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Few-Body Systems

Recent Developments in one and two Pion Production in Elementary Reactions and Few Body Systems (Talk given at 'XVth European Conference on Few-Body Problems in Physics, FB XV)

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Abstract. In this talk we cover several issues concerning pion production. The first one is the $pp \to pp\pi^0$ reaction where an alternative explanation based on the off shell extrapolation of the πN amplitude is offered. A recent model for the $\gamma N \to \pi\pi N$ reaction is presented and a new kind of exchange current is constructed from it which allows one to address problems like double Δ photoproduction from the deuteron. Finally the $(\gamma, \pi\pi)$ reaction in nuclei is studied and shown to be highly sensitive to the renormalization of the pions in nuclei.

1 The $pp \to pp\pi^0$ reaction at threshold.

This reaction has been addressed in different works [1, 2] and shown to be in strong disagreement with very precise data obtained at the Indiana Cyclotron [3]. The starting point is a model which contains the impulse approximation, diagram 1a) and the rescattering term, diagram 1b).

The first mechanism involves the usual Yukawa vertex and is only possible due to the initial and final state interaction. The second one involves the πN scattering amplitude in one of the vertices which is given in terms of an effective Lagrangian

$$\delta H = 4\pi \frac{\lambda_1}{m_\pi} \bar{\psi} \boldsymbol{\phi}\boldsymbol{\phi} \psi + 4\pi \frac{\lambda_2}{m_\pi^2} \bar{\psi} \boldsymbol{\tau}\boldsymbol{\phi} \times \dot{\boldsymbol{\phi}} \psi \qquad (1)$$

where $\lambda_1 = 0.0075$ and $\lambda_2 = 0.053$ for on shell pions at threshold. The $\phi_3 \times \dot{\phi}_3$ combination is null and hence the second term in eq. (1) does not contribute for the $\pi^0 p \to \pi^0 p$ amplitude.

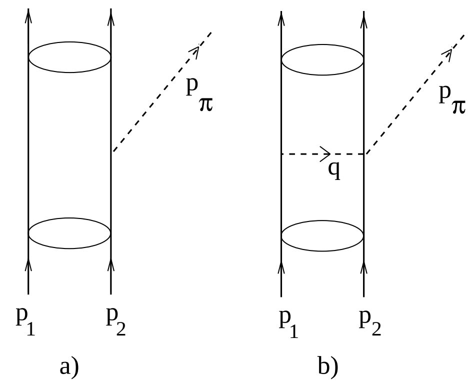

Figure 1. Feynman diagrams considered in the $pp \to pp\pi^0$ reaction near threshold. a) Born term; b) rescattering term.

Hence only the λ_1 term contributes, which gives a small contribution given the smallness of λ_1. Actually λ_1 is strictly zero in the limit of exact chiral symmetry. A way out of the puzzle was offered in [4] in terms of a relativistic exchange current involving σ exchange between positive and negative energy components. This exchange is tied to the scalar potential, V_s, of the relativistic potential of Walecka type $V_S + \gamma^0 V_V$. This interpretation which can not be excluded a priori, does not have however a strong base, because the exchange current is generated from a relativistic form of the $NN \to NN$ amplitude which is unique for on shell nucleons. However, there are an infinite amount of possible forms of the amplitude which are equivalent for on shell nucleons but which provide quite different off shell extrapolations or extensions to the states of negative energy, as is the case in [4]. One typical example is the γ_5 matrix which is equivalent to $\not{q}\gamma_5/2m$ for on shell nucleons but which provides different extensions in the negative energy sector. The work of [4] is based on a particular choice for the NN amplitude but one can obtain quite different results for the $pp \to pp\pi^0$ reaction if one starts from other expressions which are equivalent for on shell nucleons. More concretely, recent models [5] which consider σ exchange as a correlated two pion exchange would provide a different answer in the negative energy sector than the ordinary σ exchange.

We have followed here a different idea suggested in [6]. The idea is that the πN amplitude appearing in the rescattering term, fig. 1b) is not the on shell πN amplitude but the half off shell amplitude and this one is much bigger than the on shell one in all existing models of the off shell extrapolation.

