Jarlskog-like invariants for theories with scalars and fermions

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Abstract

Within the framework of theories where both scalars and fermions are present, we develop a systematic prescription for the construction of CP-violating quantities that are invariant under basis transformations of those matter fields. In theories with Spontaneous Symmetry Breaking, the analysis involves the vevs' transformation properties under a scalar basis change, with a considerable simplification of the study of CP violation in the scalar sector. These techniques are then applied in detail to the two Higgs-doublet model with quarks. It is shown that there are new invariants involving scalar-fermion interactions, besides those already derived in previous analyses for the fermion-gauge and scalar-gauge sectors.

1 Introduction

When a theory has several fields with the same quantum numbers, one can rewrite the lagrangian in terms of new fields, gotten from the original ones by a simple basis transformation. Obviously, the result of a physical process does not depend on such redefinitions; it can only depend on basis invariant quantities. The construction of such basis invariant quantities is especially useful in the context of CP violation, since the mere multiplication of a field by a phase (rephasing) will originate spurious phases in the lagrangian.

This "fuzziness" of CP was already stressed in 1966 by Lee and Wick [1], who pointed out that: *i)* CP is properly defined for some portion of the total lagrangian ($\mathcal{L}_G$) for which CP is a good symmetry; *ii)* any internal symmetry of $\mathcal{L}_G$ may be included in the definition of CP. It has become customary to use for these the name of Generalized CP (GCP) transformations. If the remaining portions of the total lagrangian are manifestly invariant under one of these GCP transformations, then the theory is CP conserving.

In the context of gauge theories, the standard procedure consists in allowing for the inclusion of any weak basis transformation in the definition of CP [2–6]. Here, the name of weak basis transformations is abusively used to denote those transformations of the matter fields that leave the gauge-matter interactions invariant, regardless of which is the gauge group. One then uses the couplings of the remaining portions of the lagrangian to build CP-violating, weak basis invariant quantities. The simplest example arises in the SM with three families, where there is only one independent CP-violating basis invariant quantity [2], J, arising in the complex Yukawa couplings (M_u for the up-type quarks, and M_d for the down-type quarks),

$$J \equiv \det[M_u M_u^\dagger, M_d M_d^\dagger] \; . \tag{1}$$

This quantity can be parametrized in terms of the Yukawa couplings and charged gauge-fermion couplings written in the mass basis – $D_u = diag(m_u, m_c, m_t)$, $D_d = diag(m_d, m_s, m_b)$ and V, respectively – as [1].

$$\begin{aligned} J &\equiv \det[V^\dagger D_u D_u^\dagger V, D_d D_d^\dagger] \\ &\propto (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \\ &\quad \times \mathrm{Im}(V_{ud} V_{cs} V_{us}^* V_{cd}^*) \, , \end{aligned} \tag{2}$$

For historical reasons, we shall use the name of Jarlskog-like invariants (or just J-invariants, for simplicity) for all CP-violating basis invariant quantities. The construction of J-invariants has been done in a variety of models. The method of choice has been to cleverly look for quantities that transform into themselves under a basis transformation but develop a minus sign under a GCP transformation. The fermion-gauge (-mass) sector was the first to be fully analyzed in a series of models [3]. Similar J-invariants were later developed for the gauge-scalar sector of multi Higgs-doublet models without fermions [4, 5].

In this article, we present the rules for the systematic construction of J-invariants in any gauge theory with fermions and scalars including the gauge-fermion, gauge-scalar but also the scalar-fermion sources of CP-violation. In Section 2 we discuss an alternative method for the construction of J-invariants inspired by perturbation theory. This enables a much cleaner study of the CP-violation in the gauge-scalar sector than that performed in Ref. [5]. In addition, it allows for the straightforward inclusion of fermions in theories with many scalars. The two Higgs-doublet model (2HDM) is worked out in detail in Section 3 where new J-invariants signaling CP violation in the scalar-gauge sector are identified. In Section 4 we draw our conclusions.

