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Nuclear effects in F_3 structure function of nucleon

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We study nuclear effects in the $F_3^A(x)$ structure function in the deep inelastic neutrino reactions on iron by using a relativistic framework to describe the nucleon spectral functions in the nucleus. The results for the ratio $R(x,Q^2) = \frac{F_3^A(x,Q^2)}{AF_3^N(x,Q^2)}$ and the Gross-Llewellyn Smith(GLS) integral $G(x,Q^2) = \int_x^1 dx F_3^A(x,Q^2)$ in nuclei are discussed and compared with the recent results available in literature from theoretical and phenomenological analyses of experimental data.

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I. INTRODUCTION

The recent experimental results reported by the NuTeV collaboration [1] on weak charged and neutral current induced (anti)neutrino processes on an iron target in the deep inelastic region have emphasized the importance of nuclear medium effects. There are many theoretical analyses of the deep inelastic scattering of charged leptons from nuclear targets where various nuclear effects like shadowing, anti-shadowing, Fermi motion and binding of the nucleons in various kinematic regions have been studied. These have been discussed in several review articles [2, 3, 4], but in the case of deep inelastic scattering of (anti)neutrinos from nuclear targets, there are few calculations where the dynamical origin of the nuclear medium effects has been studied [5, 6, 7, 8, 9]. In some theoretical analyses, nuclear medium effects have been phenomenologically described in terms of a few parameters which are determined from fitting the experimental data of charged leptons and (anti)neutrino deep inelastic scattering from various nuclear targets [10, 11, 12, 13, 14].

The differential scattering cross section for the deep inelastic scattering of (anti)neutrinos from unpolarized nucleons in the limit of lepton mass $m_l \rightarrow 0$, is described in terms of three structure functions, $F_1^\nu(x,Q^2)$, $F_2^\nu(x,Q^2)$ and $F_3^\nu(x,Q^2)$, where $x = \frac{Q^2}{2M\nu} = -\frac{q^2}{2M\nu}$ is the Bjorken variable, ν and q being the energy and momentum transfer of leptons. In the asymptotic region of Bjorken scaling i.e. $Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, x finite, all the structure functions depend only on the Bjorken variable x. In this scaling limit, $F_1^\nu(x)$ and $F_2^\nu(x)$ are related by the Callan-Gross relation [15] leading to only two independent structure functions $F_2^\nu(x)$ and $F_3^\nu(x)$ which are determined from the experimental data on deep inelastic scattering of (anti)neutrinos in the asymptotic region. Scaling violation effects have been studied in deep inelastic scattering of (anti)neutrino from nucleon targets using methods of perturbative and non-perturbative QCD. The Q^2 dependence of the structure functions is determined from the Q^2 evolution given by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [16] obtained in perturbative QCD calculations. Non perturbative Q^2 corrections to the structure functions have been also studied by many authors [17, 18].

Since most of the experimental data on deep inelastic scattering of high energy (anti)neutrinos are obtained on nuclear targets over a wide range of Q^2, it is important to study the nuclear medium modification effects on these structure functions $F_2^\nu(x,Q^2)$ and $F_3^\nu(x,Q^2)$ specially in iron where most recent, high statistics, high precision data are available [1]. The nuclear medium modification effects on the average structure function $F_3^A(x,Q^2)=\frac{1}{2}(F_3^{\nu A}(x,Q^2) + F_3^{\bar{\nu} A}(x,Q^2))$ on isoscalar nuclear targets arise mainly due to modification of the valence quark parton distributions in the nuclear medium. The contributions of sea quarks and gluons in a quark parton picture vanish in the case of a symmetric sea and arise only when the various sea quark parton distribution are taken to be different. Of much interest is the estimate of the size of the nuclear medium effects on the GLS sum rule [19] which is measured over a wide range of Q^2 for deep inelastic scattering of (anti)neutrinos on nuclear targets and has been used to determine the QCD coupling constant α_s [20].

In this paper, we study some nuclear medium effects on the nucleon structure function $F_3^A(x,Q^2)$ in iron. We use a theoretical spectral function to describe the momentum distribution of nucleons in the nucleus. The spectral function has been calculated using the Lehmann's representation for the relativistic nucleon propagator and nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter. A local density approximation is then applied to translate these results to finite nuclei. The method has been earlier used successfully to describe the EMC and other nuclear effects in deep inelastic scattering of charged leptons from nuclei [21]. Our approach

describes an alternative method to calculate nuclear medium effects in the nucleon structure functions using nuclear many body theory different from the approach used by Kulagin [5]. Our model has certain limitations as it only attempts to consider the modifications of nucleonic contributions to $F_3^N(x,Q^2)$ arising due to binding energy, off mass shell and Fermi motion of the nucleon in the nuclear medium which dominate in the region of $x \geq 0.3$. In the region of $0.3 > x > 0.1$, corresponding to the anti-shadowing region, the nuclear medium modification effects on $F_3^A(x,Q^2)$ are expected to be small due to vanishing of the pion contribution in this model unlike $F_2^{Al}(x,Q^2)$ where contributions of pion and rho mesons play a dominant role [21]. We do not consider the shadowing region of $0.0 < x < 0.1$ in this paper and that will be considered in the extension of this work. Therefore, the results presented here should be able to describe the dominant contribution of nuclear medium effects to $F_3^A(x,Q^2)$ in the range of $0.1 < x < 1$.

In sections 2 and 3, we describe briefly the formalism used to calculate nuclear effects and present our results and discussion in section 4.

II. DEEP INELASTIC NEUTRINO NUCLEON SCATTERING

The cross section for the reaction

$$\nu_l(\bar{\nu}_l) + N \rightarrow l^-(l^+) + X, \tag{1}$$

is given by:

$$\begin{aligned} \sigma \;=\; & \frac{1}{v_{rel}} \frac{2m_\nu}{2E_\nu(\mathbf{k})} \frac{2M}{2E(\mathbf{p})} \int \frac{d^3k'}{2\pi^3} \frac{2m_l}{2E_l(\mathbf{k}')} \\ & \times \prod_{i=1}^{N} \int \frac{d^3p'_i}{2\pi^3} \prod_{l\epsilon f} \left(\frac{2M'_l}{2E'_l}\right) \prod_{j\epsilon b} \left(\frac{1}{2\omega'_j}\right) \bar{\sum} \sum |T|^2 (2\pi)^4 \\ & \times \delta^4 \left(p + k - k' - \sum_{i=1}^{N} p'_i\right) \end{aligned} \tag{2}$$

where f stands for fermions and b for bosons in the final state X. The index i is split into l and j for fermions and bosons respectively,

