Heavy quark symmetry constraints on semileptonic form factors and decay widths of doubly heavy baryons

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We show how heavy quark symmetry constraints on doubly heavy baryon semileptonic decay widths can be used to test the validity of different quark model calculations. The large discrepancies in the results observed between different quark model approaches can be understood in terms of a severe violation of heavy quark spin symmetry constraints by some of those models.

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I. INTRODUCTION

In hadrons with a single heavy quark the dynamics of the light degrees of freedom becomes independent of the heavy quark flavor and spin when the mass of the heavy quark is made arbitrarily large. This is known as heavy quark symmetry (HQS) [1, 2, 3, 4]. This symmetry can be developed into an effective theory (HQET) [5] that allows a systematic, order by order, evaluation of corrections in inverse powers of the heavy quark mass. Ordinary HQS can not be applied directly to hadrons containing two heavy quarks. There, the kinetic energy term, needed to regulate infrared divergences, breaks heavy flavor symmetry [6]. Only the spin symmetry for each of the heavy quark flavor is preserved. The symmetry that survives is heavy quark spin symmetry (HQSS), which amounts to the decoupling of the heavy quark spins for infinite heavy quark masses. In that limit one can consider the total spin of the two heavy quark subsystem (S_h) to be well defined. HQSS is sufficient to derive relations between form factors for the decay of hadrons containing two heavy quarks. That was first shown in Ref. [7], where the authors adopted an approach where the two heavy quarks bind into a color anti-triplet which appears as a pointlike color source to the light degrees of freedom. Applying the "superflavor" formalism of Georgi and Wise [8, 9, 10] allowed the matrix elements of the heavy-flavor-changing weak current to be evaluated between different baryon states. Semileptonic decays of the B_c meson were also studied using HQSS in Ref. [11]. The formalism employed in [11] has been recently extended to describe semileptonic decays of bc baryons to cc baryons [12]. The scheme presented in [12] does not rely on the "superflavor" formalism and HQSS is naturally implemented in it. In agreement with Ref. [7], the authors[1] of Ref. [12] found that spin symmetry for two heavy quarks enormously simplifies heavy to heavy semileptonic baryon transitions in the heavy quark limit and near the zero recoil point. As a result it is shown how an unique function, called the Isgur-Wise (IW) function, describes an entire family of decays involving doubly heavy baryons with total spin 1/2 and 3/2. This imposes limitations to any quark model calculated form factors. Besides, the fact that all baryon matrix elements are given in term of just one function induces relations among different decay widths that, to our knowledge,

Baryon	Quark content	S_h	J^π	Mass [MeV]	Baryon	Quark content	S_h	J^π	Mass [MeV]
Ξ_{cc}	c c l	1	$1/2^+$	3612	Ω_{cc}	c c s	1	$1/2^+$	3702
Ξ^*_{cc}	c c l	1	$3/2^+$	3706	Ω^*_{cc}	c c s	1	$3/2^+$	3783
Ξ_{bb}	b b l	1	$1/2^+$	10197	Ω_{bb}	b b s	1	$1/2^+$	10260
Ξ^*_{bb}	b b l	1	$3/2^+$	10236	Ω^*_{bb}	b b s	1	$3/2^+$	10297
Ξ_{bc}	b c l	1	$1/2^+$	6919	Ω_{bc}	b c s	1	$1/2^+$	6986
Ξ'_{bc}	b c l	0	$1/2^+$	6948	Ω'_{bc}	b c s	0	$1/2^+$	7009
Ξ^*_{bc}	b c l	1	$3/2^+$	6986	Ω^*_{bc}	b c s	1	$3/2^+$	7046

TABLE I: Quantum numbers of doubly heavy baryons analyzed in this study. J^P is the spin parity of the baryon, and S_h is the spin of the heavy degrees of freedom. l denotes a light u or d quark. Mass predictions from Ref. [14] obtained using the AL1 interquark potential of Ref.[15] are also given.

[1] They find two differences with the results of Ref. [7], which cannot be eliminated by redefining the phases of the physical states. One difference was already pointed out in [13].

have not been exploited before to check the validity of different quark model calculations. This is the main purpose of this letter.

