complete fourth generation and one right handed neutrino ν_R with a Majorana mass term [22, 26, 48, 49]. We denote the new charged lepton E and the new neutrino ν_E. The relevant part of the Lagrangian is

$$\mathcal{L}_Y = -\bar{\ell} Y_e e_R \phi - \bar{\ell} Y_\nu \nu_R \tilde{\phi} - \frac{1}{2} \overline{\nu^c_R} m_R \nu_R + \text{H.c.} \,, \tag{3.1}$$

[3]See however [47] for a model with three generations, one right-handed neutrino singlet and two Higgs doublets which can accommodate neutrino masses and mixings.

where ℓ represents the left-handed lepton SU(2) doublets, e_R the right-handed charged leptons, ν_R the right-handed singlet and flavour indices are omitted. In generation space ℓ and e_R are organized as column vectors with four components. Thus, Y_e is a general, 4×4 matrix, Y_ν is a general four-component column vector whose elements we denote by y_α with $\alpha = e, \mu, \tau, E$, and m_R is a Majorana mass term. The standard kinetic terms, not shown in eq. (3.1), are invariant under general unitary transformations $\ell \to V_\ell \ell$, $e_R \to V_e e_R$ and $\nu_R \to e^{i\alpha}\nu_R$. One can use those transformations, V_ℓ and V_e, to choose Y_e diagonal and positive and also m_R can be taken positive by absorbing its phase in ν_R. Y_ν is in general arbitrary; however, there is still a rephasing invariance in ℓ and e_R that will allow us to remove all phases in Y_ν.

After spontaneous symmetry breaking (SSB) the mass matrix for the neutral leptons is a 5×5 Majorana symmetric matrix which has the standard see-saw structure with only one right-handed neutrino Majorana mass term. Therefore, it leads to two massive Majorana and three massless Weyl neutrinos. From the Lagrangian it is clear that only the linear combination of left-handed neutrinos $\nu'_4 \propto y_e \nu_e + y_\mu \nu_\mu + y_\tau \nu_\tau + y_E \nu_E$ will pair up with ν_R to acquire a Dirac mass term. Thus, it is convenient to pass from the flavour basis $(\nu_e, \nu_\mu, \nu_\tau, \nu_E)$ to a new one $\nu'_1, \nu'_2, \nu'_3, \nu'_4$ where the first three states will be massless at tree level and only ν'_4 will mix with ν_R. If V is the orthogonal matrix that passes from one basis to the other we will have $\nu_\alpha = \sum_i V_{\alpha i} \nu'_i$ $(i = 1, \cdots, 4,\ \alpha = e, \mu, \tau, E)$ with $V_{\alpha 4} \equiv N_\alpha = y_\alpha / \sqrt{\sum_\beta y_\beta^2}$. Since ν'_1, ν'_2, ν'_3 are massless, we are free to choose them in any combination of $\nu_e, \nu_\mu, \nu_\tau, \nu_E$ as long as they are orthogonal to ν'_4, i.e., $\sum_\alpha V_{\alpha \mathrm{i}} N_\alpha = 0$ for i=1,2,3. The orthogonality of V almost fixes all its elements in terms of N_α, but still leaves us some freedom to set three of them to zero. Following [22, 48] we choose $V_{\tau 1} = V_{E1} = V_{E2} = 0$ for convenience.

After this change of basis, we are left with a non-trivial 2×2 mass matrix for ν'_4 and ν_R which can easily be diagonalized and leads to two Majorana neutrinos

$$\nu_4 = i \cos\theta(-\nu'_4 + \nu'^c_4) + i \sin\theta(\nu_R - \nu^c_R),$$

$$\nu_{\bar{4}} = -\sin\theta(\nu'_4 + \nu'^c_4) + \cos\theta(\nu_R + \nu^c_R),$$

$$m_{4,\bar{4}} = \frac{1}{2}\left(\sqrt{m_R{}^2 + 4m_D^2} \mp m_R\right), \tag{3.2}$$

where $m_D = v\sqrt{\sum_i y_i^2}$, with $v = \langle \phi^{(0)} \rangle$, and $\tan^2\theta = m_4/m_{\bar{4}}$. The factor i and the relative signs in ν_4 are necessary to keep the mass terms positive and preserve the canonical Majorana condition $\nu_4 = \nu_4^c$. If $m_R \ll m_D$, we have $m_4 \approx m_{\bar{4}}$, $\tan\theta \approx 1$, and we say we are in the pseudo-Dirac limit while when $m_R \gg m_D$, $m_4 \approx m_D^2/m_R$ and $m_{\bar{4}} \approx m_R$, $\tan\theta \approx m_D/m_R$ and we say we are in the see-saw limit.

Since lepton number is broken by the ν_R Majorana mass term, there is no symmetry which prevents the tree-level massless neutrinos from gaining Majorana masses at higher order. In fact, Majorana masses for the light neutrinos, ν'_1, ν'_2, ν'_3, are generated at two loops by the diagram of figure 1, and are given by

$$M_{ij} = -\frac{g^4}{m_W^4} m_R m_D^2 \sum_\alpha V_{\alpha i} V_{\alpha 4} m_\alpha^2 \sum_\beta V_{\beta j} V_{\beta 4} m_\beta^2 I_{\alpha\beta}, \tag{3.3}$$

where the sums run over the charged leptons $\alpha, \beta = e, \mu, \tau, E$ while $i, j = 1, 2, 3$, and

$$I_{\alpha\beta} = J(m_4, m_{\bar{4}}, m_\alpha, m_\beta, 0) - \frac{3}{4} J(m_4, m_{\bar{4}}, m_\alpha, m_\beta, m_W), \tag{3.4}$$

with $J(m_4, m_{\bar{4}}, m_\alpha, m_\beta, m_W)$ the two-loop integral defined and computed in appendix A.

When $m_R = 0$, $M_{ij} = 0$, as it should, because in that case lepton number is conserved. Also when $m_D = 0$ we obtain $M_{ij} = 0$, since then the right-handed neutrino decouples completely and lepton number is again conserved.

To see more clearly the structure of this mass matrix we can take, for the moment, the limit $m_e = m_\mu = m_\tau = 0$; then, since we have chosen $V_{\tau 1} = V_{E1} = V_{E2} = 0$, the only non-vanishing element in M_{ij} is M_{33} and it is proportional to $V_{E3}^2 N_E^2 m_E^4 I_{EE}$. Keeping all the masses one can easily show that the eigenvalues of the light neutrino mass matrix are proportional to m_μ^4, m_τ^4, m_E^4 which gives a huge hierarchy between neutrino masses. Moreover, for $m_E \gg m_{4,\bar{4}} \gg m_W$, the loop integrals in eq. (3.4) can be well approximated by (see appendix A):

$$I_{EE} \approx \frac{-1}{(4\pi)^4 2 m_E^2} \ln \frac{m_E}{m_{\bar{4}}} \tag{3.5}$$

and

$$I_{\mu\mu} \approx I_{\tau\tau} \approx \frac{-1}{(4\pi)^4 2 m_{\bar{4}}^2} \ln \frac{m_{\bar{4}}}{m_4}\ , \tag{3.6}$$

leading to only two light neutrino masses, since the mass matrix in eq. (3.3) has rank 2 if the three light charged lepton masses are neglected in $I_{\alpha\beta}$. The third light neutrino mass is generated when at least m_τ is taken into account in the loop integral, leading to a further suppression. Within the above approximation, the following ratio of ν_2 and ν_3 masses is obtained [50]:

$$\frac{m_2}{m_3} \lesssim \frac{1}{4N_E^2} \left(\frac{m_\tau}{m_E}\right)^2 \left(\frac{m_\tau}{m_{\bar{4}}}\right)^2 \lesssim \frac{10^{-7}}{N_E^2}\ , \tag{3.7}$$

where we have taken $\ln(m_{\bar{4}}/m_4) \approx \ln(m_E/m_{\bar{4}}) \approx 1$ and in the last step we used that $m_E, m_{\bar{4}} \gtrsim 100\,\mathrm{GeV}$. To overcome this huge hierarchy one would need very small values of N_E which would imply that the heavy neutrinos are not mainly ν_E but some combination of the three known neutrinos ν_e, ν_μ, ν_τ; but this is not possible since it would yield observable effects in a variety of processes, like $\pi \to \mu\nu$, $\pi \to e\nu$, $\tau \to e\nu\nu$, $\tau \to \mu\nu\nu$. This requires that $y_{e,\mu,\tau} \lesssim 10^{-2} y_E$ [51, 52] and then $N_E \approx 1$.

