The Heavy Quark Spin Symmetry Partners of the $X(3872)$

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We explore the consequences of heavy quark spin symmetry for the charmed meson-antimeson system in a contact-range (or pionless) effective field theory. As a trivial consequence, we theorize the existence of a heavy quark spin symmetry partner of the $X(3872)$, with $J^{PC} = 2^{++}$, which we call $X(4012)$ in reference to its predicted mass. If we additionally assume that the $X(3915)$ is a 0^{++} heavy spin symmetry partner of the $X(3872)$, we end up predicting a total of six $D^{(*)}\bar{D}^{(*)}$ molecular states. We also discuss the error induced by higher order effects such as finite heavy quark mass corrections, pion exchanges and coupled channels, allowing us to estimate the expected theoretical uncertainties in the position of these new states.

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I. INTRODUCTION

The discovery of the $X(3872)$ resonance [1] might have confirmed a well-known theoretical expectation of hadronic physics, heavy meson molecules [2–6]. The $X(3872)$, with a mass of $m_X = 3871.57 \pm 0.25\,\text{MeV}$ [7], is extremely close to the $D^{0*}\bar{D}^0$ threshold ($m_{D^0} + m_{D^{*0}} = 3871.73 \pm 0.21\,\text{MeV}$), a feature suggesting that its nature is mostly molecular, where the role of other more compact components (e.g. tetraquark, $c\bar{c}$) will be less important in comparison. The J^{PC} quantum numbers of the $X(3872)$ are either 1^{++} or 2^{-+} [8–10], with a slight experimental preference for the second set of quantum numbers [10]. However, only the 1^{++} assignment is compatible with the molecular interpretation. If the 2^{-+} possibility turns out to be the correct one, the $X(3872)$ would be much more exotic than expected. In this regard, the analysis of Hanhart et al. [11] suggests that the experimental data, though too scarce to draw definitive conclusions, might indeed be more compatible with 1^{++} after all (see, however, Ref. [12] for a different opinion). Meanwhile, in the bottom sector, the recent discovery of the $Z_b(10610)$ and $Z_b(10650)$ isovector states by Belle [13, 14] also provides two new strong candidates for molecular states, as the Z_b's lie close to the $B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds respectively.

Actually, not all the heavy meson molecules are necessarily as shallow as the $X(3872)$ or the two Z_b resonances. Several theoretical works have suggested the molecular interpretation of other XYZ states: the $X(3915)$ [15] and the $Y(4140)$ [16] have been identified as a $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ bound state respectively by different theoretical approaches [17–19]. The $Y(4260)$ [20] has even been proposed to have the three body structure $J/\Psi K\bar{K}$ [21]. Moreover other states have been predicted, especially in the isoscalar bottom sector [22–24], but have not yet been experimentally confirmed or discarded.

Heavy meson molecules are a natural thing to expect on the basis of the similarity between the meson-meson interaction and the nuclear force that binds the deuteron [2, 4]. From this analogy, we expect that the effective field theory (EFT) formulation of nuclear physics [25–29] will also represent a constructive approach to the description of heavy meson systems at low-energies. As in the nucleon-nucleon system, the low energy interaction between a pair of heavy mesons is mediated by pion exchanges, which in turn are constrained by chiral symmetry. In contrast, the nature of the short range interaction remains unknown, but we can parametrize it in terms of contact-range operators between the nucleon / heavy meson fields. However, in the case of heavy meson molecules there is a particularly simplifying feature: pion exchanges are weaker than in the nuclear case, owing to the smaller light quark content of the heavy mesons in comparison to the nucleons. This means that pions are amenable to a perturbative treatment in a larger range of energies than in the nucleon-nucleon system. From this it is expected that the EFT description of heavy meson molecules will simplify at lowest order to a contact range theory, at least for a certain binding energy window [30]. A nice illustration of this idea is provided by X-EFT [31], which considers the low energy description of the $X(3872)$ state as a $D^0\bar{D}^{0*}$ molecule.

On a different level, the presence of a charm quark/antiquark in the heavy meson and antimeson conforming the $X(3872)$ dictates that heavy quark spin symmetry (HQSS) [32–35] is relevant for this system. In the context of the EFT description of heavy meson molecules, HQSS constrains the form of the contact range operators of the theory in a very specific way [36]. What this means is that there should be HQSS partners of the $X(3872)$, in analogy with the theorized HQSS partners of the $Z_b(10610)$ and $Z_b(10650)$ that have been already discussed in Refs. [37, 38]. The purpose of this paper is to investigate the HQSS structure of charm meson-antimeson (D, D* and $\bar{\text{D}}$, $\bar{\text{D}}^*$) molecules, with the intention of iden-

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tifying the possible HQSS partners of the $X(3872)$. For that we will assume that certain XYZ states are molecular, in particular the $X(3915)$ [15]. This identification will span a total of six states, some of which have also been predicted in other schemes.

The article is structured as follows: in Sect. II we present the EFT description of heavy meson molecules that we advocate, which is in turn inspired on the ideas of Ref. [30]. We explore the consequences of this EFT in Sect. III, where we deduce from HQSS the existence of the 2^{++} $D^*\bar{D}^*$ partner of the $X(3872)$, and of other four additional states if we identify the $X(3915)$ as a 0^{++} $D^*\bar{D}^*$ molecule. In Sect. IV we probe the robustness of the previous results by considering the effect of subleading order contributions, namely finite heavy quark mass corrections, pion exchanges and particle coupled channel effects. In Sect. V, we briefly discuss some HQSS constraints on the total decay widths of the states found in this work. In Sect. VI we discuss the significance of the results. Finally, the technical details of the manuscript can be consulted in Appendices A, B and C.

II. THE EFT DESCRIPTION AT LOWEST ORDER

In this section we review the EFT framework that we use for the description of heavy meson-antimeson molecules at lowest order. The presentation is simple and schematic, centered in the conceptual issues rather than the specific details, which can be consulted in Ref. [30]. The EFT we are advocating is in fact identical to the one presented in Ref. [38] for the isovector bottom sector. Following the findings of Ref. [30], we assume pion exchanges and particle coupled channel effects to be a subleading correction entering at next-to-leading order (NLO) and next-to-next-to-leading order ($\mathrm{N^2LO}$) respectively. Nevertheless, we will perform explicit calculations to test these assumptions.

A. Overview of the EFT Formalism

The EFT approach provides the possibility of constructing generic and systematic descriptions of arbitrary low energy processes. They are particularly useful when the system we are interested in cannot be easily explained in terms of a more fundamental description at higher energies. The EFT idea is simple: we identify the fields and symmetries that are relevant at low energies and construct all possible interactions compatible with them. Even though the number of interactions is infinite, they can be classified according to their importance at low energies by means of power counting, the ordering principle of EFT. If Q is the soft (low energy) scale of the system we are describing and Λ_0 the hard (high energy) scale, power counting allows us to express any physical quantity as a power series in terms of the small parameter $x_0 = Q/\Lambda_0$.

For illustrating this idea let us consider a physical quantity A that we want to compute in the EFT framework. This quantity receives in principle contributions from all the relevant diagrams involving the low energy fields and compatible with the low energy symmetries:

$$A(Q,\Lambda_0) = \sum_D A^{(D)}(Q,\Lambda_0)\,. \tag{1}$$

However, the different diagrams have different scaling properties that can be used for ordering the sum above. For example, we have the canonical dimension of A, which is defined as

$$A^{(D)}(\lambda\, Q, \lambda\, \Lambda_0) = \lambda^{d_A}\, A^{(D)}(Q,\Lambda_0)\,, \tag{2}$$

and is the same for all the EFT contributions to A. But the interesting scaling property is power counting, which refers to the behaviour under a transformation of the type $Q \to \lambda\, Q$

$$A^{(D)}(\lambda\, Q,\Lambda_0) = \lambda^{\nu_D}\, A^{(D)}(Q,\Lambda_0)\,, \tag{3}$$

where ν_D is the order of the contribution D, which is bounded from below (i.e. $\nu_D \geq \nu_0$). The sum of diagrams above can be reorganized as an expansion in terms of increasing scaling dimension:

$$A(Q,\Lambda_0) = \sum_{\nu\geq\nu_0} A^{(\nu)}(Q,\Lambda_0)\,, \tag{4}$$

where, for simplicity, we have get rid of the D subscripts and superscripts. For each order ν there is only a finite number of diagrams that contributes to the quantity A. Combining the scaling laws of Eqs. (2) and (3) we obtain a well-defined power series for A

$$\begin{aligned} A(Q,\Lambda_0) &= \Lambda_0^{d_A} \sum_{\nu\geq\nu_0} \left(\frac{Q}{\Lambda_0}\right)^{\nu} \hat{A}^{(\nu)}(\frac{Q'}{Q}) \\ &= \Lambda_0^{d_A} \sum_{\nu\geq\nu_0}\; x_0^{\nu}\, \hat{A}^{(\nu)}(\frac{Q'}{Q})\,, \end{aligned} \tag{5}$$

with $\hat{A}^{(\nu)}$ a dimensionless quantity that we expect to be of the order of unity (i.e. Q^0). Notice that $\hat{A}^{(\nu)}$ does not depend on the hard scale Λ_0 and is related to $A^{(\nu)}$ via Eqs. (2) and (3). In the formula above, Q' is an auxiliary soft scale we use to express $\hat{A}^{(\nu)}$ as a function of a dimensionless ratio. Provided there is a clear scale separation in the system, that is, $\Lambda_0 \geq Q$, the power series above will be convergent. Not only that, if we consider only contributions from diagrams with $\nu < \nu_{\rm max}$, the error of the EFT calculation will be $x_0^{\nu_{\rm max}+1}$. In this work we will only perform calculations at the lowest order and we expect a relative error of the order of x_0 in the calculations to follow.

If we are interested in the low energy description of heavy meson-antimeson bound states, the relevant physical object we want to expand is the (non-relativistic)

potential between the heavy meson and antimeson:

$$V = \sum_{\nu=\nu_0}^{\nu_{\rm max}} V^{(\nu)} + \mathcal{O}(x_0^{\nu_{\rm max}+1}) \,. \tag{6}$$

The expansion starts at order $\nu_0 \geq -1$, where x_0 is the ratio of the soft and hard scales of the system. The low energy degrees of freedom we consider are the heavy meson and antimeson fields and the pion field. The pion-meson vertices are constrained by chiral symmetry and the corresponding Feynman rules can be derived from heavy hadron chiral perturbation theory (HHChPT) [39]. In turn HQSS generates strong constraints on the form of the heavy meson-antimeson interactions [36]. This means that the EFT potential includes two kind of contributions: contact range interactions, i.e. four meson vertices, and pion exchanges. The set of soft scales Q includes in principle the pion mass m_π and the center-of-mass momenta $\vec{p}$ and $\vec{p}\,'$ of the meson and antimeson. The hard scale Λ_0 can represent the momentum scale at which we expect the low energy symmetries to break down, i.e. the chiral symmetry breaking scale $\Lambda_\chi = 4\pi f_\pi \sim 1\,\text{GeV}$ (with f_π the pion decay constant) for chiral symmetry and the heavy quark mass m_Q for HQSS, but it can also stand for the momentum scale at which the composite structure of the heavy mesons starts to be resolved.

At lowest (or leading) order (Q^0) the heavy meson-antimeson EFT potential is local and only depends on the momentum exchanged by the meson and anti-meson

$$\langle \vec{p}\,|V^{(0)}|\vec{p}\,'\,\rangle = V^{(0)}(\vec{p}-\vec{p}\,'\,) \,, \tag{7}$$

where we have

$$V^{(0)}(\vec{q}\,) = C_0^{(0)} + \eta \frac{g^2}{2f_\pi^2} \frac{(\vec{a}\cdot\vec{q}\,)\,(\vec{b}\cdot\vec{q}\,)}{\vec{q}^{\,2}+m_\pi^2} \,. \tag{8}$$

As can be seen, the potential is the sum of a contact and a finite range contribution. The contact range operator C_0 is a free parameter of the theory. The finite range contribution is the well-known one pion exchange (OPE) potential, where $g \simeq 0.6$ is the axial coupling between the heavy meson and the pion (we have particularized its value for the charmed meson case, see Refs. [40, 41] for a determination), $f_\pi \simeq 132\,\text{MeV}$ the pion decay constant, and $\vec{q}$ the momentum exchanged by the heavy meson and antimeson. The sign η and the polarization operators $\vec{a}$ and $\vec{b}$ depend on whether the initial/final states is $\text{P}\bar{\text{P}}$, $\text{P}\bar{\text{P}}^*$, $\text{P}^*\bar{\text{P}}$ or $\text{P}^*\bar{\text{P}}^*$. A more detailed account can be found in Appendices A, B and C, where the leading order (LO) potential is derived.