In a recent work [14] we have studied, within a nonrelativistic quark model framework, static properties of doubly heavy baryons and their semileptonic decays driven by the $b \to c$ transition at the quark level. For the semileptonic decays we limited ourselves to spin 1/2 to spin 1/2 baryon transitions[2]. While we have shown that our wave functions have the correct limit for infinite heavy quark masses[3], we did not check HQSS constraints on the form factors or decay widths. Here we would like to extend our previous study on doubly heavy baryon $b \to c$ semileptonic decays to include also doubly heavy spin 3/2 baryons and test our model and others against HQSS predictions. These type of decays have been studied in different relativistic quark model approaches [16, 17, 18], with the use of HQET [19], using QCD sum rules [20] and three-point nonrelativistic QCD sum rules [21], or in the framework of the operator product expansion using the inverse heavy quark mass technique [22]. Discrepancies between the results obtained in different quark model are sometimes very large. Therefore, it is worthwhile to use HQSS relations among decay widths to check the validity of the different calculations.

In Table I we summarize the quantum numbers of the doubly heavy baryons considered in this study[4].

II. FORM FACTOR DECOMPOSITION

Hadronic matrix elements can be parameterized in terms of form factors. For $1/2 \to 1/2$ transitions the commonly used form factor decomposition reads

$$\begin{aligned}\Big\langle B'(1/2), r'\ \vec{p}\,'\Big|\overline{\Psi}^c(0)\gamma^\mu(1-\gamma_5)\Psi^b(0)\Big|B(1/2), r\ \vec{p}\Big\rangle \;=\; & \bar{u}^{B'}_{r'}(\vec{p}\,')\Big\{\gamma^\mu\left(F_1(w)-\gamma_5 G_1(w)\right)+v^\mu\left(F_2(w)-\gamma_5 G_2(w)\right)\\ & +v'^\mu\left(F_3(w)-\gamma_5 G_3(w)\right)\Big\}u^B_r(\vec{p})\end{aligned} \tag{1}$$

with $|B(S), r\ \vec{p}\,\rangle$ representing a baryon state with three-momentum $\vec{p}$, total spin S, and spin third component r. The baryon states are normalized such that $\langle B(S), r'\ \vec{p}\,'\,|\,B(S), r\ \vec{p}\rangle = (2\pi)^3(E_B/m_B)\,\delta_{rr'}\,\delta^3(\vec{p}-\vec{p}\,')$ being E_B, m_B the baryon energy and mass. The u^B_r are dimensionless Dirac spinors, normalized as $\bar{u}_{r'}u_r = \delta_{rr'}$. v^μ, v'^μ are the four velocities of the initial and final baryon. The three vector F_1, F_2, F_3, and three axial G_1, G_2, G_3 form factors are functions of the velocity transfer $\omega = v\cdot v'$ or equivalently of the four momentum transfer ($q = p - p'$) square $q^2 = m_B^2 + m_{B'}^2 - 2m_B m_{B'}\omega$. In the decay ω [q^2] ranges from $\omega = 1$ $\left[q^2 = q^2_{\rm max} = (m_B - m_{B'})^2\right]$, corresponding to zero recoil of the final baryon, to a maximum value given by $\omega = \omega_{\rm max} = (m_B^2 + m_{B'}^2 - m_l^2)/(2m_B m_{B'})$ $\left[q^2 = m_l^2\right]$, which depends on the transition, and where m_l stands for the final charged lepton mass (we neglect neutrino masses).

