Can one detect new physics in $I = 0$ and/or $I = 2$ contributions to the decays $B \to \pi\pi$?

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(Dated: July 26, 2013)

We study the effects of new-physics contributions to $B \to \pi\pi$ decays, which can be parametrized as four new complex quantities. A simple analysis is provided by utilizing the reparametrization invariance of the decay amplitudes. We find that six quantities can be reabsorbed into the definitions of Standard Model-like parameters. As a result, the usual isospin analysis provides only two constraints on new physics which are independent of estimates for the Standard Model contributions. In particular, we show that one is not sensitive to new physics affecting the $I = 0$ amplitudes. On the other hand, $I = 2$ new physics can be detected, and its parameters can be measured by using independent determinations of the weak phases. We obtain constraints on these new-physics parameters through a fit to the current experimental data.

PACS numbers: 11.30.Er, 12.15.Hh, 13.25.Hw, 14.40.-n.

I. INTRODUCTION

The purpose of B-physics experiments is the detection of new physics. Because CP violation appears in the Standard Model (SM) through one single irremovable phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], early strategies involved determining the various incarnations of this phase (β, γ, or $\alpha \equiv \pi - \beta - \gamma$), looking for discrepancies. Several techniques were proposed to sidestep the need to deal with the amplitude magnitudes and with the CP-even strong phases, since these are affected by uncertain hadronic matrix elements – reviews can be found, for example, in [2, 3, 4].

In one such proposal, due to Gronau and London, one uses the isospin symmetry between different $B \to \pi\pi$ decays [5]. Their proposal can be worded in several different ways. We may take it as a measurement of $\beta+\gamma$, to be compared with the values allowed for this quantity by current CKM constraints on the Wolfenstein ρ–η plane [6]; we may use the measurement of β from $B_d \to \psi K$ decays, and view this as a measurement of γ; or, one may take $\gamma_{\rm ckm}$ and $\beta_{\rm ckm}$ from the fit to the ρ–η plane, looking for inconsistencies in the overall fit of the SM parameters (including all CP-odd and CP-even quantities) to the experimental observables in $B \to \pi\pi$ decays.

In this article, we follow the last approach with respect to the weak phases (dropping the subscript "ckm"), but we will consider the most general type of new physics that could affect these decays. Our objective is to find which types of new physics can be probed in $B \to \pi\pi$ decays without making any assumptions about the hadronic matrix elements of the SM contributions to these decays, and which cannot. We show that:

1. there are only two probes of new physics in $I = 2$ contributions: one probes the presence of a new weak phase in A_2; the other compares the value of $\gamma_{\pi\pi}$ extracted from the isospin analysis with that obtained independently through CKM unitarity or some other decay;

2. one *cannot* probe for new physics in $I = 0$ contributions.

We show how these conclusions follow simply from the "reparametrization invariance" introduced by two of us (Botella and Silva) in [7]. In addition, if a new weak phase in A_2 is seen, we show that it is possible to *measure* the new-physics parameters using independent determinations of the weak phases.

In section II, we explain the generic features of "reparametrization invariance" relevant for this problem. In section III, we perform a general analysis of the $B \to \pi\pi$ decays valid in the presence of new physics and we prove that the conclusions announced above follow simply from reparametrization invariance. In section IV we perform a fit of the relevant new-physics parameters to the current experimental data. These constraints on new physics do *not* depend on any assumptions about the SM contributions, which are also independently extracted from our fit. We present our conclusions in section V.

II. CONSEQUENCES OF REPARAMETRIZATION INVARIANCE

Let us consider the decay of a B meson into some specific final state f. For the moment, B stands for B^+, B_d^0 or B_s^0. When discussing generic features of the decay amplitudes without reference to any particular model, it has become commonplace to parametrize the decay amplitudes as

$$A_f = M_1 e^{i\phi_{A1}} e^{i\delta_1} + M_2 e^{i\phi_{A2}} e^{i\delta_2}, \tag{1}$$
$$\bar{A}_{\bar{f}} = M_1 e^{-i\phi_{A1}} e^{i\delta_1} + M_2 e^{-i\phi_{A2}} e^{i\delta_2}, \tag{2}$$

where ϕ_{A1} and ϕ_{A2} are two CP-odd weak phases; M_1 and M_2 are the magnitudes of the corresponding terms; and δ_1 and δ_2 are the corresponding CP-even strong phases [8]. These expressions apply to the decays of a (neutral or charged) B meson into the final state f and the charge-conjugated decay, respectively. For the decay of a neutral B meson into a CP eigenstate with CP eigenvalue $\eta_f = \pm 1$, the right-hand-side of Eq. (2) appears multiplied by η_f.

