Heavy Quark Spin Symmetry and Heavy Baryons: Electroweak Decays*

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Heavy quark spin symmetry is discussed in the context of single and doubly heavy baryons. A special attention is paid to the constraints/simplifications that this symmetry imposes on the non-relativistic constituent quark model wave functions and on the $b \to c$ semileptonic decays of these hadrons.

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I. INTRODUCTION

Heavy Quark Spin Symmetry (HQSS) has proved to be a useful tool to understand the bottom and charm physics [1, 2], and it has been extensively used to describe the dynamics of systems containing a heavy quark c or b. HQSS is an approximate symmetry of Quantum Chromodynamics (QCD), which appears in systems containing heavy quarks with masses (m_Q) that are much larger than the typical quantities ($q_{\rm light} = \Lambda_{QCD},\ m_u, m_d, m_s \cdots$) that set up the energy scale of the dynamics of the remaining (light) degrees of freedom. HQSS predicts that all type of spin interactions vanish for infinitely massive quarks: the dynamics is unchanged under arbitrary transformations on the spin of the heavy quark (Q). The spin-dependent interactions are proportional to the chromomagnetic moment of the heavy quark, and so are of the order of $1/m_Q$. The total angular momentum of the hadron $\vec{J}$ is a conserved operator, the spin of the heavy quark $\vec{S}_Q$ is conserved in the $m_Q \to \infty$ limit, and therefore the spin of the light degrees of freedom $\vec{S}_l = \vec{J} - \vec{S}_Q$ is also conserved in the heavy quark limit. Heavy hadrons come in doublets (unless $s_l = 0$, with $\vec{S}_l^2 = s_l(s_l+1)$) containing states with total spin $J_\pm = s_l \pm 1/2$ obtained by combining the spin of the light degrees of freedom with the spin of the heavy quark $s_Q = 1/2$. These doublets are degenerated in the $m_Q \to \infty$ limit. HQSS has not been systematically employed in the context of non-relativistic constituent quark models (NRCQM's). NRCQM's, based upon simple quark-quark potentials, partially inspired by QCD, lead to reasonably good descriptions of hadrons as bound states of constituent quarks. Most of the quark-quark interactions include a term with a shape and a color structure determined from the one gluon exchange contribution and a confinement potential, which is assumed to come from the long-range nonperturbative features of QCD.

In this contribution, we will study masses and electroweak decays of various single and double heavy baryon within a NRCQM scheme, but taking explicitly into account HQSS constraints, which, as we will show, lead to great simplifications in these systems.

II. SINGLE HEAVY BARYONS

Up to corrections of the order $\mathcal{O}(q_{\rm light}/m_Q)$, HQSS guaranties that the heavy baryon light degrees of freedom quantum numbers, compiled in Table I, are always well defined. The symmetry also predicts that the baryon pair Σ, Σ^* (or the Ξ', Ξ^* pair, or the Ω, Ω^* one) become degenerated for an infinitely massive heavy quark, since both baryons have the same cloud of light degrees of freedom. In this section, we first describe a variational approach [5] for the solution of the non-relativistic three-body problem in baryons with a heavy quark. Thanks to HQSS, the proposed method turns out to be quite simple, leads to simple and manageable wave functions and reproduces previous results (baryon masses, charge and mass radii, $\cdots$) obtained by solving the Faddeev equations [6]. We will also discuss in this context, the semileptonic decay $\Lambda_b \to \Lambda_c^+ l^- \bar{\nu}_l$ [7].

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TABLE I: Quantum numbers, experimental and lattice QCD masses of the baryons containing a single heavy quark. I, and $S^{\pi}_{\rm light}$ are the isospin, and the spin parity of the light degrees of freedom and S, J^P are strangeness and the spin parity of the baryon ($l = u, d$).