For $1/2 \to 3/2$ transitions we follow Llewellyn Smith [23] to write

$$\begin{aligned}&\langle B'(3/2), r'\vec{p}\,'|\ \overline{\Psi}^c(0)\gamma^\mu(1-\gamma_5)\Psi^b(0)\,|B(1/2), r\,\vec{p}\,\rangle = \bar{u}^{B'}_{\lambda\, r'}(\vec{p}\,')\Gamma^{\lambda\,\mu}u^B_r(\vec{p})\\ \Gamma^{\lambda\,\mu} = &\left(\frac{C^V_3(\omega)}{m_B}(g^{\lambda\,\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C^V_4(\omega)}{m_B^2}(g^{\lambda\,\mu}qp' - q^\lambda p'^\mu) + \frac{C^V_5(\omega)}{m_B^2}(g^{\lambda\,\mu}qp - q^\lambda p^\mu) + C^V_6(\omega)g^{\lambda\,\mu}\right)\gamma_5\\ &+\left(\frac{C^A_3(\omega)}{m_B}(g^{\lambda\,\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C^A_4(\omega)}{m_B^2}(g^{\lambda\,\mu}qp' - q^\lambda p'^\mu) + C^A_5(\omega)g^{\lambda\,\mu} + \frac{C^A_6(\omega)}{m_B^2}q^\lambda q^\mu\right)\end{aligned} \tag{2}$$

with p, p' the four-momenta of the initial, final baryon, and where we use the convention $g^{\mu\mu} = (+,-,-,-)$. $u^{B'}_{\lambda\, r'}$ is a dimensionless Rarita-Schwinger spinor normalized as $\bar{u}_{\lambda r'}\,u^\lambda_r = -\delta_{rr'}$.

For $3/2 \to 1/2$ transitions we have

$$\begin{aligned}\langle B'(1/2), r'\vec{p}\,'|\ \overline{\Psi}^c(0)\gamma^\mu(1-\gamma_5)\Psi^b(0)\,|B(3/2), r\vec{p}\,\rangle \;&=\; \bar{u}^{B'}_{r'}(\vec{p}\,')\hat{\Gamma}^{\lambda\,\mu}u^B_{\lambda\, r}(\vec{p})\\ \hat{\Gamma}^{\lambda\,\mu} = \gamma^0[\Gamma^{\lambda\,\mu}(m_B \longrightarrow m_{B'}, p &\longleftrightarrow p', q \longrightarrow -q)]^\dagger\gamma^0\end{aligned} \tag{3}$$

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[2] In that reference we missed a factor $1/\sqrt{2}$ that affected our results for form factors. Decay widths were thus affected by a factor 2. An erratum has been sent.
[3] In the infinite heavy quark mass limit the baryon should look like a meson composed of a light quark and a heavy diquark.
[4] Note that the definitions of Ξ_{bc} and Ξ'_{bc} are interchanged in some references, with Ξ_{bc} having $S_h = 0$ and Ξ'_{bc} having $S_h = 1$. The same applies to Ω_{bc} and Ω'_{bc}. In tables we always quote the results corresponding to our convention (see Table I).

Finally for $3/2 \to 3/2$ transitions, we believe there are 25 vector plus 25 axial form factors. The amount of form factors suggest a different strategy in this case and thus we do not show the form factor decomposition.

In Ref.[14] we presented results for $1/2 \to 1/2$ transition form factors. In a similar way one can evaluate all form factors for $1/2 \longleftrightarrow 3/2$ transitions. It is not the purpose of this work to present results for all individual form factors. Instead, we would like to study to what extend they obey the restrictions imposed by HQSS.

III. HQSS CONSTRAINTS ON FORM FACTORS FOR SEMILEPTONIC DOUBLY HEAVY BARYON DECAY

We quote in what follows the results obtained in Ref.[12], using HQSS and near zero recoil, for the semileptonic $bc \to cc$ baryon decay with the initial baryon at rest. There it was found that all hadronic matrix elements were given in terms of just one universal function ($\eta(\omega)$), known as the IW function. Indeed HQSS predicts

$$B_{bc} \to B_{cc} \qquad \frac{1}{\sqrt{2}}\,\eta\;\bar{u}'_{r'}(-\vec{q})(2\gamma^\mu - \frac{4}{3}\gamma^\mu\gamma_5)u_r(\vec{0}) \tag{4}$$

$$B'_{bc} \to B_{cc} \qquad \frac{1}{\sqrt{2}}\,\frac{-2}{\sqrt{3}}\eta\;\bar{u}'_{r'}(-\vec{q})(-\gamma^\mu\gamma_5)u_r(\vec{0}) \tag{5}$$