In a recent work [7] we have carried out detailed calculations of the $pp \to pp\pi^0$ cross section using two different off shell extrapolations for λ_1, one from Hamilton [8] which contains a σ exchange with the $\sigma\pi\pi$ and σNN vertices and a short range piece, which approximately cancel on shell. The other one, from Banerjee [9], is based on PCAC and has also been extensively used (we use a regularized version of it to have a well behaved Fourier Transform). The two

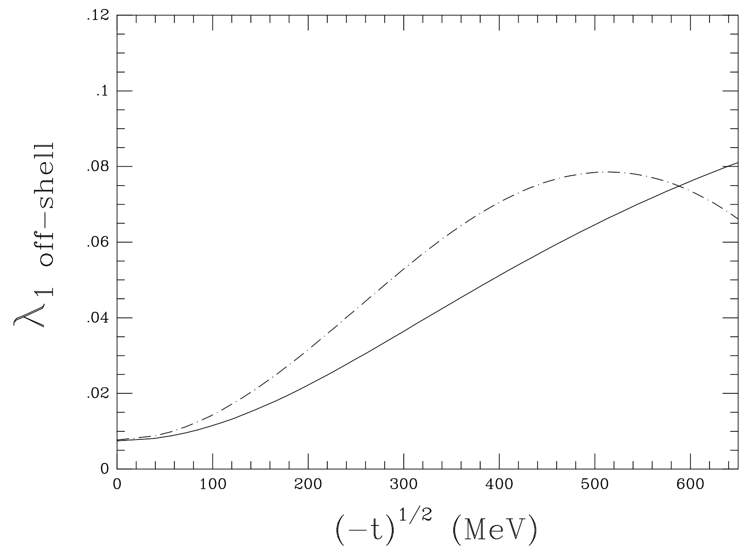

Figure 2. Off shell extrapolation of the πN isoscalar amplitudes in the Hamilton [8] (solid line) and current algebra model [9] (dashed-dotted line).

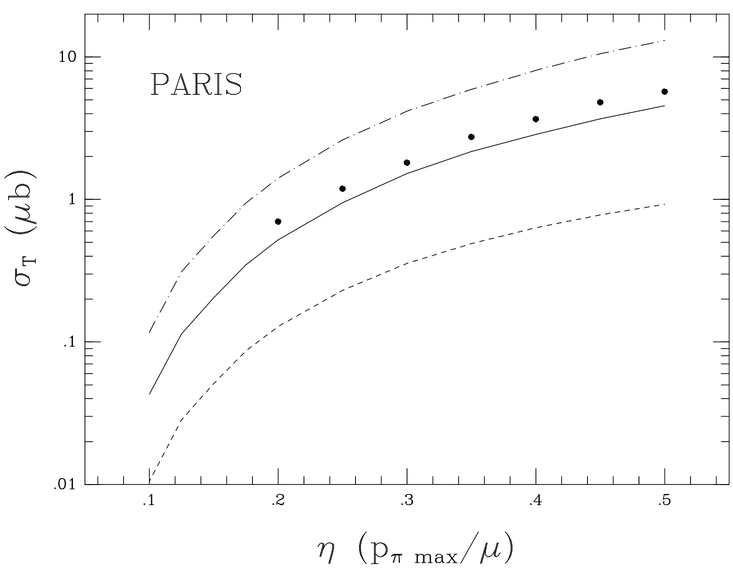

Figure 3. Cross section for $pp \to pp\pi^0$ near threshold using the Paris NN potential for the initial and final state interaction. Dashed line: with in shell πN isoscalar amplitude. Solid line: with off shell πN amplitude from Hamilton's model [8]. Dashed dotted line: same with the current algebra πN extrapolation [9].

curves can be seen in fig. 2, where we appreciate the large enhancement of the off shell amplitude for momenta around 400 MeV which are the relevant ones for the present problem (the upper curve corresponds to [9] and the lower one to [8]).

In fig. 3 we show the results which we obtain for the cross section of the $pp \to pp\pi^0$ reaction close to threshold. The dashed curve is the result obtained using the $\pi N \to \pi N$ on shell amplitude, while the dash-dotted curve and the solid line give the results obtained using the Banerjee and Hamilton extrapolations respectively. The results are obtained using the Paris potential for the initial and final state interaction and the results are similar, although a little higher, if the Bonn potential is used.

It is clear from the results that one can not be very assertive, because there are still uncertainties on the off shell extrapolation, but the results plotted there clearly show that the off shell extrapolation by itself can easily explain the experimental data, offering a down to earth explanation of the discrepancies with experiment based on the use of the on shell πN amplitude.

