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Order p^6 chiral couplings from the scalar $K\pi$ form factor

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Abstract: Employing results from a recent determination of the scalar $K\pi$ form factor $F_0^{K\pi}$ within a coupled channel dispersion relation analysis [1], in this work we calculate the slope and curvature of $F_0^{K\pi}(t)$ at zero momentum transfer. Knowledge of the slope and curvature of the scalar $K\pi$ form factor, together with a recently calculated expression for $F_0^{K\pi}(t)$ in chiral perturbation theory at order p^6, enable to estimate the $\mathcal{O}(p^6)$ chiral constants $C_{12}^r(M_\rho) = (0.3 \pm 5.4) \cdot 10^{-7}$ and $(C_{12}^r + C_{34}^r)(M_\rho) = (3.2 \pm 1.5) \cdot 10^{-6}$. Our findings also allow to estimate the contribution coming from the C_i to the vector form factor $F_+^{K\pi}(0)$ which is a crucial ingredient for a precise determination of $|V_{us}|$ from K_{l3} decays. Our result $F_+^{K\pi}(0)|_{C_i^r} = -0.018 \pm 0.009$, though inflicted with large uncertainties, is in perfect agreement with a previous estimate by Leutwyler and Roos already made twenty years ago.

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1 Introduction

Chiral Perturbation Theory (χPT) [2–6] provides a very powerful framework to study the low-energy dynamics of the lightest pseudoscalar octet. After having been developed to order p^4 in the energy expansion in the fundamental papers by Gasser and Leutwyler [3,4], the increasing demand for higher precision in the low-energy description of QCD suggested to extend this expansion to the next order p^6 [7–9].

However, the predictive power of χPT decreases when one tries to increase the accuracy, because the chiral symmetry constraints are less powerful at higher orders. While the number of allowed operators is 10 at order p^4, parameterised by the chiral constants L_i^r, it already grows to 90 at the next order p^6. Nevertheless, the situation is not as hopeless as it might seem, since to a given physical observable, only a few of the chiral couplings contribute, thus in certain cases allowing to determine all appearing couplings from phenomenology.

One such set of observables are the strangeness-changing form factors which parametrise the weak $K\pi$ transition amplitude. The vector form factor $F_+^{K\pi}(t)$ plays a crucial role in the description of K_{l3} decays, whereas the scalar form factor $F_0^{K\pi}(t)$ corresponds to the S-wave projection of the $K\pi$ transition matrix element. At order p^4, both form factors were already calculated almost twenty years ago by Gasser and Leutwyler [10].

The value $F_+^{K\pi}(0)$ is an indispensable ingredient in the determination of the quark-mixing matrix element $|V_{us}|$ from K_{l3} decays, and therefore good knowledge of this quantity is required in order to determine $|V_{us}|$ with high precision. Very recently, the calculation of $F_+^{K\pi}$ and $F_0^{K\pi}$ has thus been extended to the next order p^6 [11,12], and it was demonstrated that at this order only two new chiral couplings, C_{12}^r and C_{34}^r, contribute to $F_+^{K\pi}(0)$. In addition, it was shown that precisely the same couplings also appear in the slope and curvature of the scalar form factor $F_0^{K\pi}(t)$. As was first pointed out in ref. [12], this would allow for a determination of the two needed couplings if $F_0^{K\pi}(t)$ was known well enough.

Actually, also recently in a different context, the scalar form factor $F_0^{K\pi}(t)$ has been determined for the first time from a dispersive coupled-channel analysis of the $K\pi$ system [1]. As an input in the dispersion integrals, S-wave $K\pi$ scattering amplitudes were used which had been extracted from fits to the $K\pi$ scattering data in the framework of unitarised χPT with explicit inclusion of resonance fields [13]. The initial motivation to calculate $F_0^{K\pi}(t)$ was the fact that it determines the strangeness-changing scalar spectral function, which was then employed to calculate the mass of the strange quark from a QCD sum rule analysis [14].

Thus we are now in a position to employ the results of ref. [1] for an estimation of the chiral couplings C_{12}^r and C_{34}^r. In section 2, we briefly review the required expressions for the vector and scalar $K\pi$ form factors and in section 3, based on our previous work [1], we then calculate the slope and the curvature of the scalar form factor $F_0(t)$. Furthermore, our results for the slope and curvature of $F_0(t)$ are employed to present an estimate of the contributions to the vector form factor at zero momentum transfer $F_+(0)$, resulting from the order p^6 chiral constants, and in section 4, we end with some concluding remarks. We have also included an appendix in which we present an analytical approach to the numerical analysis followed in section 3 and discuss additional alternatives to determine $F_+(0)$.

2 $K\pi$ form factors

In the Standard Model, the decay of K mesons into a pion and a lepton pair (K_{l3} decay) is mediated by the strangeness changing vector current $\bar{s}\gamma_\mu u$. The corresponding hadronic matrix element, which parametrises the decay $K^0 \to \pi^- l^+ \nu_l$ has the general form

$$\langle \pi^-(p')|\bar{s}\gamma_\mu u|K^0(p)\rangle \;=\; (p+p')_\mu \, F_+^{K\pi}(t) + (p-p')_\mu \, F_-^{K\pi}(t)\,, \tag{2.1}$$

where $t = (p-p')^2$. In the following, we shall work in the isospin limit, and thus the matrix element in eq. (2.1) is equal to the corresponding one which describes the decay $K^+ \to \pi^0 l^+ \nu_l$, up to a global normalisation factor.[1] Therefore, different charge states for kaon and pion will not be distinguished, and to further simplify the notation, below we shall also drop the superscript on the form factors and set $F_\pm(t) \equiv F_\pm^{K\pi}(t)$.

The form factor $F_+(t)$ is also referred to as the vector form factor, because it specifies the P-wave projection of the crossed-channel matrix element $\langle 0|\bar{s}\gamma_\mu u|K\pi\rangle$. The corresponding S-wave projection is described by the scalar form factor

$$F_0(t) \;\equiv\; F_+(t) + \frac{t}{(M_K^2 - M_\pi^2)} \, F_-(t)\,. \tag{2.2}$$

At order p^4 in χPT, both the vector as well as the scalar form factor were calculated by Gasser and Leutwyler in [10]. The corresponding expressions can be found in the original paper, and will not be repeated here.

At the next order p^6, both form factors were calculated very recently in refs. [11, 12].[2] However, the two calculations used different forms of the order p^6 chiral Lagrangian, and

[1] Isospin breaking corrections resulting from both order e^2 as well as $(m_u - m_d)$ terms have been calculated in [15], and need to be included for a complete phenomenological analysis of K_{l3} decays.