[1] Of course, one can break the lagrangian in a different way, defining CP at the level of the kinetic and neutral gauge interactions. These are invariant under general basis transformations, which, in particular, may transform the left-handed fields u_L and d_L differently. One then uses the transformations of both the charged gauge-fermion and the yukawa couplings to build the CP-violating quantity $\det[V^\dagger M_u M_u^\dagger V, M_d M_d^\dagger]$ which is explicitly invariant under *any* basis transformation. In a weak basis ($V = 1$) this expression reduces to Eq. (1), while in the mass basis it reduces to Eq. (2). The reason behind using transformations that leave the gauge-matter interactions is apparent; if one does not, one must also consider the transformation properties of the gauge-matter couplings, significantly complicating the analysis.

2 Systematic construction of J-invariants

We will try to motivate our general prescription for the construction of J-invariants with some examples. We start with a generic lagrangian of the form,

$$\mathcal{L}_I = g_{ij} \alpha_i \beta_j \Phi + h_{kl} \alpha_k \gamma_l \Phi + h.c. \ , \tag{3}$$

where g and h are coupling constants and α_i, β_i and γ_i are field operators defining the ith direction in the respective U(α), U(β) and U(γ) flavour spaces, and transforming like some multiplet of the gauge group, G. As an example, for the three family SM we have, after SSB,

$$\mathcal{L}_I = (\bar{u}_L, \bar{d}_L)_i \ M_{uij} \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_{Rj} + (\bar{u}_L, \bar{d}_L)_i \ M_{dij} \begin{pmatrix} 0 \\ 1 \end{pmatrix} d_{Rj} + h.c. \ , \tag{4}$$

with the flavour spaces being U(3)$_L$, U(3)$_{uR}$ and U(3)$_{dR}$, respectively. In perturbation theory, one can generate interactions mediated by any power of $\mathcal{L}_I$. For example, to second order in perturbation theory, we will find interactions mediated by

$$(g_{ij} \alpha_i \beta_j \Phi) \ (h_{kl} \alpha_k \gamma_l \Phi) \ . \tag{5}$$

Hence, a given property of the theory (say CP violation) may show up at some order of perturbation theory as a suitable product of couplings.

Under a basis transformation the couplings transform as,

$$\begin{aligned} g_{ij} & \rightarrow U(\alpha)_{ki} \ g_{kl} \ U(\beta)_{lj} \ , \\ h_{ij} & \rightarrow U(\alpha)_{ki} \ h_{kl} \ U(\gamma)_{lj} \ . \end{aligned} \tag{6}$$

The strategy in looking for basis invariant quantities consists in taking products of couplings (as in the perturbative expansion), contracting over the internal flavour spaces and taking a trace at the end. For example, the quantities

$$H_u = M_u M_u^\dagger \quad , \quad H_d = M_d M_d^\dagger \quad , \quad H_u H_d \tag{7}$$

are tensors in the U(3)$_L$ space, whose traces are weak basis invariant. The same is true for the trace of the U(3)$_{uR}$ tensor $M_u^\dagger M_u$.

In so doing, we have already traced over the basis transformations that could lead to the spurious phases that we alluded to in the introduction. Therefore, the imaginary parts of such traces are unequivocal signs of CP violation [6]. For example, the three family J-invariant can be rederived as [6],

$$J \ \propto \ \mathrm{Im}\{Tr(H_u H_d H_u^2 H_d^2)\} \ . \tag{8}$$

This method has already been used to discover an ϵ-type contribution to baryogenesis in SU(5) with two Higgs fiveplets [7], in addition to the ϵ'-type contribution found earlier [8].