Therefore, although the idea is very attractive, the simplest version is unable to accommodate the observed spectrum of neutrino masses and mixings. However, notice that whenever a new generation and a right-handed neutrino with Majorana mass at (or below) the TeV scale are added to the SM, the two-loop contribution to neutrino masses is always present and provides an important constraint for this kind of SM extensions.

In the following we modify the original idea by adding one additional generation and one additional fermion singlet. We will see that this minimal modification is able to accommodate all current data.

4. Five generation working example

4.1 The five generations model

We add two generations to the SM and two right-handed neutrinos. We denote the two charged leptons by E and F and the two right-handed singlets by ν_{4R} and ν_{5R}. The Lagrangian is exactly the same we used for four generations (3.1) but now ℓ and e are organized as five-component column vectors while ν_R is a two-component column vector containing ν_{4R} and ν_{5R}. Thus, Y_e is a general, 5×5 matrix, Y_ν is a general 5×2 matrix and m_R is now a general symmetric 2×2 matrix. The kinetic terms are invariant under general unitary transformations $\ell \to V_\ell \ell$, $e_R \to V_e e_R$ and $\nu \to V_\nu \nu$, which can be used to choose Y_e diagonal and positive and m_R diagonal with positive elements m_{4R} and m_{5R}. After this choice, there is still some rephasing invariance $\ell_i \to e^{i\alpha_i}\ell_i$, $e_{iR} \to e^{i\alpha_i} e_{iR}$ broken only by Y_ν, which can be used to remove five phases in Y_ν. Therefore

$$Y_\nu = \Big(y,\, y' \Big)\,, \tag{4.1}$$

where y and y' are five-component column vectors with components y_α and y'_α respectively ($\alpha = e,\mu,\tau,E,F$), one of which can be taken real while the other, in general, will contain phases. The model, contrary to the four-generation case, has additional sources of CP violation in the leptonic sector. However, since at the moment we are not interested in CP violation, for simplicity we will take all y_α and y'_α real.

Much as in the four-generation case, the linear combination $\nu'_4 \propto \sum_\alpha y_\alpha \nu_\alpha$ only couples to ν_{4R} and the combination $\nu'_5 \propto \sum_\alpha y'_\alpha \nu_\alpha$ only couples to ν_{5R}. Therefore, the tree-level spectrum will contain three massless neutrinos (the linear combinations orthogonal to ν'_4 and ν'_5) and four heavy Majorana neutrinos. Unfortunately, since in the general case ν'_4 and ν'_5 may not be orthogonal to each other, the diagonalization becomes much more cumbersome than in the four-generation case. Since we just want to provide a working example, we choose ν'_4 and ν'_5 orthogonal to each other, i.e., $\sum_\alpha y_\alpha y'_\alpha = 0$. This simplifies enormously the analysis of the model and allows us to adopt a diagonalization procedure analogous to the one followed in the four-generation case.

We change from the flavour fields $\nu_e,\nu_\mu,\nu_\tau,\nu_E,\nu_F$ to a new basis $\nu'_1,\nu'_2,\nu'_3,\nu'_4,\nu'_5$ where ν'_1,ν'_2,ν'_3 are massless at tree level, so we are free to choose them in any combination of the flavour states as long as they are orthogonal to ν'_4 and ν'_5. Thus, if V is the orthogonal matrix that passes from one basis to the other $\nu_\alpha = \sum_i V_{\alpha i}\nu'_i$ ($i = 1,\cdots,5$, $\alpha = e,\mu,\tau,E,F$) we have $V_{\alpha 4} = N_\alpha = y_\alpha/\sqrt{\sum_\beta y_\beta^2}$, $V_{\alpha 5} = N'_\alpha = y'_\alpha/\sqrt{\sum_\beta y'^2_\beta}$, and $\sum_\beta N_\beta N'_\beta = 0$. The rest of the elements in $V_{\alpha i}$ can be found by using the orthogonality of V, which gives us 12 equations (9 orthogonality and 3 normalization conditions, because N_α and N'_α are already normalized and orthogonal), therefore we still can choose at will three elements of $V_{\alpha i}$; for instance we could choose $V_{F1} = V_{F2} = V_{E1} = 0$. In this case

$$V = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} & N_e & N'_e \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} & N_\mu & N'_\mu \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} & N_\tau & N'_\tau \\ 0 & V_{E2} & V_{E3} & N_E & N'_E \\ 0 & 0 & V_{F3} & N_F & N'_F \end{pmatrix} . \tag{4.2}$$

Moreover, since $\sum_\alpha N_\alpha N'_\alpha = 0$, the 4×4 mass matrix of ν'_4, ν_{4R}, ν'_5 and ν_{5R} is block-diagonal and can be separated in two 2×2 matrices (for ν'_4 and ν_{4R} and ν'_5 and ν_{5R} respectively) with the same form found in the four-generation case. Its diagonalization leads to four Majorana massive fields:

$$\begin{aligned} \nu_a &= i\cos\theta_a(-\nu'_a + \nu'^c_a) + i\sin\theta_a(\nu_{aR} - \nu^c_{aR}), \\ \nu_{\bar{a}} &= -\sin\theta_a(\nu'_a + \nu'^c_a) + \cos\theta_a(\nu_{aR} + \nu^c_{aR}), \\ m_{a,\bar{a}} &= \frac{1}{2}\left(\sqrt{m^2_{aR} + 4m^2_{aD}} \mp m_{aR}\right), \end{aligned} \tag{4.3}$$

with $a = 4, 5$, $\tan^2\theta_a = m_a/m_{\bar{a}}$, $m_{4D} = v\sqrt{\sum_\alpha y^2_\alpha}$ and $m_{5D} = v\sqrt{\sum_\alpha y'^2_\alpha}$.

4.2 Two-loop neutrino masses

As in the case of four generations, the diagrams of figure 1 (now with the four massive neutrinos running in the loop) will generate a non-vanishing mass matrix for the three neutrinos ν'_1, ν'_2, ν'_3 given by

$$M_{ij} = -\frac{g^4}{m^4_W} \sum_{a=4,5} m_{aR} m^2_{aD} \sum_\alpha V_{\alpha i} V_{\alpha a} m^2_\alpha \sum_\beta V_{\beta j} V_{\beta a} m^2_\beta I^{(a)}_{\alpha\beta}, \tag{4.4}$$

with $I^{(a)}_{\alpha\beta}$ given by (3.4) with a labeling the contribution of the 4^{th} and 5^{th} generations.

To analyze this mass matrix first we will impose several phenomenological constraints:

a) The model should be compatible with the observed universality of fermion couplings and have small rates of lepton flavour violation in the charged sector. This requires $y_e, y_\mu, y_\tau, y'_e, y'_\mu, y'_\tau \ll y_E, y_F, y'_E, y'_F$.

b) The model should fit the observed pattern of masses and mixings. A good starting point would be to have expressions able to reproduce the tribimaximal (TBM) mixing structure.

The constraint **a)** together with the orthogonality condition implies that $y_E y'_E + y_F y'_F \approx 0$, which can be satisfied, for instance, if $y_F = y'_E = 0$, that is, ν_E only couples to ν_{4R} and ν_F only couples to ν_{5R}. Then, one can define $y_\alpha = y_E(\epsilon_e, \epsilon_\mu, \epsilon_\tau, 1, 0)$, $y'_\alpha = y'_F(\epsilon'_e, \epsilon'_\mu, \epsilon'_\tau, 0, 1)$, where ϵ_i and ϵ'_i are at least $\mathcal{O}(10^{-2})$ in order to satisfy universality constraints[4] (see section 4.3.2 for more details). Thus, to order ϵ, $N_\alpha \approx (\epsilon_e, \epsilon_\mu, \epsilon_\tau, 1, 0)$, $N'_\alpha \approx (\epsilon'_e, \epsilon'_\mu, \epsilon'_\tau, 0, 1)$,

[4]This pattern of couplings can easily be enforced by using a discrete symmetry which is subsequently broken at order ϵ.

and since for $i \neq 4,5$ $\sum_\alpha V_{\alpha i} N_\alpha = \sum_\alpha V_{\alpha i} N'_\alpha = 0$, all the entries $V_{\alpha i}$ with $\alpha = e, \mu, \tau$, $i = 1,2,3$ can be order one. Now if we choose $V_{F1} = V_{F2} = V_{E1} = 0$ one can see that V_{E2}, V_{E3} are $\mathcal{O}(\epsilon)$ while V_{F3} is $\mathcal{O}(\epsilon')$.