[2] The diagonal $\pi\pi$ and KK form factors have been also computed to order p^6 in refs. [16–20].

therefore it is difficult to compare the results. A comparison was attempted in ref. [12], and differences in some parts of the results were found, but at present no definite conclusions are reached. Awaiting a clarification of these issues, we decided to employ the more recent analysis [12], which is based on the formulation of the order p^6 chiral Lagrangian presented in [8].

The value of the vector form factor at $t = 0$, $F_+(0)$, plays a crucial role in the determination of the Cabibbo-Kobayashi-Maskawa (CKM) or quark mixing matrix element V_{us} from K_{l3} decays [21]. Thus, for a high precision determination of V_{us} from K_{l3} it is mandatory to know the value of $F_+(0)$ as accurately as possible, since already at the moment the experimental and theoretical uncertainties to V_{us} are of the same magnitude. With the upcoming new information on K_{l3} from KLOE [22] and NA48 [23], actually the uncertainty on $F_+(0)$ will become the limiting factor in the V_{us} determination.

The χPT result at order p^6 for $F_+(0)$ presented in [12] was found to take the following form:

$$F_+(0) \;=\; 1 + \Delta(0) - \frac{8}{F_\pi^4}\,(C_{12}^r + C_{34}^r)\Delta_{K\pi}^2\,, \tag{2.3}$$

where $\Delta_{K\pi} \equiv M_K^2 - M_\pi^2$, and $\Delta(0)$ is the correction which arises from order p^4 and p^6, but is independent of the order p^6 chiral constants C_i^r. The order p^4 chiral constants L_i^r only appear at $\mathcal{O}(p^6)$, and a numerical value, based on recent fit results for the L_i^r, was given in [12]:

$$\Delta(0) \;=\; -\,0.0080 \pm 0.0057\,[\mathrm{loops}] \pm 0.0028\,[L_i^r]\,. \tag{2.4}$$

It should be pointed out that both, the chiral couplings C_{12}^r and C_{34}^r as well as $\Delta(0)$, depend on the chiral renormalisation scale. The value (2.4) of [12] corresponds to the scale $\mu = M_\rho$, and we shall adopt this choice for the rest of our work. Eq. (2.3) demonstrates, that the value of $F_+(0)$ only depends on the particular combination of the two $\mathcal{O}(p^6)$ chiral constants $(C_{12}^r + C_{34}^r)$.

The expression for the scalar form factor $F_0(t)$ at order p^6 in χPT, on the other hand, reads:

$$F_0(t) \;=\; F_+(0) + \overline{\Delta}(t) + \frac{(F_K/F_\pi - 1)}{\Delta_{K\pi}}\,t + \frac{8}{F_\pi^4}\,(2C_{12}^r + C_{34}^r)\,\Sigma_{K\pi}t - \frac{8}{F_\pi^4}\,C_{12}^r\,t^2\,. \tag{2.5}$$

Here, $\Sigma_{K\pi} \equiv M_K^2 + M_\pi^2$, and $\overline{\Delta}(t)$ is a function which receives contributions from order p^4 and p^6, but like $\Delta(0)$ it is independent of the C_i^r, and the order p^4 chiral constants L_i^r only appear at order p^6. Again, a fit to K_{e3}^0 and K_{e3}^+ decay data was presented in ref. [12]. A good fit over the entire phase space $0 \leq t \leq 0.13$ (t in GeV^2) is given by

$$\overline{\Delta}(t) \;=\; \overline{\Delta}_1\,t + \overline{\Delta}_2\,t^2 + \overline{\Delta}_3\,t^3 + \mathcal{O}(t^4) \;=\; -\,0.259(9)\,t + 0.840(31)\,t^2 + 1.291(170)\,t^3\,. \tag{2.6}$$

Our errors on the expansion coefficients $\overline{\Delta}_i$ have been estimated from the fit differences to the two K_{e3} channels, and from the uncertainty due to different sets of L_i^r, which at $t = 0.13\,\mathrm{GeV}^2$ was found to be around 0.0013 [12].

As should be obvious from eq. (2.5), the relation $F_0(0) = F_+(0)$, which immediately follows from the definition (2.2), is satisfied. In addition, up to order p^6, also the full scalar form factor $F_0(t)$ only depends on the two $\mathcal{O}(p^6)$ chiral couplings C_{12}^r and C_{34}^r, with different dependencies on the couplings at linear and quadratic order in t. Thus, if the t-dependence of the scalar form factor would be known from experiment or theory, the two couplings C_{12}^r and C_{34}^r could be determined, enabling us to also predict a value for $F_+(0)$. In the next section, we will show that such an analysis is actually possible employing a recent determination of the scalar $K\pi$ form factor $F_0(t)$ from a dispersive coupled-channel analysis of the $K\pi$ system [1].

3 Scalar $K\pi$ form factor and $F_+(0)$

The scalar $K\pi$ form factor has been obtained recently in ref. [1] from a coupled-channel dispersion-relation analysis. The S-wave $K\pi$ scattering amplitudes which are required in the dispersion relations were available from a description of S-wave $K\pi$ scattering data in the framework of unitarised χPT with resonances [13]. The dominant uncertainties in the scalar $K\pi$ form factor are due to two integration constants which emerge while solving the coupled channel dispersion relations.

The two integration constants can be fixed by demanding values for $F_0(0)$ as well as $F_0(\Delta_{K\pi})$. Since $F_0(0) = F_+(0)$, for this input we can invoke the most recent result of [12], $F_+(0) = 0.976 \pm 0.010$. Of course, the value of $F_+(0)$ at order p^6 in χPT also depends on the chiral couplings C_{12}^r and C_{34}^r which we aim to determine. Therefore, in the end our determination can be viewed as a consistency check that the resulting value for $F_+(0)$ is compatible with the input used for $F_0(0)$ in the calculation of the scalar $K\pi$ form factor [1]. In order to fix the second integration constant, we also require a value for $F_0(\Delta_{K\pi})$, which, to a very good approximation, is equal to F_K/F_π [24]:

$$F_0(\Delta_{K\pi}) \;=\; \frac{F_K}{F_\pi} + \Delta_{\mathrm{CT}}\,. \tag{3.1}$$

The correction Δ_{CT} is of order m_u or m_d and has been estimated to be $\Delta_{\mathrm{CT}} = -\,3 \cdot 10^{-3}$ within χPT at the next-to-leading order [10].