A problem with the EFT potential above is its behaviour for larges values of the exchanged momentum, $|\vec{q}\,| \gg m_\pi$. In this limit the LO potential tends to a constant value. At higher orders in the EFT expansion the problem worsens and the potential actually diverges. This feature can be easily deduced from power counting

$$V^{(\nu)}(\lambda Q) = \lambda^\nu V^{(\nu)}(Q) \,, \tag{9}$$

which admits a solution of the type $V^{(\nu)}(\vec{q}\,) \propto |\vec{q}\,|^\nu$ for $|\vec{q}\,| \gg m_\pi$. Of course, if we are only considering tree level amplitudes involving no heavy meson-antimeson loops, then there is no conceptual problem with the previous divergences: the EFT potential is only expected to make sense at low energies. However, if we iterate the potential in the Schödinger or Lippmann-Schwinger equation (a necessary step for the description of bound states) the divergences will require renormalization.

In non-relativistic EFTs the renormalization process is straightforward. First, we begin by regularizing the potential:

$$\langle \vec{p}\,|V_\Lambda|\vec{p}\,'\rangle = f(\frac{\vec{p}}{\Lambda})\,\langle \vec{p}\,|V|\vec{p}\,'\rangle\, f(\frac{\vec{p}\,'}{\Lambda}) \,, \tag{10}$$

where V_Λ is the regularized potential that we will employ in actual calculations, V is the unregularized (i.e. the original) potential, $f(x)$ a regulator function and Λ an ultraviolet cut-off. In the calculations to follow, we will use a gaussian regulator of the type $f(x) = e^{-x^2}$. At this point physical predictions can still depend strongly on the value of the cut-off, a problem we must solve. Thus there is a second step in the renormalization process: we allow the contact range operators to depend on the cut-off. At lowest order this implies that $C^{(0)} = C^{(0)}(\Lambda)$. If we have included all the counterterms required by power counting, they will be able to absorb all the divergences of the theory. The calculations will still contain a residual cut-off dependence, but this does not represent a problem: its size is expected to be a higher order effect, at least for a judicious choice of the cut-off. Even though there is not a well-established criterion for choosing the cut-off, calculations in nuclear EFT suggest that the optimal value of the cut-off should never be much larger than the hard scale Λ_0 [42]. Here, for avoiding the breakdown of the low energy symmetries in loops, we advocate the slightly more stringent condition $\Lambda \leq \Lambda_0$. We employ the actual values $\Lambda = 0.5\,\text{GeV}$ and $1\,\text{GeV}$, for which we have checked that the results do not change considerably. This indicates that renormalization has been correctly implemented.

B. Power Counting and Bound States

The description of bound states requires the iteration of the EFT potential in the bound state equation

$$|\Psi_B\rangle = G_0(E) V |\Psi_B\rangle \,, \tag{11}$$

where $|\Psi_B\rangle$ is the wave function, $G_0(E) = 1/(E-H_0)$ the resolvent operator and V the potential. If we require the bound state equation to be compatible with the power counting of the EFT potential, then successive iterations of the $G_0 V$ combination must be of the same order:

$$\mathcal{O}\,(V) = \mathcal{O}\,(V G_0 V) \,. \tag{12}$$

By taking into account that the G_0 operator scales as Q in loops [1], we can see that only the order Q^{-1} contribution to the potential should be iterated. Thus the presence of shallow bound states or large scattering lengths in a two-body system requires the non-perturbative treatment of a piece of the effective potential. There exists a problem then: the EFT potential we obtain from HHChPT starts at order Q^0 and is therefore incompatible with the EFT description of a low energy bound state.

The solution is to redefine power counting by promoting the C_0 contact range operator from order Q^0 to Q^{-1} [43–47]. This is equivalent to assume that the C_0 operator is contaminated by a low energy scale. We can confirm this assumption *a posteriori* by solving the bound state equation with the C_0 operator alone: if we regularize the EFT potential with a cut-off Λ, and set the value of $C_0(\Lambda)$ to reproduce the position of the bound state, we obtain the generic result

$$\frac{1}{C_0(\Lambda)} \sim \frac{\mu}{2\pi}\left(\gamma_B - \frac{2}{\pi}\Lambda\right), \tag{13}$$

where μ is the reduced mass of the two-body system and $\gamma_B = \sqrt{-2\mu E_B}$ the wave number (with E_B the bound state energy). Of course, the exact form of the relation depends on the specific regularization scheme employed – the expression above corresponds to power divergence subtraction [43, 44] – but will be in the line of the previous form. From the equation above it is evident that we can only have $C_0 \sim Q^{-1}$ as long as the cut-off Λ scales like $\mathcal{O}(Q)$. Note that this is consistent with our requirement that all involved momenta should be soft and smaller than the charm quark mass to make sense of HQSS.

C. The EFT Potential at Lowest Order

As we have seen, the S-wave LO interaction ($\mathcal{O}(Q^{-1})$) between a heavy meson ($\mathrm{H} = \mathrm{P}, \mathrm{P}^*$) and antimeson ($\bar{\mathrm{H}} = \bar{\mathrm{P}}, \bar{\mathrm{P}}^*$) only contains contact operators (i.e. four heavy meson vertices). The contact range interactions are in turn constrained by HQSS and depend on the particle channel under consideration ($\mathrm{H}\bar{\mathrm{H}} = \mathrm{D}\bar{\mathrm{D}}, \mathrm{D}\bar{\mathrm{D}}^*/\mathrm{D}^*\bar{\mathrm{D}}, \mathrm{D}\bar{\mathrm{D}}$), the total isospin I and the value of the J^{PC} quantum numbers. In particular HQSS limits the number of independent LO counter-terms to two per isospin channel at LO [36]. We will only consider the isoscalar $I = 0$ channels, to which the $X(3872)$ and the $X(3915)$ belong, in the isospin symmetric limit.

[1] This can be trivially checked by considering the rescaling transformation

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\, G_0(\lambda^2 E) = \lambda \int \frac{d^3\vec{q}}{(2\pi)^3}\, G_0(E)\,,$$

where the energy rescales as λ^2 as we are considering a non-relativistic system.

The HQSS contact range interaction can mix different particle channels with the same J^{PC} quantum numbers. Therefore, for writing the LO potential we considering the set of particle coupled channel basis

$$\mathcal{B}(0^{++}) = \left\{|P\bar{P}\rangle, |P^*\bar{P}^*(0)\rangle\right\}, \tag{14}$$

$$\mathcal{B}(1^{+-}) = \left\{\frac{1}{\sqrt{2}}\left(|P\bar{P}^*\rangle + |P^*\bar{P}\rangle\right), |P^*\bar{P}^*(1)\rangle\right\}, \tag{15}$$

$$\mathcal{B}(1^{++}) = \left\{\frac{1}{\sqrt{2}}\left(|P\bar{P}^*\rangle - |P^*\bar{P}\rangle\right)\right\}, \tag{16}$$

$$\mathcal{B}(2^{++}) = \left\{|P^*\bar{P}^*(2)\rangle\right\}, \tag{17}$$

where the number in parenthesis in the $|P^*\bar{P}^*(S)\rangle$ states is the total intrinsic spin S to which the vector meson-antimeson system couples. In this basis, the EFT potential is independent of momentum and reads (see Appendices A,B and C)

$$V^{\mathrm{LO}}(\vec{q}, 0^{++}) = \begin{pmatrix} C_{0a} & \sqrt{3}\,C_{0b} \\ \sqrt{3}\,C_{0b} & C_{0a} - 2\,C_{0b} \end{pmatrix}, \tag{18}$$

$$V^{\mathrm{LO}}(\vec{q}, 1^{+-}) = \begin{pmatrix} C_{0a} - C_{0b} & 2\,C_{0b} \\ 2\,C_{0b} & C_{0a} - C_{0b} \end{pmatrix}, \tag{19}$$

$$V^{\mathrm{LO}}(\vec{q}, 1^{++}) = C_{0a} + C_{0b}\,, \tag{20}$$

$$V^{\mathrm{LO}}(\vec{q}, 2^{++}) = C_{0a} + C_{0b}\,, \tag{21}$$

where C_{0a} and C_{0b} are the two independent counter-terms that we expect from HQSS for a given isospin sector. This contact potential V behaves as $\mathcal{O}(Q^{-1})$, but for avoiding confusions with the notation we have renamed $V^{(-1)}$ to V^{LO}, as the negative exponent could be confused by the inverse of the potential. As can be appreciated, the 1^{++} and 2^{++} cases are uncoupled and their potential is identical, a strong hint that we should expect a 2^{++} $\mathrm{D}^*\bar{\mathrm{D}}^*$ partner of the $X(3872)$.

At this point we notice that the heavy pseudoscalar and vector mesons P and P* are only degenerate in the heavy quark limit $m_Q \to \infty$. For finite m_Q there is a mass splitting between the heavy mesons

$$M_{P^*} - M_P = \Delta_Q\,, \tag{22}$$

that scales as $1/m_Q$. As a consequence of this gap, the two $\mathrm{H}\bar{\mathrm{H}}$ thresholds in the 0^{++} and 1^{+-} coupled channel happen at different energies. If we are interested in low-lying bound states, the energy difference between the two thresholds may actually be considerably larger than the binding energy of the state. Within the EFT framework this means that we may very well be entitled to ignore the coupled channel effects. The momentum scale associated with the coupled channels is

$$\Lambda_C(0^{++}) = \sqrt{2\mu\,(2\Delta_Q)}\,, \tag{23}$$

$$\Lambda_C(1^{+-}) = \sqrt{2\mu\,\Delta_Q}\,, \tag{24}$$

for the 0^{++} and 1^{+-} cases respectively, where μ is the reduced mass of the $\mathrm{H}\bar{\mathrm{H}}$ heavy meson system and Δ_Q is the energy gap. In the particular case of the charm

mesons, direct evaluation yields $\Lambda_{C(0^{++})} \sim 750\,\text{MeV}$ and $\Lambda_{C(1^{+-})} \sim 520\,\text{MeV}$. If we have a 0^{++} $\text{D}\bar{\text{D}}$ (1^{+-} $\text{D}^*\bar{\text{D}}$) bound state at threshold, the corresponding 0^{++} $\text{D}^*\bar{\text{D}}^*$ (1^{+-} $\text{D}^*\bar{\text{D}}^*$) component will have a wave number at least of the order of the hard scale $\Lambda_0 \sim 0.5 - 1\,\text{GeV}$. Thus there is no problem in ignoring the coupled channel structure and treating the two particle channels in the 0^{++} and 1^{+-} cases as independent. From a more formal EFT viewpoint what we are doing is to consider the coupled channel momentum scale as $\Lambda_C \sim \mathcal{O}(Q^0)$, from which we expect the G_0 operator involved in the particle mixing to scale like Q^3 (see for instance Ref. [30]). This translates into a suppression of particle coupled channel effects by two orders in the chiral expansion: if we count the C_{0b} operator as Q^{-1}, then particle coupled channels do not enter until order Q, that is, at least one order beyond pion exchanges.

Taking into account that we can distinguish between different particle channels in the EFT, the LO potentials for the 0^{++} and 1^{+-} cases finally simplify to

$$V^{\text{LO}}_{\text{P}\bar{\text{P}}}(\vec{q}, 0^{++}) = C_{0a}\,, \tag{25}$$

$$V^{\text{LO}}_{\text{P}^*\bar{\text{P}}^*}(\vec{q}, 0^{++}) = C_{0a} - 2\,C_{0b}\,, \tag{26}$$

$$V^{\text{LO}}_{\text{P}^*\bar{\text{P}}/\text{P}\bar{\text{P}}^*}(\vec{q}, 1^{+-}) = C_{0a} - C_{0b}\,, \tag{27}$$

$$V^{\text{LO}}_{\text{P}^*\bar{\text{P}}^*}(\vec{q}, 1^{+-}) = C_{0a} - C_{0b}\,, \tag{28}$$

where we have added the particle channel as a subscript for distinguishing states with the same J^{PC} quantum numbers. Even though the particle coupled channel structure of the 0^{++} and 1^{+-} molecules can be ignored in the LO description of these states, we expect the 0^{++} and 1^{+-} $\text{P}^*\bar{\text{P}}^*$ states to have a strong tendency to decay to $\text{P}\bar{\text{P}}$ and $\text{P}\bar{\text{P}}^*$ respectively. On the contrary, the 2^{++} $\text{P}^*\bar{\text{P}}^*$ will have a smaller partial decay width to a heavy meson-antimeson pair, as this process does not happen via a Q^{-1} counterterm, and therefore will be suppressed by one order in the EFT expansion.