As shown in reference [7], the fact that any third weak phase may be written in terms of the first two means that one may write any amplitude, with an arbitrary number N of distinct weak phases, in terms of only two. Indeed,

$$A_f = \tilde{M}_1 e^{i\phi_{A1}} e^{i\tilde{\delta}_1} + \tilde{M}_2 e^{i\phi_{A2}} e^{i\tilde{\delta}_2} + \sum_{k=3}^{N} \tilde{M}_k e^{i\phi_{Ak}} e^{i\tilde{\delta}_k} \tag{3}$$

and

$$\bar{A}_{\bar{f}} = \tilde{M}_1 e^{-i\phi_{A1}} e^{i\tilde{\delta}_1} + \tilde{M}_2 e^{-i\phi_{A2}} e^{i\tilde{\delta}_2} + \sum_{k=3}^{N} \tilde{M}_k e^{-i\phi_{Ak}} e^{i\tilde{\delta}_k} \tag{4}$$

may be written as in Eqs. (1) and (2), respectively, through the choices

$$\begin{aligned} M_1 e^{i\delta_1} &= \tilde{M}_1 e^{i\tilde{\delta}_1} + \sum_{k=3}^{N} a_k \tilde{M}_k e^{i\tilde{\delta}_k}, \\ M_2 e^{i\delta_2} &= \tilde{M}_2 e^{i\tilde{\delta}_2} + \sum_{k=3}^{N} b_k \tilde{M}_k e^{i\tilde{\delta}_k}, \end{aligned} \tag{5}$$

with

$$\begin{aligned} a_k &= \frac{\sin(\phi_{Ak} - \phi_{A2})}{\sin(\phi_{A1} - \phi_{A2})}, \\ b_k &= \frac{\sin(\phi_{Ak} - \phi_{A1})}{\sin(\phi_{A2} - \phi_{A1})}. \end{aligned} \tag{6}$$

Notice that, in addition, the phases ϕ_{A1} and ϕ_{A2} may be chosen completely at will. This property, which we refer to as "reparametrization invariance", has very unusual consequences, which were explored at length in [7].

Sometimes it is useful to consider the sums of all new contributions to B and $\overline{B}$ decays,

$$\begin{aligned} N &= \sum_{k=3}^{N} \tilde{M}_k e^{i\phi_{Ak}} e^{i\tilde{\delta}_k}, \\ \bar{N} &= \sum_{k=3}^{N} \tilde{M}_k e^{-i\phi_{Ak}} e^{i\tilde{\delta}_k}. \end{aligned} \tag{7}$$

With this notation, the proof that we may use only two weak phases as our basis follows simply from

$$N = N_{\phi_{A1}} e^{i\phi_{A1}} + N_{\phi_{A2}} e^{i\phi_{A2}}, \tag{8}$$
$$\bar{N} = N_{\phi_{A1}} e^{-i\phi_{A1}} + N_{\phi_{A2}} e^{-i\phi_{A2}}, \tag{9}$$

where

$$\begin{aligned} N_{\phi_{A1}} &= \frac{N e^{-i\phi_{A2}} - \bar{N} e^{i\phi_{A2}}}{2i \sin(\phi_{A1} - \phi_{A2})} \equiv \sum_{k=3}^{N} a_k \tilde{M}_k e^{i\tilde{\delta}_k}, \\ N_{\phi_{A2}} &= \frac{N e^{-i\phi_{A1}} - \bar{N} e^{i\phi_{A1}}}{2i \sin(\phi_{A2} - \phi_{A1})} \equiv \sum_{k=3}^{N} b_k \tilde{M}_k e^{i\tilde{\delta}_k}. \end{aligned} \tag{10}$$

Notice that, as required, the *same* complex numbers $N_{\phi_{A1}}$ and $N_{\phi_{A2}}$ appear in Eqs. (8) and (9). Said otherwise, $N_{\phi_{A1}}$ and $N_{\phi_{A2}}$ carry only magnitudes and CP-even phases, since the CP-odd phases, ϕ_{A1} and ϕ_{A2}, have been factored out explicitly in Eqs. (8) and (9).