The description of the scalar $K\pi$ form factor of ref. [1] now allows to calculate the first and second derivatives of the scalar form factor at zero momentum transfer. The different

$F_0(0)$	$F_0(\Delta_{K\pi})$	$F_0'(0)$ [GeV^{-2}]	$F_0''(0)$ [GeV^{-4}]
	1.21	0.804	1.674
0.966	1.22	0.837	1.745
	1.23	0.871	1.815
	1.21	0.770	1.603
0.976	1.22	0.804	1.674
	1.23	0.837	1.744
	1.21	0.737	1.532
0.986	1.22	0.770	1.603
	1.23	0.804	1.673

Table 1: Average values $F_0'(0)$ and $F_0''(0)$ for the unitarised chiral plus K-matrix fits (6.10K2–4) and (6.11K2–4) of ref. [1], for three values of $F_0(0)$ as well as $F_0(\Delta_{K\pi})$.

fits to the S-wave $K\pi$ scattering data have already been discussed in detail in [1, 13] and average results for the derivatives are presented in table 1 for three different values of $F_0(0)$ as well as $F_0(\Delta_{K\pi})$. The variation with respect to the different fits is very minor and has therefore not been displayed explicitly. The dominant uncertainties stem from the used ranges for $F_0(0)$ and $F_0(\Delta_{K\pi})$. We also observe that the variation of $F_0(0)$ and $F_0(\Delta_{K\pi})$ in the ranges given above leads to the same uncertainty for both derivatives $F_0'(0)$ and $F_0''(0)$. Adding these two uncertainties in quadrature, we obtain:

$$F_0'(0) \;=\; 0.804 \pm 0.048\,\mathrm{GeV}^{-2}\,, \qquad F_0''(0) \;=\; 1.67 \pm 0.10\,\mathrm{GeV}^{-4}\,. \tag{3.2}$$

The same physical content as the derivative $F_0'(0)$ can also be represented in two other constants, the scalar $K\pi$ squared radius as well as the slope parameter λ_0. From our value for $F_0'(0)$ of eq. (3.2), we then find

$$\langle r^2_{K\pi}\rangle \;=\; 6\,\frac{F_0'(0)}{F_0(0)} \;=\; (0.192 \pm 0.012)\;\mathrm{fm}^2\,, \quad \lambda_0 \;=\; M_\pi^2\,\frac{F_0'(0)}{F_0(0)} \;=\; 0.0157 \pm 0.0010\,. \tag{3.3}$$

These results are in perfect agreement to the results by Gasser and Leutwyler obtained in χPT at $\mathcal{O}(p^4)$, $\langle r^2_{K\pi}\rangle = (0.20 \pm 0.05)\,\mathrm{fm}^2$ and $\lambda_0 = 0.017 \pm 0.004$, though about a factor of four more precise. On the other hand, the recent finding by Ynduráin, $\langle r^2_{K\pi}\rangle = 0.31 \pm 0.06$ [25], being 2σ higher, is not supported by our result. In ref. [25], it was argued that the larger value found there arises due to the presence of the light κ resonance. However, also in our fits to the S-wave $K\pi$ scattering data [13], a dynamically generated light resonance, which can be identified with the κ, was found. Thus the approach of [25] to parametrise the κ resonance with an effective Breit-Wigner form appears controversial [26].

On the experimental side, the situation about the slope parameter λ_0 is rather confusing. For the two decay modes $K^0_{\mu 3}$ as well as $K^\pm_{\mu 3}$, the most recent Particle Data Group averages are given by [27]:

$$\lambda_0 \;=\; \left\{ \begin{array}{lll} 0.030 \pm 0.005 & (S = 2.0) & [K^0_{\mu 3}] \\ 0.004 \pm 0.009 & (S = 1.8) & [K^\pm_{\mu 3}] \end{array} \right. , \tag{3.4}$$

being in clear disagreement with each other.[3] Furthermore, up to now, in most extractions of λ_0 only a linear function was fitted to the data. As shown in [12] for the vector form factor, the inclusion of a curvature term could produce a sizeable shift in the slope parameter. The average of the two values (3.4) would be compatible with our findings, but in view of the inconsistence, at present we shall disregard the experimental information on λ_0.

The results for $F_0'(0)$ and $F_0''(0)$ of eq. (3.2) can now be employed in order to determine the couplings C_{12}^r as well as $(C_{12}^r + C_{34}^r)$ appearing in the order p^6 chiral Lagrangian. We prefer to calculate the combination $(C_{12}^r + C_{34}^r)$, rather than C_{34}^r itself, because precisely this combination appears in the $\mathcal{O}(p^6)$ contribution to $F_+(0)$. Comparing eq. (2.5) with the Taylor expansion for $F_0(t)$ around $t = 0$, one finds the following two relations:

$$C_{12}^r \;=\; \Big[2\overline{\Delta}_2 - F_0''(0) \Big] \frac{F_\pi^4}{16} , \tag{3.5}$$

$$(C_{12}^r + C_{34}^r) \;=\; \left[F_0'(0) - \overline{\Delta}_1 - \frac{(F_K/F_\pi - 1)}{\Delta_{K\pi}} - \frac{8\Sigma_{K\pi}}{F_\pi^4} C_{12}^r \right] \frac{F_\pi^4}{8\Sigma_{K\pi}} . \tag{3.6}$$

Inserting the given values for $F_0''(0)$ and $\overline{\Delta}_2$ into eq. (3.5), we obtain the following estimate for C_{12}^r:

$$C_{12}^r(M_\rho) \;=\; (0.3 \pm 3.3 \pm 4.3) \cdot 10^{-7} \;=\; (0.3 \pm 5.4) \cdot 10^{-7} , \tag{3.7}$$

where the first error corresponds to the variation of $F_0(0)$ and the second to the remaining uncertainties. Separate results for the three different inputs for $F_0(0)$ are also given in table 2 below. The huge uncertainty on C_{12}^r results from the fact that there is an almost complete cancellation between the two terms in eq. (3.5).