T is the invariant matrix element for the above reaction and is, written as

$$-iT = \left(\frac{iG}{\sqrt{2}}\right) \bar{u}_l(k')\gamma^\alpha(1-\gamma_5)u_l(k) \left(\frac{m_W^2}{q^2 - m_W^2}\right) \langle X|J_\alpha|N\rangle \,. \tag{3}$$

After performing the phase space integration in Eq.(2), the double differential scattering cross section evaluated for a nucleon target in its rest frame is expressed as,

$$\frac{d^2\sigma^N_{\nu,\bar{\nu}}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar{\nu}} \, W^N_{\alpha\beta} \,, \tag{4}$$

where G_F is the Fermi coupling constant, m_W is the mass of the W boson, $l(= e, \mu)$ is a lepton, k is the incoming neutrino four momentum and k' is the outgoing four momentum of the lepton, $q = k - k'$ is the four momentum transfer and Ω', E' refer to the outgoing lepton. N is a nucleon, X is a jet of n hadrons consisting of fermions(f) and bosons(b) in the final state labeled by l and j in the following.

The lepton tensor for antineutrino(neutrino) scattering $L^{\alpha\beta}$ is given by

$$L^{\alpha\beta} = k^\alpha k'^\beta + k^\beta k'^\alpha - k.k' g^{\alpha\beta} \pm i\epsilon^{\alpha\beta\rho\sigma} k_\rho k'_\sigma \,, \tag{5}$$

and the hadronic tensor $W^N_{\alpha\beta}$ is defined as

$$W^N_{\alpha\beta} = \frac{1}{2\pi} \bar{\sum_{s_N}} \sum_X \sum_{s_i} \prod_{i=1}^{n} \int \frac{d^3p'_i}{(2\pi)^3} \prod_{l\epsilon f} \left(\frac{2M'_l}{2E'_l}\right) \prod_{j\epsilon b} \left(\frac{1}{2\omega'_j}\right) \langle X|J_\alpha|N\rangle \langle X|J_\beta|N\rangle^* (2\pi)^4 \delta^4(p + q - \sum_{i=1}^{n} p'_i) \,, \tag{6}$$

where q is the momentum of the virtual W, s_N the spin of the nucleon and s_i the spin of the fermions in X. In the case of antineutrino $\langle X|J_\alpha|N\rangle$ is replaced by $\langle X|J^\dagger_\alpha|N\rangle$.

The most general form of the hadronic tensor $W^N_{\alpha\beta}$ is expressed as [22]:

$$W^N_{\alpha\beta} = \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar{\nu})} + \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^{\nu(\bar{\nu})} - \frac{i}{2M^2}\epsilon_{\alpha\beta\rho\sigma}p^\rho q^\sigma W_3^{\nu(\bar{\nu})} + \frac{1}{M^2}q_\alpha q_\beta W_4^{\nu(\bar{\nu})} + \frac{1}{M^2}(p_\alpha q_\beta + q_\alpha p_\beta)W_5^{\nu(\bar{\nu})} + \frac{i}{M^2}(p_\alpha q_\beta - q_\alpha p_\beta)W_6^{\nu(\bar{\nu})}\,, \tag{7}$$

where M is the nucleon mass and W_i^N are the structure functions, which depend on the scalars q^2 and $p.q$. The terms depending on W_4, W_5 and W_6 in Eq. (7) do not contribute to the cross section in Eq. (4) in the limit of lepton mass $m_l \to 0$.

In terms of the Bjorken variables x and y defined as

$$x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E_\nu}, \quad Q^2 = -q^2, \quad \nu = \frac{p.q}{M} \tag{8}$$

we can write the expression for the differential scattering cross section (in the limit of lepton mass $m_l \to 0$) as

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{{G_F}^2 M E_\nu}{\pi}\left\{xy^2 F_1^{\nu(\bar{\nu})}(x,Q^2) + \left(1 - y - \frac{xyM}{2E_\nu}\right)F_2^{\nu(\bar{\nu})}(x,Q^2) \pm xy(1-y/2)F_3^{\nu(\bar{\nu})}(x,Q^2)\right\}\,, \tag{9}$$

where the + (−) sign stands for the neutrino (antineutrino) cross section, and the $F_i^{\nu,\bar{\nu}}(x,Q^2)$ are dimensionless structure functions defined as

$$\begin{aligned} F_1^{\nu(\bar{\nu})}(x,Q^2) &= MW_1^{\nu(\bar{\nu})}(\nu,Q^2) \\ F_2^{\nu(\bar{\nu})}(x,Q^2) &= \nu W_2^{\nu(\bar{\nu})}(\nu,Q^2) \\ F_3^{\nu(\bar{\nu})}(x,Q^2) &= \nu W_3^{\nu(\bar{\nu})}(\nu,Q^2). \end{aligned} \tag{10}$$

In the Bjorken limit of scaling valid in the asymptotic region i.e. $Q^2 \to \infty$, $\nu \to \infty$, x finite, the structure functions $F_i^{\nu(\bar{\nu})}(x,Q^2)$ are independent of Q^2 and depend only on the single dimensionless variable x, and satisfy the Callan-Gross relation [15] given as $2xF_1(x) = F_2(x)$. Using this relation, the cross section in Eq.(9) is described in terms of two independent structure functions $F_2(x)$ and $F_3(x)$. In the quark parton model of deep inelastic scattering, in the Bjorken scaling limit, these structure functions are determined in terms of parton distribution functions for quarks and antiquarks.

Specifically, the structure function $F_3(x)$ is given as:

$$F_3^{\nu p} = 2[d(x) + s(x) - \bar{u}(x) - \bar{c}(x)]\,,$$

$$F_3^{\nu n} = 2[u(x) + s(x) - \bar{d}(x) - \bar{c}(x)]\,,$$

$$F_3^{\bar{\nu} p} = 2[u(x) + c(x) - \bar{d}(x) - \bar{s}(x)]\,,$$

$$F_3^{\bar{\nu} n} = 2[d(x) + c(x) - \bar{u}(x) - \bar{s}(x)]\,,$$

The average structure function $F_3^N(x)$ on isoscalar nucleon target defined as

$$F_3^N(x) = \frac{1}{2}\left(F_3^{\nu N} + F_3^{\bar{\nu} N}\right)$$

is given by

$$F_3^N(x) = [u_v(x) + d_v(x) + s(x) - \bar{s}(x) + c(x) - \bar{c}(x)],$$

where $u_v(x) = u(x) - \bar{u}(x)$ and $d_v(x) = d(x) - \bar{d}(x)$ are the valence quark parton distributions.