III. PARAMETRIZING THE $B \to \pi\pi$ DECAY AMPLITUDES

We may parametrize the $B \to \pi\pi$ decay amplitudes according to the isospin of the final state as

$$\begin{aligned} -\sqrt{2}A(B^+ \to \pi^+\pi^0) = -\sqrt{2}A_{+0} &= 3A_2, \\ -A(B^0 \to \pi^+\pi^-) = -A_{+-} &= A_2 + A_0, \\ -\sqrt{2}A(B^0 \to \pi^0\pi^0) = -\sqrt{2}A_{00} &= 2A_2 - A_0, \end{aligned} \tag{11}$$

and

$$\begin{aligned} -\sqrt{2}A(B^- \to \pi^-\pi^0) = -\sqrt{2}\bar{A}_{+0} &= 3\bar{A}_2, \\ -A(\overline{B^0} \to \pi^+\pi^-) = -\bar{A}_{+-} &= \bar{A}_2 + \bar{A}_0, \\ -\sqrt{2}A(\overline{B^0} \to \pi^0\pi^0) = -\sqrt{2}\bar{A}_{00} &= 2\bar{A}_2 - \bar{A}_0. \end{aligned} \tag{12}$$

In writing Eqs. (11) and (12), some coefficients and signs have been absorbed into the definition of the amplitudes for $I = 0$ (A_0 and $\bar{A}_0$) and $I = 2$ (A_2 and $\bar{A}_2$); this choice is not universal and great care should be exercised when comparing with other sources.

The right-hand-sides of Eqs. (11) and (12) contain seven independent parameters: four magnitudes ($|A_2|$, $|\bar{A}_2|$, $|A_0|$, and $|\bar{A}_0|$); and three relative phases ($\bar{\delta}_2 - \delta_2$, $\bar{\delta}_0 - \delta_0$, and $\delta_2 - \delta_0$). An overall phase can be rotated away. These seven quantities may be extracted from experiments detecting the average branching ratios (B_{+0}, B_{+-}, and B_{00}), the direct CP violation (C_{+0}, C_{+-}, and C_{00}), and the interference CP violation (S_{+-} and S_{00}) of $B \to \pi\pi$ decays, where the sub-indices refer to the charges of the physical pions in the final state. It turns out that S_{00} may be written as a function of the other observables, up to discrete ambiguities. Therefore, there are seven independent measurements in $B \to \pi\pi$ decays, allowing the determination of the seven physical parameters present on the right-hand-sides of Eqs. (11) and (12).

A different decomposition is sometimes utilized within the SM. This is related to a diagrammatic analysis and it involves two weak phases (β and γ) which appear naturally within the SM:

$$\begin{aligned} -\sqrt{2}A_{+0} &= (T + C)\, e^{i\gamma}, \\ -A_{+-} &= Te^{i\gamma} + Pe^{-i\beta}, \\ -\sqrt{2}A_{00} &= Ce^{i\gamma} - Pe^{-i\beta}. \end{aligned} \tag{13}$$

Here T, C, and P contain only magnitudes and CP-even (strong) phases. Similar relations hold for the conjugated (barred) amplitudes, by changing the signs of the CP-odd phases γ and $-\beta$. The relation between the two decompositions is

$$\begin{aligned} A_2 &= \frac{1}{3}(T + C)e^{i\gamma}, \\ \bar{A}_2 &= \frac{1}{3}(T + C)e^{-i\gamma}, \\ A_0 &= \frac{1}{3}(2T - C)e^{i\gamma} + Pe^{-i\beta}, \\ \bar{A}_0 &= \frac{1}{3}(2T - C)e^{-i\gamma} + Pe^{i\beta}. \end{aligned} \tag{14}$$

For simplicity, in writing Eqs. (14) we have neglected the SM electroweak penguin contributions, but these can be included in a straightforward way by shifting gamma roughly by 1.5°, following references [9].