A final detail concerns spontaneous symmetry breaking (SSB). After SSB, the physical degrees of freedom of the neutral scalars are described by the shifted fields (η_i) related to the original ones (ϕ_i) by the vevs (v_i) as,

$$\phi_i = v_i + \eta_i \ , \tag{9}$$

which reparametrizes a lagrangian term such as

$$a_{ij}\phi_i^\dagger \phi_j \ , \tag{10}$$

into

$$a_{ij}v_i^* v_j + a_{ij}v_i^*\eta_j + a_{ij}\eta_i^\dagger v_j + a_{ij}\eta_i^\dagger \eta_j \ , \tag{11}$$

where v_i becomes an integral part of some new couplings. Thus, for the scalar sector the construction of the invariants must also include the vacuum expectation values. As we will point out in the next section, this greatly simplifies the study of the scalar sector over the previous analysis of Ref. [5]. In addition, the minimization conditions provide relations between the couplings in the scalar potential which must be used in identifying the correct number of independent CP violating invariants.

This discussion motivates the following prescription for the construction of J-invariants:

- identify all the scalar and fermion flavour spaces in the theory;

- make a list of all the couplings according to their transformation properties under weak basis transformations, including the vacuum expectation values (which transform as vectors under the scalar basis change), and make use of the stationarity conditions of the scalar potential to reduce the number of parameters;

- construct invariants by contracting over internal flavour spaces in all possible ways, taking traces at the end (to be systematic it is best to do this first in the fermion sector, say, and then use this to define new scalar tensors, performing the scalar analysis as a next step);

- take the imaginary part to obtain a basis invariant signal of CP violation.

Note that, in general, a minimal set of CP violating quantities is not easy to find since one could in principle go to arbitrary order in perturbation theory. That analysis is best done on a case by case basis through a careful study of the CP violation sources in the model. Moreover, different particular cases of a model may require different choices for the fundamental J-invariants. This is the case even in models as simple as the four family SM [9].

One should notice the generality of the proposed method. In fact, this scheme applies to any gauge group, G. In addition, there is no renormalizability requirement. The method is applicable to any effective field theory with renormalizable, as well as nonrenormalizable interactions.

3 The two Higgs-doublet model with quarks

We will now look at an SU(2)⊗U(1) gauge theory with two Higgs-doublets and with n quark families. The particle content of the theory consists of two scalar doublets

$$\Phi_1 \quad , \quad \Phi_2 \ , \tag{12}$$

to which, without loss of generality, we can attribute the vevs $v_1/\sqrt{2}$ and $v_2 e^{i\alpha}/\sqrt{2}$, with v_1 and v_2 real, and the quark fields

$$\bar{q}_L = (\bar{p}_L \ , \ \bar{n}_L) \quad , \quad p_R \quad , \quad n_R \ , \tag{13}$$

which are n-plets in the corresponding flavour spaces: respectively, the spaces of SU(2) doublets, charged 2/3 singlets and charged $-1/3$ singlets. The Yukawa lagrangian is,

$$-\mathcal{L}_Y = \bar{q}_L \Gamma_i n_R \Phi_i + \bar{q}_L \Delta_i^* p_R \tilde{\Phi}_i + h.c. \ , \tag{14}$$

where the $n \times n$ matrices Γ_i and Δ_i^* [2] contain the Yukawa couplings to the scalar Φ_i, and a sum over the scalar space ($i = 1, 2$) is implicit. The scalar potential may be written as:

$$V_H = a_{ij} (\Phi_i^\dagger \Phi_j) + l_{ij,kl} (\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l) \ , \tag{15}$$

where hermiticity implies

$$\begin{aligned} a_{ij} &= a_{ji}^* \ , \\ l_{ij,kl} \equiv l_{kl,ij} &= l_{ji,lk}^* \ . \end{aligned} \tag{16}$$

The stationarity conditions are

$$v_i^* \left[a_{i\alpha} + 2 v_k^* \ l_{i\alpha,kl} \ v_l \right] = 0 \qquad (\text{for } \alpha = 1, 2) \ . \tag{17}$$

We have used boldfaced characters to remind us that these are tensors in the scalar space.