D. Bound States at Lowest Order

Finally, for completeness, we briefly discuss the solution of the bound state equation for the contact range potentials that appear in the LO description of heavy molecular states. Even though well-known, it will be of help for the calculations in the next section. We solve the bound state equation with the regularized potential V_Λ that reads

$$\langle \vec{p}\,|V^{\text{LO}}_\Lambda|\vec{p}\,'\rangle = f(\frac{\vec{p}}{\Lambda})\,C_0(\Lambda)\,f(\frac{\vec{p}}{\Lambda})\,, \tag{29}$$

where $f(x)$ is the regulator, and C_0 the appropriate counterterm for each J^{PC} / particle channel combination. For this potential, the wave function admits the ansatz

$$\langle \vec{p}\,|\Psi_B\rangle = \mathcal{N}\,\frac{2\mu}{p^2+\gamma^2}\,f(\frac{\vec{p}}{\Lambda})\,, \tag{30}$$

where $\mathcal{N}$ is a normalization constant, μ the reduced mass of the two body system and $\gamma = \sqrt{-2\mu E_B}$ the wave number of the state, with E_B the binding energy. Direct substitution into the bound state equation (Eq. (11)) yields

$$-\frac{1}{C_0(\Lambda)} = \int \frac{d^3\vec{q}}{(2\pi)^3}\,f^2(\frac{\vec{q}}{\Lambda})\,\frac{2\mu}{q^2+\gamma^2}\,. \tag{31}$$

This is the eigenvalue equation of the contact range theory. For the $\text{H}\bar{\text{H}}$ system there are six of these equations corresponding to the six possible S-wave states: in each case, we simply need to particularize the values of C_0 and μ as appropriate. For further details we refer the reader to Ref. [24], where the equations above were derived and discussed in the context of heavy meson-antimeson molecules.

III. THE PARTNERS OF THE $X(3872)$

We start by considering the 1^{++} $\text{D}\bar{\text{D}}^*$ state, the $X(3872)$, within the EFT formalism described in the previous section. From HQSS we expect that the heavy meson-antimeson interaction in the 1^{++} $\text{D}\bar{\text{D}}^*$ and 2^{++} $\text{D}^*\bar{\text{D}}^*$ channels will be identical. In terms of the EFT potential we have

$$V^{\text{LO}}(1^{++}) = V^{\text{LO}}(2^{++})\,, \tag{32}$$

as deduced from Eqs. (20-21). From this, we automatically anticipate the existence of a isoscalar 2^{++} $\text{D}^*\bar{\text{D}}^*$ bound state with a binding energy similar to that of the $X(3872)$.

To pinpoint the exact location of the 2^{++} partner of the $X(3872)$ we begin by determining the counterterm combination $C_{0a} + C_{0b}$. We will use a gaussian regulator $f(x) = e^{-x^2}$ and the cut-off values $\Lambda = 0.5\,\text{GeV}$ and $1\,\text{GeV}$. As we are working in the isospin symmetric limit [2], we can consider the $X(3872)$ to be a $\text{D}^*\bar{\text{D}}$ bound state with a binding energy of $B_X(1^{++}) \simeq 4.2\,\text{MeV}$, from which we obtain $C_{0a} + C_{0b} = -1.94\,\text{fm}^2$ ($-0.79\,\text{fm}^2$) for $\Lambda = 0.5\,\text{GeV}$ ($1\,\text{GeV}$). Now we can predict that the mass of the 2^{++} state lies in the vicinity of $4012\,\text{MeV}$ (a value rather independent of the cut-off), corresponding to a binding energy of $5\,\text{MeV}$. We call this state the $X(4012)$, and stress that this prediction is independent of any assumptions about the molecular nature of any other XYZ states, relying on HQSS alone. We mention in passing that isospin breaking effects, even though crucial for understanding certain decay properties of the $X(3872)$ (see, for example, Refs. [48, 49]), has not an appreciable effect in the spectroscopy problem [3].

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[2] In what follows, we use the isospin averaged masses $m_D = 1867.2\,\text{MeV}$ and $m^*_D = 2008.6\,\text{MeV}$.

[3] If we take into account isospin breaking within the formalism of Ref. [49], we find that the position of the $X(4012)$ moves by about

J^{PC}	$H\bar{H}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]
0^{++}	$D\bar{D}$	$^1S_0$	C_{0a}	3706 ± 10	3712^{+13}_{-17}	–
1^{++}	$D^*\bar{D}$	$^3S_1$	$C_{0a} + C_{0b}$	Input	Input	3872
1^{+-}	$D^*\bar{D}$	$^3S_1$	$C_{0a} - C_{0b}$	3814 ± 17	3819^{+24}_{-27}	–
0^{++}	$D^*\bar{D}^*$	$^1S_0$	$C_{0a} - 2\,C_{0b}$	Input	Input	3917
1^{+-}	$D^*\bar{D}^*$	$^3S_1$	$C_{0a} - C_{0b}$	3953 ± 17	3956^{+25}_{-28}	3942
2^{++}	$D^*\bar{D}^*$	$^5S_2$	$C_{0a} + C_{0b}$	4012 ± 3	4012^{+4}_{-9}	–

TABLE I. Predicted masses (in MeV) of the $X(3872)$ HQSS partners for two different values of the gaussian cutoff. We use as input 3871.6 MeV and 3917.4 MeV for the $X(3872)$ and $X(3915)$ masses, respectively. From this we find that the value of the LO couplings are $C_{0a} = -3.53$ fm^2 and $C_{0b} = 1.59$ fm^2 for $\Lambda = 0.5\,\text{GeV}$ and $C_{0a} = -1.06$ fm^2 and $C_{0b} = 0.27$ fm^2 for $\Lambda = 1\,\text{GeV}$. Errors in our predicted masses are obtained by varying the strength of the contact interaction in each channel by $\mp 15\%$, which corresponds to the expected violations of HQSS for the charm quark mass.

For predicting states beyond the $X(4012)$, we have to identify a particular XYZ state as a further molecular partner of the $X(3872)$. In this way we will be able to determine the two contact range couplings (C_{0a} and C_{0b}) and obtain the full spectrum of molecular states. Two interesting candidates are the $X(3915)$ [15] and the $X(3940)$ [50, 51], from which the first is the most promising. The $X(3915)$ has been theorized to be a 0^{++} or 2^{++} $\text{D}^*\bar{\text{D}}^*$ molecule in Refs. [17, 18]. Even though the work of Refs. [17, 18] cannot discriminate between the 0^{++} or 2^{++} quantum numbers [4], HQSS suggests that the most probable J^{PC} value is 0^{++} instead of 2^{++}, as the later would imply a remarkable violation of HQSS. On the contrary, the accommodation of the $X(3940)$ [50, 51] within the HQSS pattern of molecular states faces a problem. The $X(3940)$ decays strongly to $\text{D}\bar{\text{D}}^*$, a feature compatible with the expected properties for a 1^{+-} $\text{D}^*\bar{\text{D}}^*$ axial state. However, the production mechanism for the $X(3940)$ is more compatible with a positive C-parity state than with a negative C-parity one: this state is produced in the reaction $e^+e^- \to J/\Psi X(3940)$, most probably via an intermediate virtual photon ($e^+e^- \to \gamma^* \to J/\Psi X$), suggesting that the C-parity is positive. Nonetheless this is not a definitive conclusion, and it may happen that the $X(3940)$ resonance is being produced via two virtual photons, see for example Refs. [52, 53] for a case in which this process is less suppressed than naively expected.

At this point we notice that the identification of the $X(3915)$ as a $\text{D}^*\bar{\text{D}}^*$ molecular state, though more promising than that of the $X(3940)$, is not free of problems either. In particular, the binding energy of the corresponding heavy vector meson-antimeson system is of the order of $\sim 100\,\text{MeV}$, which translates into a wave number of $\sim 450\,\text{MeV}$. This means that the $X(3915)$ lies not too far away from the limits of what can be described within the EFT. Its wave number indicates that a description in terms of mesons alone may be incomplete and that the explicit inclusion of shorter range components (e.g. tetraquark or charmonium-like) may be necessary [5]. However, the EFT framework is very helpful and convenient in this regard. Abusing the limits of the EFT translates into a noticeable cut-off dependence and a lack of convergence of the EFT expansion, that is, subleading order corrections will be able to completely alter the LO results. As we will explain in the next paragraph, the cut-off dependence is numerically small, and as we will check in the next section, the subleading order corrections are moderate, but nonetheless under control. All this indicates that the $X(3915)$ is probably more amenable to an EFT treatment than naively expected.

From the assumption that the $X(3915)$ is a 0^{++} molecule we can determine the counterterm combination $C_{0a} - 2\,C_{0b}$ and consequently the location of the six possible HQSS partners of the $X(3872)$. The masses of the molecular states resulting from the previous identification can be consulted in Table I. We obtain a 0^{++} $\text{D}\bar{\text{D}}$, 1^{+-} $\text{D}\bar{\text{D}}^*$ and 1^{+-} $\text{D}^*\bar{\text{D}}^*$ state which we call $X(3710)$, $X(3820)$ and $X(3955)$. The errors in Table I refer to uncertainties owing to violations of HQSS in the charm sector (see Section IV A for a detailed explanation). The location of these three hadronic molecules is rather independent of the cut-off. If (instead of a gaussian regulator) we use a sharp cut-off, there are small variations of about $\sim 1\,\text{MeV}$ in the location of the states. Curiously, the $X(3955)$ state we obtain is not far away from the aforementioned $X(3940)$ molecular candidate. A possible identification is suggestive but contingent on the eventual determination of the quantum numbers (especially the C-parity) of the $X(3940)$.

$\sim 1\,\text{MeV}$, to $4013\,\text{MeV}$. In this calculation we have ignored the $I = 1$ counterterm in the $J^{PC} = 1^{++}$ channel, which (in absence of additional information) is considered to be of order Q^0 and hence subleading.

[4] Branz et al. [18] notice that the (little) known decay properties of the $X(3915)$ are compatible with both assignments

[5] This is based on the observation that the mean quadratic separation of the mesons is $\sqrt{\langle r^2 \rangle} = 0.5 - 0.8\,\text{fm}$ depending on the cut-off.

J^{PC}	$H\bar{H}$	$^{2S+1}L_J$	E ($\Lambda = 0.5\,\mathrm{GeV}$)	E ($\Lambda = 1\,\mathrm{GeV}$)	Exp [7]
0^{++}	$D\bar{D}$	$^1S_0$	3708	3720	–
1^{++}	$D^*\bar{D}$	$^3S_1$-$^3D_1$	Input	Input	3872
1^{+-}	$D^*\bar{D}$	$^3S_1$-$^3D_1$	3816	3823	–
0^{++}	$D^*\bar{D}^*$	$^1S_0$-$^5D_2$	Input	Input	3917
1^{+-}	$D^*\bar{D}^*$	$^3S_1$-$^3D_1$	3954	3958	3942
2^{++}	$D^*\bar{D}^*$	$^1D_2$-$^5S_2$-$^5D_2$-$^5G_2$	4015	4014	–

TABLE II. Predicted masses (in MeV) of the $X(3872)$ HQSS partners when the OPE potential is included. We display results for two different values of the gaussian cutoff. Now, we find $C_{0a} = -3.46$ fm^2 and $C_{0b} = 1.98$ fm^2, and $C_{0a} = -0.98$ fm^2 and $C_{0b} = 0.69$ fm^2, for $\Lambda = 0.5$ and 1 GeV, respectively.