2 Double pion photoproduction on the nucleon.

A recent model for the $\gamma p \to \pi^+\pi^- p$ reaction has been presented in [10]. The model is schematically depicted in fig. 4, where the baryonic intermediate states stand for $N, \Delta, N^*(1440)$ and $N^*(1520)$. In total 67 Feynman diagrams appear when this states are considered, although some terms found to be negligible are not taken into account. At energies below $E_\gamma = 800$ MeV many of those terms are also negligible and a simplified model, fig. 5, which contains only 20 terms, is sufficiently good, and this is the one used for the study of the other isospin channels.

The results for the $\gamma p \to \pi^+\pi^- p$ reaction are shown in fig. 6. Four different lines are depicted there which we explain below.

The dominant term in the $\gamma p \to \pi^+\pi^- p$ amplitude is the one involving the $\gamma N \Delta \pi$ Kroll-Ruderman term (diagram (i) in fig. 5). It just happens that the amplitude depicted in fig. 5(p), with the excitation of the $N^*(1520)$ resonance which later on decays in $\Delta^{++}\pi^-$, has a piece in the amplitude with exactly the same spin and momentum structure as the dominant term of fig. 5(i) and gives rise to a strong interference even if individually it is a small term. This is one of the surprising and interesting accidents of this reaction which allows one to determine the signs of the s- and d-wave amplitudes for the $N^*(1520) \to \Delta\pi$ reaction, something which poses new challenges to quark models of the hadrons [11]. To envisage this we write here the basic ingredients of these amplitudes which are the couplings $N^*(1520) \to N\gamma$ and $N^*(1520) \to \Delta\pi$:

$$-i\delta H_{N^*N\gamma} = i g_\gamma \boldsymbol{S}^\dagger \cdot \boldsymbol{\varepsilon} + g_\sigma (\boldsymbol{\sigma} \times \boldsymbol{S}) \cdot \boldsymbol{\varepsilon}$$

$$-i\delta H_{N^*\Delta\pi} = -[f + \frac{g}{\mu^2}(\boldsymbol{S}_\Delta \cdot \boldsymbol{q})^2] T^{\dagger\lambda} \tag{2}$$

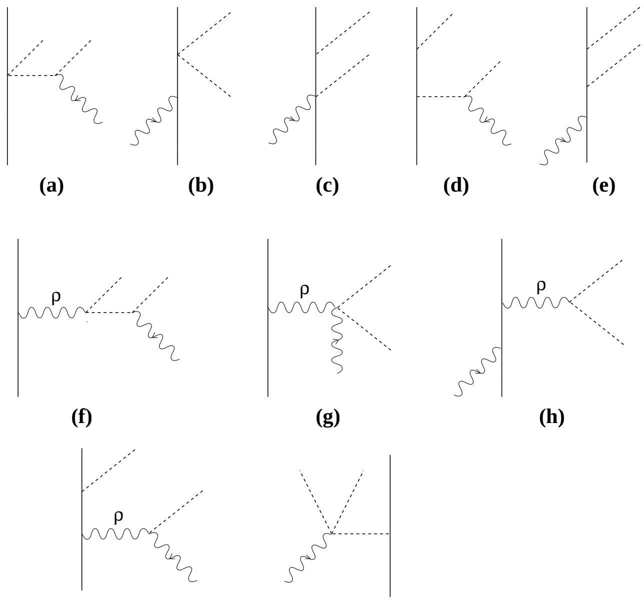

Figure 4. Classification of the Feynman diagrams into one point, two point and three point diagrams. Continuous straight lines: baryons. Dashed lines: pions. Wavy lines: photons and ρ-mesons (marked explicitly).

where $\boldsymbol{S}, \boldsymbol{T}$ are spin, isospin transition operators from spin, isospin 1/2 to 3/2, $\boldsymbol{S}_\Delta$ the ordinary spin matrix for spin 3/2 and $\boldsymbol{\epsilon}$ is the photon polarization vector. The amplitudes g_γ and g_σ are fitted to the experimental $N^*(1520)$ helicity amplitudes, and the strength of the s-wave and d-wave partial widths in the $N^* \to \Delta\pi$ decay are also taken from experiment. This gives, however, four possible solutions:

$$
\begin{aligned}
&a)\ f = -0.19\ ,\ g = 0.18 \quad ; \quad b)\ f = 1.03\ ,\ g = -0.18\ ; \\
&c)\ f = -1.03\ ,\ g = 0.18 \quad ; \quad d)\ f = 0.19\ ,\ g = -0.18
\end{aligned} \tag{3}
$$