A further simplification occurs if we assume that $V_{E3} = 0$, since in that case E only couples to ν'_2 and and F only couples to ν'_3. Then, in the limit $m_e = m_\mu = m_\tau = 0$ the neutrino mass matrix M_{ij} in eq. (4.4) is already diagonal and we can easily estimate the size of the two larger eigenvalues by neglecting the masses of the known charged leptons in front of m_E and m_F. We find[5]

$$M_{22} \sim \epsilon^2 m_{4R} \frac{g^4 m_{4D}^2 m_E^2}{m_W^4 (4\pi)^4 2} \ln \frac{m_E}{m_{\bar{4}}}, \quad M_{33} \sim \epsilon'^2 m_{5R} \frac{g^4 m_{5D}^2 m_F^2}{m_W^4 (4\pi)^4 2} \ln \frac{m_F}{m_{\bar{5}}} \,. \tag{4.5}$$

Taking $m_F \sim m_{5D} \sim m_W/g$ and $\epsilon' \sim 10^{-2}$ we find $M_{33} \sim 2 \times 10^{-9} m_{5R}$, therefore, to obtain $M_{33} \sim 0.05\,\mathrm{eV}$ we need $m_{5R} \sim 20\,\mathrm{MeV}$ (or $\epsilon' \lesssim 10^{-3}$ for $m_{5R} \sim 1\,\mathrm{GeV}$). Since m_{5R} and m_{4R} control the splitting between the two heavy Majorana neutrinos we are naturally in the pseudo-Dirac regime unless the ϵ's are below 10^{-4}. We also see that the higher $m_{4D(5D)}$ or $m_{E(F)}$, the lower the ϵ (ϵ') that is needed for a given $m_{4R(5R)}$. On the other hand, it is clear that the required hierarchy between M_{33} and M_{22} can be easily achieved both in the normal and the inverted hierarchy cases, while the degenerate case cannot be fitted within this scheme since the third neutrino mass is proportional to m_τ^4. After discussing the phenomenology of the model with more detail in section 4.3, we present the allowed regions of the parameter space in figure 4.

Now let us turn to constraint **b)**, that is, the light neutrino mixings. With our simplifying choices the diagonal entries of the light neutrino mass matrix are proportional to m_τ^4, m_E^4, m_F^4, whereas the off-diagonal ones are proportional to $m_\tau^2 m_E^2$ and $m_\tau^2 m_F^2$. Therefore the neutrino states ν'_1, ν'_2, ν'_3 are very close to being the true mass eigenstates and the first 3×3 elements of V, $V_{\alpha i}$, with $\alpha = e, \mu, \tau\,, i = 1,2,3$ give us directly the PMNS mixing matrix (up to permutations). Then, by using the orthogonality conditions it is easy to find the structure of Yukawas that reproduce a given pattern for the PMNS matrix. Let us study separately the two phenomenologically viable cases, normal hierarchy (NH) and inverted hierarchy (IH).

4.2.1 Normal hierarchy

In the normal hierarchy case ($m_1 < m_2 < m_3$), the experimental data tell us that $m_3 \approx \sqrt{|\triangle m_{31}^2|} \approx 0.05$ eV, $m_2 \approx \sqrt{\triangle m_{21}^2} \approx 0.01$ eV and allow for $m_1 \ll m_2$. The structures we have found (by choosing $V_{F1} = V_{F2} = V_{E1} = V_{E3} = 0$) automatically fall in this scheme, since (for $m_{E,F} \gg m_{4,\bar{4},5,\bar{5}} \gg m_W$) we obtain $(m_1, m_2, m_3) \propto (m_\tau^4/m_{\bar{4}}^2, m_E^2, m_F^2)$. Is there any choice of the Yukawa couplings y_α and y'_α that leads naturally to some phenomenologically successful structure, for instance TBM? If we impose TBM in $V_{\alpha i}$ ($\alpha = e, \mu, \tau$, $i = 1,2,3$), given the structure of N_α and N'_α, the orthogonality of V (at order ϵ^2) immediately tells us that $\epsilon_e = \epsilon_\mu = -\epsilon_\tau \equiv \epsilon$, $\epsilon'_e = 0, \epsilon'_\mu = \epsilon'_\tau \equiv \epsilon'$, and finally $V_{E2} = -\epsilon\sqrt{3}$,

[5] Note that the position of the eigenvalues in M_{ij} depends on the position of the zeros in $V_{\alpha i}$. The choice we made is very convenient to reproduce the normal hierarchy spectrum.

$V_{F3} = -\epsilon'\sqrt{2}$. Therefore, a successful choice of the Yukawas will be

$$\begin{aligned} y_\alpha &= y_E(\epsilon, \epsilon, -\epsilon, 1, 0), \\ y'_\alpha &= y'_F(0, \epsilon', \epsilon', 0, 1), \end{aligned} \tag{4.6}$$

which, keeping only terms up to order ϵ^2, leads to

$$V \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\epsilon^2 & 0 & \epsilon & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\epsilon^2 & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\epsilon'^2 & \epsilon & \epsilon' \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}\epsilon^2 & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\epsilon'^2 & -\epsilon & \epsilon' \\ 0 & -\epsilon\sqrt{3} & 0 & 1 - \frac{3}{2}\epsilon^2 & 0 \\ 0 & 0 & -\epsilon'\sqrt{2} & 0 & 1 - \epsilon'^2 \end{pmatrix} + \mathcal{O}(\epsilon^3). \tag{4.7}$$

Assuming that $m_{E,F} \gg m_{4,\bar{4},5,\bar{5}} \gg m_W$, we find:

$$m_2 = -\frac{3g^4}{m_W^4}\epsilon^2 m_{4D}^2 m_{4R} m_E^4 I_{EE} \approx \frac{3g^4}{2(4\pi)^4 m_W^4}\epsilon^2 m_{4D}^2 m_{4R} m_E^2 \ln\frac{m_E}{m_{\bar{4}}}, \tag{4.8}$$

$$m_3 = -\frac{2g^4}{m_W^4}\epsilon'^2 m_{5D}^2 m_{5R} m_F^4 I_{FF} \approx \frac{g^4}{(4\pi)^4 m_W^4}\epsilon'^2 m_{5D}^2 m_{5R} m_F^2 \ln\frac{m_F}{m_{\bar{5}}}\,, \tag{4.9}$$

and the required ratio $m_3/m_2 \approx 5$ can be easily accommodated, for instance if the fifth generation is heavier than the fourth one or $\epsilon' > \epsilon$.

4.2.2 Inverted hierarchy

In the inverted hierarchy case ($m_3 < m_1 \lesssim m_2$), we have $m_2 \approx m_1 \approx \sqrt{|\triangle m_{31}^2|} \approx 0.05$ eV and $m_3 \ll m_1$ is allowed. Therefore now we need $(m_1, m_2, m_3) \propto (m_E^2, m_F^2, m_\tau^4/m_{\bar{4}}^2)$, which is just a cyclic permutation of the three eigenvalues. This ordering cannot be obtained directly with our previous choice for the zeroes in $V_{\alpha i}$, so now it is convenient to choose $V_{F1} = V_{F3} = V_{E3} = V_{E2} = 0$ instead. Following the same procedure as above, we find that

$$\begin{aligned} y_\alpha &= y_E(-2\epsilon, \epsilon, -\epsilon, 1, 0), \\ y'_\alpha &= y'_F(\epsilon', \epsilon', -\epsilon', 0, 1) \end{aligned} \tag{4.10}$$

will reproduce the desired TBM pattern, leading to

$$V \approx \begin{pmatrix} \sqrt{\frac{2}{3}} - \sqrt{6}\epsilon^2 & \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\epsilon'^2 & 0 & -2\epsilon & \epsilon' \\ -\frac{1}{\sqrt{6}} + \sqrt{\frac{3}{2}}\,\epsilon^2 & \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\epsilon'^2 & \frac{1}{\sqrt{2}} & \epsilon & \epsilon' \\ \frac{1}{\sqrt{6}} - \sqrt{\frac{3}{2}}\,\epsilon^2 & -\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}\epsilon'^2 & \frac{1}{\sqrt{2}} & -\epsilon & -\epsilon' \\ \epsilon\sqrt{6} & 0 & 0 & 1 - 3\,\epsilon^2 & 0 \\ 0 & -\epsilon'\sqrt{3} & 0 & 0 & 1 - \frac{3}{2}\epsilon'^2 \end{pmatrix} + \mathcal{O}(\epsilon^3)\ . \tag{4.11}$$

Assuming that $m_{E,F} \gg m_{4,\bar{4},5,\bar{5}} \gg m_W$, we get:

$$m_1 \approx \frac{3g^4}{(4\pi)^4 m_W^4}\epsilon^2 m_{4D}^2 m_{4R} m_E^2 \ln\frac{m_E}{m_{\bar{4}}}, \tag{4.12}$$

$$m_2 \approx \frac{3g^4}{2(4\pi)^4 m_W^4}\epsilon'^2 m_{5D}^2 m_{5R} m_F^2 \ln\frac{m_F}{m_{\bar{5}}}\ , \tag{4.13}$$

while the ratio of masses between the heaviest neutrinos, $m_1/m_2 \approx 1$ can be obtained by choosing the different parameters in their natural range.