Under weak basis redefinitions of scalars (through a unitary matrix U), and of fermions (through matrices X), the Yukawa couplings get transformed as [10]

$$\begin{aligned} \Gamma_i &\rightarrow X_L^\dagger \Gamma_j U_{ji} X_{dR} \ , \\ \Delta_i^* &\rightarrow X_L^\dagger \Delta_j^* U_{ji}^* X_{uR} \ , \end{aligned} \tag{18}$$

with the scalar potential parameters and the vevs transforming as

$$\begin{aligned} a_{ij} &\rightarrow U_{ki}^* a_{kl} U_{lj} \ , \\ l_{ij,kl} &\rightarrow U_{mi}^* U_{ok}^* l_{mn,op} U_{nj} U_{pl} \ , \end{aligned} \tag{19}$$

$$v_i \rightarrow U_{ji}^* v_j \ . \tag{20}$$

[2] The reason behind the noncanonical definition of matrices Δ_i with the complex conjugation will become apparent in Eq. (22).

Following our general scheme, we first build fermion basis invariants such as the traces

$$\begin{aligned} T^{\Gamma}_{ij} &= Tr^f(\Gamma_i \Gamma_j^\dagger) \\ T^{\Delta}_{ij} &= Tr^f(\Delta_i \Delta_j^\dagger) \end{aligned} \tag{21}$$

where Tr^f indicates that a trace has been taken over the relevant fermion flavour space. These tensors, which are second order in the Yukawa couplings, transform under a scalar basis change as

$$T_{ij} \to U^*_{ki} T_{kl} U_{lj} \ . \tag{22}$$

One may now combine the scalar basis tensors in Eqs. (19), (20) and (21), taking a trace at the end and thus obtaining weak basis independent quantities. If these have a nonzero imaginary part, we will have a sign of CP violation.

Being the only vector, v_i must always appear in the scalar traces in the combination

$$V_{ij} = v_i v_j^* \ . \tag{23}$$

Further, all these second rank tensors are hermitian and hence one needs either three different ones or the repetition of two of them in order to get a complex trace. For example

$$Tr(Va) \ , \qquad Tr(V^2 a) \ , \tag{24}$$

are clearly real, while

$$J_a = \mathrm{Im}\ Tr(VaT^{\Gamma}) \ , \qquad J_b = \mathrm{Im}\ Tr(VaT^{\Delta}) \ , \tag{25}$$

which mix the scalar and fermion sectors, or

$$J_1 = \mathrm{Im}\ (v_i^* v_j^* a_{i\alpha} a_{j\beta} l_{\alpha k,\beta l} v_k v_l) \ , \qquad J_3 = \mathrm{Im}\ (v_i^* a_{ij} l_{jk,kl} v_l) \ , \tag{26}$$

which depend exclusively on the scalar sector, may be nonzero.

In this form the invariants are difficult to interpret and, moreover, it is not always clear how to include in them the stationarity conditions (17). A much clearer picture arises if one transforms the scalars into a basis in which only the first scalar has a vacuum expectation value.

3.1 The Higgs basis

Indeed, since in this model all the scalars are in the same representation of SU(2), the vev may be rotated all into the first scalar through the transformation

$$\Phi_i = U_{ij} H_j \tag{27}$$

with

$$U^\dagger = \frac{1}{v} \begin{pmatrix} v_1 & v_2 e^{-i\alpha} \\ v_2 & -v_1 e^{-i\alpha} \end{pmatrix} \ . \tag{28}$$

Therefore, the new scalars may be parametrized as

$$H_1 = \begin{pmatrix} G^+ \\ (v + H^0 + iG^0)/\sqrt{2} \end{pmatrix} \qquad H_2 = \begin{pmatrix} H^+ \\ (R + iI)/\sqrt{2} \end{pmatrix} , \tag{29}$$

where G^+ and G^0 are the Goldstone bosons, which, in the unitary gauge, become the longitudinal components of the W^+ and of the Z^0, and H^0, R and I are real neutral fields, with H^0 coupling to fermions proportionally to their masses (in the fermion mass basis). Note that these features remain the same if one multiplies H_2 by a phase. All that does is to rotate R and I through an orthogonal transformation.