The interference of the amplitudes is between the Kroll-Ruderman term of fig. 5(i), and the N^* term of fig. 5(p) through the s-wave decay amplitude of the $N^* \to \Delta\pi$. Thus one finds a combined amplitude

$$
i\frac{f^*}{\mu}\boldsymbol{S}\cdot\boldsymbol{q}_+ D_\Delta \left[e\frac{f^*}{\mu} - (g_\gamma - g_\sigma)\left(f + \frac{5}{4}g\frac{\boldsymbol{q}^{\,2}}{\mu^2}\right) D_{N^*}\right] \boldsymbol{S}^\dagger \cdot \boldsymbol{\varepsilon} \tag{4}
$$

where $\boldsymbol{q}_+$ is the momentum of the π^+ and D_Δ, D_{N^*} are the propagators of the Δ and $N^*(1520)$ respectively.

One can see in fig. 6 that out of the four possible solutions of eq. (3) only one, solution (b), gives rise to good agreement with experiment [12] . The

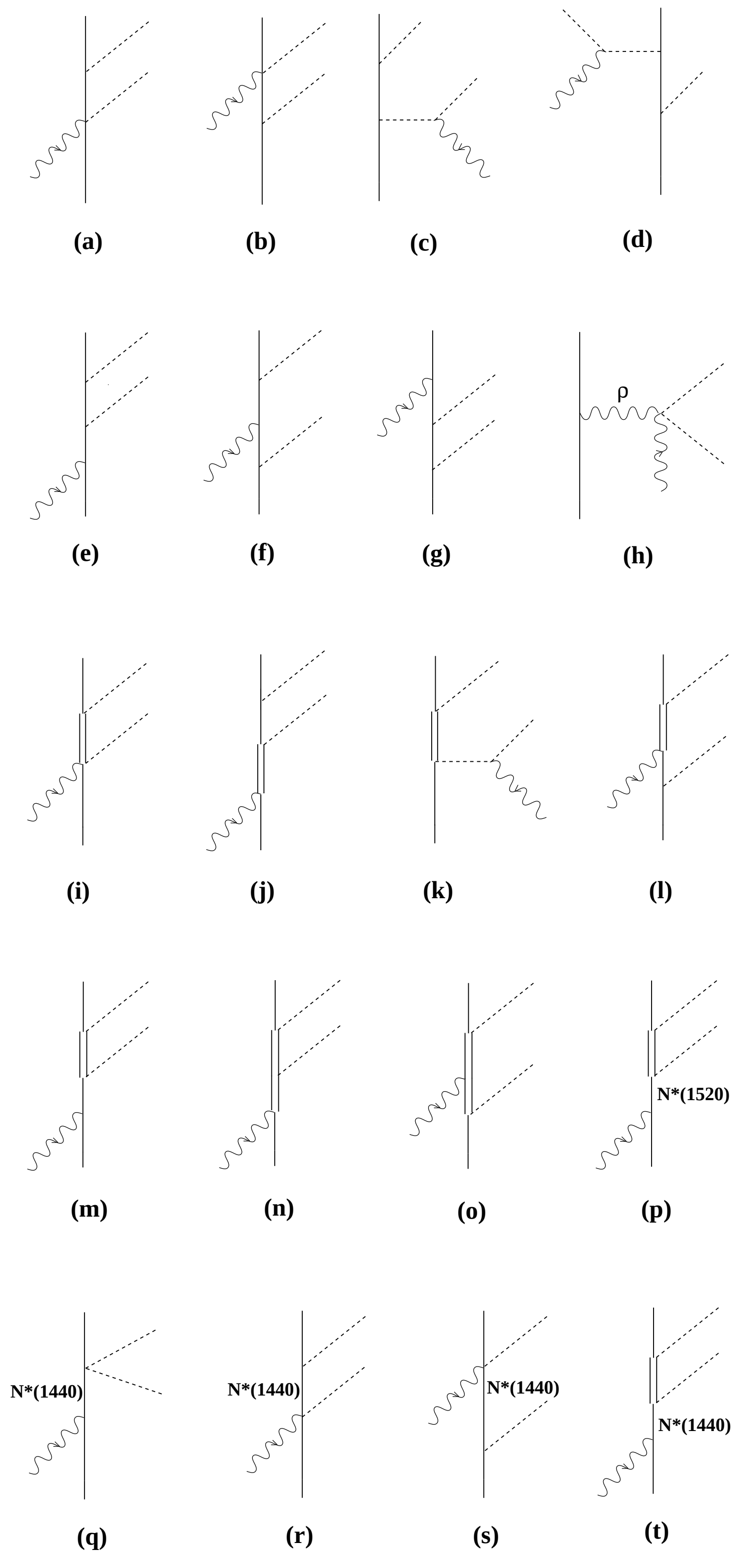

Figure 5. Feynman diagrams for the $\gamma N \to \pi\pi N$ reactions.

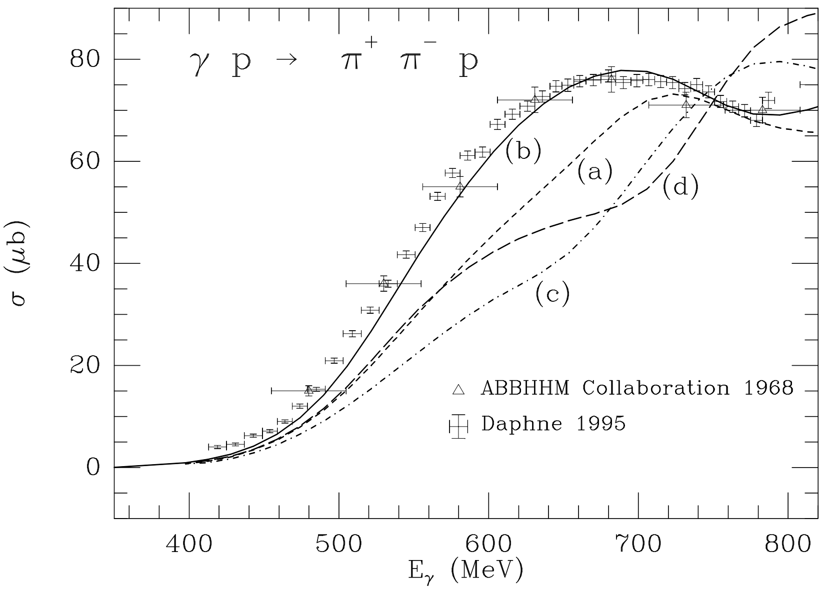

Figure 6. Total cross section for the $\gamma p \to \pi^+\pi^- p$ reaction. See text for explanation.

reason of the interference can be seen in eq. (4). Since $g_\gamma - g_\sigma = 0.157 > 0$ and $(f + \frac{5}{4} g \boldsymbol{q}^2/\mu^2) > 0$ one finds constructive interference below the $N^*(1520)$ energy and destructive interference above that energy and this interference is what gives rise to the peak in the cross section, which does not appear without the $N^*(1520)$ contribution.

We have calculated the cross section for other channels. One of them, particularly interesting, is the $\gamma p \to \pi^0\pi^0 p$ also measured in [12]. We can see in fig. 7 that again only the solution (b) is the one that best fits the data. We should mention that the TAPS collaboration has also preliminary data [13] which agree even better with our results in this channel.

For reasons of space we shall not show here the results for other channels. Only we mention that we find disagreement with the experimental results in the $\gamma p \to \pi^+\pi^0 n$ channel for which we do not yet envisage an explanation.

3 Double Δ production in the $\gamma d \to pn\pi^+\pi^-$ reaction

In a recent paper [14] we have tackled the reaction $\gamma d \to \Delta^{++}\Delta^-$ which has been measured recently [15].

The excitation of two Δ necessarily requires the participation of two nucleons. We need some exchange currents mechanism which generate this transition. The previous model for the $\gamma p \to \pi^+\pi^- p$ reaction offers us a clue on how to proceed. Let us take the diagrams of fig. 5(i), 5(p), which together with the pion pole term (diagram (k) of fig. 5), give already a fair description of the reaction.