Thus, for an isoscalar target and a symmetric sea, $F_3^N(x)$ structure function is given in terms of valence quarks u_v and d_v which satisfies the Gross-Llewellyn Smith sum rule [19]:

$$\int_0^1 F_3^N(x)dx = 3. \tag{11}$$

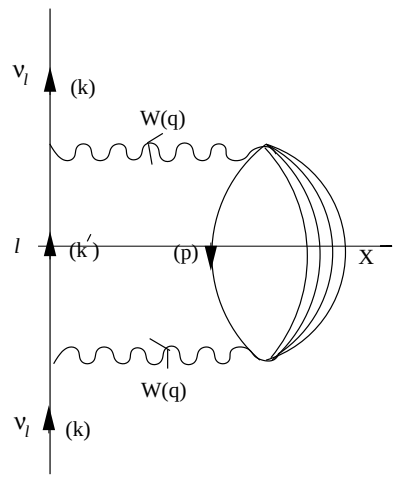

FIG. 1: Self-energy diagram of the neutrino in the nuclear medium associated with the process of deep inelastic neutrino-nucleon scattering. The imaginary part is calculated by cutting along the horizontal line and applying the Cutkosky rule for putting the particles on the mass shell.

In the non-asymptotic region, scaling violations occur and the structure function $F_i(x)$ acquire Q^2 dependence which are calculated with the DGLAP equations of Q^2 evolution obtained using perturbative QCD. As a consequence, the Callan Gross relation and the Gross-Llewellyn Smith sum rule are modified as follows:

$$2xF_1(x,Q^2) = F_2(x,Q^2)\frac{1+\frac{4M^2x^2}{Q^2}}{1+R(x,Q^2)}, \tag{12}$$

where R=$\frac{\sigma_L}{\sigma_T}$ is the ratio of the cross section of longitudinally to transversely polarized W bosons [23] and

$$S(Q^2) \;=\; \int_0^1 F_3(x,Q^2)dx = 3\left(1-\frac{\alpha_s}{\pi}-a(n_f)\left(\frac{\alpha_s}{\pi}\right)^2-b(n_f)\left(\frac{\alpha_s}{\pi}\right)^3\right)-\frac{\Delta_{HT}}{Q^2}, \tag{13}$$

where $a(n_f)$ and $b(n_f)$ are functions of the number of quark flavors accessible at a given Q^2 and Δ_{HT} is the higher twist correction [24].

III. NUCLEAR EFFECTS IN NEUTRINO SCATTERING

When the reaction given by Eq.(1) takes place on a nucleon in the nucleus, nuclear effects have to be considered. There are two main nuclear effects. Firstly, a kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus, leading to a Lorentz contraction of the incident flux used in deriving Eq.(4). Secondly, the more important dynamic effects which arise due to Fermi motion, Pauli blocking and strong interaction of the initial nucleon in the nuclear medium.

In a nuclear medium the expression for the cross section given in Eq.(4) is modified as:

$$\frac{d^2\sigma^A_{\nu,\bar{\nu}}}{d\Omega' dE'} = \frac{G_F^2}{(2\pi)^2}\,\frac{|\mathbf{k}'|}{|\mathbf{k}|}\,\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar{\nu}}\,W^A_{\alpha\beta}\,, \tag{14}$$

where $W^A_{\alpha\beta}$ is the nuclear hadronic tensor defined in terms of nuclear hadronic structure functions $W_{iA}(x,Q^2)$ through Eq.(7). Accordingly, the dimensionless structure functions $F_i^A(x,Q^2)$ are defined in terms of $W_i^A(x,Q^2)$ through Eq.(10). In our formalism the neutrino nuclear cross sections are obtained in terms of neutrino self energy $\Sigma(k)$ in the nuclear medium which also defines the dimensionless nuclear structure functions $F_i^A(x,Q^2)$. A comparison with Eq.(36) gives the nuclear hadronic tensor $W^A_{\alpha\beta}$. With proper choice of tensor components α,β, the dimensionless nuclear structure functions $F_i^A(x,Q^2)$ are obtained[21].

The neutrino self-energy in nuclear matter corresponding to Fig.1 is given by,

$$\Sigma(k) = \quad (-i)\frac{G_F}{\sqrt{2}}\frac{4}{m_\nu}\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2-m_l^2+i\epsilon}\left(\frac{m_W}{q^2-m_W^2}\right)^2\;L_{\alpha\beta}\;\;\Pi^{\alpha\beta}(q)\,, \tag{15}$$

where $L_{\alpha\beta}$ is given by Eq.(5) and $\Pi^{\alpha\beta}(q)$ is the W self-energy in the nuclear medium and is written with the help of Fig.2 as:

$$-i\Pi^{\alpha\beta}(q) = (-)\int \frac{d^4p}{(2\pi)^4} iG(p) \sum_X \sum_{s_p,s_i} \prod_{i=1}^{n} \int \frac{d^4p'_i}{(2\pi)^4} \prod_l iG_l(p'_l) \prod_j iD_j(p'_j) \left(\frac{-G_F m_W^2}{\sqrt{2}}\right)$$
$$\times \langle X|J^\alpha|N\rangle \langle X|J^\beta|N\rangle^* (2\pi)^4 \delta^4(q+p-\Sigma_{i=1}^n p'_i)\,. \tag{16}$$

In the above expression $G_l(p'_l)$ and $D_j(p'_j)$ are respectively the nucleon and meson relativistic propagators in the final state which are taken as the standard free relativistic propagators [25]. $G(p)$ is the nucleon propagator with mass M and energy $E(\mathbf{p})$ in the initial state, which is calculated for a relativistic nucleon in the interacting Fermi sea.

For this we start with the relativistic Dirac propagator G(p) for a free nucleon, which is written in terms of the contribution from the positive and negative energy components of the nucleon described by the Dirac spinors $u(\mathbf{p})$ and $v(\mathbf{p})$ using their appropriate normalisations [25] as

$$G^0(p) = \frac{\not p + M}{p^2 - M^2 + i\epsilon} = \frac{M}{E(p)} \left\{ \frac{\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})}{p^0 - E(\mathbf{p}) + i\epsilon} + \frac{\sum_r v_r(-\mathbf{p})\bar{v}_r(-\mathbf{p})}{p^0 + E(\mathbf{p}) - i\epsilon} \right\} \tag{17}$$