Baryon	S	J^P	I	$S^{\pi}_{\rm light}$	Quark content	$M_{exp.}$ [3] [MeV]	$M_{Latt.}$ [4] [MeV]
Λ_c	0	$\frac{1}{2}^+$	0	0^+	udc	2286.46 ± 0.14	2270 ± 50
Σ_c	0	$\frac{1}{2}^+$	1	1^+	llc	2453.5 ± 0.5	2460 ± 80
Σ_c^*	0	$\frac{3}{2}^+$	1	1^+	llc	2517.5 ± 0.5	2440 ± 70
Ξ_c	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	lsc	2469.5 ± 1.5	2410 ± 50
Ξ_c'	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	lsc	2577 ± 3	2570 ± 80
Ξ_c^*	-1	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	lsc	2645.9 ± 0.6	2550 ± 80
Ω_c	-2	$\frac{1}{2}^+$	0	1^+	ssc	2695.2 ± 1.7	2680 ± 70
Ω_c^*	-2	$\frac{3}{2}^+$	0	1^+	ssc	2765.9 ± 2.0	2660 ± 80
Λ_b	0	$\frac{1}{2}^+$	0	0^+	udb	5620.2 ± 1.6	5640 ± 60
Σ_b	0	$\frac{1}{2}^+$	1	1^+	llb	5811 ± 4	5770 ± 70
Σ_b^*	0	$\frac{3}{2}^+$	1	1^+	llb	5832 ± 4	5780 ± 70
Ξ_b	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	lsb	5790.5 ± 2.7	5760 ± 60
Ξ_b'	-1	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	lsb		5900 ± 70
Ξ_b^*	-1	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	lsb		5900 ± 80
Ω_b	-2	$\frac{1}{2}^+$	0	1^+	ssb	6071 ± 40	5990 ± 70
Ω_b^*	-2	$\frac{3}{2}^+$	0	1^+	ssb		6000 ± 70

In the Laboratory (LAB) frame (see left panel of Fig. 1), the Hamiltonian (H) of the three quark (q, q', Q, with $q, q' = l$ or s and $Q = c$ or b) system reads:

$$H = \sum_{i=q,q',Q} \left(m_i - \frac{\vec{\nabla}^2_{x_i}}{2m_i} \right) + V_{qq'} + V_{Qq} + V_{Qq'} \tag{1}$$

where $m_q, m_{q'}$ and m_Q are the quark masses, and the quark-quark interaction terms, V_{ij}, depend on the quark spin-flavor quantum numbers and the quark coordinates ($\vec{x}_1, \vec{x}_2$ and $\vec{x}_h$ for the q, q' and Q quarks respectively). The nabla operators in the kinetic energy stand for derivatives with respect to the spatial variables $\vec{x}_1, \vec{x}_2$ and $\vec{x}_h$. To separate the Center of Mass (CM) free motion, we go to the heavy quark frame: $\vec{R}, \vec{r}_1, \vec{r}_2$; $\vec{R}$ and $\vec{r}_1$ ($\vec{r}_2$) are the CM position in the LAB frame and the relative position of the quark q (q') with respect to the heavy quark Q. The Hamiltonian now reads

$$H = -\frac{\vec{\nabla}^2_{\vec{R}}}{2M} + H^{\rm int} \tag{2}$$

$$H^{\rm int} = -\sum_{i=1,2} \frac{\vec{\nabla}^2_i}{2\mu_i} - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_Q} + V_{qq'}(\vec{r}_1 - \vec{r}_2) + V_{Qq}(\vec{r}_1) + V_{Qq'}(\vec{r}_2) + M \tag{3}$$

where $M = (m_q + m_{q'} + m_Q)$, $\mu_{1,2} = (1/m_{q,q'} + 1/m_Q)^{-1}$ and $\vec{\nabla}_{1,2} = \partial/\partial_{\vec{r}_1,\vec{r}_2}$. The intrinsic Hamiltonian $H^{\rm int}$ describes the dynamics of the baryon, and it can be rewritten as the sum of two single particle Hamiltonians (h^{sp}_i), which describe the dynamics of the light quarks in the mean field created by the heavy quark, plus the light–light interaction term, which includes the Hughes-Eckart term ($\vec{\nabla}_1 \cdot \vec{\nabla}_2$).

$$H^{\rm int} = \sum_{i=q,q'} h^{sp}_i + V_{qq'}(\vec{r}_1 - \vec{r}_2, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_Q} + M \tag{4}$$

$$h^{sp}_1 = -\frac{\vec{\nabla}^2_1}{2\mu_1} + V_{Qq}(\vec{r}_1, spin), \qquad h^{sp}_2 = -\frac{\vec{\nabla}^2_2}{2\mu_2} + V_{Qq'}(\vec{r}_2, spin) \tag{5}$$