In this basis, the Yukawa coupling matrices become

$$\Gamma_i^H = \Gamma_j U_{ji} \ , \qquad \Delta_i^{H*} = \Delta_j^* U_{ji}^* \ , \tag{30}$$

with the scalar couplings transformed into μ_{ij} and $\lambda_{ij,kl}$, given in terms of the original ones by the right hand side of Eq. (19), with the specific form of U written in Eq. (28). With an obvious change of notation ($\mu_{11} = \mu_1$, $\mu_{12} = \mu_3$, etc.) we can write the scalar potential in the familiar form

$$\begin{aligned} V_H &= \mu_1 H_1^\dagger H_1 + \mu_2 H_2^\dagger H_2 + (\mu_3 H_1^\dagger H_2 + h.c.) \\ &\quad + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\ &\quad + \left[\lambda_5 (H_1^\dagger H_2)^2 + \lambda_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7 (H_2^\dagger H_2)(H_1^\dagger H_2) + h.c. \right] , \end{aligned} \tag{31}$$

in which all the coupling constants, except μ_3, λ_5, λ_6, and λ_7, are real by hermiticity.

In this basis, the stability conditions of the vacuum in Eq. (17) take the very simple form

$$\mu_1 = -\lambda_1 v^2 \ , \qquad \mu_3 = -\lambda_6 v^2/2 \ . \tag{32}$$

Using this last equation one can eliminate μ_3 so that only the phases of λ_5, λ_6 and λ_7 remain. The simplicity of these relations, and the fact that only H_1 has a vev, greatly simplify the form of the invariants. For example, in this basis the invariants of Eq. (26) take the very simple form [5]

$$J_1 \propto \mathrm{Im}(\lambda_6^2 \lambda_5^*) \ , \qquad J_3 \propto \mathrm{Im}(\lambda_6 \lambda_7^*) \ . \tag{33}$$

These are the only two independent CP-violating basis invariant quantities of the most general 2HDM without fermions. The other possible invariant,

$$J_2 \propto \mathrm{Im}(\lambda_7^2 \lambda_5^*) \ , \tag{34}$$

is just a combination of the previous two.

These quantities were derived in Ref. [5] by looking for CP-violating quantities, invariant under a phase transformation of H_2. This required the expression of V_H in terms of the component field of Eq. (29), and the subsequent analysis of the transformation properties of each term in V_H under that rephasing, a procedure that is rather tedious. With the methods we have developed above, this analysis becomes straightforward, mainly due to the inclusion of the vevs' transformation properties. Further, it becomes clear that these are invariants under any weak basis change, and not just under a rephasing of H_2. They just happen to be written in the Higgs basis for simplicity.

3.2 The quark mass basis

As happens in the SM for the Jarlskog invariant, the physical interpretation of invariants involving fermions is clearer when the Yukawa matrices are parametrized in terms of their values in the quark mass basis. This is done through transformations that diagonalize the couplings of the quarks to the Higgs (H_1) with the vev $v/\sqrt{2}$:

$$\begin{aligned} v/\sqrt{2}\ X_{dL}^{\dagger}\Gamma_1^H X_{dR} &= D_d = diag(m_d, m_s, \ldots)\ , \\ v/\sqrt{2}\ X_{uL}^{\dagger}\Delta_1^{H*} X_{uR} &= D_u = diag(m_u, m_c, \ldots)\ , \end{aligned} \tag{35}$$

while the couplings to the vevless Higgs (H_2) become

$$\begin{aligned} v/\sqrt{2}\ X_{dL}^{\dagger}\Gamma_2^H X_{dR} &= N_d\ , \\ v/\sqrt{2}\ X_{uL}^{\dagger}\Delta_2^{H*} X_{uR} &= N_u\ , \end{aligned} \tag{36}$$

where $V = X_{uL}^{\dagger}X_{dL}$ is the CKM matrix. The Yukawa lagrangian may then be written as