The nucleon propagator G(p) is then calculated by making a perturbative expansion of $G(p)$ in terms of $G^0(p)$ given in Eq.(17) by retaining the positive energy contributions only(the negative energy components are suppressed). This perturbative expansion is summed in ladder approximation to give [21]:

$$\begin{aligned} G(p) &= \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p}) \frac{1}{p^0 - E(\mathbf{p})} \\ &+ \frac{M}{E(\mathbf{p})} \sum_r \frac{u_r(\mathbf{p})\bar{u}_r(\mathbf{p})}{p^0 - E(\mathbf{p})} \sum (p^0, \mathbf{p}) \frac{M}{E(\mathbf{p})} \sum_s \frac{u_s(\mathbf{p})\bar{u}_s(\mathbf{p})}{p^0 - E(p)} + ... \\ &= \frac{M}{E(\mathbf{p})} \sum_r \frac{u_r(\mathbf{p})\bar{u}_r(\mathbf{p})}{p^0 - E(\mathbf{p}) - \bar{u}_r(\mathbf{p}) \sum^N (p^0, \mathbf{p}) u_r(\mathbf{p}) \frac{M}{\mathbf{E(p)}}} \end{aligned} \tag{18}$$

where $\Sigma^N(p^0, p)$ is the nucleon self energy in nuclear matter taken from Ref. [26]. The relativistic nucleon propagator G(p) in a nuclear medium is then expressed as [21]:

$$G(p) = \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p}) \left[\int_{-\infty}^{\mu} d\,\omega \frac{S_h(\omega,)}{p^0 - \omega - i\eta} + \int_{\mu}^{\infty} d\,\omega \frac{S_p(\omega, p)}{p^0 - \omega + i\eta} \right], \tag{19}$$

where $S_h(\omega, p)$ and $S_p(\omega, p)$ being the hole and particle spectral functions respectively, which are given by [21, 26]:

$$S_h(\omega, \mathbf{p}) = \frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p)}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma^N(p^0, p))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p))^2} \tag{20}$$

for $p^0 \leq \mu$

$$S_p(\omega, \mathbf{p}) = -\frac{1}{\pi} \frac{\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p)}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma^N(p^0, p))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma^N(p^0, p))^2} \tag{21}$$

for $p^0 > \mu$.

The normalisation of this spectral function is obtained by imposing the baryon number conservation following the method of Frankfurt and Strikman [27]. For this purpose we evaluate the electromagnetic form factor at q=0, assuming baryons have unit charge for the purpose of normalisation, corresponding to Fig.2a, i.e.

$$\langle N|B^\mu|N\rangle \equiv \bar{u}(p)\gamma^\mu u(p) \;=\; B\frac{p^\mu}{M};\; B = 1,\; p^\mu \equiv (E(\mathbf{p}), \mathbf{p}) \tag{22}$$

When the nucleons are in the nuclear medium the many-body diagram shown by Fig.2b is evaluated [21] and gives

$$\langle A|B^\mu|A\rangle = -\int \frac{d^4p}{(2\pi)^4} ViTr[G(p^0, p)\gamma^\mu] e^{ip^0\eta}. \tag{23}$$

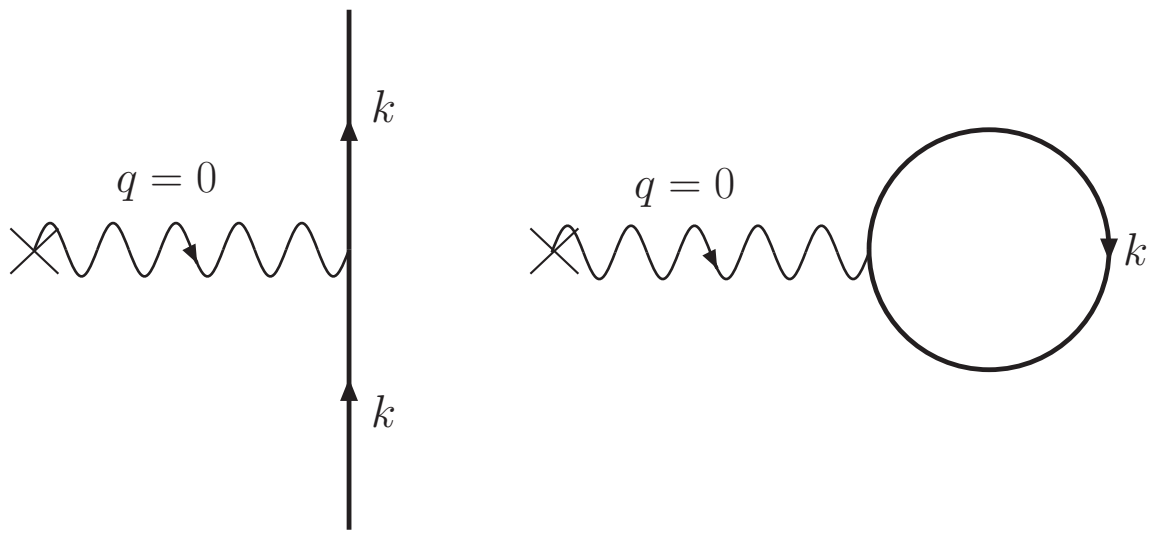

FIG. 2: Electromagnetic form factors for the case (a) free nucleon, (b) Fermi sea with B baryons.

where V is the volume of the normalisation box and $exp(ip^0\eta)$, with $\eta \to 0^+$, is the convergence factor for loops appearing at equal times.

Using the expression given by Eq.(19), it can be seen that the convergence factor limits the contribution of the hole spectral function and this gives

$$\begin{aligned}\langle A|B^\mu|A\rangle &= V\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}Tr\left[\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})\gamma^\mu\right]\int_{-\infty}^{\mu} S_h(\omega,p)d\omega \\ &= V\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}Tr\left[\frac{(\not{p}+M)_{onshell}}{2M}\gamma^\mu\right]\int_{-\infty}^{\mu} S_h(\omega,p)d\omega \\ &= 2V\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\frac{p^\mu_{onshell}}{M}\int_{-\infty}^{\mu} S_h(\omega,p)d\omega \equiv B\frac{P^\mu_A}{M_A}. \end{aligned} \tag{24}$$

It is to be noted that in the last step we have imposed that this matrix element gives the right current with B baryons, in analogy to the expression given by Eq. (22). P^μ_A is the four momentum of the nucleus. The operator $(\not{p}+M)$ comes from $\sum u_r(\mathbf{p})\bar{u}_r(\mathbf{p})$ which depends only on $\mathbf{p}$, and that corresponds to a free particle with $(p^\mu = E(\mathbf{p}), \mathbf{p})$, therefore, the operator $(\not{p}+M)$ is written on shell. The last step in Eq. (24)) is obtained after performing the trace. Evaluating Eq. (24) in the rest frame of the nucleus we obtain the normalisation as

$$2V\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,p)d\omega = B \ = 1. \tag{25}$$