Thus, the model accommodates the light neutrino masses and mixings. In the next section we will analyse current phenomenological bounds on the mixings between the new generations and the first three, ϵ, ϵ'.

4.3 The parameters of the model

We have seen above that neutrino masses are proportional to $\epsilon^2 m_{4R}$ (or $\epsilon'^2 m_{5R}$) and a product of masses, $m_{4D}^2 m_E^2$ (or $m_{5D}^2 m_F^2$), which come from the Higgs mechanism and are proportional to Yukawa couplings. As discussed in section 2 the values of these masses cannot vary too much; perturbative unitarity requires they are smaller than about 1 TeV [42] while lower limits for charged leptons masses from colliders are about 100 GeV. Lower limits for neutral fermions are a bit less uncertain. In the case of unstable pure Dirac neutrinos ($m_{aR} = 0$, $a = 4, 5$) the neutrino masses are basically m_{aD} and the lower limits are about 90 GeV, therefore, in that case, $m_{aD} \gtrsim 90$ GeV. If neutrinos have both Dirac and Majorana mass terms ($m_{aR} \neq 0$) the masses are given by eq. (4.3) and the lower limits are[6] $m_{\bar{a}} \geq m_a > 63$ GeV, then the upper limits on $m_{aD} < 1$ TeV automatically imply $m_{aR} \lesssim 16$ TeV and therefore $m_{\bar{a}} \lesssim 16$ TeV. More generally in figure 2 we present the allowed regions in the plane m_{aR} vs m_{aD} given the lower bound on $m_a > 63$ GeV and the upper limit on $m_{aD} < 1000$ GeV. We also plot the lines corresponding to $m_a = 200$, 400, 600 and 800 GeV.

[6]Notice that in our scenario there is a lower bound on the mixing ϵ, in order to obtain the correct scale of light neutrino masses, which implies that the heavy neutrinos would have decayed inside the detector at LEP.

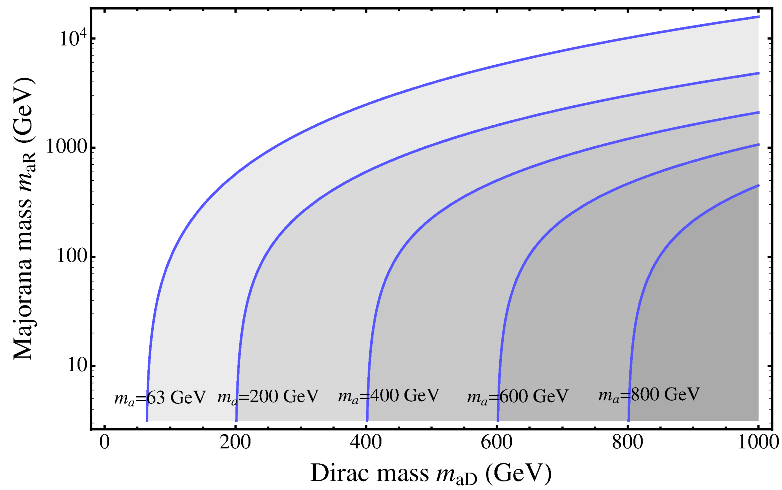

Figure 2: Allowed region in m_{aR}–m_{aD} given the present lower limit ($m_a > 63\,\mathrm{GeV}$) on the mass of an extra generation neutrino ($a = 4, 5$ refers the 4^{th} or 5^{th} generation).

To be definite we will take

$$100\,\mathrm{GeV} < m_{4D}, m_{5D}, m_E, m_F < 1000\,\mathrm{GeV}\,,$$

$$63\,\mathrm{GeV} \lesssim m_a \leq m_{aD} \leq m_{\bar{a}} \lesssim 16\,\mathrm{TeV}\,,$$

with $m_a m_{\bar{a}} = m_{aD}^2$ and $m_{\bar{a}} - m_a = m_{aR}$. In addition there are strong constrains from the electroweak oblique parameters which in the pure Dirac case require some degeneracy of masses, $m_{4D} \simeq m_E$ ($m_{5D} \simeq m_F$). However, these constraints depend on the complete spectrum of the theory (masses of quarks and leptons from new generations and the Higgs boson mass) and are less certain. In fact, contributions from the splitting of masses in the quark sector can be compensated in part by lepton contributions with large m_{aR} [53, 54], which, if we do take into account the constraints set by LEP II can vary from essentially zero (Dirac case) to 16 TeV.

The other parameters that enter neutrino masses are the ϵ's, which characterize the mixing of light neutrinos with heavy neutrinos, and the m_R's, which characterize the amount of total lepton number breaking. The ϵ parameters will produce violations of universality and flavour lepton number conservation in low energy processes. The combination of data from these processes will allow us to constrain both ϵ and ϵ'. On the other hand, to obtain information on the m_R's we will use the light neutrino masses, which in the model are Majorana particles. We will also study the contributions of the heavy neutrinos to neutrinoless double beta decay.

4.3.1 Lepton flavour violation processes ($\mu \to e\gamma$ and μ–e conversion)

The general expression for the branching ratio of $\mu \to e\gamma$ produced through a virtual pair W-neutrino is:

$$B(\mu \to e\gamma) = \frac{3\alpha}{2\pi}\,|\delta_\nu|^2\,, \tag{4.14}$$

where

$$\delta_\nu = \sum_i U_{ei} U^*_{\mu i}\, H\left({}^{m^2_{\chi_i}}/{}_{m^2_W}\right) \tag{4.15}$$

and H is the loop function for this process [51]

$$H(x) = \frac{x\left(2x^2+5x-1\right)}{4(x-1)^3} - \frac{3x^3\log(x)}{2(x-1)^4},$$

with m_{χ_i} the masses of all heavy neutrinos running in the loop and U_{ei} and $U_{\mu i}$ their couplings to the electron and the muon respectively. In (4.14) we have used the unitarity of the mixing matrix and neglected the light neutrino masses to rewrite the final result only in terms the heavy neutrino contributions. Then, as the mixings of the heavy neutrinos with the light leptons are different in normal and inverted hierarchy, so are the $\mu \to e\gamma$ amplitudes generated; one can see just by inspection of the mixing matrices, eqs. (4.7) and (4.11) that in NH only the pair $\nu_4, \nu_{\bar{4}}$ couples to both the electron and the muon, whereas in IH the four heavy neutrinos contribute to the process. The predicted branching ratios are

$$\text{NH:} \qquad B(\mu \to e\gamma) = \frac{3\alpha}{2\pi}\, \bar{H}_4^2\, \epsilon^4, \tag{4.16}$$

$$\text{IH:} \qquad B(\mu \to e\gamma) = \frac{3\alpha}{2\pi}\, \left[\bar{H}_5\, \epsilon'^2 - 2\, \bar{H}_4\, \epsilon^2\right]^2 , \tag{4.17}$$

where

$$\bar{H}_a \equiv \cos^2\theta_a H(m_a^2/m_W^2) + \sin^2\theta_a H(m_{\bar{a}}^2/m_W^2)\,.$$

Now since $H(x)$ is a monotonically increasing function and $m_{\bar{a}} \geq m_a > 63\,\text{GeV}$ we have $\bar{H}_a \geq H(m_a^2/m_W^2) > 0.09$ which gives the less stringent constraint on ϵ and ϵ'. The experimental bound reads $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$, and it is translated into

$$\text{NH}: \qquad \epsilon < 0.03, \tag{4.18}$$

$$\text{IH}: \qquad |\epsilon'^2 - 2\,\epsilon^2| < 7 \times 10^{-4}. \tag{4.19}$$

To see how these bounds depend on the masses of the heavy neutrinos we display in figure 3 $B(\mu \to e\gamma)$ against the mass of the heavy neutrino m_4 in the NH case. For IH we expect similar results unless there are strong cancellations. We also display as horizontal lines present limits [17] $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ and near future limits [55]. From the figure we can extract a conservative bound of the order of the one quoted above, $\epsilon < 0.03$.