Our result of eq. (3.7) for C_{12}^r can be directly compared with an estimate given in ref. [12], based on assuming that the scalar pion form factor is dominated by the lowest

[3] After completion of our work, we became aware of a very recent high statistics study of the $K^- \to \pi^0 \mu^- \nu$ decay [28], which is in good agreement to our result of eq. (3.3).

lying scalar resonance:

$$C_{12}^r|_{\rm SMD} \;=\; -\,\frac{F_\pi^4}{8M_S^4} \;\approx\; -\,1.0\cdot 10^{-5}\,. \tag{3.8}$$

For this estimate it was assumed that the lowest lying scalar resonance can be identified with the $a_0(980)$. However, recently there appears mounting evidence that the $a_0(980)$ is of dynamical origin and that the lowest preexisting scalar resonance is in fact the $a_0(1450)$ [29–35]. Furthermore, the estimate of eq. (3.8) does not take into account the scale dependence of the chiral coupling C_{12}^r. In general, the scale dependence of the $C_i^r(\mu)$ can be deduced from ref. [9], and is found to be

$$C_i^r(\mu_2) \;=\; C_i^r(\mu_1) - \frac{1}{(4\pi)^4}\Big(\Gamma_i^{(2)}\ln^2\frac{\mu_1}{\mu_2} + (4\pi)^2\Big(2\,\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu_1)\Big)\ln\frac{\mu_1}{\mu_2}\Big)\,, \tag{3.9}$$

with the coefficients $\Gamma_i^{(2)}$, $\Gamma_i^{(1)}$ and $\Gamma_i^{(L)}(\mu)$ being presented in table II of [9].

Although the scale of the lowest-resonance approximation is not determined, it appears natural to assume that the relevant scale is close to the scalar mass M_S. Employing $M_S = 1.45\,\text{GeV}$ and evolving C_{12}^r to M_ρ, we obtain $C_{12}^r|_{\rm SMD}(M_\rho) = 4.0\cdot 10^{-6}$, somewhat smaller and with opposite sign compared to eq. (3.8). For comparison, the corresponding result for $M_S = 1\,\text{GeV}$ would be $C_{12}^r|_{\rm SMD}(M_\rho) = -\,7.8\cdot 10^{-6}$, close to the estimate (3.8). Complete consistency of the resonance estimate and our result of eq. (3.7) would be obtained for a scalar mass around $M_S = 1.25\,\text{GeV}$. From this observation, we conclude that the scalar meson dominance approximation for the $\mathcal{O}(p^6)$ chiral constant C_{12}^r is compatible with our findings, but in view of the strong scale dependence, we are unable to draw more definite conclusions.

Now, we have all the quantities needed for the determination of $(C_{12}^r + C_{34}^r)$ from eq. (3.6). For the ratio F_K/F_π, we have employed the value $F_K/F_\pi = 1.22\pm 0.01$ from ref. [21]. Furthermore, our result (3.2) for the derivative $F_0'(0)$ is required as an input. As discussed above, half of the uncertainty on this value is given by the variation of $F_0(\Delta_{K\pi})$. On the other hand, because of eq. (3.1), the values for F_K/F_π and $F_0(\Delta_{K\pi})$ are strongly correlated and this correlation should be taken into account for our determination of $(C_{12}^r + C_{34}^r)$. What we have then done to estimate the uncertainty was to take half the error on $F_0'(0)$ as given in (3.2) to be 100% correlated with F_K/F_π, but added the remaining half due to $F_0(0)$ fully uncorrelated. With this treatment of uncertainties, we arrive at the main result of our work:

$$(C_{12}^r + C_{34}^r)(M_\rho) \;=\; (3.2\pm 1.4\pm 0.6)\cdot 10^{-6} \;=\; (3.2\pm 1.5)\cdot 10^{-6}\,. \tag{3.10}$$

Again, the first error corresponds to the variation of $F_0(0)$ and the second to the remaining parameters. Separate values for the three inputs for $F_0(0)$ are also listed in table 2 below.

Also our result of eq. (3.10) can be compared directly with the scalar-resonance estimate of $(C_{12}^r + C_{34}^r)$. Employing the corresponding expression for C_{34}^r [36],

$$C_{34}^r|_{\rm SMD} \;=\; \frac{17}{64}\,\frac{F_\pi^4}{M_S^4}\,, \tag{3.11}$$

and evolving the result to the scale M_ρ, we find $(C_{12}^r + C_{34}^r)|_{\rm SMD}(M_\rho) = 5.8 \cdot 10^{-6}$. Thus, in this case, the resonance estimate is 1.7σ larger than our result of eq. (3.10), but in view of the strong scale dependence, which is also present for the combination $(C_{12}^r + C_{34}^r)$, the difference should be considered as an error estimate of the scalar-resonance approximation. However, as will be discussed further below, demanding consistency between our input value for $F_0(0)$ and the resulting output for $F_+(0)$, inspection of table 2 provides some indication that the true value for $(C_{12}^r + C_{34}^r)(M_\rho)$ might be somewhat larger than our result (3.10), although the large uncertainties make it impossible to draw more definite conclusions.

As a cross check, our results of eqs. (3.7) and (3.10) can be used to verify if the value for $\Delta_{\rm CT}$ in χPT at $\mathcal{O}(p^6)$ is compatible with the order p^4 result given above. Evaluating $F_0(t)$ of eq. (2.5) at $t = \Delta_{K\pi}$, one finds

$$F_0(\Delta_{K\pi}) - \frac{F_K}{F_\pi} \;=\; \Delta(0) + \overline{\Delta}(\Delta_{K\pi}) + 16\,\frac{M_\pi^2}{F_\pi^4}\,\Delta_{K\pi}(2C_{12}^r + C_{34}^r) \;=\; -\,0.006 \pm 0.007\,, \tag{3.12}$$

which is in reasonable agreement to the value for $\Delta_{\rm CT}$ give above. However, one should emphasise that it is not clear whether the expansion of eq. (2.6) can still be trusted for $\overline{\Delta}(\Delta_{K\pi})$.

Our result of eq. (3.10) for $(C_{12}^r + C_{34}^r)$ can readily be translated into an estimate of the order p^6 contribution to $F_+(0)$ resulting from the chiral constants C_i^r:

$$\begin{aligned} F_+(0)|_{C_i^r} \;&=\; -\,\frac{8}{F_\pi^4}\,(C_{12}^r + C_{34}^r)\Delta_{K\pi}^2 \qquad (3.13)\\ &=\; \left[\frac{(F_K/F_\pi - 1)}{\Delta_{K\pi}} + \overline{\Delta}_1 - F_0'(0) + \Big(\overline{\Delta}_2 - \tfrac{1}{2}F_0''(0)\Big)\Sigma_{K\pi}\right]\frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} \;=\; -\,0.018 \pm 0.009\,. \end{aligned}$$

This result is in perfect agreement with an estimate of the same contribution already given in the pioneering work by Leutwyler and Roos [21], $F_+(0)|_{C_i^r} = -\,0.016 \pm 0.008$. Inspection of table 2 shows that in this case the uncertainty is dominated by the variation of $F_0(0)$. The remaining parameters only give a small contribution to the error.