In the local density approximation, we do not have a box of constant density, and the reaction takes place at a point $\mathbf{r}$, lying inside a volume element d^3r with local density $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ corresponding to the proton and neutron. Therefore, the upper limit in the integration over nucleon momentum in Eq. (25) is the local Fermi momentum $k_{F_{p,n}}(\mathbf{r})$ of the nucleon given by:

$$k_{F_p}(\mathbf{r}) = \left[3\pi^2\rho_p(\mathbf{r})\right]^{1/3}, k_{F_n}(\mathbf{r}) = \left[3\pi^2\rho_n(\mathbf{r})\right]^{1/3}. \tag{26}$$

This makes the spectral function $S_h(\omega,p)$ density dependent i.e. $S_h(\omega,p,k_F(\mathbf{r}))$ and the normalisation condition given in Eq. (25) is modified to

$$2\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,p,k_{F_{p,n}}(\mathbf{r}))d\omega = \rho_{p,n}(\mathbf{r}) \tag{27}$$

For a symmetric nuclear matter of density $\rho(\mathbf{r})$, there is a unique Fermi momentum given by $k_F(\mathbf{r}) = \left[\frac{3}{2}\pi^2\rho(\mathbf{r})\right]^{1/3}$ for which we obtain

$$4\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,p,k_F(\mathbf{r}))d\omega = \rho(\mathbf{r}), \tag{28}$$

where $\rho(r)$ is the baryon density for the nucleus.

This leads to the normalisation condition [28] given by

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega,p,\rho(r))\, d\omega = A \tag{29}$$

In the antineutrino case the expressions obtained are very similar. $L_5^{\alpha\beta}$ appears, as in Eq. (5), with a minus sign in front and in the W self-energy, Eq. (16), we have $\langle X|J_\alpha^\dagger|N\rangle$, instead of $\langle X|J_\alpha|N\rangle$.

The probability per unit time for the neutrino to collide with nucleons when traveling through nuclear matter is:

$$\Gamma(k) = -\frac{2m_\nu}{E_\nu(k)}\,\mathrm{Im}\,\Sigma(k)\,, \tag{30}$$

and the cross section for neutrino scattering from an element of volume d^3r and surface dS in the nucleus is given by

$$d\sigma \;=\; \Gamma dt dS = \Gamma\frac{dt}{dl}\,dl dS = \frac{\Gamma}{v}\,d^3r = \Gamma\,\frac{E_\nu(k)}{|k|}\,d^3r = -\frac{2m_\nu}{|k|}\,\mathrm{Im}\,\Sigma\,d^3r\,. \tag{31}$$

Using Eq.(31) in Eq.(15), we get the expression for the total scattering cross section in the local density approximation as

$$\sigma = \frac{4\sqrt{2}G_F}{|\mathbf{k}|}\mathrm{Im}\int d^3r\int\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2-m_\mu^2+i\epsilon}\left(\frac{m_W}{q^2-m_W^2}\right)^2 L_{\alpha\beta}\Pi^{\alpha\beta}(q)\,. \tag{32}$$

The imaginary part of the neutrino self energy in Eq.(32) is evaluated by means of the Cutkosky rules [25] by cutting the Feynman diagram shown in Fig. 3 along the dotted line which puts the particles corresponding to the cut propagators on the mass shell by replacing the fermion and meson propagators by their imaginary parts as

$$\begin{aligned}
\Sigma(k) &\rightarrow 2iIm\Sigma(k)\\
D(p_j{}') &\rightarrow 2i\theta(p_{0j})\;ImD(p_j{}')\\
G(p_l{}') &\rightarrow 2i\theta(p_{0l}{}')\;ImG(p_l{}')\\
\frac{1}{k'^2-m_l^2+i\epsilon} &\rightarrow 2\pi\delta(k'^2-m_l^2).
\end{aligned} \tag{33}$$

After performing the $k_0{}'$, $p_0{}'$ and p_0 integrations for all momenta in Eq.(32) using Eqs.(19) and (33), we get the differential scattering cross section which is written in the local density approximation as:

$$\frac{d^2\sigma^A_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{{G_F}^2}{(2\pi)^2}\,\frac{|\mathbf{k}'|}{|\mathbf{k}|}\,\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu}\,W^A_{\alpha\beta}\,, \tag{34}$$

where

$$W^A_{\alpha\beta} = 4\int d^3r\int\frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} dp^0\frac{M}{E(\mathbf{p})}S_h(p^0,p,\rho(r))W^N_{\alpha\beta}(p,q),\;. \tag{35}$$

with $W^N_{\alpha\beta}(p,q)$ is given by Eq.(6).

Note that the factor $\frac{M}{E(\mathbf{p})}$ in Eq. (35) comes naturally in our formalism, when we perform the various momentum integrations in Eq. (32) to calculate the imaginary part using Eqs. (19) and (33). This can be physically understood as a kinematic factor which appears in the cross section defined in Eq. (2) for a nucleon moving with momentum $p=(E,\mathbf{p})$ in the rest frame of the nucleus leading to the equation:

$$\frac{d^2\sigma^A_{\nu,\bar\nu}}{d\Omega' dE'} = \frac{G_F^2}{(2\pi)^2}\,\frac{|\mathbf{k}'|}{|\mathbf{k}|}\,\frac{M}{E(\mathbf{p})}\,\left(\frac{m_W^2}{q^2-m_W^2}\right)^2 L^{\alpha\beta}_{\nu,\bar\nu}\,W^N_{\alpha\beta}\,. \tag{36}$$

When this cross section for a nucleon target moving with momentum p^μ in the rest frame of the nucleus of density $\rho(r)$ and weighted with the spectral function $S_h(p^0,p,\rho(r))$, is summed over all the nucleons in the nucleus, it leads to Eq. (34). Similar equations are also obtained in Refs. [3, 9, 29, 30].

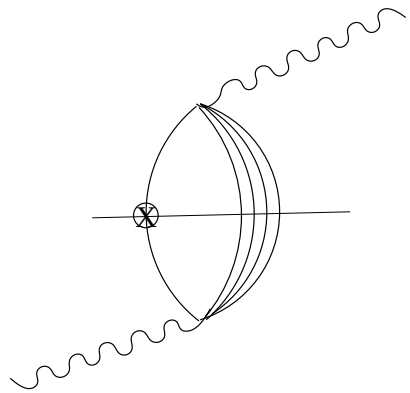

FIG. 3: Self-energy diagram of the W-boson in the nuclear medium. The imaginary part is calculated by cutting along the horizontal line and applying the Cutkosky rules when putting the particle on mass shell.