$$\begin{aligned} -\mathcal{L}_Y &= \left(\bar{u}_L V\ , \bar{d}_L\right)\left[D_d\left(\frac{\sqrt{2}}{v}H_1\right) + N_d\left(\frac{\sqrt{2}}{v}H_2\right)\right]\ d_R \\ &+ \left(\bar{u}_L\ , \bar{d}_L V^{\dagger}\right)\left[D_u\left(\frac{\sqrt{2}}{v}\tilde{H}_1\right) + N_u\left(\frac{\sqrt{2}}{v}\tilde{H}_2\right)\right]\ u_R + h.c.\ . \end{aligned} \tag{37}$$

One can still perform equal rephasings on the left- and right-handed components of each quark without affecting these properties. Usually this is used to remove $2n-1$ unphysical phases from the CKM matrix. The matrices N_d and N_u are, however, perfectly arbitrary complex $n \times n$ matrices.

In this basis the fermion traces of Eq. (21) are

$$\begin{aligned} \frac{v^2}{2}T_{11}^{\Gamma} &= \sum_{i=1}^{n} m_{di}^2\ , \\ \frac{v^2}{2}T_{22}^{\Gamma} &= \sum_{i,k=1}^{n} |N_{dik}|^2\ , \\ \frac{v^2}{2}T_{12}^{\Gamma} \equiv \frac{v^2}{2}T_{21}^{\Gamma*} &= \sum_{i=1}^{n} m_{di}N_{dii}\ , \end{aligned} \tag{38}$$

with similar expression for the up sector. The invariants of Eq. (25) thus become

$$\begin{aligned} J_a &= \mathrm{Im}(\mu_{12}T_{21}^{\Gamma}) \\ &= \sum_{i=1}^{n} \mathrm{Im}(m_{di}\ \mu_3\ N_{dii}^{*})\ , \\ J_b &= \sum_{i=1}^{n} \mathrm{Im}(m_{ui}^{*}\ \mu_3\ N_{uii})\ . \end{aligned} \tag{39}$$

At first sight this is a surprising result since only one power of the mass is involved, contrary to our SM acquired intuition. The point is that, in the SM there is only one

Yukawa matrix for the down-type quarks, say, and therefore it needs to appear twice to preclude changes in the invariants arising from different rephasings of the right and left-handed components of some quark. Here, as is seen explicitly in Eq. (39), the existence of two matrices allows for the construction of invariants in which the phase change in one of those matrices is compensated by the same phase change in the other. To be more specific, imagine that we perform a phase change on the d_R quark,

$$d_R \to e^{i\delta} d_R. \tag{40}$$

Then,

$$\begin{aligned} D_{d11} &\to D_{d11} e^{i\delta} , \\ N_{d11} &\to N_{d11} e^{i\delta} , \end{aligned} \tag{41}$$

and their rephasings get cancelled in the Eq. (39). It was to emphasize this point that the masses (which are real positive numbers in the mass basis) were kept inside the imaginary part in these equations.

The appearance of μ_3 (or better, λ_6, once the stationarity conditions are used) is also easy to understand. If, for example, one rephases H_2 by

$$H_2 \to e^{i\xi} H_2 , \tag{42}$$

one gets

$$\begin{aligned} \mu_3 &\to \mu_3 e^{i\xi} , \\ N_d &\to N_d e^{i\xi} , \\ N_u &\to N_u e^{-i\xi} , \end{aligned} \tag{43}$$

and hence μ_3 must appear in combination with $N_d^\dagger$ (or N_u). A similar rephasing of H_1 leads to the conclusion that D_d (or $D_u^\dagger$) must also be involved. We have thus succeeded in constructing weak basis invariant quantities that control the feeding of phases between the fermion and the scalar sectors, through the rephasings of either quarks or scalars.