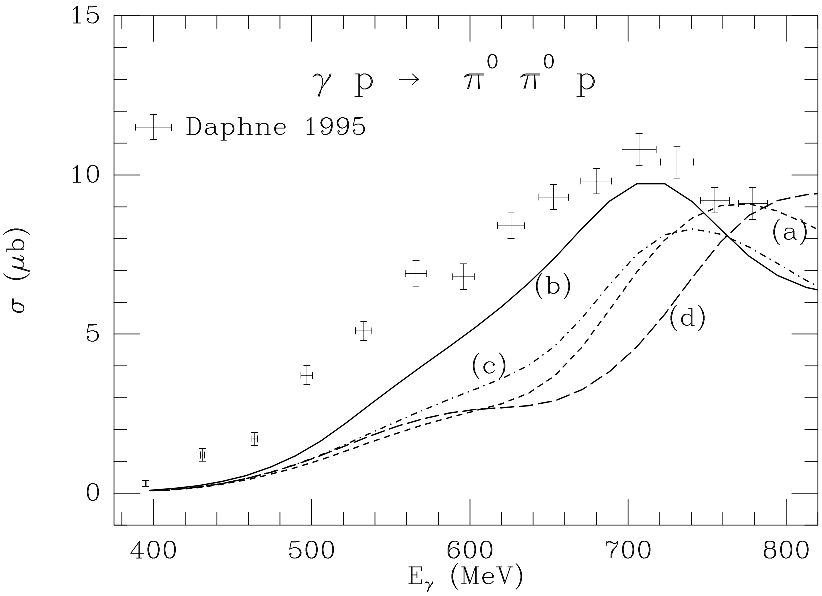

Figure 7. Total cross section for the $\gamma p \to \pi^0\pi^0 p$ reaction. See text for explanation.

We can attach the π^- produced to a second nucleon and produce a second Δ. The diagrams which appear are shown in fig. 8, providing a fair model for this reaction up to energies of around $E_\gamma \simeq 800$ MeV, where the elementary model for the $\gamma p \to \pi^+\pi^- p$ reaction starts having deficiencies.

The results for the $\gamma d \to \Delta^{++}\Delta^-$ reaction can be seen in fig. 9 compared with the experiment. The dashed line omits the contribution of the $N^*(1520)$ while the solid one takes it into account. We can see that there is a good agreement with experiment up to around $E_\gamma = 800$ MeV and the data hint at discrepancies from there on. The consideration of the $N^*(1520)$ term seems to improve the results a bit.

4 Double pion photoproduction in nuclei.

We have studied the $(\gamma, \pi^+\pi^-)$ inclusive reaction in nuclei in order to see renormalization effects due to the interaction of pions with the nucleus. Such effects were predicted in [16] and observed in [17] for the $(\pi, 2\pi)$ reaction in nuclei and they could only be magnified here since the photons penetrate deeper than the pions. We evaluate the cross section by assuming the nucleus to be made of bits of infinite nuclear matter at each point of the nucleus. This is the essence of the local density approximation, which given the fact that the photons explore the whole nuclear volume, is a very good approximation [18].

The cross section for this process in the "improved" impulse approximation would be

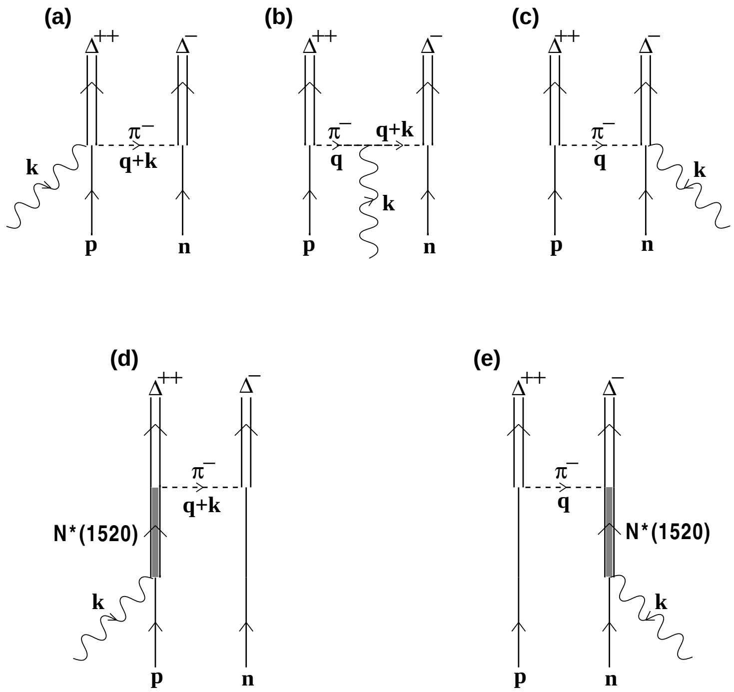

Figure 8. Terms considered in our model for the $\gamma d \rightarrow \Delta^{++}\Delta^{-}$ reaction.