Some extra information could be extracted from $\tau \to e\gamma$ and $\tau \to \mu\gamma$. Thus, from $B(\tau \to e\gamma) < 3.3 \times 10^{-8}$ we obtain $\epsilon < 0.3$ in the case of NH and $|\epsilon'^2 - 2\,\epsilon^2| < 0.08$, limits that show exactly the same dependence on ϵ and ϵ' as the one obtained in $\mu \to e\gamma$, but which are roughly one order of magnitude worse. From $B(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$, although the bounds are of the same order of magnitude as for $B(\tau \to e\gamma)$, we obtain different combinations of ϵ's, $|\epsilon'^2 - \epsilon^2| < 0.09$ for NH and $\epsilon'^2 + \epsilon^2 < 0.09$ for IH.

Another very interesting process which gives information on ϵ is μ–e conversion in nuclei. From present data [17] one obtains bounds similar to the limit obtained from $\mu \to e\gamma$. However, there are plans to improve the sensitivity in μ–e conversion in 4 and

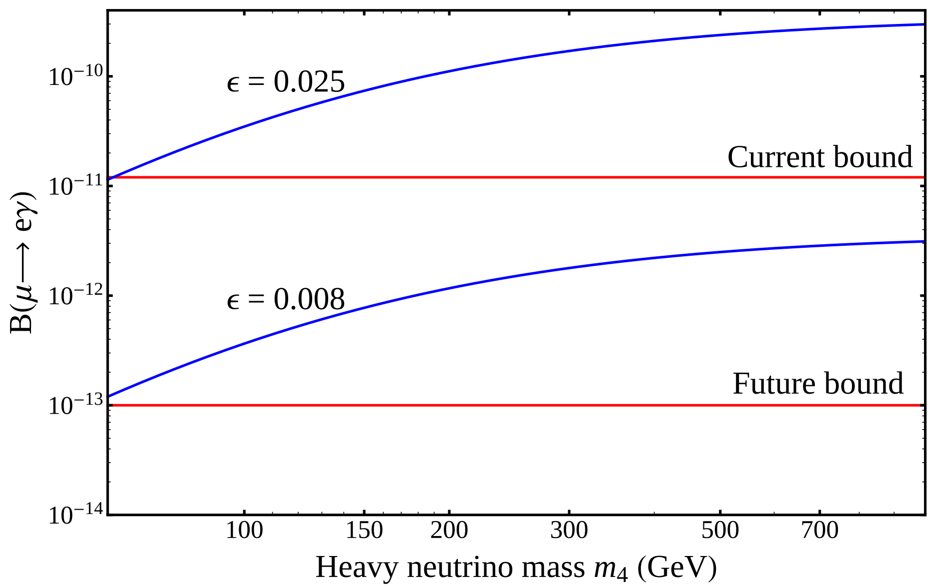

Figure 3: $B(\mu \to e\gamma)$ against m_4 for different values of ϵ in the NH case. We also display present and future limits on $B(\mu \to e\gamma)$ as horizontal lines.

even 6 orders of magnitude [56], therefore we expect much stronger bounds in the future coming from μ–e conversion. Strong correlations between both processes exist, as can be seen in [51].

4.3.2 Universality bounds

New heavy generations that couple to the observed fermions can potentially lead to violations of universality in charged currents because of the "effective" lack of unitarity in the mixings when the heavy generations cannot be produced. Data from neutrino oscillation experiments can also be used to constrain deviations from unitarity of some of the elements of the leptonic mixing matrix [57], however, in our scenario they lead to weaker bounds than the ones obtained here.

There are different types of universality bounds which constrain the mixings of light fermions with new generations:

- Lepton-hadron universality. One compares weak couplings of quarks and leptons using muon decay and nuclear β decay, which are very well tested. In our case this involves mixings both in the quark and lepton sectors and they are not useful to test individually the lepton mixings we are interested in.

- Relations between muon decay, m_Z, m_W and the weak mixing angle $\sin^2\theta_W$. These are very well-determined relations in the SM, and in our case they are modified because the heavy neutrinos cannot be produced in ordinary muon decay. Unfortunately, these relations depend strongly on the ρ parameter, which receives contributions from the Higgs and very large contributions from the heavy fermions of the new generations.

Therefore, although these type of relations could be used to set bounds on the ϵ's, they would depend on other unknown parameters.

- Ratios of decay widths of similar processes. The bounds obtained from this type of processes are very robust because most of the uncertainties cancel in the ratios. We will only consider the most precise among these ratios, which are well measured and can be computed accurately [58–60]:

$$R_{\pi\to e/\pi\to\mu} \equiv \frac{\Gamma(\pi\to e\bar{\nu})}{\Gamma(\pi\to \mu\bar{\nu})}, \tag{4.20}$$

$$R_{\tau\to e/\tau\to\mu} \equiv \frac{\Gamma(\tau\to e\bar{\nu}\nu)}{\Gamma(\tau\to \mu\bar{\nu}\nu)}, \tag{4.21}$$

$$R_{\tau\to e/\mu\to e} \equiv \frac{\Gamma(\tau\to e\bar{\nu}\nu)}{\Gamma(\mu\to e\bar{\nu}\nu)} = B_{\tau\to e}\frac{\tau_\mu}{\tau_\tau}, \tag{4.22}$$

$$R_{\tau\to \mu/\mu\to e} \equiv \frac{\Gamma(\tau\to \mu\bar{\nu}\nu)}{\Gamma(\mu\to e\bar{\nu}\nu)} = B_{\tau\to \mu}\frac{\tau_\mu}{\tau_\tau}, \tag{4.23}$$

where $B_{\tau\to f} = \Gamma(\tau\to f\bar{\nu}\nu)/\Gamma(\tau\to \text{all})$ is the branching ratio of the tau decay to the fermion f, and $\tau_f = 1/\Gamma(f\to\text{all})$ its lifetime. In our model there are corrections to these ratios because ν_e, ν_μ and ν_τ have a small part of $\nu_{4,\bar{4}}$ and $\nu_{5,\bar{5}}$, which are heavy and cannot be produced. This leads to an additional violation of universality which depends on the mixings of ν_e, ν_μ and ν_τ with $\nu_{4,\bar{4}}$ and $\nu_{5,\bar{5}}$. For $R_{\pi\to e/\pi\to\mu}$, and using the $V_{\alpha i}$ in (4.7) and (4.11), we find that

$$\frac{R_{\pi\to e/\pi\to\mu}}{R^{\rm SM}_{\pi\to e/\pi\to\mu}} = \frac{|V_{e1}|^2+|V_{e2}|^2+|V_{e3}|^2}{|V_{\mu1}|^2+|V_{\mu2}|^2+|V_{\mu3}|^2} = \begin{cases} 1+\epsilon'^2 & \text{NH} \\ 1-3\epsilon^2 & \text{IH}\end{cases} . \tag{4.24}$$

$R_{\tau\to e/\tau\to\mu}/R^{\rm SM}_{\tau\to e/\tau\to\mu}$ tests exactly the same couplings, therefore the result is the same as in (4.24).

$R_{\tau\to e/\mu\to e}$ gives a different information because it tests τ/μ universality; however, we find that for both NH and IH $R_{\tau\to e/\mu\to e}/R^{\rm SM}_{\tau\to e/\mu\to e} = 1$, and since it is independent of the ϵ's, this process does not give any further information. This is a consequence of our choice for the Yukawa couplings[7], which, up to signs, are equal for the τ and μ neutrinos.

Finally for $R_{\tau\to\mu/\mu\to e}$, using our mixing matrices, we find

$$\frac{R_{\tau\to \mu/\mu\to e}}{R^{\rm SM}_{\tau\to \mu/\mu\to e}} = \frac{|V_{\tau1}|^2+|V_{\tau2}|^2+|V_{\tau3}|^2}{|V_{e1}|^2+|V_{e2}|^2+|V_{e3}|^2} = \begin{cases} 1-\epsilon'^2 & \text{NH} \\ 1+3\epsilon^2 & \text{IH}\end{cases} , \tag{4.25}$$

which gives exactly the inverse combinations of those obtained from e/μ universality tests.