The impact of a generic new-physics model in $B \to \pi\pi$ decays will show up in both $I = 0$ and $I = 2$ amplitudes, with a variety of weak phases. This can be parametrized as

$$\begin{aligned} A_2 &= \frac{1}{3}(T + C)e^{i\gamma} + N_2, \\ \bar{A}_2 &= \frac{1}{3}(T + C)e^{-i\gamma} + \bar{N}_2, \end{aligned}$$

$$
\begin{aligned}
A_0 &= \frac{1}{3}(2T - C)e^{i\gamma} + Pe^{-i\beta} + N_0, \\
\bar{A}_0 &= \frac{1}{3}(2T - C)e^{-i\gamma} + Pe^{i\beta} + \bar{N}_0,
\end{aligned}
\tag{15}
$$

where N_0, $\bar{N}_0$, N_2, and $\bar{N}_2$ are complex numbers. We may use the consequences of reparametrization invariance in Eqs. (8)–(10) in order to rewrite Eqs. (15) as

$$
\begin{aligned}
A_2 &= \frac{1}{3}(t + c)e^{i\gamma} + N_{2,o} , \\
\bar{A}_2 &= \frac{1}{3}(t + c)e^{-i\gamma} + N_{2,o} , \\
A_0 &= \frac{1}{3}(2t - c)e^{i\gamma} + pe^{-i\beta}, \\
\bar{A}_0 &= \frac{1}{3}(2t - c)e^{-i\gamma} + pe^{i\beta}.
\end{aligned}
\tag{16}
$$

Here

$$
t + c = T + C + 3N_{2,\gamma} , \tag{17}
$$

$$
2t - c = 2T - C + 3N_{0,\gamma} , \tag{18}
$$

$$
p = P + N_{0,-\beta} , \tag{19}
$$

where

$$
\begin{aligned}
N_{2,\gamma} &= i\frac{\bar{N}_2 - N_2}{2\sin\gamma}, \\
N_{2,o} &= \frac{\bar{N}_2 + N_2}{2} - i\frac{\bar{N}_2 - N_2}{2\tan\gamma}, \\
N_{0,\gamma} &= \frac{\bar{N}_0 + N_0}{2}\frac{\sin\beta}{\sin(\beta+\gamma)} + i\frac{\bar{N}_0 - N_0}{2}\frac{\cos\beta}{\sin(\beta+\gamma)}, \\
N_{0,-\beta} &= \frac{\bar{N}_0 + N_0}{2}\frac{\sin\gamma}{\sin(\beta+\gamma)} - i\frac{\bar{N}_0 - N_0}{2}\frac{\cos\gamma}{\sin(\beta+\gamma)},
\end{aligned}
\tag{20}
$$

are obtained from Eqs. (8)–(10) with $\{\phi_{A1}, \phi_{A2}\} = \{\gamma, 0\}$ for the $I = 2$ contributions, and with $\{\phi_{A1}, \phi_{A2}\} = \{\gamma, -\beta\}$ for the $I = 0$ contributions.

We stress that our choice of $\{\phi_{A1}, \phi_{A2}\} = \{\gamma, 0\}$ for the $I = 2$ contributions is not mandatory. We could equally well have chosen a more general basis $\{\phi_{A1}, \phi_{A2}\} = \{\gamma, \phi\}$, as long as the phase ϕ was *known* and did not have to be fitted for [10]. For example, we could take $\phi = 5°$, or $\phi = 10°$, or even $\phi = \beta$, with β determined from $B_d \to \psi K$ decays.

The main results of our paper arise by comparing Eqs. (16), valid in the presence of generic new-physics contributions to $B \to \pi\pi$ decays, with Eqs. (14), valid within the SM. First, we notice that the expressions for A_0 and $\bar{A}_0$ have exactly the same form in Eqs. (14) and in Eqs. (16). This means that, without specific assumptions made about the hadronic matrix elements involved in the SM contributions T, C, and P, the measurements of A_0 and $\bar{A}_0$ cannot be used to test for the presence of new physics in $I = 0$ (or lack thereof). This is one of our main points. It is *impossible* to detect new physics in $I = 0$ without specific assumptions about the hadronic matrix elements involved in the SM contributions. Note that the impossibility of detecting $I = 0$ new physics has long been suspected; reparametrization invariance offers a *proof* of this fact.