IV. SUBLEADING ORDER CORRECTIONS

In this section we explore the impact of the main subleading order contributions. For this we must take into account the existence of two different, unrelated expansions in the EFT formalism we are proposing. The first is the expansion in terms of inverse powers of the heavy quark mass and the second the standard power counting expansion. In the previous section we have assumed exact, instead of approximate, HQSS. The existence of $1/m_Q$ deviations from the heavy quark limit implies that the location of the molecular partners of the $X(3872)$ will be subjected to uncertainties. Apart from this, the power counting expansion indicates that we should take into account two important subleading order corrections appearing at order Q^0 and Q^1 respectively: the OPE potential and the particle coupled channel effects. As we will see, the induced shifts in the position of the molecular states from these subleading interactions will in general agree with (and in the case of the OPE potential be smaller than) the *a priori* EFT expectations, thus confirming the robustness of the molecular spectrum we have deduced so far.

A. The $1/m_Q$ Corrections

The LO potentials for the 1^{++} and 2^{++} heavy meson molecules – the $X(3872)$ and the theorized $X(4012)$ – are only identical in the heavy quark limit. Thus the existence of the $X(4012)$ may be contingent on the size of HQSS violations stemming from the finite charm quark mass. In general, we expect the heavy meson-antimeson potentials of Eqs. (20-21) and (25-28) to deviate from the heavy quark limit by a quantity of the order of

$$V^{\mathrm{LO}}_{(m_Q=m_c)} = V^{\mathrm{LO}}_{(m_Q=\infty)} \left(1 + \mathcal{O}(\frac{\Lambda_{\mathrm{QCD}}}{m_c})\right) , \quad (33)$$

where m_c is the charm quark mass ($\sim 1.5\,\mathrm{GeV}$) and $\Lambda_{\mathrm{QCD}} \sim 200\,\mathrm{MeV}$, translating into an expected 15% violation of HQSS for the LO contact range potentials. The exceptions are the potentials of the 1^{++} $\mathrm{D}\bar{\mathrm{D}}^*$ and 0^{++} $\mathrm{D}^*\bar{\mathrm{D}}^*$ channels, as they are fixed to reproduce the position of the $X(3872)$ and $X(3915)$ states.

Assuming this 15% uncertainty in the contact range interactions, we obtain the error bars of Table I. As can be seen, the prediction of the 2^{++} partner of the $X(3872)$ is robust with respect to this theoretical error source. Other molecular states show moderate uncertainties of the order of $10 - 20\,\mathrm{MeV}$ in the binding energies. As we will see in the following subsections, these uncertainties are a bit smaller (but yet of the same order) as the subleading order corrections coming from pion exchanges and the particle coupled channels.

B. One Pion Exchange

A possible issue with the present EFT treatment of the $X(3915)$ as a $\mathrm{D}^*\bar{\mathrm{D}}^*$ molecule is whether the OPE potential is really perturbative in this case. With a wave number of $\gamma \simeq 450\,\mathrm{MeV}$, the wave function of the $X(3915)$ probes the intermediate distances where the tensor component of the OPE potential is stronger than the central one. In this regard and according to the arguments in Ref. [30], we expect tensor OPE to become non-perturbative at a critical center-of-mass momentum of $k_{\mathrm{crit}} \simeq 420\,\mathrm{MeV}$ for the 0^{++} $\mathrm{D}^*\bar{\mathrm{D}}^*$ channel [6]. Consequently the $X(3915)$ lies at the edge of the domain of validity of the EFT description we are using, in which pion exchanges are perturbative (i.e. small), and may require a more sophisticated EFT with non-perturbative pions. If we were predicting the binding energy of this molecular state, the previous observation would translate into a large uncertainty in the calculations of the order of 100%. The cause of this uncertainty will be high energy fluctuations in the meson-antimeson loops generated by the pion exchanges. However, as the binding energy is used as the input of the calculation, the uncertainty can manifest either in the value of the C_{0a} and C_{0b} counterterms, thus subjecting the predictions of Table I to large

[6] As we will comment later, this number is subjected to large uncertainties.

uncertainties, or in a failure of the theory to converge once subleading order corrections are included.

Yet there are two mitigating circumstances that may increase the expected breakdown scale of the present EFT calculation, thus turning the predictions much more reliable. On the one side, the critical momenta above which the tensor OPE is no longer perturbative [30] are subjected to considerable uncertainties stemming from the far-from-perfect separation of scales in heavy meson molecules. In particular the value of the critical momenta may be up to 50% larger than expected, in which case the $X(3915)$ will lie well within the range of applicability of the EFT with perturbative pions. On the other, we are limiting ourselves to the spectroscopy problem. It is worth noticing in this respect that the spin-spin structure of the C_{0b} contact operator is very similar to the one we obtain from pseudoscalar meson exchange for the non-tensor piece of the interaction. We thus expect the C_{0b} operator to be able to partially absorb the shift in the binding energy generated by the pion exchanges. However, the effect is restricted to the central component of OPE, which is much better behaved than the tensor one at short distances. Of course, the definitive test is to recalculate the position of the predicted molecular states after the non-perturbative inclusion of the OPE potential.

The distinctive feature of the OPE potential is that it can mix different partial waves. For the non-perturbative calculation we follow the formalism of Ref. [24], where we considered the partial wave decomposition of tensor OPE in detail for the $\mathrm{P}\bar{\mathrm{P}}^*$ particle channel. The extension to the $\mathrm{P}^*\bar{\mathrm{P}}^*$ case is trivial and only entails a change in the partial waves that are actually coupled, which can be consulted in Table II. For determining the value of the C_{0a} and C_{0b} counterterms we fix the binding energies of the $X(3872)$ and $X(3915)$ respectively. This procedure generates again a total of six molecular states, four of them predictions, as can be seen in Table II. We can appreciate that the binding energies of the states are relatively stable with respect to the iteration of the OPE potential. The most affected state is the 0^{++} $\mathrm{D}\bar{\mathrm{D}}$ molecule, the $X(3710)$ state, for which the binding energy can increase by almost $10\,\mathrm{MeV}$ for $\Lambda = 1\,\mathrm{GeV}$ with respect to the pionless case. Curiously, for this state there is no OPE contribution to the finite range potential: two pseudoscalar objects cannot exchange a single pion, so the first non-trivial contribution to the finite range potential comes from two pion exchange, which we have not considered here. The dynamics of this state is solely controlled by the C_{0a} counterterm. In this regard, when the contact range potential is adjusted to reproduce the binding energies of the $X(3872)$ and $X(3915)$ states in presence of the OPE potential, the change in the value of the C_{0a} coupling is not counterbalanced by a pion exchange contribution in the 0^{++} $\mathrm{D}\bar{\mathrm{D}}$ case. Hence, we find a larger shift in the location of this molecular state in comparison to the others.

Indeed we find relatively small shifts in the energy of states other than the $X(3710)$ (see Tables I and II). The aforementioned observation seem to confirm the suspicions about the role of the C_{0b} operator which could effectively accommodate most of the OPE effects. Equivalently, the inclusion of the OPE potential produces a larger change in the C_{0b} coupling than in the C_{0a} one, and actually we find that the bulk of the change in C_{0b} is given by the size of the contact piece of OPE

$$C_{0b} \to C_{0b} - \frac{g^2}{2f_\pi^2}\,, \tag{34}$$

where $g^2/2f_\pi^2 \sim 0.4\,\mathrm{fm}^2$, see Appendix C for details

The natural conclusion of the previous calculations is that OPE is indeed perturbative in heavy meson molecules. There is however a caveat: if the spectroscopy problem is rather insensitive to the OPE potential, how can we appreciate whether OPE is perturbative or not in the molecular states? A partial answer is that C_{0b} is only expected to absorb effects coming from central OPE, but not from tensor OPE, which is the problematic piece. But there is another answer, that lies in the observation that non-perturbative OPE would entail a significant change in the power counting of the contact range operators. As happens in nuclear EFT, the LO counterterms that are able to renormalize the scattering amplitude with perturbative pions [43, 44] are not enough to renormalize the corresponding non-perturbative formulation [54]. In particular, for heavy meson EFT we have that while perturbative OPE requires two counterterms (C_{0a} and C_{0b}), for non-perturbative OPE this number is at least five [30]. As the shifts in the binding energies are only weakly cut-off dependent, we do not require new counterterms and we can confidently conclude that OPE is perturbative.

We mention in passing that an alternative possibility to check whether OPE is perturbative is the description of bound state properties that depend on the existence of a D-wave component of the wave function, which is a typical signature of the tensor force. It turns out that D-wave probabilities of the molecular states are quite small (from $1-4\%$, with a strong cut-off dependence [7]). Unfortunately, it looks difficult to find experimental observations that could depend on (and consequently constraint) the D-wave components of the molecular wave-functions. Nevertheless, the small size of the D-wave probabilities is consistent with the expectation of them to be a second order correction in perturbation theory.

C. Particle Coupled Channels

As previously discussed particle coupled channel effects are suppressed by two orders in the EFT expan-

[7] One should keep in mind that the D-wave probability is not *per se* an observable quantity, meaning that we should not be worried by its moderate cut-off dependence.

J^{PC}	$H\bar{H}$	$E - i\Gamma/2$ ($\Lambda = 0.5$ GeV)	$E - i\Gamma/2$ ($\Lambda = 1$ GeV)	Exp [7]
0^{++}	$D\bar{D}, D^*\bar{D}^*$	3658	3669	–
1^{++}	$D^*\bar{D}$	Input	Input	3872
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	3730	3739	–
0^{++}	$D\bar{D}, D^*\bar{D}^*$	$3917 - \frac{i}{2}\,23$	$3917 - \frac{i}{2}\,50$	$3917 \pm 3 - \frac{i}{2}\,28^{+10}_{-9}$
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	$3979 - \frac{i}{2}\,24$	$3979 - \frac{i}{2}\,39$	$3942 \pm 9 - \frac{i}{2}\,37^{+27}_{-17}$
2^{++}	$D^*\bar{D}^*$	4012	4012	–

TABLE III. Predicted masses and widths (in MeV) of the $X(3872)$ HQSS partners when coupled channels effects are included. The contact terms are adjusted to reproduce the $X(3872)$ and $X(3915)$ masses, while OPE effects are neglected. We find $C_{0a} = -4.16$ fm^2 and $C_{0b} = 2.21$ fm^2, and $C_{0a} = -1.14$ fm^2 and $C_{0b} = 0.35$ fm^2, for $\Lambda = 0.5$ and 1 GeV, respectively.

J^{PC}	$H\bar{H}$	$E - i\Gamma/2$ ($\Lambda = 0.5$ GeV)	$E - i\Gamma/2$ ($\Lambda = 1$ GeV)	Exp [7]
0^{++}	$D\bar{D}, D^*\bar{D}^*$	3690	3694	–
1^{++}	$D^*\bar{D}$	Input	Input	3872
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	3782	3782	–
0^{++}	$D\bar{D}, D^*\bar{D}^*$	$3939 - \frac{i}{2}\,12$	$3937 - \frac{i}{2}\,31$	$3917 \pm 3 - \frac{i}{2}\,28^{+10}_{-9}$
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	$3984 - \frac{i}{2}\,17$	$3982 - \frac{i}{2}\,29$	$3942 \pm 9 - \frac{i}{2}\,37^{+27}_{-17}$
2^{++}	$D^*\bar{D}^*$	4012	4012	–

TABLE IV. Predicted masses and widths (in MeV) of the $X(3872)$ HQSS partners when coupled channels effects are included. The contact terms are fixed to the values given in the caption of Table I (i.e., they are adjusted to reproduce the $X(3872)$ and $X(3915)$ masses neglecting coupled channel effects). Moreover, OPE interactions are not taken into account either.

sion. From this we expect coupled channel effects in the binding energies of the 0^{++} and 1^{+-} states to scale as

$$|\Delta E_B| \simeq |E_B| \left(\frac{\gamma_B}{\Lambda_C}\right)^2 , \tag{35}$$

where E_B is the binding energy, $\gamma_B = \sqrt{-2\mu E_B}$ the wave number of the bound state and Λ_C the typical momentum scale of the coupled channel under consideration, which we consider to be a hard scale $\Lambda_C \sim \Lambda_0$. The estimation above translates into an uncertainty of around 30 MeV (40 MeV) for the 0^{++} (1^{+-}) coupled channel, where we have employed the wave number of the deepest bound state within the coupled channel. It should be noticed that in the case of the $D^*\bar{D}^*$ molecular states the energy shift is complex, as the 0^{++} (1^{+-}) state can decay into a $D\bar{D}$ ($D^*\bar{D}$) meson-antimeson pair.