$$B_{bc} \to B^*_{cc} \qquad \frac{1}{\sqrt{2}}\,\frac{-2}{\sqrt{3}}\eta\;\bar{u}'^{\mu}_{r'}(-\vec{q})u_r(\vec{0}) \tag{6}$$

$$B'_{bc} \to B^*_{cc} \qquad \frac{1}{\sqrt{2}}\,(-2)\,\eta\;\bar{u}'^{\mu}_{r'}(-\vec{q})u_r(\vec{0}) \tag{7}$$

$$B^*_{bc} \to B_{cc} \qquad \frac{1}{\sqrt{2}}\,\frac{-2}{\sqrt{3}}\eta\;\bar{u}'_{r'}(-\vec{q})u^{\mu}_{r}(\vec{0}) \tag{8}$$

$$B^*_{bc} \to B^*_{cc} \qquad \frac{1}{\sqrt{2}}\,(-2)\,\eta\;\bar{u}'^{\lambda}_{r'}(-\vec{q})(\gamma^\mu - \gamma^\mu\gamma_5)u_{\lambda r}(\vec{0}) \tag{9}$$

where here B stands for a Ξ or Ω baryons. The IW function which controls the Ξ decays is different to that appearing in Ω decays since the IW function depends on the light degrees of freedom. The IW function η is approximately one at zero recoil ($\eta(\omega = 1) \approx 1$), as can be deduced from vector conservation in the limit of degenerate b and c quarks[5].

Similar results can be obtained for semileptonic $bb \to bc$ baryon decays, but there will have different IW functions because of the heavy flavor symmetry (HFS) breaking in hadrons with two heavy quarks.

Let us see the implications of the above relations for the form factors calculated in quark models.

A. $1/2 \to 1/2$ transitions

Near zero recoil the three vector structures γ^μ, v^μ and v'^μ present in Eq.(1) give, up to corrections proportional to $|\vec{q}|$ that cancel near zero recoil, the same contribution. On the other hand, the Dirac's structure of the axial form factors G_2 and G_3, and due to the anti-diagonal nature of γ_5, give contributions that are again proportional to $|\vec{q}|$ and thus cancel near zero recoil. To the extent that for the actual heavy quark masses we are close enough to the infinite heavy quark mass limit, Eqs.(4,5) imply the following restrictions on form factors

$$B_{bc} \to B_{cc} \qquad \frac{1}{\sqrt{2}}(F_1 + F_2 + F_3) = \frac{3}{2\sqrt{2}}G_1 = \eta \tag{10}$$

$$B'_{bc} \to B_{cc} \qquad (F_1 + F_2 + F_3) = 0 \;\;;\;\; -\sqrt{\frac{3}{2}}G_1 = \eta \tag{11}$$

The same relations can be derived respectively for $B_{bb} \to B_{bc}$ and $B_{bb} \to B'_{bc}$ decays.

In Fig.1 we show our results for the above quantities evaluated for the $\Xi_{bc} \to \Xi_{cc}$, $\Xi'_{bc} \to \Xi_{cc}$ and $\Xi_{bb} \to \Xi_{bc}$, $\Xi_{bb} \to \Xi'_{bc}$ transitions. We have used the AL1 interquark potential of Ref. [15] and actual heavy quark masses (baryon masses are given in Table I). We do not show very similar results obtained for transitions involving Ω baryons. We

[5] Note the authors of Ref.[12] missed a global normalization factor $1/\sqrt{2}$. An erratum has been sent.