Conversely, if one makes assumptions about the quantities involved in the SM contributions $2T - C$ and/or P, then the deviations $(2t-c)_{\rm exp} - (2T-C)$ and $p_{\rm exp} - P$ *can* indeed be used to probe the $I = 0$ contributions $N_{0,\gamma}$ and $N_{0,-\beta}$, respectively. This contradicts an analysis performed earlier by two of us (Baek and London) in references [11, 12]. The imprecision had to do with a very subtle question related to rephasing. It is only in the language of reparametrization invariance that this issue becomes simple to understand, illustrating how powerful reparametrization invariance is as a tool to organize the new-physics contributions.

Second, we notice that the expressions for A_2 and $\bar{A}_2$ do not have the same form in Eqs. (14) and in Eqs. (16). One piece of the new-physics contribution, $N_{2,\gamma}$, can indeed be reabsorbed into the definition of $t + c$, as in Eq. (17). (As with the $I = 0$ contributions, the presence of the new $I = 2$ contribution $N_{2,\gamma}$ may only be tested for under specific assumptions for the SM contributions to $T + C$.) But the other piece, $N_{2,o}$, cannot be reabsorbed by a redefinition of SM-like parameters. This means that the presence of some types of new physics in $I = 2$ *can* be detected, even without specific assumptions made about the hadronic matrix elements involved in the SM contributions T and C. Because $N_{2,o}$ is a complex number, we expect two such tests; these are related with the magnitude of $N_{2,o}$, and (once this magnitude is nonzero) with the difference between its (strong) phase and that of $t + c$.

To understand the first test, let us start by considering the case in which the (strong) phase of $N_{2,o}$ coincides with that of $t + c$, δ_{t+c}. In that case the $I = 2$ amplitudes may be written as

$$\begin{aligned} A_2 &= e^{i\delta_{t+c}} \left[\frac{1}{3}|t+c|e^{i\gamma} + |N_{2,o}|\right] = e^{i\delta_{t+c}} e^{i\gamma_{\pi\pi}} |A_2|, \\ \bar{A}_2 &= e^{i\delta_{t+c}} \left[\frac{1}{3}|t+c|e^{-i\gamma} + |N_{2,o}|\right] = e^{i\delta_{t+c}} e^{-i\gamma_{\pi\pi}} |A_2|, \end{aligned} \tag{21}$$

where

$$\tan\gamma_{\pi\pi} = \frac{\sin\gamma}{\cos\gamma + 3\frac{|N_{2,o}|}{|t+c|}}. \tag{22}$$

This type of new physics will be seen as a difference between the phase $\gamma_{\pi\pi}$ obtained from the isospin analysis of $B \to \pi\pi$ decays and the phase $\gamma_{\rm ckm}$ obtained from the current CKM constraints on the Wolfenstein ρ–η plane. Naturally, this signal of new physics disappears as $N_{2,o}$ vanishes. Moreover, in this case, because the same $|A_2|$ appears on both lines of Eq. (21), $|\bar{A}_{+0}|^2 - |A_{+0}|^2 \propto |\bar{A}_2|^2 - |A_2|^2 = 0$, and there is no direct CP violation in $B^\pm \to \pi^\pm\pi^0$ decays. So, the (one) test of new physics possible when $C_{+0} = 0$ is

$$\left|\frac{N_{2,o}}{t+c}\right| = \frac{\sin(\gamma_{\rm ckm} - \gamma_{\pi\pi})}{3\sin\gamma_{\pi\pi}}. \tag{23}$$

The second test on $N_{2,o}$ arises if it carries a strong phase which differs from δ_{t+c}. In that case $|\bar{A}_2|$ differs from $|A_2|$, and this will be reflected in the appearance of direct CP violation in $B^\pm \to \pi^\pm\pi^0$ decays.