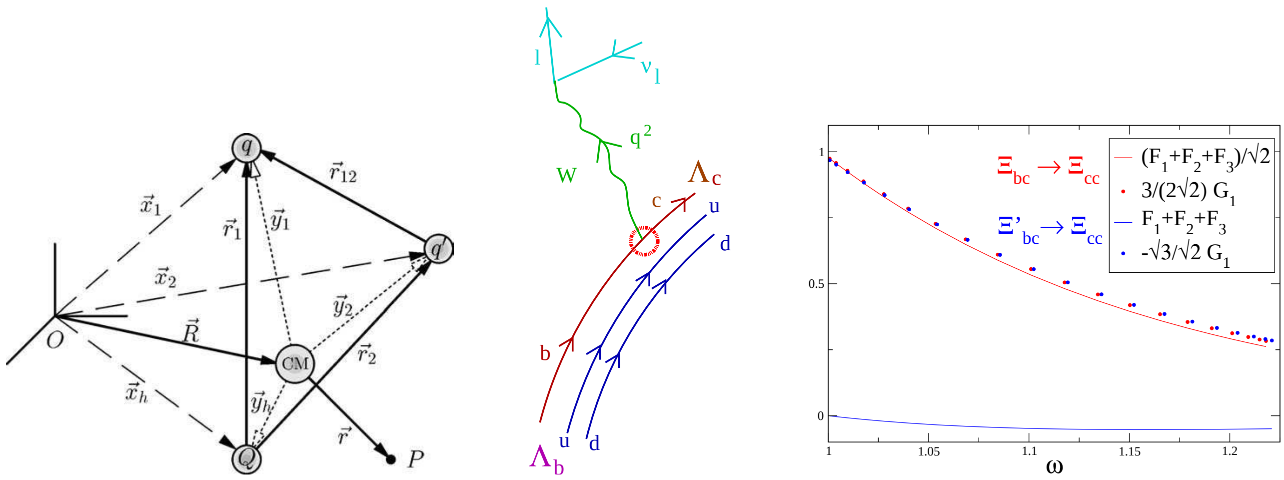

FIG. 1: Definition of different coordinates (left) and sketch of the $\Lambda_b \to \Lambda_c$ decay (middle). Besides, in the right panel, taken from Ref. [15], we show the form factor combinations $(F_1 + F_2 + F_3)/\sqrt{2}$ and $3G_1/\sqrt{8}$ of the $\Xi_{bc} \to \Xi_{cc}$ transition (red), and $(F_1 + F_2 + F_3)$ and $-\sqrt{3}G_1/\sqrt{2}$ of the $\Xi'_{bc} \to \Xi_{cc}$ transition (blue) evaluated using the AL1 interquark potential of Ref. [6].

In what respects to the quark–quark interactions, we use some phenomenological ones [6] obtained from quark–antiquark potentials fitted to a large sample of meson states in every flavor sector[1]. The general structure is as follows ($i, j = l, s, c, b$):

$$V_{ij}^{q\bar q}(r) \;=\; -\frac{\kappa f_c(r)}{r} + \lambda r^p - \Lambda + \left\{\frac{a_0\kappa}{m_i m_j}\frac{e^{-r/r_0}}{r r_0^2} + \frac{2\pi\kappa' f_c(r)}{3 m_i m_j}\frac{e^{-r^2/x_0^2}}{\pi^{\frac{3}{2}} x_0^3}\right\}\vec\sigma_i\vec\sigma_j, \tag{6}$$

with $\vec\sigma$ the spin Pauli matrices, $f_c(r) = 1 - e^{-r/r_c}$ and $x_0(m_i, m_j) = A\left(\frac{2m_i m_j}{m_i+m_j}\right)^{-B}$.