Now to evaluate F_3^A, we calculate the components xy on both sides of Eq. (35). We get by taking q along the z axis:

$$W_{xy}^A = -\frac{i}{2M_A} q_z W_3^A \,, \tag{37}$$

and for the right hand side we have for the nucleon moving with momentum p:

$$W_{xy}^N = \frac{p_x p_y}{M^2} W_2^N(p,q) + \frac{i}{2M^2} W_3^N [p_z q_0 - p_0 q_z] \,. \tag{38}$$

Since we have

$$q_0 W_3^A = F_3^A(x) \,, \tag{39}$$

$$\frac{p.q}{M} W_3^N(p,q) = F_3^N(x_N) \,, \tag{40}$$

with x as defined in Eq. (8) and x_N is the Bjorken variable expressed in terms of the nucleon variables , $(p^0, \mathbf{p})$, in the nucleus

$$x_N = \frac{Q^2}{2p.q} = \frac{Q^2}{2(p_0 q_0 - p_z q_z)}, \tag{41}$$

we obtain the expression for $F_3^A(x)$ in the Bjorken limit

$$\begin{aligned} F_3^A(x,Q^2) \;&=\; 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp^0 S_h(p^0,p,\rho(r)) \\ &\times \frac{x_N}{x} \left[\frac{p_0 q_z - p_z q_0}{M q_z} \right] F_3^N(x_N, Q^2) \,, \end{aligned} \tag{42}$$

where the contribution of W_2 appearing in Eq. (38) vanishes after momentum integration. Defining γ as

$$\gamma = \frac{q_z}{q^0} = \left(1 + \frac{4M^2x^2}{Q^2} \right)^{1/2} \,, \tag{43}$$

we get

$$F_3^A(x,Q^2) \;=\; 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 S_h(p^0,p,\rho(r)) F(p,Q^2) F_3^N(x_N,Q^2), \tag{44}$$

where $F(p,Q^2) = \frac{M}{E(\mathbf{p})} \left(\frac{p_0\gamma - p_z}{(p_0 - p_z\gamma)\gamma} \right)$.

This is our main equation which describes the modification on $F_3^A(x,Q^2)$ due to nuclear medium effects and was earlier obtained in Ref.[6].

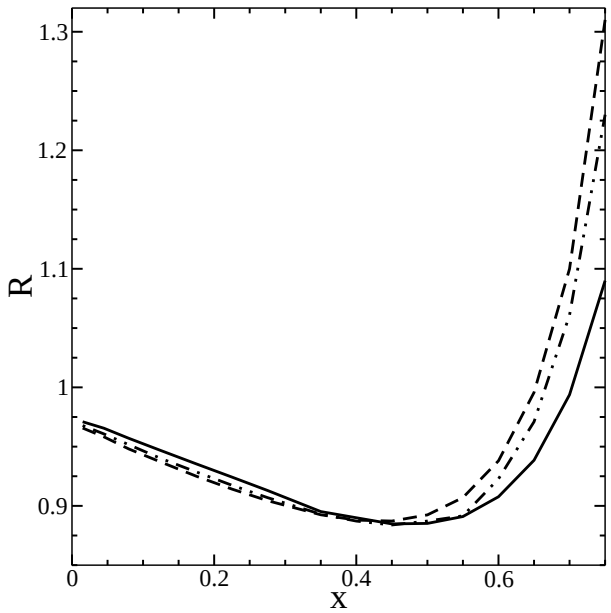

FIG. 4: Results for the ratio $R = \frac{F_3^A(x)}{AF_3^N(x)}$ at different Q^2 using MRST NNLO [33] parton distribution function. Solid line at Q^2=$10GeV^2$, dashed-dotted line at Q^2=$100GeV^2$ and dashed line at Q^2=$1000GeV^2$.

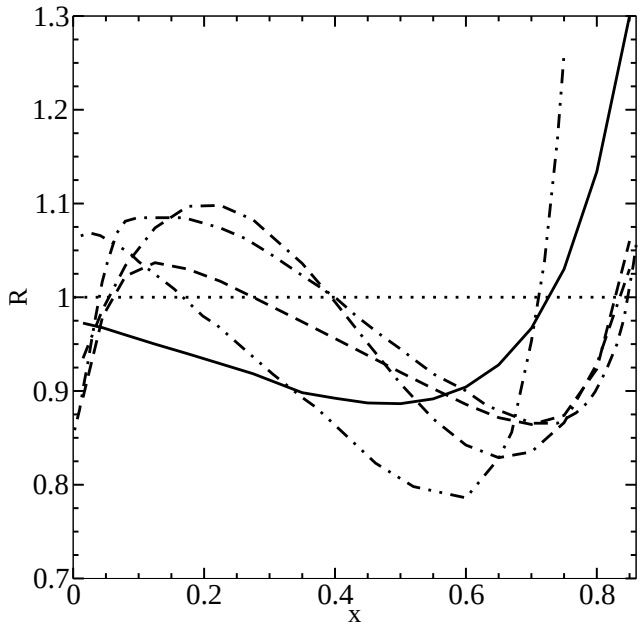

FIG. 5: Results for the ratio $R = \frac{F_3^A(x)}{AF_3^N(x)}$ at $Q^2 = 5$ GeV^2 by different authors. Solid line: this work using MRST2004 NNLO parton distribution function; double dashed-dotted line: Hirai et al. [10]; short dashed line: NuTeV collaboration [1], dashed-double dotted line the results of Kulagin [5] and the results of Kulagin and Petti [9] shown by dashed-dotted line.

IV. RESULTS AND DISCUSSION

We obtain the numerical results for $F_3^A(x, Q^2)$ using Eq.(44) and S_h given by Eq. (20). The density for ^{56}Fe nucleus is taken to be a two Fermi parameter distribution i.e.

$$\rho(r) = \frac{\rho_0}{1 + exp(\frac{r-c1}{c2})}, \tag{45}$$

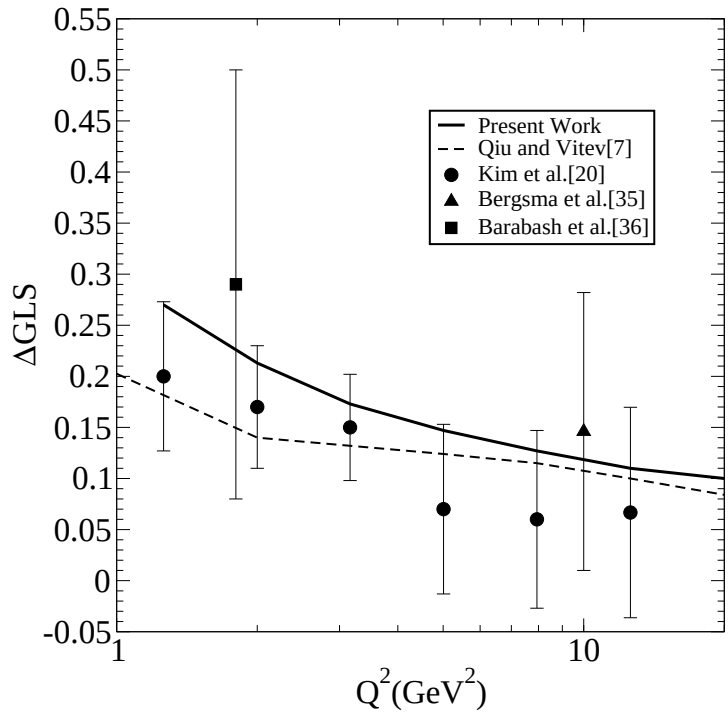

FIG. 6: ΔGLS=$\frac{1}{3}(3-\int_0^1 F_3^A(x,Q^2)dx)$ vs Q^2.