In both cases, if we take the values of γ and $-\beta$ from independent measurements, the number of observables in $B \to \pi\pi$ decays is equal to the number of theoretical parameters. Thus, it is not only possible to detect a nonzero $N_{2,o}$; one can also measure its parameters. Up to now, this has not been realized; as above, it is only by using reparametrization invariance that one sees this.

We conclude that there are only two independent tests for new physics in $B \to \pi\pi$ decays which do not depend on hadronic estimates for the SM contributions. New physics in $I = 0$ contributions and $N_{2,\gamma}$ pieces in $I = 2$ cannot be tested for. In contrast, $N_{2,o}$ contributions can be tested for, and they appear as $\gamma_{\pi\pi} - \gamma_{\rm ckm} \neq 0$, or $C_{+0} \neq 0$. In addition, if the weak phases are assumed to be known independently, one can measure the parameters of $N_{2,o}$. Further tests and measurements are possible if one makes specific assumptions about the hadronic matrix elements of the SM.

IV. CONSTRAINING NEW-PHYSICS CONTRIBUTIONS WITH CURRENT DATA

The present $B \to \pi\pi$ measurements are detailed in Table I. The phase β is taken from the measurements of

TABLE I: Branching ratios, direct CP asymmetries C_f, and interference CP asymmetries S_f (if applicable) for the three $B \to \pi\pi$ decay modes. Data comes from Refs. [13, 14, 15]; averages (shown) are taken from Ref. [16].

	$BR[10^{-6}]$	C_f	S_f
$B^+ \to \pi^+\pi^0$	5.5 ± 0.6	0.02 ± 0.07	
$B^0 \to \pi^+\pi^-$	4.6 ± 0.4	-0.37 ± 0.10	-0.50 ± 0.12
$B^0 \to \pi^0\pi^0$	1.51 ± 0.28	-0.28 ± 0.39	

interference CP violation in $B \to \psi K$ decays: $\sin 2\beta = 0.725 \pm 0.037$ [17]. Thus, 2β is determined up to a twofold ambiguity. We assume that $\beta \sim 23.5^\circ$, in agreement with the SM. The value of γ is taken from independent measurements [18]. For the purposes of the fit, we assume symmetric errors, and take $\gamma = (58.2 \pm 6.0)^\circ$.

Using the independent determinations of the SM CP phases, along with the latest $B \to \pi\pi$ measurements, we obtain the values for the isospin amplitudes. The fit to present data yields four solutions, presented in Table II. We get

TABLE II: Results of a fit of the isospin amplitudes to current $B \to \pi\pi$ data. We have factored out the (unphysical) overall phase $\bar{\delta}_0$. The magnitudes are measured in eV and the phases in degrees.

$\|A_2\|$	$\|A_0\|$	$\|\bar{A}_2\|$	$\|\bar{A}_0\|$	$\delta_2 - \bar{\delta}_0$	$\delta_0 - \bar{\delta}_0$	$\bar{\delta}_2 - \bar{\delta}_0$
11.4 ± 0.7	6.8 ± 1.6	11.2 ± 0.7	19.3 ± 2.0	-35.1 ± 80.1	-35.1 ± 134	-59.5 ± 9.3
11.4 ± 0.7	6.8 ± 1.6	11.2 ± 0.7	19.3 ± 2.0	7.6 ± 80.1	7.6 ± 134	59.5 ± 9.3
11.4 ± 0.7	6.8 ± 1.6	11.2 ± 0.7	19.3 ± 2.0	79.8 ± 80.1	79.8 ± 134	-59.5 ± 9.3
11.4 ± 0.7	6.8 ± 1.6	11.2 ± 0.7	19.3 ± 2.0	122 ± 80.1	122 ± 134	59.5 ± 9.3

$\chi^2_{min}/d.o.f. = 0.0049/0$, which is larger than expected. This occurs because the current data are slightly inconsistent with the isospin $\{A_0, A_2\}$ description. Indeed, we have for the central values

$$\cos(\delta_2 - \delta_0) = \frac{\frac{2}{3}|A_{+0}|^2 + |A_{+-}|^2 - 2|A_{00}|^2}{2\sqrt{2}|A_{+0}||A_0|} = 1.07, \tag{24}$$

where $|A_0|$ is given by

$$|A_0|^2 = \frac{2}{3}\left(-\frac{2}{3}|A_{+0}|^2 + |A_{+-}|^2 + |A_{00}|^2\right). \tag{25}$$

This explains why our fit gives the same values for δ_2 and δ_0.