In a baryon, the singlet color wave function is completely anti-symmetric under the exchange of any of the three quarks. Within the SU(3) quark model, we assume a complete symmetry of the wave function under the exchange of the two light quarks (u, d, s) flavor, spin and space degrees of freedom. On the other hand, for the interactions described above, we have that both the total spin of the baryon, $\vec S_{\rm B} = (\vec\sigma_q + \vec\sigma_{q'} + \vec\sigma_Q)/2$, and the orbital angular momentum of the light quarks with respect to Q, $\vec L$ [$= \vec l_1 + \vec l_2$, with $\vec l_k = -{\rm i}\;\; \vec r_k \times \vec\nabla_k$, $\;\; k = 1, 2$] commute with $H^{\rm int}$. We will assume that the ground states of the baryons are in s–wave, $L = 0$, which implies that the spatial wave function can only depend on the relative distances r_1, r_2 and $r_{12} = |\vec r_1 - \vec r_2|$. Note that when the heavy quark mass is infinity, the total spin of the light degrees of freedom, $\vec S_{\rm light} = (\vec\sigma_q + \vec\sigma_{q'})/2$, commutes with $H^{\rm int}$, since the $\vec\sigma_Q\cdot\vec\sigma_{q,q'}/(m_Q m_{q,q'})$ terms vanish in this limit. With all these ingredients, and taking into account the quantum numbers of the light degrees of freedom for each baryon, compiled in Table I and that in general are always well defined in the static limit mentioned above, we have constructed the wave functions in our variational approach. Thus, for instance for $\Lambda-$type baryons ($\Lambda-$type baryons: $I = 0,\; S_{\rm light} = 0$), we use[2]

$$|\Lambda_Q; J = \frac{1}{2}, M_J\,\rangle \;=\; \Big\{|00\rangle_I \otimes |00\rangle_{S_{\rm light}}\Big\}\Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) \otimes |Q; M_J\rangle \tag{7}$$

where $\Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) = \Psi_{ll}^{\Lambda_Q}(r_2, r_1, r_{12})$ to guaranty a complete symmetry of the wave function under the exchange of the two light quarks (u, d) flavor, spin and space degrees of freedom, and finally M_J is the baryon total angular momentum third component. Note, that SU(3) flavor symmetry (SU(2), in the case of the Λ_Q baryon) would also allow for a component in the wave function of the type

$$\sum_{M_S M_Q}(\frac{1}{2}1\frac{1}{2}|M_Q M_S M_J)\Big\{|00\rangle_I \otimes |1M_S\rangle_{S_{\rm light}}\Big\}\Theta_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) \otimes |Q; M_Q\rangle \tag{8}$$

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[1] The usual $V_{ij}^{qq} = V_{ij}^{q\bar q}/2$ prescription is assumed here.

[2] An obvious notation has been used for the isospin–flavor ($|I, M_I\rangle_I$, $|ls\rangle$ or $|sl\rangle$) and spin ($|S, M_S\rangle_{S_{\rm light}}$) wave functions of the light degrees of freedom.

with $\Theta_{ll}^{\Lambda_Q}(r_1,r_2,r_{12}) = -\Theta_{ll}^{\Lambda_Q}(r_2,r_1,r_{12})$ (for instance terms of the type r_1-r_2) and, the real numbers $(j_1 j_2 j|m_1 m_2 m)$ are Clebsh-Gordan coefficients. This component is forbidden by HQSS in the limit $m_Q \to \infty$, where $S_{\rm light}$ turns out to be well defined and set to zero for Λ_Q-type baryons. The most general SU(2) Λ_Q wave function will involve a linear combination of the two components, given in Eqs. (7) and (8). Neglecting $\mathcal{O}(q/m_Q)$, HQSS imposes an additional constraint, which justifies the use of a wave function of the type of that given in Eq. (7) with the obvious simplification of the three body problem. One can benefit [5] from similar simplifications induced from HQSS for all the rest of baryons compiled in Table I.

The spatial wave function, $\Psi_{qq'}^{B_Q}$, is determined by the variational principle:

$$\delta\langle B_Q|H^{\rm int}|B_Q\rangle = 0 \tag{9}$$

For simplicity, we use a Jastrow–type functional form for the spatial wave function, as in the context of the similar problem of double Λ hypernuclei [8],

$$\Psi_{qq'}^{B_Q}(r_1,r_2,r_{12}) \;=\; F^{B_Q}(r_{12})\phi_q^Q(r_1)\phi_{q'}^Q(r_2) \tag{10}$$

For simplicity, we do not entirely determine the functions ϕ_q^Q and $\phi_{q'}^Q$ from the variational principle, but we rather fix the bulk of these functions to the $s-$wave ground states $(\varphi_{i=q,q'}^Q)$ of the single particle Hamiltonians, $h_{i=q,q'}^{sp}$, defined in Eq. (5), and modify their behavior at large distances. Thus, we take

$$\phi_q^Q(r_1) = (1+\alpha_q r_1)\varphi_q^Q(r_1), \qquad \phi_{q'}^Q(r_2) = (1+\alpha_{q'} r_2)\varphi_{q'}^Q(r_2) \tag{11}$$

with only one (two) free parameter for a ll or ss (ls) baryon light quark content. Finally, we construct the light–light correlation function, F^{B_Q}, from a linear combination of gaussians, with a total of eleven free parameters to be determined by the variational principle. The mass of the baryon is just the expected value of the intrinsic Hamiltonian.