[7] Which, in turn, is a consequence of the TBM structure we wanted to reproduce.

Observable	R [17]	$R^{\rm SM}$	$R/R^{\rm SM}$
$R_{\pi\to e/\pi\to\mu}$	$(1.230 \pm 0.004) \times 10^{-4}$	$(1.2352 \pm 0.0001) \times 10^{-4}$	$0.996 \pm 0.003.$
$R_{\tau\to e/\tau\to\mu}$	1.028 ± 0.004	1.02821 ± 0.00001	1.000 ± 0.004
$R_{\tau\to\mu/\mu\to e}$	$(1.31 \pm 0.06) \times 10^{6}$	$(1.3086 \pm 0.0006) \times 10^{6}$	1.001 ± 0.004

Table 2: Relevant universality tests

Therefore, if we combine the three results, $R_{\pi\to e/\pi\to\mu}/R^{\rm SM}_{\pi\to e/\pi\to\mu}$, $R_{\tau\to e/\tau\to\mu}/R^{\rm SM}_{\tau\to e/\tau\to\mu}$ and $(R_{\tau\to\mu/\mu\to e}/R^{\rm SM}_{\tau\to\mu/\mu\to e})^{-1}$ and use the data collected in table (2), we obtain

$$0.998 \pm 0.002 = \begin{cases} 1+\epsilon'^2 & \text{NH} \\ 1-3\epsilon^2 & \text{IH} \end{cases}, \tag{4.26}$$

which translates into the following upper 90% C.L. limits on ϵ' and ϵ

$$\text{NH}: \qquad \epsilon' < 0.04, \tag{4.27}$$

$$\text{IH}: \qquad \epsilon < 0.04. \tag{4.28}$$

Notice that although in the IH case we have more sensitivity than in the NH case because of the factor of 3 we finally obtain similar limits in the two cases. This is because in the NH case the deviation from 1 obtained in the model is always positive while the present measured value is slightly smaller than 1 (in both cases we used the Feldman & Cousins prescription [61] to set 90% C.L. limits).

Now we can use all data from LFV and universality and conclude that in the NH case we have $\epsilon < 0.03$, basically from $\mu \to e\gamma$, and $\epsilon' < 0.04$, basically from universality tests. In the IH case we obtain that, except in a narrow band around $\epsilon'^2 \simeq 2\epsilon^2$, $\epsilon \lesssim 0.02$ and $\epsilon' \lesssim 0.03$ basically from $\mu \to e\gamma$; if $\epsilon'^2 \simeq 2\epsilon^2$ there is a cancellation in $\mu \to e\gamma$ but still one can combine these data with the universality limits to obtain $\epsilon \lesssim 0.04$ and $\epsilon' \lesssim 0.06$.

4.3.3 Neutrinoless double beta decay ($0\nu2\beta$)

As commented above m_{4R}, m_{5R} are the relevant parameters which encapsulate the non-conservation of total lepton number and they control, together with the ϵ's, the neutrino masses. Therefore, it would be useful to have additional independent information on these parameters. The most promising experiments to test the non-conservation of total lepton number are neutrinoless double beta decay experiments. The standard contribution, produced by light neutrinos, to $0\nu2\beta$ has largely been studied (for a recent review see for instance [35]) and, given the expected future sensitivity, $m_{ee} = 0.01\,\text{eV}$, it will be very difficult to see it unless the neutrino spectrum is inverted or degenerate. However, if heavy neutrinos from new families are Majorana particles, they lead to tree-level effects in neutrinoless double beta decay [62], while, in our scenario, light neutrino masses are generated at two loops; thus, in principle, it is possible that these new contributions dominate over the standard ones.

The contribution of new generation heavy neutrinos (with mass larger than about the proton mass, m_p) to the rate of neutrinoless double beta decay can be written in terms of an effective mass,

$$\langle M_N \rangle^{-1} = \sum_a U_{ea}^2 M_a^{-1}, \tag{4.29}$$

where U_{ea} is the coupling of the electron to the left-handed component of the heavy neutrino a. The non-observation of neutrinoless double beta decay implies that [63]

$$\langle M_N \rangle > 10^8 \, \mathrm{GeV} \ . \tag{4.30}$$

In the case of NH the electron only couples to ν_4 and $\nu_{\bar{4}}$ (see eqs. (4.7) and (4.3)), thus

$$\langle M_N \rangle^{-1} = 2\epsilon^2 \left(\frac{\cos^2\theta}{m_4} - \frac{\sin^2\theta}{m_{\bar{4}}}\right) = \epsilon^2 \frac{m_{4R}}{m_{4D}^2} \ , \tag{4.31}$$

and using (4.30), we get:

$$m_{4R}\epsilon^2/m_{4D}^2 \, < 10^{-8} \, \mathrm{GeV}^{-1}. \tag{4.32}$$

This is the same combination of the relevant parameters ($m_{4R}\epsilon^2$) that appears in (4.8) for m_2, therefore we can use neutrino data to set bounds on the heavy neutrino contribution to neutrinoless double beta decay written in terms of the effective mass. We obtain

$$\langle M_N \rangle = \frac{3g^4 m_{4D}^4 m_E^2 \ln \dfrac{m_E}{m_{\bar{4}}}}{2m_2 (4\pi)^4 m_W^4} \gtrsim 2 \times 10^{11} \, \mathrm{GeV} \ , \tag{4.33}$$

where we have used $m_2 \sim 0.01 \, \mathrm{eV}$, typical values for $m_{4D} \sim m_E \sim 100 \, \mathrm{GeV}$ and $\ln(m_E/m_{\bar{4}}) \sim 1$. This is far from present, eq. 4.30, and future sensitivities.

In the case of IH, the effective mass is given by

$$\langle M_N \rangle^{-1} = 4\epsilon^2 m_{4R}/m_{4D}^2 + \epsilon'^2 m_{5R}/m_{5D}^2 \ , \tag{4.34}$$

leading also to unobservable effects in $0\nu 2\beta$ decay.

To summarize all phenomenological constraints on the model, in figure 4 we show in blue the allowed region in the $\epsilon - m_{4R}$ plane, which leads to $M_{33} \sim 0.05 \, \mathrm{eV}$ varying the charged lepton masses m_E (m_F) and the Dirac neutrino masses m_{4D} (m_{5D}) between 100 GeV–1 TeV, and imposing the LEP bound on the physical neutrino mass, $m_4 > 63 \, \mathrm{GeV}$. We also plot the present bounds on the mixings ϵ (ϵ') from $\mu \to e\gamma$ and future limits from μ–e conversion if expectations are attained.

5. Collider signatures

As we mentioned before, the LHC offers a unique opportunity to discover (or exclude) new sequential generations of quarks and leptons. For instance, with 1 fb^{-1} at 7 TeV, the exclusion bound on b' would reach 500 GeV via $b' \to Wt$ decay channel, close to the partial

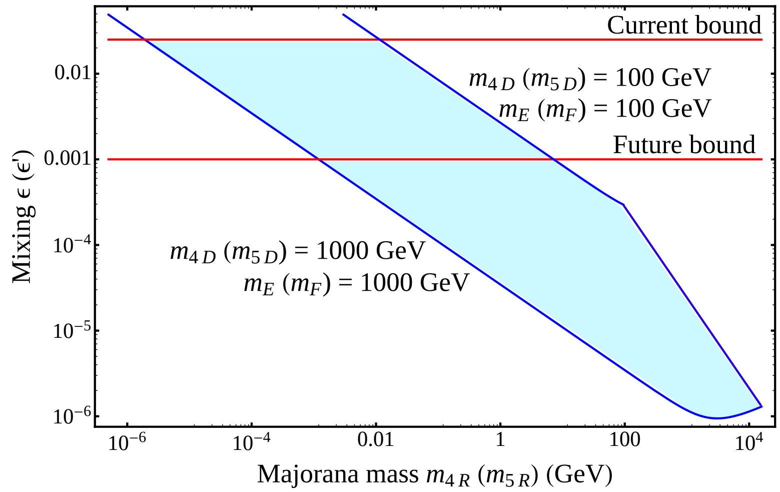

Figure 4: Parameter space that predicts the right scale for heavy and light neutrinos (blue region between the curves). As a comparison we also present the current bound from $\mu \to e\gamma$ and future limits from μ–e conversion experiments.

wave unitarity bound. Even if the t' and b' are too heavy to be seen directly, their effects may be manifest at LHC, since they induce a large $gg \to ZZ$ signal [64]. See also [41] for prospects of detecting very long-lived fourth generation quarks, i.e., in the case of extremely small mixings with the lighter three generations.