where the parameters c1=4.106fm and c2=0.519fm and are taken from Ref [31]. While using the spectral function $S_h(\omega,\mathbf{p},\rho(r))$ in Eq.(44), we ensure that it gives the correct binding energy for the iron nucleus by varying the free piece of the real part of the nucleon self energy Σ^N in Eq.(20).

This is done by calculating the average kinetic and total nucleon energy given by:

$$< T >= \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}(E(\mathbf{p})-M)\int_{-\infty}^{\mu} S_h(p^0,p,\rho(r))\, dp^0\,, \tag{46}$$

$$< E >= \frac{4}{A}\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(p^0,p,\rho(r))\, p^0 dp^0\,, \tag{47}$$

and the binding energy per nucleon given by [21]:

$$|E_A| = -\frac{1}{2}(< E-M > + \frac{A-1}{A-2} < T >) \tag{48}$$

After the free parameter in Σ^N is fixed, the expected values of the kinetic energy $< T >$ and the total nucleon energy $< E >$ are obtained as 30MeV and 48MeV for the case of ^{56}Fe which are in good agreement with other models and experiment (See e.g. Table II of Ref.[32]). A similar agreement is also obtained with other nuclei. The structure function $F_3^A(x,Q^2)$ is numerically evaluated. For $F_3^N(x_N,Q^2)$ we use the MRST2004 NNLO parton distribution functions [33]. The Q^2 evolution of $F_3^N(x,Q^2)$ is assumed to be given by the Q^2 evolution of GLS integral $S(Q^2)$ given in Eq. (12). The strong coupling constant $\alpha_S(Q^2)$ is calculated using the variable flavour number evolution equation with $\Lambda = 251MeV$ for $n_f = 4$ and $\Lambda = 178MeV$ for $n_f = 5$ [24] which gives $\alpha_S(M_Z)$=.1153. The constants $a(n_f)$ and $b(n_f)$ in Eq. (12) are taken from Larin and Vermarseren [24]. The target mass corrections are incorporated using the prescription of Petti and Kulagin [3] following the work of Georgi and Politzer [34].

In Fig.4, we show the results for the ratio R(x,Q^2)=$\frac{F_3^A(x,Q^2)}{AF_3^N(x,Q^2)}$ as a function of x for various values of Q^2. For Q^2=$10GeV^2$, we get a suppression for $x \leq x_{min}$=0.7, beyond which we get an enhancement. This value of x_{min} decreases with increasing Q^2. For $x > x_{min}$, the ratio increases very fast and becomes larger than unity as $x \to 1$. This is mainly due to the Fermi motion of the nucleons. In the region of 0.3< x <1, this behavior is very similar to that seen in the EMC effect with charged lepton and (anti)neutrino scattering from the nuclear targets in the analyses of the structure function $F_2^{A,l}(x,Q^2)$ and $F_2^{A,\nu(\bar{\nu})}(x,Q^2)$.

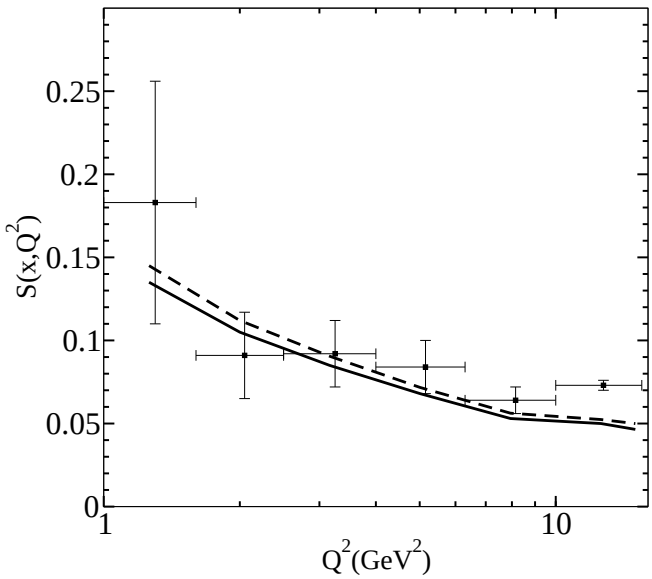

FIG. 7: The contribution to the GLS integral from the region x=0.5 to x=1 for different Q^2 using MRST NNLO[33] parton distribution function(PDF) and its comparison from the results of Kim et al.[20]. The dashed line is the result with the free case, solid line is the result with nuclear medium effects obtained in our model with spectral function including QCD and TMC corrections, and the solid square with error bars are the results of Kim et al.[20]

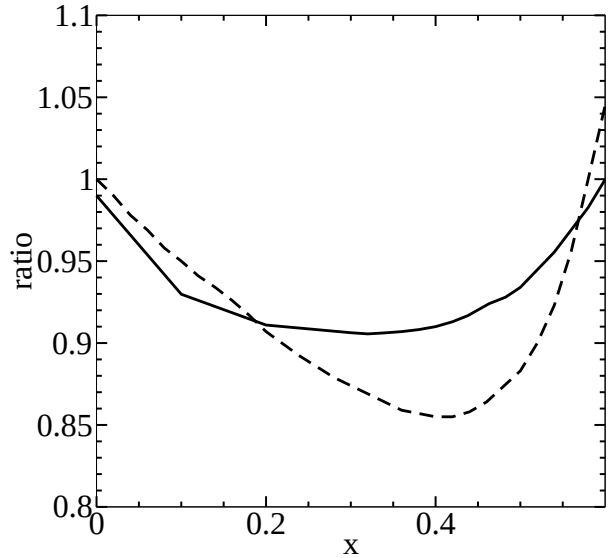

FIG. 8: The x dependence of the nucleus/nucleon ratio of the GLS integral from the nuclear effects to the free case i.e. the ratio $\frac{\int_{x_{min}}^{1} F_3^A(x)dx}{\int_{x_{min}}^{1} F_3^N(x)dx}$ at $Q^2 = 5GeV^2$ using MRST NNLO[33] parton distribution function(PDF). The dashed line is the result of Kulagin [5].