Provided with this family of Jastrow type functions constrained by HQSS and using several inter-quark interactions, we have calculated masses, charge and mass radii of all bottom and charm baryons compiled in Table I (see [5]). For the baryons considered in [6], we agree remarkably well with the results of this latter reference, obtained by solving involved Faddeev equations, but thanks to HQSS, the baryon wave functions are significantly simpler and more manageable than those derived in [6].

Using the semi–analytical wave functions found here, we have also studied the semileptonic decays [7] (see middle panel of Fig. 1)

$$\Lambda_b^0 \to \Lambda_c^+ l^- \bar\nu_l, \qquad \Xi_b^0 \to \Xi_c^+ l^- \bar\nu_l \tag{12}$$

with $l = e, \mu$. We work on coordinate space, and develop a novel expansion of the electroweak current operator, which supplemented with heavy quark effective theory constraints, allows us to predict the baryon form factors and the decay distributions for all q^2 values accessible in the physical decays. Our results for the partially integrated longitudinal and transverse decay widths, in the vicinity of the $q^2 = (m_{\Lambda_b} - m_{\Lambda_c})^2$ point, are in excellent agreement with lattice calculations [9]. Comparison of our integrated Λ_b-decay width to experiment[3, 10] allows us to extract the V_{cb} Cabbibo-Kobayashi-Maskawa matrix element for which we obtain [7] a value of

$$|V_{cb}| = 0.040 \pm 0.005 \text{ (stat) } {}^{+0.001}_{-0.002} \text{ (theory)} \tag{13}$$

also in excellent agreement with a recent determination, $|V_{cb}| = 0.0414 \pm 0.0012\,({\rm stat}) \pm 0.0021\,({\rm syst}) \pm 0.0018\,({\rm theory})$ from the exclusive $\bar{\rm B}^0_{\rm d} \to {\rm D}^{*+} l^- \bar\nu_l$ decay [11]. Besides for the $\Lambda_b(\Xi_b)-$decay, the longitudinal and transverse asymmetries, and the longitudinal to transverse decay ratio are $\langle a_L\rangle = -0.954 \pm 0.001$ (-0.945 ± 0.002) , $\langle a_T\rangle = -0.665 \pm 0.002$ (-0.628 ± 0.004) and $R_{L/T} = 1.63 \pm 0.02$ (1.53 ± 0.04), respectively.

III. DOUBLY HEAVY BARYONS

We briefly review now static properties, semileptonic and electromagnetic decays of doubly heavy baryons within the HQSS constrained NRCQM scheme sketched in the previous section. We will consider the ground states of baryons containing two charm, two bottom or one charm and one bottom quarks (see Table II). In these systems, HQSS amounts to the decoupling of the heavy quark spins in the infinite heavy quark mass limit. In that limit one can consider the total spin of the two heavy quark subsystem (S_h) to be well defined [12]. To solve the baryon three–quark problem, we will again use a variational approach that leads to simple and manageable wave functions thanks to the

TABLE II: Summary of the quantum numbers of the baryons containing two heavy quarks. $S^\pi_{hh'}$ stands for the spin parity of the heavy subsystem.

Baryon	S	J^P	I	S^π_{hh}	Quark Content
Ξ_{cc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ξ^*_{cc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ω_{cc}	-1	$\frac{1}{2}^+$	0	1^+	ccs
Ω^*_{cc}	-1	$\frac{3}{2}^+$	0	1^+	ccs
Ξ_{bb}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ξ^*_{bb}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ω_{bb}	-1	$\frac{1}{2}^+$	0	1^+	bbs
Ω^*_{bb}	-1	$\frac{3}{2}^+$	0	1^+	bbs