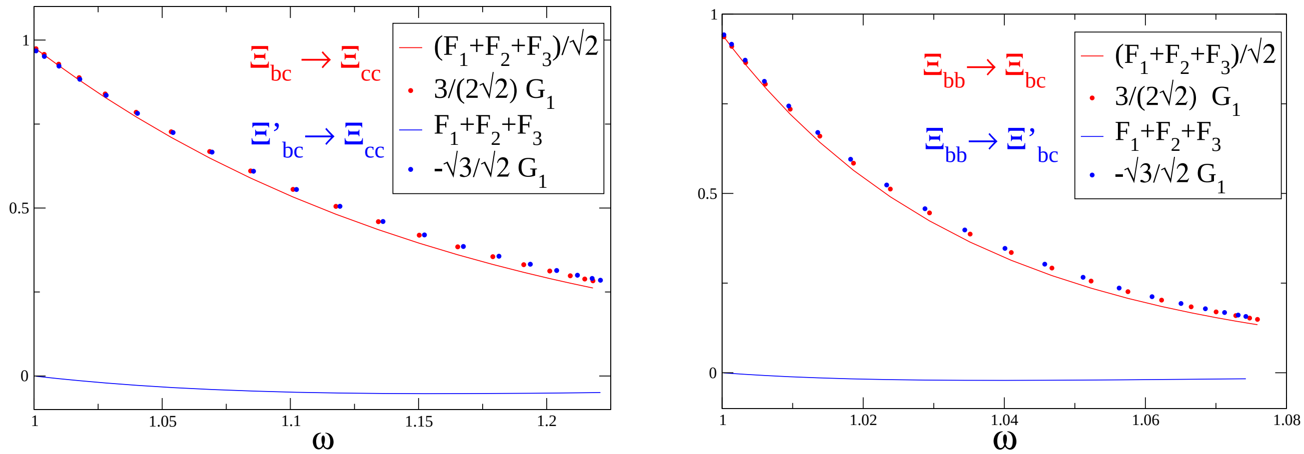

FIG. 1: Left panel: $(F_1 + F_2 + F_3)/\sqrt{2}$ and $3\,G_1/2\sqrt{2}$ of the $\Xi_{bc} \to \Xi_{cc}$ transition (in red), and $F_1 + F_2 + F_3$ and $-\sqrt{3}\,G_1/\sqrt{2}$ of the $\Xi'_{bc} \to \Xi_{cc}$ transition (in blue) evaluated using the AL1 interquark potential of Ref.[15]. Right panel: same as left panel for $\Xi_{bb} \to \Xi_{bc}$ and $\Xi_{bb} \to \Xi'_{bc}$ transitions.

see in the figure that the above restrictions are, to a good approximation, satisfied by our calculation over the entire ω region.

B. $1/2 \longleftrightarrow 3/2$ transitions

In this case one can see that all contributions generated by the C_3^V, C_4^V, C_5^V, C_6^V, and C_6^A form factors are proportional to $|\vec{q}\,|$, cancelling thus near zero recoil. The same happens for the $q^\lambda \gamma^\mu$ dependence of the C_3^A form factor and the $q^\lambda p'^\mu$ dependence of the C_4^A form factor. On the other hand the $g^{\lambda\mu}$ dependence of the axial part of the current survives near zero recoil. The restrictions imposed by Eqs.(6,7,8) are in this case

$$B_{bc} \to B^*_{cc} \qquad -\sqrt{\frac{3}{2}}\left(\frac{C_3^A}{m_{B_{bc}}}(m_{B_{bc}} - m_{B^*_{cc}}) + \frac{C_4^A}{m^2_{B_{bc}}}(m_{B_{bc}} E_{B^*_{cc}} - m^2_{B^*_{cc}}) + C_5^A\right) = \eta \tag{12}$$

$$B'_{bc} \to B^*_{cc} \qquad -\frac{1}{\sqrt{2}}\left(\frac{C_3^A}{m_{B'_{bc}}}(m_{B'_{bc}} - m_{B^*_{cc}}) + \frac{C_4^A}{m^2_{B'_{bc}}}(m_{B'_{bc}} E_{B^*_{cc}} - m^2_{B^*_{cc}}) + C_5^A\right) = \eta \tag{13}$$

$$B^*_{bc} \to B_{cc} \qquad -\sqrt{\frac{3}{2}}\left(-\frac{C_3^A}{m_{B_{cc}}}(m_{B^*_{bc}} - m_{B_{cc}}) - \frac{C_4^A}{m^2_{B_{cc}}}(m^2_{B^*_{bc}} - m_{B^*_{bc}} E_{B_{cc}}) + C_5^A\right) = \eta \tag{14}$$

For $B_{bb} \to B^*_{bc}$, $B^*_{bb} \to B_{bc}$ and $B^*_{bb} \to B'_{bc}$ the relations obtained are given respectively, and with obvious changes, by Eqs.(12), (14) and again (14) but in the latter case with the factor $\sqrt{3/2}$ changed to $1/\sqrt{2}$.