For the simplest case of just one quark family, it is easy to see that we have all the invariants we need. In fact, in the mass basis, there are five complex quantities: λ_5, λ_6 and λ_7 in the scalar sector (μ_3 is related to λ_6 through the stationarity conditions); and N_d and N_u in the Yukawa couplings, which are now just complex numbers. The freedom to rephase H_2 allows us to set one of these quantities real and only four J-invariants remain. We can take these to be J_1, J_3, J_a and J_b. However, depending on the particular case in question, other combinations might be more useful. For instance, in a model with $\mu_3 = 0 = \lambda_6$, all of the above are zero but there are still three CP-violating phases. One is J_2 and the others may be chosen as

$$J_x = \mathrm{Im}(m_u m_d N_d^* N_u^*) , \tag{44}$$

$$J_y = \mathrm{Im}(m_d \lambda_7 N_d^*) . \tag{45}$$

The invariant J_x appears for example in the charged Higgs boson contribution to the decay rate asymmetry $\Gamma[\bar{b} \to \bar{s}\gamma] - \Gamma[b \to s\gamma]$ and to the dipole moment of the neutron, in the approximation in which the third quark family decouples from the first two [11].

The situation is similar for more quark families. In general one has J_1 and J_3 from the scalar sector, and one needs to find an extra $2n^2$ (from the phases in the N matrices) plus $(n-1)(n-2)/2$ (from the irremovable phases in the CKM matrix) invariants, using the methods described above. As a simple illustration we can look at the 2HDM with two families. There are now 8 phases in the couplings to H_2 and yet no phase in the CKM matrix (in the parametrization we described above). Besides the fermion traces of Eq. (21), we now need also the tensors

$$\begin{aligned} T^{\Gamma\Gamma}_{ijkl} &= Tr(\Gamma_i \Gamma^\dagger_j \Gamma_k \Gamma^\dagger_l) \ , \\ T^{\Delta\Delta}_{ijkl} &= Tr(\Delta_i \Delta^\dagger_j \Delta_k \Delta^\dagger_l) \ , \end{aligned} \tag{46}$$

and the quantities

$$T^{\Gamma\Delta}_{ijkl} = Tr(\Gamma_i \Gamma^\dagger_j \Delta^*_k \Delta^{*\dagger}_l) \ , \tag{47}$$

which transforms in the last two indices as U^* and $U^\top$, because of the required inclusion of Δ^* to compensate for the transformation properties of Γ under the quark left-handed flavour transformations. Of course, one must keep this in mind when performing the scalar traces. It is easy to check that the phases N_{d11} and N_{d22} are directly related to T^{Γ}_{12} and $T^{\Gamma\Gamma}_{1112}$. The phase of $N_{d12}N_{d21}$ can then be easily related to $T^{\Gamma\Gamma}_{1212}$ or $T^{\Gamma\Gamma}_{2212}$. Similarly for the up quark sector. The final two independent combinations of phases can be related, for example, to

$$\begin{aligned} T^{\Gamma\Delta}_{1112} &= Tr(D_d D^\dagger_d V^\dagger D_u N^\dagger_u V) \\ &= m_d^2 m_c N^*_{u22} + m_s^2 m_u N^*_{u11} \\ &+ (m_s^2 - m_d^2) c_\theta \left[m_c (N^*_{u22} + s_\theta N^*_{u12}) - m_u (N^*_{u11} - s_\theta N^*_{u21}) \right] \ , \\ T^{\Gamma\Delta}_{1211} &= Tr(D_d N^\dagger_d V^\dagger D_u D^\dagger_u V) \\ &= m_u^2 m_s N^*_{d22} + m_c^2 m_d N^*_{d11} \\ &+ (m_c^2 - m_u^2) c_\theta \left[m_s (N^*_{d22} - s_\theta N^*_{d12}) - m_d (N^*_{d11} + s_\theta N^*_{d21}) \right] \ , \end{aligned} \tag{48}$$

where the mass basis parametrization was used, and s_θ and c_θ are the sine and cosine of the Cabbibo angle, respectively. Clearly the J_a and J_b of Eq. (39) are still invariants. In addition, 6 new invariants can be made out in the same way as in Eq. (26), substituting l_{ijkl} for the tensors of Eqs. (46) and (47). These have interpretations similar to the simpler ones of the one family, 2HDM above. The game to be played for three families is now obvious, with the added feature that in that case the CKM matrix has an irremovable phase leading to the original invariant: the gauge-fermion (-mass) Jarlskog invariant [2].