Regarding the lepton sector, the standard searches for a fourth generation have to be restricted to the parameter space which leads to the correct light neutrino mass scale, depicted in figure 4. The expected signatures depend on the nature (Dirac or Majorana) of the neutrinos, which are generally assumed to be the lightest states.

Most theoretical analysis of fourth generation Majorana neutrino at hadron colliders have focused on the process $q\bar{q}' \to W^\pm \to \nu_4 \ell^\pm$, where the fourth generation neutrino is produced in association with a light charged lepton [65–68]. Subsequently, if neutrinos are Majorana, they will decay through $\nu_4 \to W^\mp \ell^\pm$, leading to the low-background like-sign dilepton signature in half the events. However in our model the cross section for this process is suppressed both by the mixing of the extra generations with the first three and by the small Majorana masses $m_{4R,5R}$, much as in the neutrinoless double beta decay discussed above, so it will not be observable at LHC for the parameter range that reproduce the correct scale of light neutrino masses.

Alternatively, the lighter neutrinos can be pair-produced via an s-channel Z boson, $q\bar{q} \to Z \to \nu_I \nu_J$ $(I, J = 4, \bar{4})$ [69]. Although the W production has a higher cross section than the Z at hadron colliders, and the mass reach is enhanced when only one heavy particle is produced, if the mixing angle between the extra generations and the light ones is less than about 10^{-6} the neutrino production rates in the W channel are so suppressed that they are unobservable [65]. However the rate of heavy neutrino pair-production via a Z boson is independent of this mixing, becoming the dominant production mechanism in the small mixing regime. Moreover, if the mass difference between ν_4 and $\nu_{\bar{4}}$ is at least 1 GeV, and the mixing so small that the decay $\nu_{\bar{4}} \to \nu_4 Z$ always dominates, the above processes also lead to like-sign leptons in half of the events [8]. See ref. [69] for a detailed study of the Tevatron dataset potential to exclude (or discover) fourth generation neutrinos with both, Dirac and Majorana masses, up to 150-175 GeV, depending on the mixing. For the LHC, only the pure Majorana case has been studied in ref. [70]. According to them, the LHC at $\sqrt{s} = 10$ TeV with 5 fb^{-1} could expect to set a 95% CL mass lower limit of $m_N > 300\,\mathrm{GeV}$ or report 3σ evidence for the ν_4 if $m_{\nu_4} < 225\,\mathrm{GeV}$. We expect a similar sensitivity in our model, in the region $m_4 - m_{\bar{4}} > 1\,\mathrm{GeV}$ and small mixing ($\epsilon, \epsilon' \lesssim 10^{-4}$) i.e., somehow complementary to the one probed in LFV processes. See also [71] for an evaluation of the LHC discovery potential for both Majorana and Dirac type fourth family neutrinos in the process $pp \to Z/H \to \nu_4 \bar{\nu}_4 \to W\mu W\mu$.

Searches for fourth generation charged leptons at the LHC have been studied in [72], also in a general framework with Dirac and Majorana neutrino masses, and assuming that the neutrino ν_4 is the lightest fourth generation lepton. For charged leptons with masses under about 400 GeV, the dominant production channel is charged lepton - neutrino, through the process $q\bar{q}' \to W^\pm \to \nu_4 E^\pm$. The neutrino ν_4 can only decay to $\nu_4 \to W\ell$, and being Majorana it can decay equally to $W^-\ell^+$ and $W^+\ell^-$. Therefore when a pair of fourth generation leptons are produced, we expect the decay products to contain like-sign di-leptons half of the time. The sensitivity study for this process in events with two like-sign charged leptons and at least two associated jets shows that with $\sqrt{s} = 7$ TeV and 1 fb^{-1} of data, the LHC can exclude fourth generation charged lepton masses up to 250 GeV. It would be interesting to study the parameter space in our model that would lead to this type of signals.

In the above searches, it was assumed that the lightest neutrino decays promptly. However, if the mixing of the lightest fourth generation neutrino with the first three generation leptons is $\epsilon \lesssim 10^{-7}$ its proper lifetime will be $\tau_4 \gtrsim 10^{-10}s$. The decay length at the LHC is given by $d = \beta c \gamma \tau_4 \sim 3\mathrm{cm}(\tau_4/10^{-10}s)\beta\gamma$, thus for $\tau_4 \gtrsim 10^{-10}s$ the fourth neutrino will either show displaced vertices in its decay or decay outside the detector, if $d \gtrsim \mathcal{O}(m)$, which is a typical detector size. In our scenario, such a tiny mixing is only compatible with large Majorana masses, $m_R \sim 1$ TeV (see figure 4), far from the pseudo-Dirac case. Searches for Majorana neutrinos stable on collider times have been discussed in [73], where it is proposed to use a quadri-lepton signal that follows from the pair production and decay of heavy neutrinos $pp \to Z \to \nu_{\bar{4}} \nu_{\bar{4}} \to \nu_4 \nu_4 ZZ$, when both Z's decay leptonically. The final state is thus 4ℓ plus missing energy. For 30 fb^{-1} of LHC data at 13 TeV, ν_4 masses can be tested in the range 100 to 180 GeV, and $\nu_{\bar{4}}$ masses from 150 to 250 GeV.

Finally, if the lightest fourth generation lepton is the charged one, there is a striking signal which to our knowledge has not been studied in the literature: lepton number violating like-sign fourth generation lepton pair-production, through $q\bar{q}' \to W^\pm \to E^\pm \nu_{4,\bar{4}} \to E^\pm E^\pm W^\mp$ or via W fusion, $q\bar{q} \to W^\pm W^\pm q' q' \to E^\pm E^\pm jj$ These processes are not suppressed by the small mixing with the first three generations, so in principle they could

[8] In the exact Dirac limit, $\nu_{\bar{4}}$ must decay to $W\ell$ and the different contributions to same sign di-lepton production cancel, since the Dirac neutrino conserves lepton number. However, as far as $\nu_{\bar{4}}$ always decays to $\nu_4 Z$ there is no interference amplitude, and same-sign di-lepton decays are unsuppressed.

be observable in our scenario. Depending on the charged lepton lifetime, they will decay promptly to same-sign light di-leptons, show displaced vertices or leave an anomalous track of large ionization and/or low velocity. A detailed phenomenological study would be very interesting, but it is beyond the scope of this work.

6. Summary and conclusions

We have analysed a simple extension of the SM in which light neutrino masses are linked to the presence of n extra generations with both left- and right-handed neutrinos. The Yukawa neutrino matrices are rank n, so if we do not impose lepton number conservation and allow for right-handed neutrino Majorana masses, at tree level there are $2n$ massive Majorana neutrinos and three massless ones. In order to obtain heavy neutrino masses above the experimental limits from direct searches at LEP, the Dirac neutrino masses should be at the electroweak scale, similar to those of their charged lepton partners, and the right-handed neutrino Majorana masses can not be too high (of order 10 TeV at most). The three remaining neutrinos get Majorana masses at two loops, therefore this framework provides a natural explanation for the tiny masses of the known SM neutrinos. On the other hand, it should be kept in mind that the two-loop contribution to the neutrino mass matrix is always present in this type of SM extensions, therefore the experimental upper limit on the absolute light neutrino mass scale leads to a relevant constraint which has to be taken into account.

We have shown that the minimal extension with a fourth generation can not fit simultaneously the ratio of the solar and atmospheric neutrino mass scales, $\Delta m^2_{sol}/\Delta m^2_{atm}$, the lower bound on the heavy neutrino mass from LEP and the limits on the mixing between the fourth generation and the first three from low energy universality tests. Then, there are two possibilities: either enlarge the Higgs sector [47] or consider a five generation extension. In this work we have analyzed the second one, while the first will be studied elsewhere [74]. Notice that five generations are still allowed by the combination of collider searches for its direct production, indirect effects in Higgs boson production at Tevatron and LHC, and precision electroweak observables [19], provided the Higgs mass is roughly $m_H > 300$ GeV. However they will be either discovered or fully excluded at LHC, making our proposal falsifiable in the very near future.