We show now In Fig.2 our results for the $\Xi_{bc} \to \Xi^*_{cc}$, $\Xi'_{bc} \to \Xi^*_{cc}$ and $\Xi^*_{bc} \to \Xi^*_{cc}$ transitions, and for the $\Xi_{bb} \to \Xi^*_{bc}$, $\Xi^*_{bb} \to \Xi_{bc}$ and $\Xi^*_{bb} \to \Xi'_{bc}$ transitions, again evaluated with the AL1 interquark potential of Ref.[15] and actual heavy quark masses. We do not show results for Ω baryons which are very similar to the ones presented. We see again our calculation is in accordance with HQSS constraints.

C. $3/2 \to 3/2$ transitions

In this case we have not evaluated explicitly individual form factors. Here we proceed as follows: we evaluate the hadronic matrix elements in Eq. (9) for different spin configurations, selecting only vector or axial components for which the matrix element does not cancel near zero recoil. The corresponding matrix elements are also evaluated in the quark model of Ref. [14]. By comparison of the two calculations we get the IW function. In this way one can

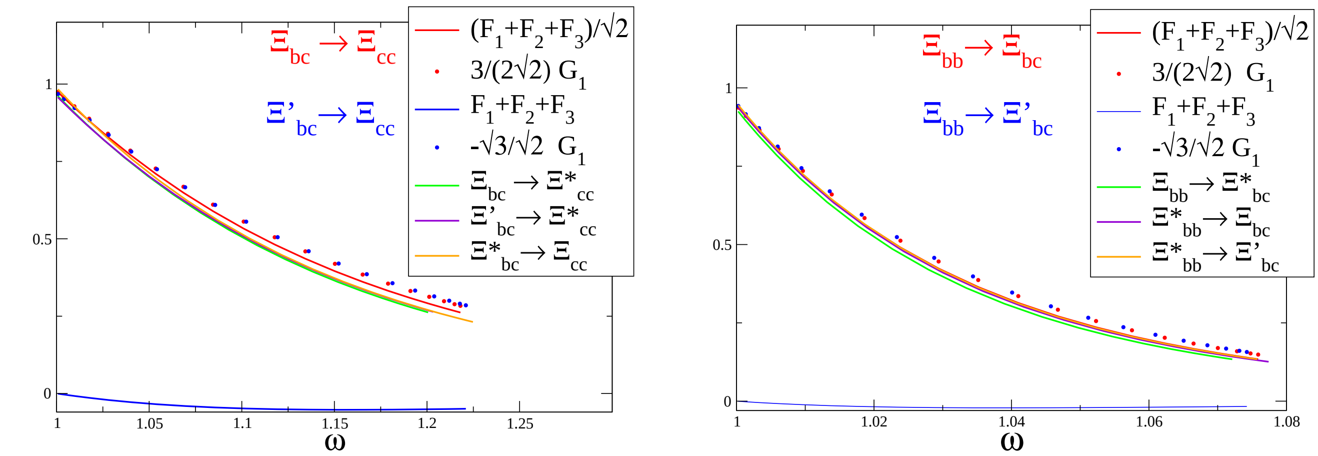

FIG. 2: Left panel: relations in Eqs. (12,13,14) for $\Xi_{bc} \to \Xi^*_{cc}$, $\Xi'_{bc} \to \Xi^*_{cc}$ and $\Xi^*_{bc} \to \Xi_{cc}$ transitions. We also show, for better comparison the results already shown in Fig. 1. Right panel: similar relations for $\Xi_{bb} \to \Xi^*_{bc}$, $\Xi^*_{bb} \to \Xi_{bc}$ and $\Xi^*_{bb} \to \Xi'_{bc}$ transitions. All the results have been obtained using the AL1 interquark potential of Ref.[15].