$$\sigma = \frac{\pi}{k}\int d^3r \int \frac{d^3p}{(2\pi)^3}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\sum_{\alpha}\overline{\sum}\sum |T_\alpha|^2$$

$$n_\alpha(\boldsymbol{p})[1-n_\alpha(\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{q}_1-\boldsymbol{q}_2)]\frac{1}{2\omega(q_1)}\frac{1}{2\omega(q_2)}$$

$$2\pi\delta(k^0+E(p)-\omega(\boldsymbol{q_1})-\omega(\boldsymbol{q_2})-E(\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{q}_1-\boldsymbol{q}_2)) \qquad (5)$$

where $\boldsymbol{p}$ is the momentum of the nucleons from the Fermi sea, $\boldsymbol{k}$ the momentum of the photon and $\boldsymbol{q}_1, \boldsymbol{q}_2, \omega(\boldsymbol{q}_1), \omega(\boldsymbol{q}_2)$ the momenta and energies of the two pions. The variable α indicates isospin channels, and $n_\alpha(\boldsymbol{p})$ is the nucleon occupation number in the Fermi sea. Since

$$2\int \frac{d^3p}{(2\pi)^3}\, n_\alpha(\boldsymbol{p}) = N \text{ or } Z \qquad (6)$$

one is essentially multiplying by N or Z (and summing) the individual $\gamma N \rightarrow \pi^+\pi^- N$ cross sections, with two important modifications: i) the Fermi motion is explicitly taken into account ii) the Pauli blocking factor $[1-n]$ is explicitly considered there. The formulation of the problem is easily done in terms of the selfenergy of the photon, $\Pi(\boldsymbol{k})$ and the relationship

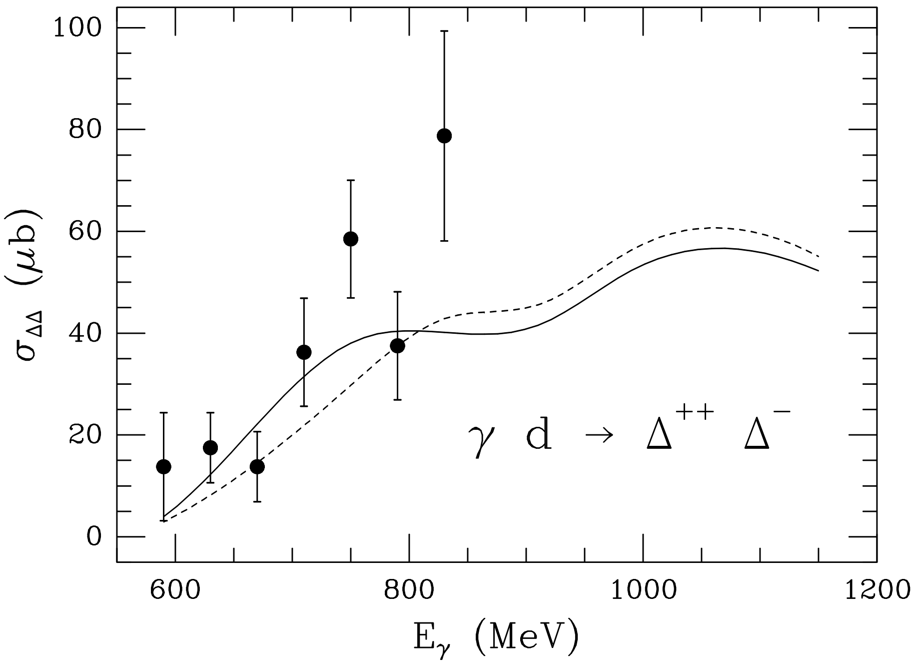

Figure 9. Results of the model compared to the data of ref. [15]. Dashed line, omitting the N^* terms. Solid line, results including all terms of the model of Fig. 8.

$$\sigma = -\frac{1}{k}\int Im\Pi\left(\boldsymbol{k}, \rho\left(\boldsymbol{r}\right)\right) d^3r \tag{7}$$

which leads to eq. (5). The formulation in terms of a selfenergy diagram allows for systematic corrections. An important one arises from the consideration of the renormalization of the pion propagators, which essentially substitutes $\omega(\boldsymbol{q})$ in eq. (5) by $\tilde{\omega}(q)$, the pion dispersion relation in the medium. Simultaneously we also renormalize the Δ and pion propagators in the amplitudes for the model of $(\gamma, \pi^+\pi^-)$. In addition one must distort the pion waves by the imaginary part of the pion selfenergy due to absorption in order to take into account the absorption of the pions in their way out of the nucleus.