In contrast to the OPE corrections, the counterterm structure stemming from HQSS is not expected to be able to absorb the kind of divergences associated with the coupled channel calculations, meaning that the actual error in the calculation will probably saturate the previous bound. We can check the EFT *a priori* estimates given above by means of a concrete calculation in which the particle coupled channel effects are fully taken into account. However, as we will see, this task is not trivial, specifically in what regards to the choice of the appropriate regularization scheme. To illustrate this point we can study the perturbative estimate of the binding energy shift induced by coupled channel dynamics. If we consider a small change in the potential

$$V \to V + \delta V \,, \tag{36}$$

the perturbative correction to the binding energy is expected to be

$$\delta E = \langle \Psi | \delta V | \Psi \rangle \,, \tag{37}$$

where $|\Psi\rangle$ is the wave function of the bound state. In the case of particle coupled channels, the δV operator reads

$$\delta V_\alpha = V_{\alpha\beta} G_{0,\beta}(E) V_{\beta\alpha} \,, \tag{38}$$

where α represents the channel we are interested in, $\beta \neq \alpha$ the other channel and $V_{\alpha\beta}$ the transition potential from channel α to β, which is proportional to the C_{0b} contact operator. We can distinguish two cases, depending on whether the unperturbed energy E is (a) above or (b) below the β channel threshold. The most interesting case is (a), corresponding to the modification of the 0^{++} or 1^{+-} $D^*\bar{D}^*$ molecular state energies (α channel) by the 0^{++} $D\bar{D}$ or 1^{+-} $D\bar{D}^*$ states (β channel), which lie in the continuum for the energies relevant for the $\alpha-$channel states. A direct calculation yields

$$\delta E_\alpha = \left[C_{\alpha\beta}(\Lambda) \int_\Lambda \frac{d^3\vec{p}}{(2\pi)^3}\, \Psi_\alpha(\vec{p}) \right]^2 \times \int_\Lambda \frac{d^3\,\vec{q}}{(2\pi)^3}\, \frac{2\mu}{k_\beta^2 - \vec{q}^{\,2} + i\epsilon} \,, \tag{39}$$

where $\Psi_\alpha(\vec{p})$ is the wave function of the α bound state, $k_\beta^2 = -\gamma_\alpha^2 + \Lambda_C^2$ the momentum of the heavy meson-antimeson pair above the threshold, γ_α the wave number of the α bound state below threshold and Λ_C the coupled channel momentum scale. The integrals are assumed to be regularized with a cut-off Λ and an arbitrary regulator function that we have not specified yet. The $C_{\alpha\beta}$ transition contact operator can be identified with $2\,C_{0b}$ ($\sqrt{3}\,C_{0b}$) in the 0^{++} (1^{+-}) molecular state.

As can be seen, if $\Lambda < k_\beta$, a natural thing to expect if the momentum separation of the coupled channels is a hard scale, the perturbative correction to the binding energy is effectively suppressed by a factor of $k_\beta^2 \sim \Lambda_C^2$:

$$\delta E_\alpha = \frac{2\mu}{k_\beta^2}\left[C_{\alpha\beta}(\Lambda)\int_\Lambda \frac{d^3\vec{p}}{(2\pi)^3}\,\Psi_\alpha(\vec{p})\right]^2 \times \int_\Lambda \frac{d^3\,\vec{q}}{(2\pi)^3}\left(1+\frac{\vec{q}^{\,2}}{k_\beta^2}+\frac{\vec{q}^{\,4}}{k_\beta^4}+\dots\right), \qquad (40)$$

that is, by two powers in the counting. However, we can also appreciate that the δE_α correction is strongly scale dependent. On the one hand, if we consider that the wave function behaves as

$$\Psi_\alpha(\vec{p}) \propto \frac{1}{\vec{p}^{\,2}+\gamma_\alpha^2}\,, \qquad (41)$$

we see that the integral in the first line of the expression for the energy shift of Eq. (39) diverges as Λ:

$$\int_\Lambda \frac{d^3\vec{p}}{(2\pi)^3}\,\Psi_\alpha(\vec{p}) \propto \Lambda\,. \qquad (42)$$

On the other, the integral related to the decay into the continuum state β in the second line of Eq. (39) diverges as Λ. However, had we re-expanded the propagator $1/(k_\beta^2-\vec{q}^{\,2})$ in inverse powers of k_β (as mandated by power counting), the power-law divergence would have worsened to Λ^3, see Eq. (40). Putting all the pieces together (and ignoring the propagator re-expansion), the total divergence in the energy shift is given by $C_{\alpha\beta}^2\,\Lambda^3$.

This divergent behaviour tell us that we are required to include counterterms to absorb them [8]. The problem is that the counterterms renormalizing the coupled channel dynamics are higher order. In principle the C_{0b} operator could do the job, but we need to take into account that this contact operator is already determined by the condition of reproducing the binding energy of a molecular state. Thus, we do not expect C_{0b} to balance for the particle coupled channel effects. The renormalization group behaviour of C_{0b} is approximately given by

$$C_{0b}(\Lambda) \propto \frac{1}{\mu\,\Lambda}\,, \qquad (43)$$

for large Λ, see Eq. (13). This means in turn that C_{0b} can absorb the piece proportional to Λ^2 of the coupled channel divergence,

$$\left[C_{\alpha\beta}(\Lambda)\int_\Lambda \frac{d^3\vec{p}}{(2\pi)^3}\,\Psi_\alpha(\vec{p})\right]^2 \propto \Lambda^0\,, \qquad (44)$$

since $C_{\alpha\beta}$ is proportional to C_{0b}. Thus, we are left with a residual cut-off dependence of Λ in the best case. The worst case scenario is however when the cut-off and the coupled channel scale coincide, $\Lambda \sim \Lambda_C$, in which case the β-channel integral peaks. This can be easily appreciated if we use a sharp cut-off regulator

$$2\pi^2\int \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{\theta(\Lambda-|\vec{q}|)}{k_\beta^2-\vec{q}^{\,2}+i\epsilon} = -\left[\Lambda + i\frac{\pi}{2}k_\beta\,\theta(\Lambda-k_\beta)\right] + \frac{k_\beta}{2}\,\log\left|\frac{\Lambda-k_\beta}{\Lambda+k_\beta}\right|, \qquad (45)$$

where the real part of the integral diverges at $\Lambda = k_\beta \sim \Lambda_C$, a very puzzling situation (see a related discussion in Ref. [55]). Of course, the problem can be solved by using a smoother regulator, as the gaussian scheme that we have been employing along this work. In this case the real part integral will show a maximum (but not diverge) at $\Lambda \sim \Lambda_C$. This signals the transition from a power counting in which Λ_C is a hard scale to a different one in which it is a soft scale.

All this indicates that one should presumably add new counterterms at $\mathcal{O}(Q)$ to soften the cutoff dependence and make the EFT renormalizable again. At this point it is worth mentioning that the EFT treatment of coupled channel dynamics has been only discussed for the case in which Λ_C is a soft scale [56, 57]. However, the corresponding analysis for the $\Lambda_C \sim \Lambda_0$ case has not been done yet and will be left for future research [58]. Independently of the exact form of the power counting for coupled channel dynamics, it is clear that higher orders will introduce new unknown constants that cannot be fixed at the moment owing to the scarce experimental data available. Nevertheless, here we will present full non-perturbartive results including coupled channel effects. Even though the energy shifts thus obtained will be cutoff (and regulator) dependent, they will not vastly deviate from the *a priori* estimates of Eq. (35), reinforcing the (qualitative) reliability of the LO predictions. Of course, had we included all the relevant $\mathcal{O}(Q)$ counterterms (and known the entire experimental spectrum of $\mathrm{D}^{(*)}\bar{\mathrm{D}}^{(*)}$ states), the deviations would have decreased. Furthermore, we notice that coupled channel effects produce changes in C_{0a} and C_{0b} comparable in magnitude to those we should expect from violations of HQSS, that is, about 15% for the charm quark mass.

The non-perturbative calculation of the coupled channel effects is presented in Table III. As in previous cases, we have adjusted the C_{0a} and C_{0b} counterterms to reproduce the $X(3872)$ and $X(3915)$ masses. We have searched for the poles of the scattering amplitude in the first and second Riemann sheets (we refer to Ref. [59]

[8] It is important to comment at this point that the different divergences we are discussing correspond to particular choices of how to expand in terms of power counting. The full, non-perturbative coupled channel calculation is free of divergences.

for further details on this subject). The former are to be interpreted as bound states, while the later correspond to the location of resonant states in the complex plane, where the real part of the pole position is the mass and the imaginary part is half the decay width ($E_{\rm pole} = M - \frac{i}{2}\,\Gamma$). As can be seen in Table III, the location of the 1^{+-} $\mathrm{D}\bar{\mathrm{D}}^*$ and $\mathrm{D}^*\bar{\mathrm{D}}^*$ states have been shifted by about $80-85\,\mathrm{MeV}$ and $25\,\mathrm{MeV}$ respectively. The correction to the binding energy of the $\mathrm{D}\bar{\mathrm{D}}^*$ state is large, saturating and even exceeding the EFT expectation. In the case of the 0^{++} $\mathrm{D}\bar{\mathrm{D}}$ partner of the $X(3915)$, the shift in the position of the state is of $40-50\,\mathrm{MeV}$, of the order of the EFT expectation. Had we use a sharp cut-off instead of a gaussian one, the location of the bound and resonant states would have drastically changed for $\Lambda = 0.5\,\mathrm{GeV}$. This is not surprising in view of Eq. (45) and the related discussion. However, for larger cut-offs such as $\Lambda = 1\,\mathrm{GeV}$, variations are significantly smaller and the results are similar to those obtained in the gaussian cut-off scheme.

To further check the uncertainties affecting our results, we have also considered the alternative option of using the counterterm values of the original uncoupled calculation to estimate the coupled channel effects, in which case we obtain the results of Table IV. In this second scheme we observe that the change in the position of the states agrees much better with the EFT expectations: the energies of the 1^{+-} $\mathrm{D}\bar{\mathrm{D}}^*$ and $\mathrm{D}^*\bar{\mathrm{D}}^*$ states consistently change by about $35\,\mathrm{MeV}$, while in the 0^{++} $\mathrm{D}\bar{\mathrm{D}}$ and $\mathrm{D}^*\bar{\mathrm{D}}^*$ states we end up with an energy shift of about $20-25\,\mathrm{MeV}$. The reason for the additional stabilization of the calculations may be that we are not forcing the reproduction of the $X(3915)$ in a cut-off window in which we may not expected to obtain this state (owing to the large coupled channel corrections). In this case the $X(3915)$ state, which we do not adjust now, shifts is mass to around $3940\,\mathrm{MeV}$, with a width of about $15-30\mathrm{MeV}$, depending on the value of the cutoff. If we consider that the uncertainties coming from the $1/m_Q$ corrections are of the order of $15-30\,\mathrm{MeV}$, the properties of the 0^{++} $\mathrm{D}^*\bar{\mathrm{D}}^*$ state could certainly be accommodated with the existing experimental data for this resonance ($M = 3917 \pm 3$ and $\Gamma = 28^{+10}_{-9}\,\mathrm{MeV}$ [7]).

V. HQSS AND DECAY PROPERTIES

The dynamics of the molecular states studied in this work is solely determined, within our approach, by the re-interaction of the open charm channels $\mathrm{D}^{(*)}\bar{\mathrm{D}}^{(*)}$. We have ignored hidden charm channels like, for example, the $J/\Psi\,\omega$ or $\eta_c\,\omega$. We expect these latter channels to have little effect on the inner structure and masses of the molecular states, as suggested by explicit calculations performed in Refs. [60–62]. Yet, within the EFT approach, it has been customary to ignore hidden charm channels in the study of the $X(3872)$ resonance, see e.g. Ref. [36]. Analogously, the hidden bottom channels have also been ignored in the recent studies [37, 38] of the $Z(10610)$ and $Z'(10650)$ molecular states in the bottom sector. Nevertheless, the hidden charm channels can play an important role in the decay of some of the states described here, especially if they are placed below the open charm $\mathrm{D}^{(*)}\bar{\mathrm{D}}^{(*)}$ thresholds. Moreover, the J/Ψ meson provides a clear experimental signature and thus its decay modes are often used in the detection of the XYZ states. The detailed study of the hidden charm decays of the molecular states described here is beyond the scope of this work and we left it for future research.