$F_0(0)$	C_{12}^r $[10^{-7}]$	$C_{12}^r + C_{34}^r$ $[10^{-6}]$	$F_+(0)\vert_{C_i^r}$	$F_+(0)$
0.966	-3.0 ± 4.3	4.6 ± 0.6	-0.026 ± 0.003	0.966 ± 0.007
0.976	0.3 ± 4.3	3.2 ± 0.6	-0.018 ± 0.003	0.974 ± 0.007
0.986	3.5 ± 4.3	1.7 ± 0.6	-0.009 ± 0.003	0.983 ± 0.007

Table 2: Results for the different quantities calculated in this work for three different inputs for $F_0(0)$. The errors correspond to a variation of all other input parameters.

4 Conclusions

Employing results of a recent determination of the scalar $K\pi$ form factor $F_0(t)$ within a coupled channel dispersion relation approach [1], in this work we were able to calculate the slope and the curvature of $F_0(t)$ at zero momentum transfer. Our corresponding results have been given in eq. (3.2).

Rather recently, the vector and scalar $K\pi$ form factors have also been calculated in chiral perturbation theory at order p^6 in the chiral expansion [11, 12]. Comparing the resulting expressions for the slope and curvature of $F_0(t)$, together with our findings, we were in a position to estimate the order p^6 chiral constants C_{12}^r and $(C_{12}^r + C_{34}^r)$ with the result:

$$C_{12}^r(M_\rho) \;=\; (0.3 \pm 5.4) \cdot 10^{-7}\,, \qquad (C_{12}^r + C_{34}^r)(M_\rho) \;=\; (3.2 \pm 1.5) \cdot 10^{-6}\,, \tag{4.1}$$

where the large uncertainties in C_{12}^r are due to numerical cancellations between the two terms in the relation (3.5).

The vector form factor at zero momentum transfer $F_+(0)$ (2.3) is a crucial ingredient in the determination of the CKM matrix element $|V_{us}|$ from K_{l3} decays and the $\mathcal{O}(p^6)$ contribution resulting from the chiral constants C_i^r happens to be just proportional to the combination $(C_{12}^r + C_{34}^r)$. Employing our estimate for $(C_{12}^r + C_{34}^r)$, we then obtained

$$F_+(0)|_{C_i^r} \;=\; -\,0.018 \pm 0.009\,, \tag{4.2}$$

being in perfect agreement with an estimate of the same contribution already given in the original work by Leutwyler and Roos [21], $F_+(0)|_{C_i^r} = -\,0.016 \pm 0.008$. Further improvement of the presented analysis would require the measurement of $F_0'(0)$, or equivalently λ_0, to better than 5%. This would then also allow to improve the value of $F_0(\Delta_{K\pi})$ from our dispersion relation approach to $F_0(t)$, and thereby to acquire independent information on the value of F_K/F_π. Vice versa, also an improvement of our knowledge on the ratio F_K/F_π from other sources would help to reduce the uncertainty on $F_+(0)$.

Compiling the information presented in reference [12] and this work, we are in a position to present an updated estimate for $F_+(0)$:

$$
\begin{aligned}
F_+(0) &= 1 - 0.0227\,[p^4] + 0.0113\,[p^6\text{-loops}] + 0.0033\,[p^6\text{-}L_i^r] - 0.018\,[p^6\text{-}C_i^r] \\
&\quad \pm 0.0057\,[\text{loops}] \pm 0.0028\,[L_i^r] \pm 0.009\,[C_i^r] \\
&= 0.974 \pm 0.011\,, \qquad (4.3)
\end{aligned}
$$

where all errors have been added in quadrature. Let us note that while using the same input parameters as in [11], the authors of ref. [12] found numerical agreement for the order p^6 loop plus L_i^r contribution, implying that this piece is reasonably well established.

In table 2, we have again presented our results for $F_+(0)$, for the three values of $F_0(0)$ separately. We observe, that for the value $F_0(0) = 0.966$, complete agreement between input and output is obtained, which seems to indicate that this value of $F_0(0)$ is preferred. This would correspond to a slightly larger value for $(C_{12}^r + C_{34}^r)(M_\rho)$, in better agreement with the scalar resonance saturation estimate presented in the last section. However, in view of the large uncertainties, we are unable to draw further conclusions from this observation. Furthermore, in the analysis presented above, isospin violation has been neglected for simplicity. Nevertheless, for a complete phenomenological analysis of K_{l3} decays, it is mandatory to include isospin violating corrections, as they are crucial to explain the differences between K_{l3}^0 and K_{l3}^+ decays [15, 21, 36]. We intend to return to these questions in the future.

Nevertheless, already at this level a qualitative discussion of the influence of our results can be given. In the original work by Leutwyler and Roos [21], the order p^6 contribution corresponding to our result (4.2), was considered to be the total correction at this order. However, as was also pointed out in ref. [12], adding the two-loop correction as well as the $\mathcal{O}(p^6)$ contribution proportional to the L_i^r, a partial cancellation takes place and the full $\mathcal{O}(p^6)$ correction turns out to be smaller. This in turn implies, that our final result (4.3) for $F_+(0)$ is larger than the corresponding value originally employed in [21], $F_+(0) = 0.961$ (already including a tiny isospin correction), and the resulting value for $|V_{us}|$ from K_{l3} decays should be smaller than the present Particle Data Group average [27].[4]

It will be very interesting to see how the upcoming improvements in the determination of $|V_{us}|$ from K_{l3} decays, both on the theoretical as well as the experimental side will compare to the determination of $|V_{us}|$ from hadronic τ decays into strange particles [38, 39], which with upcoming more precise experimental results on the relevant τ decay rate should also

[4]Depending on the treatment of the experimental K_{e3} data, also larger values for $|V_{us}|$ can be obtained [36]. Furthermore, larger values can be accommodated in the framework of generalised χPT [37].

be extremely promising. This should also shed light on the question of a possible violation of unitarity in the first row of the CKM matrix.

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A Analytic dependence on $F_0(0)$ and $F_0(\Delta_{K\pi})$

In this appendix, we derive analytical formulae which explicitly show the dependence of C_{12}^r, $C_{12}^r + C_{34}^r$ and $F_+(0)$ on the values of $F_0(0)$ and $F_0(\Delta_{K\pi})$. Furthermore, we discuss the dependence of our results on the input taken for F_K/F_π.