Baryon	S	J^P	I	$S^\pi_{hh'}$	Quark Content
Ξ'_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	bcl
Ξ_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ξ^*_{bc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ω'_{bc}	-1	$\frac{1}{2}^+$	0	0^+	bcs
Ω_{bc}	-1	$\frac{1}{2}^+$	0	1^+	bcs
Ω^*_{bc}	-1	$\frac{3}{2}^+$	0	1^+	bcs

simplifications introduced in the problem by considering S_h to be well defined. Details of this approach run in parallel to those described in the previous section for single heavy baryons. Results for masses and semileptonic $b \to c$ decay widths of these baryons, calculated with different quark-quark interactions, can be found in [13]. We will focus here first, on the constraints that HQSS imposes to the weak matrix elements that described these semileptonic decays, and second on the hyperfine mixing in the physical $bc-$baryons, and the consequences of this mixing, in conjunction with HQSS, for semileptonic and electromagnetic decays of the actual $bc-$baryon states.

Spin symmetry for both the b and c quarks enormously simplifies the description of all semileptonic $\Xi^{(\prime *)}_{bc} \to \Xi^{(*)}_{cc}\, l\, \bar\nu_l$, $\Omega^{(\prime *)}_{bc} \to \Omega^{(*)}_{cc}\, l\, \bar\nu_l$ decays[3] in the limit $m_{b,c} \gg \Lambda_{\rm QCD}$ and close to the zero recoil point $[q^2 = (m_{bc} - m_{cc})^2]$. All the weak transition matrix elements are given in terms of a single universal function [14]. Lorentz covariance alone allows a large number of form factors (six to describe $\Xi_{bc} \to \Xi_{cc}$, another six for $\Xi'_{bc} \to \Xi_{cc}$, eight each for $\Xi_{bc} \to \Xi^*_{cc}$, $\Xi'_{bc} \to \Xi^*_{cc}$ and $\Xi^*_{bc} \to \Xi_{cc}$, and even more for $\Xi^*_{bc} \to \Xi^*_{cc}$). Let us consider, f.i., the $\Xi_{bc} \to \Xi_{cc}$ and $\Xi'_{bc} \to \Xi_{cc}$ decays, in each case the weak matrix element can be expressed in terms of three vector $(F's)$ and three axial $(G's)$ form factors (r, r' are baryon helicity indices, $u's$ are Dirac spinors, $v^\mu = p^\mu/m_{\Xi^{(\prime)}_{bc}}$ and $v'^\mu = p'^\mu/m_{\Xi_{cc}}$ are initial and final baryon velocities),

$$\left\langle \Xi_{cc}, r'\ \vec p\,'\,|\,\bar c\gamma^\mu(1-\gamma_5)b(0)|\,\Xi^{(\prime)}_{bc}, r\ \vec p \right\rangle \;=\; \bar u^{\Xi_{cc}}_{r'}(\vec p\,')\Big\{\gamma^\mu\left(F_1(w) - \gamma_5 G_1(w)\right) + v^\mu\left(F_2(w) - \gamma_5 G_2(w)\right) + v'^{\,\mu}\left(F_3(w) - \gamma_5 G_3(w)\right)\Big\} u^{\Xi^{(\prime)}_{bc}}_r(\vec p), \quad w = v\cdot v' \tag{14}$$

In the $m_Q \to \infty$ limit, the above 12 form factors are not independent and are all related to a single universal function. Finite c and b quark mass corrections turn out to be small, as it can be appreciated in the right panel of Fig. 1 (note that there, some combinations of form factors that, in the $m_Q \to \infty$ limit, should be equal or vanish [14] are plotted). An extensive analysis of HQSS constraints on NRCQM semileptonic form factors and decay widths of doubly heavy baryons can be found in [15].

To end this section, we will discuss the hyperfine mixing for $bc-$baryons. In Table II we showed the $J^\pi = \frac{1}{2}^+$ ground states of these baryons classified so that S_h is well defined (S_h-basis). Due to the finite value of the heavy quark masses, the hyperfine spin interaction ($\vec\sigma \cdot \vec\sigma$ term in Eq. (6)) between the light quark and any of the heavy quarks can admix both $S_h = 0$ and $S_h = 1$ spin components into the wave function [16]. This mixing should be negligible for bb and cc doubly heavy baryons as the antisymmetry of the wave function would require radial excitations and/or higher orbital angular momentum in the $S_h = 0$ component. However, in the bc sector one expects the actual physical Ξ (Ω) particles to be admixtures of the Ξ_{bc}, Ξ'_{bc} (Ω_{bc}, Ω'_{bc}) states listed in Table II. This requires to diagonalize the Hamiltonian, calculated for instance in the S_h-basis. This is easily done and details can be found in [17]. Qualitatively[4], the physical eigenstates ($\Xi^{(1,2)}_{bc}$) turn out be quite close to those ($\hat\Xi_{bc}, \hat\Xi'_{bc}$) in which the light quark q