However, the generic decay properties of the molecular states can be discussed at the qualitative level in the basis of HQSS. If we ignore phase space effects, HQSS predicts [37, 38] for the total widths:

$$\begin{aligned}\Gamma(1^{++}) &= \Gamma(2^{++}) \\ &= \frac{3}{2}\,\Gamma_{\mathrm{D}\bar{\mathrm{D}}}(0^{++}) - \frac{1}{2}\,\Gamma_{\mathrm{D}^*\bar{\mathrm{D}}^*}(0^{++})\,, \quad (46)\\ \Gamma_{\mathrm{D}\bar{\mathrm{D}}^*}(1^{+-}) &= \Gamma_{\mathrm{D}^*\bar{\mathrm{D}}^*}(1^{+-})\,, \quad (47)\end{aligned}$$

where we denote each molecular state by its quantum number J^{PC} and additionally its particle content if necessary. As noticed in Ref. [38], the relations above can also be obtained within the EFT framework we advocate by promoting the C_{0a} and C_{0b} couplings to complex values. In this way, one can implicitly take into account the multiple decay channels of the molecular states (as with an optical potential). In contrast with the bottom sector, where the previous relationships were derived, we expect however noticeable corrections to Eqs. (46) and (47) in the charm sector. The reason is that both HQSS violations and phase space corrections are larger in the charm sector than in the bottom one. The relations above involve total widths and do not necessarily hold for decays into open charm channels, where phase space corrections are crucial and indeed forbid some decays. For instance, if we pay attention to Eq. (46), and since $\Gamma_{\mathrm{D}\bar{\mathrm{D}}}(0^{++}) = 0$ for an open charm decay into $\mathrm{D}^*\bar{\mathrm{D}}^*$ [9], we will have to conclude that $\Gamma(1^{++}) = \Gamma(2^{++}) = \Gamma_{\mathrm{D}^*\bar{\mathrm{D}}^*}(0^{++}) = 0$. However, we find a partial decay width of the order of tens of MeV for the $X(3915)$ state into $D\bar{D}$.

Nonetheless, a clear implication of the relationships above is that the $X(4012)$ should be a relatively narrow state, just like the $X(3872)$. In addition, if we assume $\Gamma(1^{++})$ and $\Gamma(2^{++})$ to be much smaller than the other decay widths, then we can estimate the total width of the $X(3710)$ resonance to be a third of the $X(3915)$ width ($\Gamma_{\mathrm{D}^*\bar{\mathrm{D}}^*}(0^{++}) = 28^{+10}_{-9}$ according to the PDG [7]), yielding $\Gamma_{\mathrm{D}\bar{\mathrm{D}}}(0^{++}) \sim 10\,\mathrm{MeV}$. For the $X(3815)$ and $X(3955)$ resonances the situation is similar: HQSS without phase space considerations predicts them to have the same width, but if one takes into account the large $\mathrm{D}^*\bar{\mathrm{D}}$

[9] Note that in the infinitely heavy quark limit the D and D^* mesons are degenerated. Thus, the $\mathrm{D}^*\bar{\mathrm{D}}^*$ decay channel could be open depending on the binding energies.

contribution to the $X(3955)$, the $X(3815)$ should be narrower than its partner.

VI. DISCUSSION AND CONCLUSIONS

In this work we have argued that the application of HQSS to the charmed meson-antimeson system, combined with the identification of the $X(3872)$ and $X(3915)$ resonances as isoscalar $D\bar{D}^*$ and $D^*\bar{D}^*$ molecules, implies the existence of four molecular partners of these two states (Table I). This prediction is subjected to a series of uncertainties, namely the approximate nature of HQSS (especially for the charm sector), the effect of the OPE potential and the impact of the particle coupled channel dynamics. We have estimated the size of these corrections within the EFT framework and concluded that the HQSS pattern of molecular states is rather stable (Tables II, III and IV). In contrast, the exact location of the molecular partners is subjected to moderate uncertainties of up to $40-50\,\text{MeV}$ for the most bound cases, in agreement with the EFT expectations.

The determination of the $D^{(*)}\bar{D}^{(*)}$ family of bound states hinges on the assumption that the $X(3872)$ and the $X(3915)$ states are molecular. In this regard we find it worth commenting that, while the identification of the $X(3872)$ as a 1^{++} loosely bound $D\bar{D}^*$ state is a widely accepted hypothesis, the case for the molecular nature of the $X(3915)$ is less compelling but nevertheless still compatible with the experimental information available for this resonance. Thus we expect the conclusions solely derived from the $X(3872)$ to be more solid and less speculative than those depending on the $X(3915)$.

In this regard the tentative 2^{++} $D^*\bar{D}^*$ partner of the $X(3872)$, which we have called the $X(4012)$ in reference to its predicted mass, see Table I, is probably the most robust and model independent prediction of the present work. The $X(4012)$ is not affected by particle coupled channel effects and its mass only varies mildly, by about $2-3\,\text{MeV}$, when the OPE potential is included. Perhaps in the real world the $1/m_Q$ effects may be larger than we have estimated or there may be a further and unexpected subleading correction that turns out to be large. In this case the $X(4012)$ state might move slightly up above the $D^*\bar{D}^*$ threshold and become virtual or might descend to a lower mass region. Be as it may, we are quite confident about the existence of a molecular state with these quantum numbers close to the $D^*\bar{D}^*$ threshold.

The prediction of new $D^{(*)}\bar{D}^{(*)}$ states beyond the $X(4012)$ requires the identification of a further XYZ state (besides the $X(3872)$) as a charmed meson-antimeson molecule. The $X(3915)$ is a good candidate, which we assume to be a 0^{++} $D^*\bar{D}^*$ bound state. Of course we notice that the molecular interpretation of the $X(3915)$ [17–19], while plausible, is not so well-established. Consequently the three additional 0^{++} $D\bar{D}$ and 1^{+-} $D\bar{D}^*$ and $D^*\bar{D}^*$ states we obtain from the $X(3915)$, which we call the $X(3710)$, $X(3820)$ and $X(3955)$ respectively, see Table I, should be granted a more conjectural status. Nevertheless, we notice that the only necessary condition for the existence of molecular states different than the $X(3872)$ and $X(4012)$ is that $C_{0b} \geq 0$.

Other theoretical approaches have also predicted several $D^*\bar{D}^*$ molecular-like states, but usually with a mass spectrum incompatible with HQSS. In the quark model of Ref. [63] there are six hidden charm diquark-antidiquark states arranged in a pattern similar to the one we find. In particular there is a tetraquark 0^{++} state at $3723\,\text{MeV}$ that could be identified with the $X(3710)$ state we obtain. However, the 2^{++} state appears at $3952\,\text{MeV}$ and is identified with the $X(3940)$ resonance [50, 51]. Unless a considerable violation of HQSS is taking place, this tensor state is too tightly bound to be considered the HQSS partner of the $X(3872)$. Curiously, the two 1^{+-} hidden charm diquark-antidiquark states of Ref. [63] are located at a similar depth below the $D^*\bar{D}^*$ and $D\bar{D}^*$ thresholds respectively and therefore respect the HQSS expectations.

Another interesting theoretical approach for the study of hidden charm resonances is the hidden gauge formalism, using an extension of the $SU(3)$ chiral lagrangians to $SU(4)$ that implements a particular pattern of $SU(4)$ flavor symmetry breaking. Within this framework, Gammerman et al. [60] have obtained a 0^{++} $D\bar{D}$ molecular state in the vicinity of $3700\,\text{MeV}$, that is to be identified with the $X(3710)$ $D\bar{D}$ molecular state we predict. The extension of the hidden gauge to axial states [61] predicts (among others) a negative C-parity state at $3840\,\text{MeV}$, not far way from the $3815-3820\,\text{MeV}$ mass range we obtain for the 1^{+-} $D\bar{D}^*$ state. Finally, the related exploration of resonances generated by the interaction of two vector mesons in Ref. [62] predicts a series of 0^{++}, 1^{+-} and 2^{++} $D^*\bar{D}^*$ states. The 0^{++} $D^*\bar{D}^+$ resonance is found in the region around $3940\,\text{MeV}$. Though not identical, this figure does not differ much from the mass of the $X(3915)$ resonance that we employ as input. The 1^{+-} $D^*\bar{D}^*$ state of Ref. [62] matches rather well with the mass of the $X(3955)$ state we obtain. However, the 2^{++} $D^*\bar{D}^*$ isoscalar resonance is considerably different from the $X(4012)$ state: its mass and width are $M = 3929 \pm 3\,\text{MeV}$ and $\Gamma = 29 \pm 10\,\text{MeV}$ respectively, where the dominant decay channel is $D\bar{D}$. This mass, which is clearly incompatible with the HQSS pattern, is the result of the remarkably strong vector-vector interaction generated by the hidden gauge model. Curiously, the properties of this tensor $D^*\bar{D}^*$ resonance are strikingly similar to those of the $\chi_{c2}(2P)$ charmonium state [64, 65] (sometimes referred to as the $Z(3940)$): $M = 3927.2 \pm 2.6$ and $\Gamma = 24 \pm 6\,\text{MeV}$, decaying mostly to $D\bar{D}$ [7].

The comparison of the HQSS spectrum with experimentally known states is however incomplete. In principle there is so far no experimental evidence in favor (or against) of the positive C-parity $X(3710)$ and $X(4012)$ states we predict. Interestingly, the properties of the predicted $X(3955)$ molecular state are not very different from what is experimentally known about the

$X(3940)$ resonance [50, 51], which have been observed to decay into $D\bar{D}^*$ (just as would have been expected for a 1^{+-} $D^*\bar{D}^*$ state). There is a problem though in this identification: the $X(3940)$ is suspected to be a positive C-parity state, while the $X(3955)$ has negative C-parity. The reason is that the usual production mechanism $e^+e^- \to \gamma^* \to J/\Psi X$ favors the generation of positive C-parity XYZ states, owing to the quantum numbers of the intermediate virtual photon and the final J/Ψ. This mechanism also implies that any prediction about negative C-parity states will be more difficult to confirm or discard experimentally. However, even though not so probable, the production of the final $J/\Psi X$ state may happen via two virtual photons, in which case the XYZ resonance may have negative C-parity. This alternative production mechanism is not always as suppressed as expected, as demonstrated in Refs. [52, 53] for $e^+e^- \to \gamma^*\gamma^* \to J/\Psi\, J/\Psi$. In principle, a similar mechanism could take place in the $X(3940)$ state, in which case the identification with a 1^{+-} $D\bar{D}^*$ molecule would be very appealing, but it may also be possible that the $X(3955)$ molecular state has simply not been observed yet.

We have also examined the role played by the OPE potential in the $D^{(*)}\bar{D}^{(*)}$ system (Table II). In agreement with the conclusions of Ref. [30], we have verified that pion exchanges can be treated perturbatively in the case of isoscalar charm meson-antimeson molecules. Curiously, the suppression of the OPE effects is larger than naively expected in terms of the power counting. The remarkable simplification is that the EFT framework we are proposing consists entirely on contact range interactions at lowest order. According to Ref. [30] this interesting simplification will also apply in the isovector charm sector, as well as the isovector bottom one [38]. In this regard non-perturbative OPE seems to be only required in the case of isoscalar bottom meson-antimeson molecules. This possibility, and the corresponding EFT, was partially explored in Ref. [24] for the $B\bar{B}^*$/ $B^*\bar{B}$ case. Lastly, particle coupled channel dynamics are suppressed by two orders in the counting, as expected, but a more complete analysis would be welcomed, specially in what regards to regulator dependence.

Even though all the previous states and their possible identification with theoretically predicted / experimentally known resonances are contingent on the validity of the molecular hypothesis for the $X(3915)$, the bottomline of the approach we advocate is that, provided we identify at least two molecular states representing two different combinations of the C_{0a} and C_{0b} counterterms, we will be able to predict the full molecular spectrum of the $D^{(*)}\bar{D}^{(*)}$ system. If the $X(3915)$ identification proves erroneous in the future, the finding of a different molecular state candidate could be used to obtain the remaining states. Owing to the contact range character of the present EFT framework at lowest order, the calculational effort involved in this task will be minimal.