In ref. [1], the $K\pi$ and $K\eta'$ scalar form factors were obtained by numerically solving the so called Muskhelishvili-Omnès problem [40, 41]. According to these references the most general scalar form factor can be expressed in terms of two linearly independent solutions $\{\mathcal{F}_{10}(s), \mathcal{F}_{11}(s)\}$ and $\{\mathcal{F}_{20}(s), \mathcal{F}_{21}(s)\}$, where the first subscript indicates the independent solution and the second the channel, 0 for $K\pi$ and 1 for $K\eta'$, so that:

$$F(s) \;=\; \alpha_1\, \mathcal{F}_1(s) + \alpha_2\, \mathcal{F}_2(s)\,, \tag{A.1}$$

where only the first subscript is indicated, and $F(s)$ is a column vector of the two form factors $F_0(s)$ and $F_1(s)$ for $K\pi$ and $K\eta'$ channels, respectively. Generally, $\alpha_{1,2}$ are polynomials [40] although in our case they are just constants since the canonical solutions $\mathcal{F}_i(s)$ vanish at infinity like $1/s$, and we require the resulting scalar form factor $F(s)$ to also vanish at infinity. The solutions $\mathcal{F}_i(s)$ are just an output from the employed T-matrices of ref. [13], once two normalisation conditions for each $\mathcal{F}_i(s)$ are imposed. We choose:

$$\mathcal{F}_{10}(0) \;=\; 1\,, \quad \mathcal{F}_{20}(0) \;=\; 0\,, \quad \mathcal{F}_{10}(\Delta_{K\pi}) \;=\; 0\,, \quad \mathcal{F}_{20}(\Delta_{K\pi}) \;=\; 1\,. \tag{A.2}$$

With this choice, $F(s)$ in eq. (A.1) can be expressed as follows:

$$F(s) \;=\; F_1(0)\, \mathcal{F}_1(s) + F_1(\Delta_{K\pi})\, \mathcal{F}_2(s)\,. \tag{A.3}$$

Taking into account the previous expression and eqs. (3.5) and (3.6), we then find:

$$C_{12}^r \;=\; \frac{F_\pi^4}{16}\Big[\, 2\overline{\Delta}_2 - F_0(0)\mathcal{F}_{10}''(0) - F_0(\Delta_{K\pi})\mathcal{F}_{20}''(0)\,\Big]\,, \tag{A.4}$$

$$C_{12}^r + C_{34}^r \;=\; \frac{F_\pi^4}{8\Sigma_{K\pi}}\left[\, F_0(0)\mathcal{F}_{10}'(0) + F_0(\Delta_{K\pi})\mathcal{F}_{20}'(0) - \overline{\Delta}_1 - \frac{(F_K/F_\pi - 1)}{\Delta_{K\pi}} - \frac{8\Sigma_{K\pi}}{F_\pi^4}\, C_{12}^r\,\right], \tag{A.5}$$

where we have made use of eq. (A.3) to express $F_0'(0)$ and $F_0''(0)$ in terms of the constants $F_0(0)$ and $F_0(\Delta_{K\pi})$. We can substitute the last expression for C_{12}^r into eq. (A.5), so that:

$$\begin{aligned} C_{12}^r + C_{34}^r \;=\; & \Bigg[\, F_0(0)\mathcal{F}_{10}'(0) + F_0(\Delta_{K\pi})\mathcal{F}_{20}'(0) - \overline{\Delta}_1 - \Sigma_{K\pi}\overline{\Delta}_2 - \frac{(F_K/F_\pi - 1)}{\Delta_{K\pi}} \\ & + \frac{\Sigma_{K\pi}}{2}\Big(F_0(0)\mathcal{F}_{10}''(0) + F_0(\Delta_{K\pi})\mathcal{F}_{20}''(0)\Big)\Bigg]\frac{F_\pi^4}{8\Sigma_{K\pi}}\,. \end{aligned} \tag{A.6}$$

This equation, together with eq. (A.4), explicitly shows the dependence of the chiral counterterms C_{12}^r and $C_{12}^r + C_{34}^r$ on the inputs $F_0(0)$ and $F_0(\Delta_{K\pi})$. On the other hand, making use of eq. (2.3), we can also write:

$$\begin{aligned} F_+(0) &= 1 + \Delta(0) - \Bigg[F_0(0)\left\{\mathcal{F}'_{10}(0) + \frac{\Sigma_{K\pi}}{2}\mathcal{F}''_{10}(0)\right\} + F_0(\Delta_{K\pi})\left\{\mathcal{F}'_{20}(0) + \frac{\Sigma_{K\pi}}{2}\mathcal{F}''_{20}(0)\right\} \\ &- \overline{\Delta}_1 - \Sigma_{K\pi}\overline{\Delta}_2 - \frac{(F_K/F_\pi - 1)}{\Delta_{K\pi}}\Bigg]\frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}} . \end{aligned} \tag{A.7}$$

It is worth stressing that eq. (A.7) is not an identity since is not valid for arbitrary values of $F_0(0)$. As discussed in section 3 and 4, imposing consistency between input and output values for $F_0(0)$ would make it feasible to fix $F_0(0)$ without employing the value given in ref. [12] as an input. Indeed, solving for $F_0(0) = F_+(0)$ in eq. (A.7) one explicitly finds:

$$\begin{aligned} F_+(0) &= \left[1 + \Delta(0) + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}}\left(\overline{\Delta}_1 + \Sigma_{K\pi}\overline{\Delta}_2 + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} - \frac{F_K}{F_\pi}\left[\mathcal{F}'_{20}(0) + \mathcal{F}''_{20}(0)\frac{\Sigma_{K\pi}}{2}\right]\right)\right] \\ &\times \left[1 + \frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}}\left(\mathcal{F}'_{10}(0) + \mathcal{F}''_{10}(0)\frac{\Sigma_{K\pi}}{2}\right)\right]^{-1} . \end{aligned} \tag{A.8}$$

The values of the derivatives $\mathcal{F}'_{i0}(0)$ and $\mathcal{F}''_{i0}(0)$ with $i = 1, 2$ slightly vary over the T-matrices used in ref. [1].[5] Nevertheless, this source of error is negligible compared with the uncertainties from the rest of inputs that enter on the right hand side of eq. (A.8), already introduced in section 3. From eq. (A.8) one then obtains:

$$F_0(0) = 0.966 \pm 0.041 . \tag{A.9}$$

Unfortunately, the large error of $\Delta(0)$ in eq. (2.4), due to lack of a precise knowledge of the L_i^r coefficients, prevents this method to be competitive since the resulting error bar is a factor of four larger than the one in refs. [12, 21] and in eq. (4.3).