Given the large number of free parameters in a five generation framework (10 neutrino Yukawa couplings, 2 charged lepton masses and 2 right-handed neutrino Majorana masses), we have considered a very simple working example assuming that i) the linear combinations of left-handed neutrinos that get Dirac masses at tree level are orthogonal to each other, ii) each extra generation left-handed neutrino couples only to one of the two right-handed SM singlet states and iii) each extra generation charged lepton couples only to one linear combination of the (tree-level) massless neutrinos. Then, we are left with 2 neutrino Yukawa couplings y_E, y_F, 2 charged lepton masses m_E, m_F, 2 right-handed neutrino Majorana masses m_{aR},$a = 4, 5$, which characterize the amount of total lepton number breaking, and two small parameters ϵ, ϵ' which determine the mixing among the first three generations and the new ones. Moreover, at leading order the two-loop neutrino masses m_2, m_3 depend only on $\epsilon, m_{4R}, y_E, m_E$ and $\epsilon', m_{5R}, y_F, m_F$, respectively (see eq. (4.5)). Even in this over-simplified case we are able to accommodate all current data, including the observed pattern of neutrino masses and mixings both for normal and inverted hierarchy spectrum. A definite prediction of the model (independent of the above simplifying assumptions) is that the three light neutrinos can not be degenerate.

We have explored the parameter space regions able to generate the correct scale of neutrino masses, ~ 0.05 eV. We find that for typical values of m_E, m_{4D} (m_F, m_{5D}) at the electroweak scale, we need $\epsilon^2 m_{4R}$ ($\epsilon'^2 m_{5R}$) $\lesssim 1$ keV to obtain the atmospheric mass scale (see figure (4)).

We have also studied the current bounds on the mixing parameters ϵ and ϵ' from the non-observation of LFV rare decays $\ell_\alpha \to \ell_\beta \gamma$, as well as from universality tests. All of them are independent of the Majorana masses m_{iR}, since they conserve total lepton number. Depending on the light neutrino mass spectrum (normal or inverted), the strongest bounds come from $\mu \to e\gamma$ and from universality tests in π decays. Combining the information from both processes we can set independent limits on ϵ and ϵ' which being quite conservative are of the order of the few percent, $\epsilon \lesssim 0.03$ and $\epsilon' \lesssim 0.04$.

Finally, we have analysed the phenomenological prospects of the model. With respect to LFV signals, future MEG data will improve the limits on the ϵ's by a factor of about 3 while, if expectations from μ–e conversion are attained the limits on the ϵ's will be pushed to 10^{-3} . This region of observable LFV effects corresponds to the pseudo-Dirac limit, $m_{aR} \lesssim 1$ GeV, i.e., two pairs of strongly degenerate heavy neutrinos. In this regime, they can only be discovered at LHC using pure Dirac neutrino signatures, which are more difficult to disentangle from the background.

On the other hand, we find that in the complementary region of very small mixing $\epsilon, \epsilon' \ll 10^{-3}$, $m_{aR} \gtrsim 1$ GeV, the lighter Majorana neutrinos $\nu_4, \nu_{\bar{4}}$ will lead to observable same-sign di-lepton signatures at LHC. A detailed study is missing, but previous results seem to indicate that a lower bound on m_4 of order 300 GeV could be set with 5 fb^{-1} of LHC data at 10 TeV [70].

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A. The neutrino mass two-loop integral

The relevant integral is

$$J \equiv J(m_4, m_{\bar{4}}, m_\alpha, m_\beta, m_W) =$$

$$= \int_{pq} \frac{p \cdot q}{((p+q)^2 - m_4^2)((p+q)^2 - m_{\bar{4}}^2)(p^2 - m_\alpha^2)(q^2 - m_\beta^2)(p^2 - m_W^2)(q^2 - m_W^2)}, \qquad \text{(A.1)}$$

where

$$\int_{pq} = \int \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \,.$$

We combine propagators with the same momentum by using

$$\frac{1}{(p^2 - m_\alpha^2)(p^2 - m_W^2)} = \frac{1}{(m_\alpha^2 - m_W^2)} \int_{m_W^2}^{m_\alpha^2} \frac{dt_1}{(p^2 - t_1)^2} \,,$$

then

$$J = \int_t \int_{pq} \frac{(pq)}{(p^2 - t_1)^2 (q^2 - t_2)^2 ((p+q)^2 - t_3)^2} \,,$$

where

$$\int_t = \frac{1}{(m_\alpha^2 - m_W^2)} \frac{1}{(m_\beta^2 - m_W^2)} \frac{1}{(m_4^2 - m_{\bar{4}}^2)} \int_{m_W^2}^{m_\alpha^2} dt_1 \int_{m_W^2}^{m_\beta^2} dt_2 \int_{m_4^2}^{m_{\bar{4}}^2} dt_3 \,.$$

Now we use the standard Feynman parametrization to combine the last two propagators and perform the integral in q, which leads to

$$J = -\frac{i}{(4\pi)^2} \int_t \int_0^1 \frac{dx}{1-x} \int_p \frac{p^2}{(p^2 - t_1)^2 (p^2 - t_3/(1-x) - t_2/x)^2} \,.$$

The integral in p can be reduced by using an additional Feynman parameter and the final result can be written as

$$J = -\frac{2}{(4\pi)^4} \int_0^1 dx \int_0^1 dy \int_t \frac{y(1-y)x}{t_3 xy + t_2 (1-x) y + t_1 x (1-x)(1-y)} \,. \qquad \text{(A.2)}$$

The integrals in t_1, t_2, t_3 can be done analytically and reduced to logarithms. The expressions obtained are complicated but can be used to feed the final numerical integration in x and y which converges smoothly for most of the parameters. The expression in (A.2) is also very useful to obtain different approximations for small masses as compared with the largest mass in the integral. For that purpose one can use

$$\lim_{a \to 0} \lim_{b \to a} \frac{1}{b-a} \int_a^b dt f(t) = \lim_{a \to 0} f(a) = f(0) \,.$$

Thus, for instance if $m_{\bar{4}}, m_4 \gg m_\alpha, m_\beta, m_W \sim 0$ we can take $t_1, t_2 \to 0$ in the integrand and perform trivially the remaining integrals,

$$J = -\frac{1}{(4\pi)^4} \frac{1}{m_{\bar{4}}^2 - m_4^2} \ln \frac{m_{\bar{4}}^2}{m_4^2}, \qquad \text{(A.3)}$$

in agreement with the result in [48].

If $m_\beta, m_{\bar{4}}, m_4 \gg m_\alpha, m_W \sim 0$ the integral can also be computed (take $t_1 \to 0$ in the integrand and perform the rest of the integrals). The result can be written in terms of the dilogarithm function $\mathrm{Li}_2(x)$ and it is rather compact,

$$J = -\frac{1}{(4\pi)^4 m_\beta^2} \left(\frac{\pi^2}{6} - \frac{m_{\bar{4}}^2}{m_{\bar{4}}^2 - m_4^2} \left(\mathrm{Li}_2 \left(1 - \frac{m_\beta^2}{m_{\bar{4}}^2} \right) - \frac{m_4^2}{m_{\bar{4}}^2} \mathrm{Li}_2 \left(1 - \frac{m_\beta^2}{m_4^2} \right) \right) \right) \,.$$

When $m_\beta \to 0$ it reduces, as it should, to (A.3) .

We are especially interested in the case $m_\alpha = m_\beta \equiv m_E$ with $m_E > m_W$, but $m_4, m_{\bar{4}}$ could be larger or smaller than m_E (and even smaller than m_W since we only know that $m_{\bar{4}} \geq m_4 > 63\,\mathrm{GeV}$). Some asymptotic expressions can be obtained when there are large hierarchies in masses

$$J \approx -\frac{1}{(4\pi)^4}\frac{1}{m_X^2}\ln\frac{m_X^2}{m_Y^2}\ ,$$

where m_X is the heaviest of $m_E, m_{\bar{4}}, m_4, m_W$ and m_Y the next to the heaviest of these masses. This expression can be used to perform analytical estimates but, since in the allowed range of masses the hierarchies cannot be huge, we do expect large corrections to these estimates. Fortunately, as commented above, the exact value of J for all values of the masses can be obtained numerically rather easily using (A.2). For fast estimates one can use

$$J \approx \frac{1}{(4\pi)^4}\frac{1}{m_{\bar{4}}^2 - m_4^2 - m_E^2}\ln\left(\frac{m_4^2+m_E^2}{m_{\bar{4}}^2}\right)\ , \qquad m_E, m_{\bar{4}}, m_4 \gg m_W,$$

which interpolates smoothly the different asymptotic expressions and reproduces the complete result with an error less than 50% in the worse case.

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