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Appendix A: The Effective Lagrangian at Lowest Order

In this appendix we write the EFT Lagrangian that describes the strong interactions of heavy mesons and antimesons containing a heavy quark Q or antiquark $\bar{Q}$ respectively. We use the matrix field $H^{(Q)}$ ($H^{(\bar{Q})}$) to denote the following combination of the pseudoscalar and vector heavy-meson (antimeson) fields

$$H_a^{(Q)} = \frac{1+\not{v}}{2}\left(P^{*(Q)}_{a\mu}\gamma^\mu - P^{(Q)}_a\gamma_5\right), \tag{A1}$$

$$H^{(\bar{Q})a} = \left(P^{*(\bar{Q})a}_{\mu}\gamma^\mu - P^{(\bar{Q})a}\gamma_5\right)\frac{1-\not{v}}{2}, \tag{A2}$$

where the pseudoscalar meson (antimeson) fields are represented by $P_a^{(Q)}$ ($P_a^{(\bar{Q})}$), while $P_a^{*(Q)}$ ($P_a^{*(\bar{Q})}$) is employed for their vector HQSS partners (see, for example, Ref. [66] for further details). Finally, v is the velocity parameter. In principle there should be a v subscript to indicate that we are defining the fields for a specific value of v, but we have omitted it to avoid complicating the notation. The fields are isospin doublets (hence the index a), where for the pseudoscalar meson and antimesons we have

$$P_a^{(Q/\bar{Q})} = (P^0, P^+), \tag{A3}$$

$$P_a^{(\bar{Q}/Q)} = (\bar{P}^0, P^-), \tag{A4}$$

plus the analogous expressions for the vector case. The heavy quark/antiquark superindex changes depending on whether we are considering charm or bottom meson fields (in the charm case, D^0 and D^+ contain the quark field, while in the bottom case $B^{(0)}$ and B^+ contain the antiquark field). The heavy vector meson and antimeson are subjected to the additional condition

$$v\cdot P_a^{*(Q)} = 0, \tag{A5}$$

$$v\cdot P^{*(\bar{Q})a} = 0, \tag{A6}$$

which in turn defines the three different polarizations of the heavy vector mesons.

The fields $H_a^{(Q)}$ and $H^{(\bar{Q})a}$ respectively transform as a $(2,\bar{2})$ and $(\bar{2},2)$ representation under the heavy quark spin $\otimes$ $SU(2)_V$ isospin symmetry [67], that is

$$H_a^{(Q)} \to S\left(H^{(Q)}U^\dagger\right)_a, \tag{A7}$$

$$H^{(\bar{Q})a} \to \left(UH^{(\bar{Q})}\right)^a S^\dagger, \tag{A8}$$

where S is the heavy quark spin transformation and U the isospin one. The hermitian conjugate fields are

$$\bar{H}^{(Q)a} = \gamma^0 \, H_a^{(Q)\dagger} \, \gamma^0 \,, \tag{A9}$$

$$\bar{H}_a^{(\bar{Q})} = \gamma^0 \, \bar{H}^{(\bar{Q})a\dagger} \, \gamma^0 \,, \tag{A10}$$

and transform as [67]

$$\bar{H}^{(Q)a} \to \left(U \bar{H}^{(Q)} \right)^a S^\dagger \,, \tag{A11}$$

$$\bar{H}^{(\bar{Q})_a} \to S \left(\bar{H}^{(\bar{Q})} U^\dagger \right)^a \,. \tag{A12}$$

Of course, the Lagrangian should be invariant under the previous symmetry transformations.

At leading order in the EFT expansion the Lagrangian can be written as the sum of two contributions

$$\mathcal{L}^{(0)} = \mathcal{L}^{(0)}_{4H} + \mathcal{L}^{(0)}_{\pi HH} \tag{A13}$$

where the first one contains a 4-meson interaction vertex and the second the meson-pion vertex. The 4-meson contact range Lagrangian consistent with HQSS and chiral symmetry [36] reads:

$$\begin{aligned}\mathcal{L}^{(0)}_{4H} = {} & D_{0a} \, \mathrm{Tr}\left[\bar{H}^{(Q)a} H_a^{(Q)} \gamma_\mu \right] \mathrm{Tr}\left[H^{(\bar{Q})b} \bar{H}_b^{(\bar{Q})} \gamma^\mu \right] + D_{0b} \, \mathrm{Tr}\left[\bar{H}^{(Q)a} H_a^{(Q)} \gamma_\mu \gamma_5 \right] \mathrm{Tr}\left[H^{(\bar{Q})b} \bar{H}_b^{(\bar{Q})} \gamma^\mu \gamma_5 \right] \\ & + E_{0a} \, \mathrm{Tr}\left[\bar{H}^{(Q)a} \, \vec{\tau}_a^{\,b} \, H_b^{(Q)} \gamma_\mu \right] \mathrm{Tr}\left[H^{(\bar{Q})r} \, \vec{\tau}_r^{\,s} \, \bar{H}_s^{(\bar{Q})} \gamma^\mu \right] + E_{0b} \, \mathrm{Tr}\left[\bar{H}^{(Q)a} \, \vec{\tau}_a^{\,b} \, H_b^{(Q)} \gamma_\mu \gamma_5 \right] \mathrm{Tr}\left[H^{(\bar{Q})r} \, \vec{\tau}_r^{\,s} \, \bar{H}_s^{(\bar{Q})} \gamma^\mu \gamma_5 \right] \,.\end{aligned} \tag{A14}$$

where τ_{ab} are the Pauli matrices, and a,b,r and s are isospin indices. We notice that for each isospin channel ($I = 0, 1$) we have only two independent constants.

On the other hand, at leading order in the chiral expansion the HHπ and $\bar{\mathrm{H}}\bar{\mathrm{H}}\pi$ couplings are determined by the Lagrangian [67]

$$\begin{aligned}\mathcal{L}^{(0)}_{\pi HH} = -\frac{g}{\sqrt{2} f_\pi} \Big\{ & \mathrm{Tr}\left[\bar{H}^{(Q)b} H_a^{(Q)} \gamma_\mu \gamma_5 \right] \\ & + \mathrm{Tr}\left[H^{(\bar{Q})b} \bar{H}_a^{(\bar{Q})} \gamma^\mu \gamma_5 \right] \Big\} (\vec{\tau} \cdot \partial_\mu \vec{\pi})_b^a \\ & + \mathcal{O}(\pi^2)\end{aligned} \tag{A15}$$

where $\vec{\pi}$ is the relativistic field that describes the pion, g is the $PP^*\pi$ coupling and $f_\pi \simeq 132\,\mathrm{MeV}$ the pion decay constant. In the charm sector, g has been determined from the D^* meson decays in Refs. [40, 41] yielding $g = 0.59 \pm 0.01 \pm 0.07$, which we approximate by $g \simeq 0.6$. In the strict heavy quark limit, the latest lattice QCD results suggest the value $g = 0.449 \pm 0.047 \pm 0.019$, see Refs. [68, 69]. In the normalization above the pion field has dimensions of [energy], while the heavy meson or antimeson fields $H^{(\bar{Q})}$ or $H^{(\bar{Q})}$ have dimensions of $[\mathrm{energy}]^{3/2}$: as usual in heavy quark physics, we employ a non-relativistic normalization for the heavy mesons that differs from the usual relativistic convention by a factor of $\sqrt{M_H}$ (see for instance Ref.[35]).

Appendix B: Projecting the Potential into the Partial Wave Basis

In this appendix we delineate how to project the heavy meson-antimeson potential into the partial wave basis. In first place we define the non-relativistic potential for the transition (not necessarily elastic)

$$\mathrm{H}(1)\bar{\mathrm{H}}(2) \to \mathrm{H}(1')\bar{\mathrm{H}}(2') \,, \tag{B1}$$

in terms of the tree level scattering amplitude

$$\mathcal{T}_{\mathrm{tree}} = -i \, \mathcal{V}(1 + 2 \to 1' + 2') \,, \tag{B2}$$

where 1,2 and 1′,2′ schematically represent the initial and final state of each of the particles. For heavy meson-antimeson scattering the initial (final) state is completely represented by the momentum exchanged between the particles $\vec{p} = \vec{p}_1 - \vec{p}_2$ ($\vec{p}\,' = \vec{p}_1\,' - \vec{p}_2\,'$) and by the total and third component of the spin of each of the particles, which we can collectively call $\sigma = \{(S_1 m_1)(S_2 m_2)\}$ ($\sigma' = \{(S_1' m_1')(S_2' m_2')\}$). If we compute $\mathcal{V}(1+2 \to 1'+2')$ in terms of the usual Feynman rules (each vertex contributes with $i\mathcal{L}$, additional i factors for each pion propagator in the case of OPE contributions, etc..), the relationship between the non-relativistic potential and the invariant scattering amplitude is

$$\langle \vec{p}\,'; \sigma' | V | \vec{p}; \sigma \rangle = \frac{1}{4} \mathcal{V}(\vec{p}, \sigma \to \vec{p}\,', \sigma') \,. \tag{B3}$$

Notice that the factor dividing the invariant scattering amplitude is 4, instead of the usual $4\sqrt{M_1 M_2 M_1' M_2'}$, owing to the $\sqrt{M_H}$ normalization factor included in the heavy meson/antimeson fields.

Now we specify the procedure for the partial wave projection of the potential. To denote the different partial waves we employ the spectroscopic notation $^{2S+1}L_J$, where S, L and J are the intrinsic, orbital and total angular momentum. With this, we define the states with good angular momentum as follows

$$\begin{aligned}|p\,; JMLS\rangle = {} & \frac{1}{\sqrt{4\pi}} \sum_{M_L, M_S} (LSJ | M_L M_S M) \\ & \times \int d\Omega(\hat{p}) Y_{L,M_L}(\hat{p}) | \vec{p}, S M_S \rangle \,,\end{aligned} \tag{B4}$$

where p is the modulus of the center-of-mass (c.m.) momentum $\vec{p}\,(= p \times \hat{p})$ of the $\mathrm{H}\bar{\mathrm{H}}$ pair and $(LSJ|M_L M_S M)$

is a Clebsch-Gordan coefficient. The normalization of the states above can be determined from the normalization of the plane wave basis, that is

$$\langle \vec{p}\,'; S'M'_S \,|\vec{p}; SM_S\rangle = (2\pi)^3\,\delta^3(\vec{p}-\vec{p}\,')\,\delta_{S,S'}\delta_{M_S M'_S}\,, \tag{B5}$$

yielding

$$\langle p'; J'M'L'S' \,|p\,; JMLS\rangle = (2\pi)^3\,\frac{\delta(p'-p)}{4\pi\,p\,p'}\,\delta_{JJ'}\delta_{MM'}\delta_{LL'}\delta_{SS'}\,. \tag{B6}$$

In this basis the partial wave projection of the potential reads

$$\begin{aligned} V^{S'S}_{JL'L}(p',p) &\equiv \langle p'; JML'S' \,|V|p\,; JMLS\rangle \\ &= \frac{1}{4\pi}\int d\Omega(\hat{p})\int d\Omega(\hat{p}')\sum_{M_L M_S M'_L M'_S}(LSJ|M_L M_S M)(L'S'J|M'_L M'_S M)Y^*_{L',M'_L}(\hat{p}')Y_{L,M_L}(\hat{p}) \\ &\times \sum_{m_1 m_2 m'_1 m'_2}(S_1S_2S|m_1m_2M_S)(S'_1S'_2S'|m'_1m'_2M'_S)\,\langle \vec{p}\,'; (S'_1m'_1)(S'_2m'_2)|V|\vec{p}\,; (S_1m_1)(S_2m_2)\rangle\,, \end{aligned} \tag{B7}$$

where thanks to rotational invariance the above matrix element is independent of the third component of the total angular momentum M.