In ref. [1], also different solutions for the strangeness changing scalar $K\pi$ and $K\eta'$ form factors were found with only one independent solution that vanishes at infinity. They correspond to the fits 6.10K1 and 6.11K1 of this reference. As discussed in ref. [1], one can pass from the case with one independent and vanishing solution at infinity to the most general one of eq. (A.1), by slightly varying three free parameters above 1.9 GeV, giving rise to very little changes in the scattering amplitudes for such high energies so that the same set of data is properly reproduced. However, the one independent solution case provides us with a tighter determination of the scalar form factors since only one unknown

[5] For the fit 6.10K3 one has: $\mathcal{F}'_{10}(0) = -3.346$, $\mathcal{F}''_{10}(0) = -7.186$, $\mathcal{F}'_{20}(0) = 3.335$ and $\mathcal{F}''_{20}(0) = 7.121$, in units of GeV^{-2} and GeV^{-4} for the first and second derivatives, respectively.

constant, namely α_1, appears. This should result in a determination of $F_0(0)$ with smaller uncertainty than the one in (A.9). The expressions obtained above from (A.1) to (A.8) for the general case can be particularised to the one independent solution case by just equating $\mathcal{F}_{2i}(s)=0$, with $i=0,1$. Thus, from eq. (A.8) one has:

$$\begin{aligned} F_+(0) &= \left[1+\Delta(0)+\frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}}\left(\overline{\Delta}_1+\Sigma_{K\pi}\overline{\Delta}_2+\frac{F_K/F_\pi-1}{\Delta_{K\pi}}\right)\right] \\ &\times \left[1+\frac{\Delta_{K\pi}^2}{\Sigma_{K\pi}}\left(\mathcal{F}'_{10}(0)+\mathcal{F}''_{10}(0)\frac{\Sigma_{K\pi}}{2}\right)\right]^{-1} . \end{aligned} \tag{A.10}$$

Taking into account that $\mathcal{F}'_{10}(0)=0.803$ GeV^{-2} and $\mathcal{F}''_{10}(0)=1.667$ GeV^{-4} for the fit 6.10K1 and $\mathcal{F}'_{10}(0)=0.800$ GeV^{-2} and $\mathcal{F}''_{10}(0)=1.661$ GeV^{-4} for the fit 6.11K1, we end up with the value:

$$F_0(0) \;=\; 0.979\pm 0.009\,. \tag{A.11}$$

This method for fixing $F_0(0)$ is competitive with the result given in eq. (4.3), and even slightly more precise. Within errors, (4.3), (A.9) and (A.11) are all found to be compatible. We can then employ eq. (3.6) and eq. (A.6), with $\mathcal{F}'_{20}(0)=\mathcal{F}''_{20}(0)=0$, to fix $F_+(0)|_{C_i^r}$ with the result

$$F_+(0)|_{C_i^r} \;=\; -0.013\pm 0.009\,, \tag{A.12}$$

compatible with our previous finding (3.13) and ref. [21].

Let us finally note that the resulting value for $F_0(0)$ in eqs. (4.3), (A.9) and (A.11) requires as an input the quotient F_K/F_π that we have fixed from ref. [21] to 1.22 ± 0.01. Nevertheless, in order to obtain this value, ref. [21] already employed $F_+(0)$ for the $K\pi$ system that was calculated from $\mathcal{O}(p^4)$ χPT, together with their estimate of $F_+(0)|_{C_i^r}$. Therefore, our numbers for $F_+(0)$ are not completely independent from the results of ref. [21]. To indicate the dependence on F_K/F_π, using eq. (A.10) we present two more results for $F_0(0)$ for other central values of F_K/F_π, with the same uncertainty of ± 0.01:[6]

$$\frac{F_K}{F_\pi} \;=\; 1.20 \;\Rightarrow\; F_0(0) \;=\; 0.965\pm 0.009\,, \quad \frac{F_K}{F_\pi} \;=\; 1.24 \;\Rightarrow\; F_0(0) \;=\; 0.993\pm 0.009\,. \tag{A.13}$$

[6]We can equate eqs. (A.8) and (A.10) and then solve for F_K/F_π in terms of $\Delta(0)$, $\overline{\Delta}_1$, $\overline{\Delta}_2$ and the first and second derivatives at the origin of the two kind of solutions of the scalar form factors, which have different $\mathcal{F}_{10}(s)$ although we have kept the same symbol. This results in fully independent evaluations of F_K/F_π and of $F_0(0)$ to those of ref. [21]. Performing such an exercise one finds: $F_K/F_\pi=1.19\pm 0.06\Rightarrow F_0(0)=0.96\pm 0.05$. Our result for F_K/F_π is compatible with that of ref. [21] although a factor of six less precise. One would definitely need to improve the precision in our knowledge of $\Delta(0)$ to have more accurate results. For a hypothetical 5% error for $\Delta(0)$ the error in F_K/F_π would turn out to be 0.2 and that for $F_0(0)$ would be 0.016.

References

[1] M. Jamin, J. A. Oller, and A. Pich, Strangeness-changing scalar form factors, *Nucl. Phys.* **B622** (2002) 279, hep-ph/0110193.

[2] S. Weinberg, Phenomenological Lagrangians, *Physica* **A96** (1979) 327.

[3] J. Gasser and H. Leutwyler, Chiral perturbation theory to one loop, *Ann. Phys.* **158** (1984) 142.

[4] J. Gasser and H. Leutwyler, Chiral perturbation theory: expansions in the mass of the strange quark, *Nucl. Phys.* **B250** (1985) 465, 517.

[5] G. Ecker, Low-energy QCD, *Prog. Part. Nucl. Phys.* **36** (1996) 71, hep-ph/9511412.

[6] A. Pich, Chiral perturbation theory, *Rept. Prog. Phys.* **58** (1995) 563, hep-ph/9502366.

[7] J. Bijnens, G. Colangelo, G. Ecker, J. Gasser, and M. E. Sainio, Pion pion scattering at low-energy, *Nucl. Phys.* **B508** (1997) 263, Erratum-ibid. **B517** (1998) 639, hep-ph/9707291.