We now wish to perform the fit in the notation of diagrammatic amplitudes. Using the rephasing freedom to set $\arg N_{2,0} = 0$, we obtain the results in Table III. We get $\chi^2_{min} = 0.0049$.

TABLE III: Results of a fit of the diagrammatic amplitudes to current $B \to \pi\pi$ data. We have factored out the (unphysical) overall phase $\delta_{N_{2,0}} = \arg N_{2,0}$. The magnitudes are measured in eV and the phases in degrees.

$\|t\|$	$\|c\|$	$\|p\|$	$\|N_{2,0}\|$	$\delta_t - \delta_{N_{2,0}}$	$\delta_c - \delta_{N_{2,0}}$	$\delta_p - \delta_{N_{2,0}}$
6.1 ± 2.7	9.9 ± 13.7	12.9 ± 3.2	9.6 ± 6.5	81.5 ± 70.5	-40.5 ± 90.0	22.3 ± 74.1
2.8 ± 2.6	19.8 ± 23.8	11.4 ± 6.6	13.2 ± 1.3	41 ± 108	-174 ± 9	-48.6 ± 64.2
22.8 ± 4.0	18.2 ± 6.7	7.3 ± 6.5	2.7 ± 9.3	-156 ± 52	155 ± 32	157 ± 20
19.6 ± 3.9	6.1 ± 22.4	6.4 ± 1.7	6.0 ± 8.4	-19.1 ± 43.9	68.9 ± 174	-127 ± 35

The results in Table III are related to those in Table II through

$$\begin{aligned} p &= \frac{\bar{A}_0\, e^{i\gamma} - A_0\, e^{-i\gamma}}{2i \sin(\beta+\gamma)}, \\ t &= -\frac{\bar{A}_2 - A_2}{2i\sin\gamma} - \frac{\bar{A}_0\, e^{-i\beta} - A_0\, e^{i\beta}}{2i\sin(\beta+\gamma)}, \\ c &= -2\,\frac{\bar{A}_2 - A_2}{2i\sin\gamma} + \frac{\bar{A}_0\, e^{-i\beta} - A_0\, e^{i\beta}}{2i\sin(\beta+\gamma)}, \\ N_{2,0} &= \frac{\bar{A}_2\, e^{i\gamma} - A_2\, e^{-i\gamma}}{2i\sin\gamma}. \end{aligned} \tag{26}$$

One could be worried by the fact that we have used the rephasing freedom in order to set $\bar{\delta}_0 = 0$ when obtaining Table II, while we have used the rephasing freedom in order to set $\arg N_{2,0} = 0$ in obtaining Table III. Nevertheless, both Tables contain only rephasing-invariant quantities which, therefore, can be related. It is easy to see how the rephasing freedom drops out from Eqs. (26) when one relates rephasing-invariant quantities in both parametrizations.

We have also performed the fit of the current experimental data to the SM, obtained by setting $N_{2,0} = 0$. The results are listed in Table IV. We find $\chi^2_{min}/d.o.f. = 0.296/2$, meaning that, if one waives any predictions for the hadronic matrix elements, then the SM provides an excellent fit to the current data.

TABLE IV: Results of a fit of the SM diagrammatic amplitudes to current $B \to \pi\pi$ data. We have factored out the (unphysical) overall phase δ_p. The magnitudes are measured in eV and the phases in degrees.

$\lvert t\rvert$	$\lvert c\rvert$	$\lvert p\rvert$	$\delta_t - \delta_p$	$\delta_c - \delta_p$
21.9 ± 1.1	18.7 ± 1.7	5.4 ± 1.5	55.6 ± 14.7	-11.9 ± 16.9

Notice that Table IV only has one solution, while Table III had four. The reason is the following: in the SM $\bar{A}_2 = A_2 e^{-2i\gamma}$, or, in term of rephasing invariant quantities,

$$|\bar{A}_2|\, e^{i(\bar{\delta}_2-\bar{\delta}_0)} = |A_2|\, e^{i(\delta_2-\bar{\delta}_0)}\, e^{-2i\gamma}. \tag{27}$$

We can see that the third solution in Table III is the one which best satisfies Eq. (27), giving the smallest χ^2 of all.