Appendix C: The Lowest Order Heavy Meson-Antimeson Potential and Its Partial Wave Projection

The lowest order $H\bar{H}$ potential contains a contact and a finite range (pion exchange) piece. The EFT potential can be derived from the tree level scattering amplitudes resulting from the $\mathcal{L}^{(0)}_{4H}$ and $\mathcal{L}^{(0)}_{\pi HH}$ Lagrangians of Eqs. (A14) and (A15). Even though the partial wave projection of the contact piece is trivial, we will start with the OPE potential in order to fix the notation. The tree level invariant amplitude can be obtained from the pion-meson $\mathcal{L}^{(0)}_{\pi HH}$ Lagrangian of Eq. (A15), taking the schematic form in the strict heavy quark limit,

$$\mathcal{V}_{\rm OPE}(\vec{p}\,',\vec{p}) \propto \frac{(\vec{a}\cdot\vec{q})(\vec{b}\cdot\vec{q})}{\vec{q}^{\,2}+m_\pi^2}\,, \tag{C1}$$

where $\vec{q}=\vec{p}-\vec{p}\,'$ (that is, the potential is local) and $\vec{a}$, $\vec{b}$ is the corresponding polarization operator in each of the $H\bar{H}\pi$ vertices. The proportionality factor is $g^2/8f_\pi^2$ times a sign that depends the pseudoscalar or vector nature of each of the particles in the initial and final states. The modifications to take into account in the above equation the mass difference between the pseudoscalar and vector heavy mesons masses are also discussed in Refs. [24, 30]. Since we will not be considering particle coupled channels with the OPE interaction, this becomes an issue in this work only for the $D\bar{D}^*\to D^*\bar{D}$ channel. In that case in Eq. (C1), m_π^2 should be substituted by $\mu_\pi^2 = m_\pi^2-(m_{D^*}-m_D)^2$. Indeed, $\mu_\pi^2\le 0$, since there is a very small absorptive contribution from the $D\bar{D}\pi$ channel. We will neglect it, as in Ref. [24], and in that case we will consider only the real part of the potential. We now continue by Fourier transforming the amplitude above into coordinate space. This step, though counter-intuitive at first sight, will enormously facilitate the calculation of the partial wave projection of the potential in momentum space. We remind that the coordinate and momentum space potentials are related by

$$\langle \vec{p}\,'; \sigma'|V|\vec{p}\,;\sigma\rangle = \int d^3r\, e^{i(\vec{p}-\vec{p}\,')\cdot\vec{r}}\,\langle\sigma'|V(\vec{r})|\sigma\rangle\,, \tag{C2}$$

and we use the symbols σ and σ' to encode all the spin indices (see the previous appendix). Then we make use of a well-known relationship

$$\int\frac{d^3q}{(2\pi)^3}\,\frac{(\vec{a}\cdot\vec{q})(\vec{b}\cdot\vec{q})}{\vec{q}^{\,2}+m_\pi^2}e^{-i\vec{q}\cdot\vec{r}} = -\vec{a}\cdot\vec{\nabla}\,\vec{b}\cdot\vec{\nabla}\left(\frac{e^{-m_\pi r}}{4\pi\,r}\right) \tag{C3}$$

from which we obtain the form of the OPE potential

$$V_{OPE}(\vec{r}) \propto \frac{\vec{a}\cdot\vec{b}}{3}\,\delta^3(\vec{r}) - \Big(v_C(r)\,\vec{a}\cdot\vec{b} + \,v_T(r)\,S_{12}(\vec{a},\vec{b})\Big)\,. \tag{C4}$$

In this equation $S_{12}(\vec{a},\vec{b})$ is the tensor operator, which we define as

$$S_{12}(\vec{a},\vec{b}) = \frac{3(\vec{a}\cdot\vec{r})(\vec{b}\cdot\vec{r})}{r^2} - \vec{a}\cdot\vec{b}\,. \tag{C5}$$

In turn, the central and tensor pieces of the potential, v_c and v_T, are given by

$$v_C(r) = \frac{m_\pi^3}{12\pi}\left(\frac{e^{-m_\pi r}}{m_\pi r}\right), \tag{C6}$$

$$v_T(r) = v_C(r)\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right). \tag{C7}$$

Of course, we are interested in the partial wave projection of the potential above, $V_{JL'L}^{S'S}(r)$, since its partial wave Fourier transform provides the multipole expansion of the potential in momentum space (Eq. (B7))

$$V_{JL'L}^{S'S}(p',p) = 4\pi i^{L-L'} \times \int_0^{+\infty} dr\, r^2 j_L(p\,r)\, j_{L'}(p'r)\, V_{JL'L}^{S'S}(r)\,, \tag{C8}$$

where $j_L(x)$ represent the spherical Bessel function of order L. The advantage of this expression is that it can be analytically evaluated with relative ease for the OPE potential. After a bit of Racah algebra and taking into account all the signs and factors we have obviated so far (again, details can be consulted in Ref. [30]), we arrive at the final expression for the OPE potential in the partial wave basis

$$(V_{OPE})_{J\,L'L}^{S'S}(r) = -\frac{g^2}{6f_\pi^2}\,\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{\delta(r)}{4\pi r^2}\,C_{12} + \frac{g^2}{2f_\pi^2}\,\vec{\tau}_1\cdot\vec{\tau}_2\,[v_C(r)\,C_{12}+v_T(r)\,S_{12}]\,, \tag{C9}$$

where all the calculational complications are conveniently hidden in C_{12} and S_{12}, the partial wave projections of the $\vec{a}\cdot\vec{b}$ and $S_{12}(\vec{a},\vec{b})$ operators: C_{12} and S_{12} depend on J, L, L', S and thus encode all the information required to determine the coordinate space potential in a particular partial wave.

The most compact way to write the C_{12} and S_{12} factors is in matrix form, where the matrix is defined in a basis formed by the set of partial waves with well-defined quantum numbers J^{PC}. We can illustrate this by considering all the J^{PC} combinations that contain S-waves, that is, the J^{PC} values we have studied in this work. First we consider the set of 0^{++} partial waves, defined as

$$\mathcal{B}(0^{++}) = \left\{D\bar{D}(^1S_0), D^*\bar{D}^*(^1S_0), D^*\bar{D}^*(^5D_0)\right\}, \tag{C10}$$

from which the $\mathbf{C}_{12}$ and $\mathbf{S}_{12}$ matrices are

$$\mathbf{C}_{12}(0^{++}) = \begin{pmatrix} 0 & & \\ -\sqrt{3} & 2 & \\ 0 & 0 & -1 \end{pmatrix}, \tag{C11}$$

$$\mathbf{S}_{12}(0^{++}) = \begin{pmatrix} 0 & & \\ 0 & 0 & \\ \sqrt{6} & \sqrt{2} & 2 \end{pmatrix}, \tag{C12}$$

where we have only shown the lower and diagonal components as the matrices are symmetric. Next we move to the 1^{++} case, for which we have the particle states $\mathrm{D\bar{D}^*}$ and $\mathrm{D^*\bar{D}}$ that we need to arrange in states with good C-parity. We thus define

$$[D\bar{D}^*(\eta)] = \frac{1}{\sqrt{2}}\left[D\bar{D}^* - \eta\, D^*\bar{D}\right]. \tag{C13}$$

In this convention the intrinsic $C-$parity of these states is independent of the isospin and equal to η. The 1^{++} basis thus reads

$$\mathcal{B}(1^{++}) = \Big\{[D\bar{D}^*(+)](^3S_1), [D\bar{D}^*(+)](^3D_1), D^*\bar{D}^*(^5D_1)\Big\}, \tag{C14}$$

for which we obtain the matrices

$$\mathbf{C}_{12}(1^{++}) = \begin{pmatrix} -1 & & \\ 0 & -1 & \\ 0 & 0 & -1 \end{pmatrix}, \tag{C15}$$

$$\mathbf{S}_{12}(1^{++}) = \begin{pmatrix} 0 & & \\ \sqrt{2} & -1 & \\ \sqrt{6} & \sqrt{3} & 1 \end{pmatrix}, \tag{C16}$$

The next case is 1^{+-}, in which we employ the basis

$$\mathcal{B}(1^{+-}) = \Big\{[D\bar{D}^*(-)](^3S_1), [D\bar{D}^*(-)](^3D_1), D^*\bar{D}^*(^3S_1), D^*\bar{D}^*(^3D_1)\Big\}, \tag{C17}$$

and get the matrices

$$\mathbf{C}_{12}(1^{+-}) = \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ -2 & 0 & 1 & \\ 0 & -2 & 0 & 1 \end{pmatrix}, \tag{C18}$$

$$\mathbf{S}_{12}(1^{+-}) = \begin{pmatrix} 0 & & & \\ -\sqrt{2} & 1 & & \\ 0 & -\sqrt{2} & 0 & \\ -\sqrt{2} & 1 & -\sqrt{2} & 1 \end{pmatrix}. \tag{C19}$$

The most complex case is 2^{++}, in which OPE mixes all the possible particle channels. We thus work in the basis

$$\mathcal{B}(2^{++}) = \left\{ D\bar{D}(^1D_2), [D\bar{D}^*(+)](^3D_2), D^*\bar{D}^*(^1D_2), D^*\bar{D}^*(^5S_2), D^*\bar{D}^*(^5D_2), D^*\bar{D}^*(^5G_2) \right\}, \tag{C20}$$

and obtain the matrices

$$\mathbf{C}_{12}(2^{++}) = \begin{pmatrix} 0 & & & & & \\ 0 & -1 & & & & \\ -\sqrt{3} & 0 & 2 & & & \\ 0 & 0 & 0 & -1 & & \\ 0 & 0 & 0 & 0 & -1 & \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \tag{C21}$$

$$\mathbf{S}_{12}(2^{++}) = \begin{pmatrix} 0 & & & & & \\ 0 & 1 & & & & \\ 0 & 0 & 0 & & & \\ \sqrt{\frac{6}{5}} & -3\sqrt{\frac{2}{5}} & \sqrt{\frac{2}{5}} & 0 & & \\ -2\sqrt{\frac{3}{7}} & \frac{3}{\sqrt{7}} & -\frac{2}{\sqrt{7}} & -\sqrt{\frac{14}{5}} & -\frac{3}{7} & \\ 6\sqrt{\frac{3}{35}} & \frac{12}{\sqrt{35}} & \frac{6}{\sqrt{35}} & 0 & -\frac{12}{7\sqrt{5}} & \frac{10}{7} \end{pmatrix}. \tag{C22}$$

Now we consider the contact range piece of the potential, which can be easily derived from the 4-meson $\mathcal{L}^{(0)}_{4H}$ Lagrangian of Eq. (A14). For that, we expand the $\mathcal{L}^{(0)}_{4H}$ Lagrangian into its explicit representation in terms of pseudoscalar and vector heavy meson fields (detailed expressions can be consulted in Ref. [30]). The subsequent partial wave projection is straightforward (owing to the simplification of working with S-waves only):

$$(V_C)^{S'S}_{J\,L'L}(r) = \frac{\delta(r)}{4\pi r^2}\left(D_{0a} - E_{0a}\,\vec{\tau}_1\cdot\vec{\tau}_2\right)\delta_{JS}\,\delta_{J'S'}\,\delta_{L0}\,\delta_{L'0} + \frac{\delta(r)}{4\pi r^2}\left(D_{0b} - E_{0b}\,\vec{\tau}_1\cdot\vec{\tau}_2\right)C_{12}\,, \tag{C23}$$

from which the potentials of Eqs. (18)–(21) are derived, Notice however that in Eqs. (18)–(21) we have specified the potential according to the J^{PC} quantum number of the heavy meson-antimeson system. The relation between the couplings of Eqs. (18)–(21) and the corresponding ones in the $\mathcal{L}^{(0)}_{4H}$ Lagrangian is provided by the expressions

$$\begin{aligned} C_{0a} &= D_{0a} + 3\,E_{0a}\,, \\ C_{0b} &= D_{0b} + 3\,E_{0b} \end{aligned} \tag{C24}$$

where we have isolated the isoscalar contribution [10]. If we are considering the full potential resulting from the sum of the contact range and OPE contribution

$$V^{S'S}_{J\,L'L}(r) = (V_C)^{S'S}_{J\,L'L}(r) + (V_{OPE})^{S'S}_{J\,L'L}(r)\,, \tag{C25}$$

we see that the $\delta(r)$ contribution within the OPE potential can be absorbed within the contact range piece by means of the replacement

$$C_{0b} \to C_{0b} - \frac{g^2}{2f_\pi^2}\,, \tag{C26}$$

which in fact is able to account for the bulk of the change of this operator when we add the OPE potential, see the related discussion in Sect. IV.

[10] For the isovector states we get $C_{0a}(I=1) = D_{0a} - E_{0a}$ and $C_{0b}(I=1) = D_{0b} - E_{0b}$

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