In fig. 10 we can see the results of the evaluation of [19]. The curve labelled 1 is the impulse approximation $N\sigma_n + Z\sigma_p$ multiplied by the pion absorption factor. The curve 2 is a pure scaling of the cross section in the deuteron $(A\sigma_d/2)$, while curve 3 is the result from our calculations. The differences between 1 and 3 indicate effects of renormalization, which increase the cross section by a factor from 4 to 5. It is also interesting to see that in spite of the absorption of the final pions, the cross section obtained is still larger than A/2 times the cross section in the deuteron.

It would be interesting to perform such experiments, easily implementable

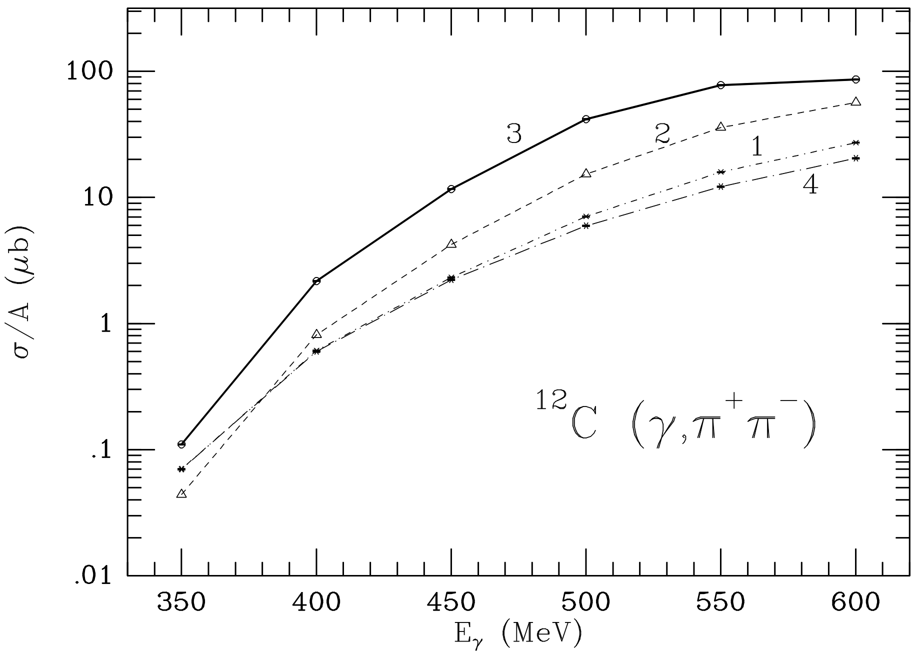

Figure 10. Cross section per nucleon for the $^{12}C(\gamma,\pi^+\pi^-)$ reaction. Dot-dashed line (labelled 1): Simple impulse approximation. Dashed line (labelled 2): Scaling of the deuteron cross section. Continuous line (labelled 3): Full model. Long dashed-dotted curve (labelled 4): cross section renormalizing the internal pion and delta lines in the amplitudes and ignoring the renormalization of the external pion lines.

at Mainz as a complement to the $(\gamma,\pi\pi)$ program on the nucleon, and in general to the whole program of reaction mechanisms in photonuclear interactions.

5 Summary:

We made a short review of reactions producing one or two pions in elementary processes, in deuteron and ^{12}C. The off shell dependence of the isoscalar πN amplitude was shown to be very important in the $pp \to pp\pi^0$ reaction. On the other hand a rather complete model for the $\gamma N \to \pi\pi N$ reaction is made, which is bound to play a similar role as the $\gamma N \to \pi N$ reaction when the energy of the experiments go up to the GeV regime as in Mainz. We offered an example of exchange currents produced from that model in the $\gamma d \to \Delta^{++}\Delta^-$ reaction. Finally we also showed important renormalization effects in the $(\gamma,\pi^+\pi^-)$ reaction in nuclei, which once more stresses the role played by pion physics in photonuclear reactions in general.

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