In the region of 0.1< x <0.3, we do not see any enhancement due to the anti-shadowing effect in $F_3^A(x,Q^2)$ as found in $F_2^{A,l}(x,Q^2)$ [21]. In the present calculation, this is due to the neglect of the mesonic contributions, responsible for the enhancement in $F_2^{A,l}(x,Q^2)$. However, for $F_3^A(x,Q^2)$ these mesonic contributions are expected to be small because of the vanishing of the dominant term due to pions. Even in the case of $F_2^{A,\nu(\bar{\nu})}(x,Q^2)$, the recent[13] estimates of nuclear medium effects in the anti-shadowing region, do not see any enhancement in this region, in disagreement with the other results available from earlier analyses [10, 11, 12]. This is not unexpected as these structure functions, at low x, probe different contributions of the parton distribution functions in the nuclear medium. This suggests that the nuclear medium modification effects could be different in $F_2^{A,l}(x,Q^2)$, $F_2^{A,\nu(\bar{\nu})}(x,Q^2)$ and $F_3^{A,\nu(\bar{\nu})}(x,Q^2)$, and calls

for a comprehensive study of nuclear medium modification effects on these structure functions specially in the region of small x.

In Fig.5, we compare our results for R(x,Q^2) with the results of Tzanov et al. [1], Kulagin and Petti [3], Kulagin [5] and Hirai et al. [10]. While the work of Kulagin [5] and Kulagin and Petti [3, 9] use a nuclear model to calculate the nuclear effects which shows a Q^2 dependence, the work of Tzanov et al. [1] and Hirai et al. [10] are phenomenological analyses, which assume the nuclear effects to be independent of Q^2. In this figure, the results for R(x,Q^2) for $Q^2 = 5GeV^2$, are presented. We find a suppression in $F_3^A(x, Q^2)$ for x <0.7 and an enhancement thereafter, which are respectively smaller than the results of Ref. [5], but are larger than the recent results of Kulagin and Petti [9]. It should be noted that these latter results [9] give suppression in the region of 0.4< x <0.8 and enhancement for x >0.8, which are smaller than the present results and the results obtained earlier in Ref. [5]. When compared with the results of Tzanov et al. [1] and Hirai et al. [10], we find a smaller suppression in the region 0.5< x <0.7. In the region 0.7< x <0.8, we find an enhancement while they obtain a suppression. The results of the phenomenological analyses shown in Fig. 5 [1, 10] use the experimental data on $F_2^{A,l}(x, Q^2)$ in charged lepton scattering to estimate the nuclear effects. It can, therefore, be inferred that we find nuclear effects in the case of $F_3^A(x, Q^2)$ different than obtained in the case of $F_2^{A,l}(x, Q^2)$. This is true for all Q^2 in our model.

The GLS sum rule (Eq.11) provides a benchmark to test various models used for the calculation of $F_3^A(x, Q^2)$. We find that in the $Q^2 \to \infty$ limit, the correction to the GLS sum rule in our model comes from the off shell modifications to the nucleon spectral function. It is easy to see this from Eq.(44). The nuclear GLS sum rule in the limit $Q^2 \to \infty$ (i.e. $\gamma \to 1$) is given by:

$$S = \int_0^1 F_3^A(x)dx = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 S_h(p^0, p, \rho(r)) \left(\frac{p_0 - p_z}{E(\mathbf{p})} \right) F_3^N(x_N)dx_N \tag{49}$$

In the limit of noninteracting nucleons this trivially reproduces the GLS sum rule for free nucleons.

The Q^2 dependent nuclear effects in the GLS integral enter through the factor γ and the Q^2 dependence of $F_3^N(x_N, Q^2)$ in Eq.(43) when it is integrated over x. The Q^2 dependent nuclear corrections to the GLS sum rule are thus linked to the Q^2 dependent perturbative and non-perturbative QCD effects appearing in $F_3^N(x_N, Q^2)$. In Fig.6, we show the Q^2 dependence of the nuclear effects of the GLS integral, where we plot ΔGLS=$\frac{1}{3}(3 - \int_0^1 F_3^A(x, Q^2)dx)$ as a function of Q^2. The experimental results from CCFR collaborations[20], CHARM collaborations[35] and IHEP-JINR collaborations[36] are also shown. The Q^2 behavior of ΔGLS has been found to be in agreement with the present available experimental results. In this figure, we have also shown the theoretical results obtained by Qiu and Vitev [7]. Our results are in agreement with the results of Qiu and Vitev [7] for $Q^2 > 5GeV^2$ where theoretically the suppression is found to be larger than the experimental results. For $Q^2 < 5GeV^2$, we find a larger suppression compared to the central value of the experimental result and both theoretical values are within the experimental errors. The nuclear corrections to the GLS sum rule found by Kulagin and Petti [9] are quite small due to the cancellation of nuclear shadowing, not included in our model, and off shell effects.

We show in Fig.7, the value of S(x,Q^2)=$\int_x^1 dx F_3^A(x, Q^2)$ for x=0.5, with and without the nuclear effects and compare them with the experimental results of Kim et al.[20]. The nuclear effects tend to reduce S(x,Q^2) but the reductions are smaller than the experimental uncertainties. In Fig.8, we show the results for S(x,Q^2)=$\int_x^1 dx F_3^A(x, Q^2)$ vs x at Q^2=$5GeV^2$, where we have also shown the result obtained by Kulagin [5]. We find that nuclear medium effects leading to a suppression in 0.2< x <0.55 region and an enhancement in x >0.55 region are smaller than obtained by Kulagin [5].

To summarize our results, we have studied nuclear effects in the structure function $F_3^A(x, Q^2)$ in the iron nucleus using a many body theory to describe the spectral function of the nucleon in the nuclear medium for all Q^2. The nuclear effects are found to decrease the value of the structure function for $x \leq x_{min}$=0.7 and increase at higher $x > x_{min}$. The parameter x_{min} is found to be Q^2 dependent which decreases with Q^2. The results are compared with other theoretical analysis of Kulagin [5] and phenomenological analyses of Tzanov et al. [1] and Hirai et al. [10]. The effect of the nuclear medium modifications on the Gross-Llewellyn Smith [19] sum rule and its Q^2-dependence has also been studied. In general nuclear medium effects decrease the value of GLS integral for all Q^2.

V. ACKNOWLEDGMENT

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