[8] J. Bijnens, G. Colangelo, and G. Ecker, The mesonic chiral lagrangian of order p^6, *J. High Energy Phys.* **02** (1999) 020, hep-ph/9902437.

[9] J. Bijnens, G. Colangelo, and G. Ecker, Renormalisation of chiral perturbation theory at order p^6, *Ann. Phys.* **280** (2000) 100, hep-ph/9907333.

[10] J. Gasser and H. Leutwyler, Low-energy expansion of meson form factors, *Nucl. Phys.* **B250** (1985) 517.

[11] P. Post and K. Schilcher, K_{l3} form-factors at order p^6 of chiral perturbation theory, *Eur. Phys. J.* **C25** (2002) 427, hep-ph/0112352.

[12] J. Bijnens and P. Talavera, K_{l3} decays in chiral perturbation theory, *Nucl. Phys.* **B669** (2003) 341, hep-ph/0303103.

[13] M. Jamin, J. A. Oller, and A. Pich, S-wave $K\pi$ scattering in chiral perturbation theory with resonances, *Nucl. Phys.* **B587** (2000) 331, hep-ph/0006045.

[14] M. Jamin, J. A. Oller, and A. Pich, Light quark masses from scalar sum rules, *Eur. Phys. J.* **C24** (2002) 237, hep-ph/0110194.

[15] V. Cirigliano, M. Knecht, H. Neufeld, H. Rupertsberger, and P. Talavera, Radiative corrections to K_{l3} decays, *Eur. Phys. J.* **C23** (2002) 121, hep-ph/0110153.

[16] J. Gasser and U.-G. Meissner, Chiral expansion of pion form-factors beyond one loop, *Nucl. Phys.* **B357** (1991) 90.

[17] J. Bijnens, G. Colangelo, and P. Talavera, The vector and scalar form factors of the pion to two loops, *J. High Energy Phys.* **05** (1998) 014, hep-ph/9805389.

[18] J. Bijnens and P. Talavera, Pion and kaon electromagnetic form factors, *J. High Energy Phys.* **03** (2002) 046, hep-ph/0203049.

[19] M. Frink, B. Kubis, and U.-G. Meissner, Analysis of the pion-kaon sigma term and related topics, *Eur. Phys. J.* **C25** (2002) 259, hep-ph/0203193.

[20] J. Bijnens and P. Dhonte, Scalar form factors in SU(3) chiral perturbation theory, *J. High Energy Phys.* **0310** (2003) 061, hep-ph/0307044.

[21] H. Leutwyler and M. Roos, Determination of the elements V_{us} and V_{ud} of the Kobayashi-Maskawa matrix, *Zeit. Phys.* **C25** (1984) 91.

[22] KLOE collaboration (B. Sciascia et al.), KLOE prospects and preliminary results for K_{l3} decay measurements, *ECONF* **C0304052** (2003) WG607, hep-ex/0307016.

[23] NA48/2 collaboration (D. Madigozhin et al.), K_{l3} prospects from NA48/2, *ECONF* **C0304052** (2003) WG605, hep-ex/0307076.

[24] F. R. Dashen and M. Weinstein, Theorem on the form-factors in K_{l3} decay, *Phys. Rev. Lett.* **22** (1969) 1337.

[25] F. J. Ynduráin, The quadratic scalar radius of the pion and the mixed $\pi - K$ radius, *FTUAM 03-15* (2003), hep-ph/0309039.

[26] J. A. Oller, On the resonant and non-resonant contributions to $B \to \rho\pi$, *ECONF* **C0304052** (2003) WG412, hep-ph/0306294.

[27] Particle Data Group (K. Hagiwara et al.), Review of particle physics, *Phys. Rev.* **D66** (2002) 010001.

[28] ISTRA+ COLLABORATION (O.P. YUSHCHENKO ET AL.), High statistic study of the $K^- \to \pi^0 \mu^- \nu$ decay, (2003).

[29] E. VAN BEVEREN ET AL., A low lying scalar meson nonet in a unitarized meson model, *Z. Phys.* **C30** (1986) 615.

[30] J. A. OLLER and E. OSET, Chiral symmetry amplitudes in the S-wave isoscalar and isovector channels and the σ, $f_0(980)$ and $a_0(980)$ scalar mesons, *Nucl. Phys.* **A620** (1997) 438, Erratum **A652** (1999) 407, hep-ph/9702314.

[31] J. A. OLLER and E. OSET, Theoretical study of the $\gamma\gamma \to$ meson meson reaction, *Nucl. Phys.* **A629** (1998) 739, hep-ph/9706487.

[32] E. VAN BEVEREN, G. RUPP, N. PETROPOULOS, and F. KLEEFELD, The light scalar mesons within quark models, *AIP Conf. Proc.* **660** (2003) 353, hep-ph/0211411.

[33] V. CIRIGLIANO, G. ECKER, H. NEUFELD, and A. PICH, Meson resonances, large N_c and chiral symmetry, *J. High Energy Phys.* **06** (2003) 012, hep-ph/0305311.

[34] J. A. OLLER, The mixing angle of the lightest scalar nonet, *Nucl. Phys.* **A727** (2003) 353, hep-ph/0306031.

[35] W. BARDEEN, E. EICHTEN, and H. THACKER, Chiral Lagrangian parameters for scalar and pseudoscalar mesons, (2003), hep-lat/0307023.

[36] V. CIRIGLIANO, H. NEUFELD, and H. PICHL, K_{e3} decays and CKM unitarity, (2004), hep-ph/0401173.

[37] N. H. FUCHS, M. KNECHT, and J. STERN, Contributions of order $m^2_{\rm quark}$ to K_{l3} form factors and unitarity of the CKM matrix, *Phys. Rev.* **D62** (2000) 033003, hep-ph/0001188.

[38] E. GÁMIZ, M. JAMIN, A. PICH, J. PRADES, and F. SCHWAB, Determination of m_s and $|V_{us}|$ from hadronic τ decays, *J. High Energy Phys.* **01** (2003) 060, hep-ph/0212230.

[39] M. JAMIN, A novel route to V_{us}, *hep-ph/0309147* (2003), hep-ph/0309147, Proc. of *2nd Workshop on the CKM Unitarity Triangle*, Durham, England, April 5-9 2003.

[40] N. I. MUSKHELISHVILI, *Singular Integral Equation*, Dover Publications, Inc., New York, 1992.

[41] O. Babelon, J.-L. Basdevant, D. Caillerie, and G. Mennessier, Unitarity and inelastic final state interactions, *Nucl. Phys.* **B113** (1976) 445.