Whether these invariants are equal to zero or proportional to each other depends on the model one considers. In the very popular 2HDM with soft breaking of the Z_2 symmetry and spontaneous CP breaking [12], there is only one CP violating phase: the relative phase between the two vacuua. Then, the Jarlskog invariant is zero, and all other invariants are, of course, either zero or proportional to $\sin\alpha$.

4 Conclusion

We have presented a systematic method for the construction of basis invariant CP-violating quantities for theories with both scalars and fermions. This procedure was inspired by perturbation theory and will reproduce the CP violations occurring perturbatively. We also point out that, due to SSB, one must take the into account the transformation properties of the vevs, and have shown how that leads to a very simple analysis of the CP violation in the scalar sector.

A simple application of this scheme was worked out in detail for the 2HDM with n quark families, for which we reproduced the earlier results for the CP-violating invariants in the gauge-fermion and gauge-scalar. In addition, we have constructed new invariants that express the CP violation in the scalar-fermion sector.

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References

[1] T. D. Lee and G. C. Wick, Phys. Rev. **148**, 1385 (1966).

[2] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); I. Dunietz, O. W. Greenberg, and D.-D. Wu, Phys. Rev. Lett. **55**, 2935 (1985); F. J. Botella and L. -L. Chau, Phys. Lett. **B181**, 123 (1986).

[3] J. Bernabéu, G. C. Branco, and M. Gronau, Phys. Lett. **169B**, 243 (1986), for more than three families; G. C. Branco and L. Lavoura, Nucl. Phys. **B278**, 738 (1986), for models with vectorlike quarks; G. C. Branco and M. N. Rebelo, Phys. Lett. **B173**, 313 (1986), for the L-R symmetric model; G. C. Branco, L. Lavoura, and M. N. Rebelo, Phys. Lett. **B180**, 264 (1986), for neutrinos with Majorana masses; G. C. Branco and V. A. Kostelecky, Phys. Rev. D **39**, 2075 (1989), for supersymmetric models.

[4] A. Méndez and A. Pomarol, Phys. Lett. **B272**, 313 (1991).

[5] L. Lavoura and J. P. Silva Phys. Rev. D **50**, 4619 (1994).

[6] J. Roldán, PhD Thesis, Universidad de València, 1991.

[7] F. J. Botella and J. Roldán, Phys. Rev. D **44**, 966 (1991).

[8] D. V. Nanopoulos and S. Weinberg, Phys. Rev. D **20**, 2484 (1979).

[9] M. Gronau, A. Kfir, and R. Loewy, Phys. Rev. Lett. **56**, 1538 (1986).

[10] These basis transformations and their corresponding GCP transformations were studied in detail, with different objectives, in a series of very clear articles by the Vienna group:
G. Ecker, W. Grimus, and W. Konetschny, Nucl. Phys. **B191**, 465 (1981); G. Ecker, W. Grimus, and H. Neufeld, Nucl. Phys. **B247**, 70 (1984); G. Ecker, W. Grimus, and H. Neufeld, J. Phys. **A20**, L807 (1987); H. Neufeld, W. Grimus, and G. Ecker, Int. J. Mod. Phys. A **3**, 603 (1988);

[11] L. Wolfenstein and Y. L. Wu, CMU preprint CMU-HEP-94-24, July 1994, unpublished.

[12] G. C. Branco and M. N. Rebelo, Phys. Lett. **B173**, 313 (1986). See also L. Lavoura, Int. J. Mod. Phys. **A9**, 1873 (1994).