[3] Similar conclusions hold for the bb into bc baryon decays.
[4] We will focus only on the $\Xi-$type baryons. The discussion is similar for the $\Omega-$type states.

and the c quark are coupled to well defined total spin $S_{qc} = 0, 1$ ($S_{qc}-$basis). Note that, the S_h and S_{qc} basis are related by a trivial rotation,

$$\hat{\Xi}_{bc} = \frac{\sqrt{3}}{2}\Xi'_{bc} + \frac{1}{2}\Xi_{bc}, \qquad \hat{\Xi}'_{bc} = -\frac{1}{2}\Xi'_{bc} + \frac{\sqrt{3}}{2}\Xi_{bc} \tag{15}$$

In the $S_{qc}-$basis, hyperfine mixing is always inversely proportional to the b quark mass, and it is thus much smaller than for the S_h-basis case. Indeed, NRCQM's calculations show that physical and $S_{qc}-$basis states differ in just a rotation of around $4^{\rm o}$ [17].

Masses (eigenvalues) are very insensitive to hyperfine mixing, since the non-diagonal terms induced by the hyperfine spin interactions are around one thousand times smaller than the diagonal ones. However, as Roberts and Pervin [16] pointed out, this mixing could greatly affect the decay widths of doubly heavy baryons. NRCQM calculations confirmed [17, 18] this strong dependence of the semileptonic $b \to c$ decay widths on the hyperfine mixing. This is not surprising, and it can be easily understood since the $b \to c$ semileptonic decay width for transitions involving the S_h-basis Ξ_{bc} state is very much different from the corresponding one involving the Ξ'_{bc} baryon. This is a straightforward prediction of HQSS [14] and its validity was corroborated in the context of NRCQM's in [15]. Indeed, HQSS predictions might be also used to experimentally obtain information on the mixing angle for bc baryons in a model independent manner [17]. The idea is to use HQSS to predict width ratios for physical states, for instance,

$$R_1^{\rm phys.} = \frac{\Gamma(\Xi_{bc}^{(2)} \to \Xi_{cc}^*)}{\Gamma(\Xi_{bc}^{(1)} \to \Xi_{cc}^*)} \sim \tan^2\theta + \mathcal{O}(\frac{m_q, \Lambda_{QCD}}{m_c}), \tag{16}$$

that will depend, in the $m_Q \to \infty$ limit, only on the small rotation angle θ between the $S_{qc}-$ and the physical states[5]. Experimental data, when available, could be used to extract information on the admixtures in the actual physical states.

Flavor conserving one-photon transitions $\Xi_{bc}^* \to \Xi_{bc}^{(1)}\gamma$, $\Xi_{bc}^{(2)}\gamma$, $\quad \Xi_{bc}^{(1)} \to \Xi_{bc}^{(2)}\gamma$ depend also on the mixing angle [19]. As in the case of $b \to c$ semileptonic decays, there are large corrections to these electromagnetic decay widths due to the hyperfine mixing. However, here next-to-leading $\mathcal{O}(1/m_Q)$ corrections turn out to be quite large [f.i., from phase space $\Gamma_{em} \propto (M_i - M_f)^3$, and because $M_i \approx M_f$, the widths are very sensitive to the actual baryon masses], and in this case, it will not be possible to determine, relying only on HQSS, the actual hyperfine mixing matrix.

IV. CONCLUDING REMARKS

We have discussed some constraints and the enormous simplifications that HQSS imposes on single and doubly heavy baryons. Within a NRCQM scheme, i) we have variationally computed baryon masses and wave functions, and used these latter ones to calculate $b \to c$ semileptonic decays of these baryons, ii) we have studied the hyperfine mixing for $bc-$baryons and shown that it greatly affects their electromagnetic and semileptonic decay widths, and iii) we have discussed how such dependence, when compared to the HQSS predictions, might be used to experimentally extract information on the admixtures in the actual physical bc baryons, in a model independent manner.

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