V. CONCLUSIONS

We have considered the most general new-physics contributions to the $I = 0$ and $I = 2$ amplitudes in $B \to \pi\pi$ decays, which involve 4 new complex parameters N_0, $\bar{N}_0$, N_2, and $\bar{N}_2$. We have shown that N_0 and $\bar{N}_0$ may be absorbed by a redefinition of the SM contributions to $B \to \pi\pi$ decays, as can $N_{2,\gamma}$, *c.f.* Eqs. (17)–(19). This means that new-physics contributions of this type – and in particular, all new-physics contributions to $I = 0$ – *cannot* be detected unless specific ranges are taken for the SM contributions. In contrast, $N_{2,o}$ allows for two tests for the new physics, related to C_{+0} and $\gamma_{\pi\pi} - \gamma_{\rm ckm}$. These are the only two probes of new physics in $B \to \pi\pi$ decays which do not involve estimates of the SM hadronic matrix elements. Furthermore, if one takes values for the weak phases from independent determinations, the $B \to \pi\pi$ observables allow one to measure the $N_{2,o}$ parameters. We have shown that all of these conclusions follow simply from the reparametrization invariance introduced in [7], thus illustrating the power of this concept in providing a clear organization of the new-physics contributions.

Acknowledgments

We thank Y. Nir, G. Raz, and L. Wolfenstein for discussions. J. P. S. is extremely grateful to Y. Nir and to the Department of Particle Physics of the Weizmann Institute of Science for their excellent hospitality, while portions of the this work were made. The work of S. B. and D. L. is supported by NSERC of Canada. F. J. B. is partially supported by the spanish M. E. C. under FPA2002-00612 and HP2003-0079 ("Accion Integrada hispano-portuguesa"). J. P. S. is supported in part by the Portuguese *Fundação para a Ciência e a Tecnologia* (FCT) under the contract CFTP-Plurianual (777), and through the project POCTI/37449/FNU/2001, approved by the Portuguese FCT and POCTI, and co-funded by FEDER.

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[8] Notice that parametrizations of the B and $\overline{B}$ decay amplitudes using the same weak and strong phases, as in Eqs. (1) and (2), only keep this form if the state kets $|B\rangle$ and $|\overline{B}\rangle$ are rephased with the *same* phase ξ_B. This type of change can be absorbed with a modification of any strong phase δ into $\delta' = \delta + \xi_B$. This rephasing freedom can be used to reduce the number of parameters on the right-hand-sides of Eqs. (1) and (2). The possibility of changing the two state vectors, $|B\rangle$ and $|\overline{B}\rangle$ with different phases brings some subtleties. Additional care must be taken with possible rephasings of the quark field operators that mediate the decays. These issues are explained in detail in [2] and [4].
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[10] To be specific, we may parametrize

$$N_2 \;=\; N_{2,\gamma}\, e^{i\gamma} + N_{2,\phi}\, e^{i\phi},$$

$$\bar{N}_2 = N_{2,\gamma}\, e^{-i\gamma} + N_{2,\phi}\, e^{-i\phi}, \tag{28}$$

in which case the first two lines of Eq. (20) become

$$\begin{aligned} N_{2,\gamma} &= -\frac{\bar{N}_2 + N_2}{2} \frac{\sin\phi}{\sin(\gamma-\phi)} + i\frac{\bar{N}_2 - N_2}{2} \frac{\cos\phi}{\sin(\gamma-\phi)}, \\ N_{2,\phi} &= \frac{\bar{N}_2 + N_2}{2} \frac{\sin\gamma}{\sin(\gamma-\phi)} - i\frac{\bar{N}_2 - N_2}{2} \frac{\cos\gamma}{\sin(\gamma-\phi)}. \end{aligned} \tag{29}$$

Clearly

$$N_{2,\phi} = \frac{\sin\gamma}{\sin(\gamma-\phi)} N_{2,0}. \tag{30}$$

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