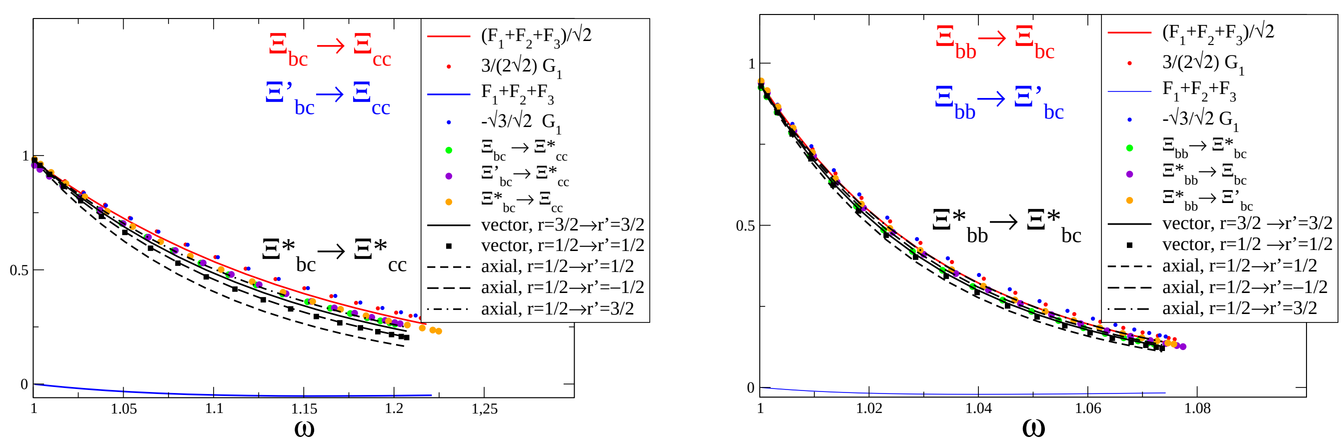

FIG. 3: Left panel: Different IW functions obtained for $\Xi^*_{bc} \to \Xi^*_{cc}$ transitions (black curves) using the vector or the axial part of the weak transition current, and for different spin configurations. For better comparison we also show the corresponding results obtained for $1/2 \to 1/2$ and $1/2 \longleftrightarrow 3/2$ transitions. All results have been obtained using the AL1 interquark potential of Ref.[15]. Right panel: same as left panel for $bb \to bc$ transitions.

obtain five different functions, two of them with the vector part of the current and three others with the axial part. In the infinite heavy quark mass limit these five functions should coincide among themselves and with the ones obtained in $1/2 \to 1/2$ and $1/2 \longleftrightarrow 3/2$ transitions.

The results are shown in Fig. 3. What we see is, that to a good approximation, better in the $bb \to bc$ case as one is closer to the infinite heavy quark mass limit, all $1/2 \to 1/2$, $1/2 \longleftrightarrow 3/2$ and $3/2 \to 3/2$ transitions are governed in terms of just one function. As mention before this function is different for the $bc \to cc$ and $bb \to bc$ cases due to HFS breaking.

IV. SEMILEPTONIC DECAY

In this section we present our results for semileptonic decay widths and compare them with the ones obtained in other quark model approaches. In some cases there are large discrepancies between different calculations. The fact that, at least for the $bb \to bc$ case, we are not far from the infinite heavy quark mass limit suggests that some calculations might be inconsistent with HQSS.

The decay width is given by

$$\Gamma = \frac{G_F^2}{2\pi^4}|V_{cb}|^2 m_{B'}^3 \int_1^{\omega_{max}} d\omega\ \sqrt{\omega^2-1}\ \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \tag{15}$$

where $G_F = 1.16637(1)\times 10^{-11}\,\mathrm{MeV}^{-2}$[24] is the Fermi decay constant and $|V_{cb}|$ is the modulus of the corresponding Cabibbo–Kobayashi–Maskawa matrix element. $\mathcal{L}^{\mu\nu}$ is the leptonic tensor defined as

$$\mathcal{L}^{\mu\nu} \;=\; \int \frac{d^3k}{2E}\frac{d^3k'}{2E'}\,\delta^{(4)}(q-k-k')\left(k'^{\mu}k^{\nu}+k'^{\nu}k^{\mu}-g^{\mu\nu}k\cdot k'+\mathrm{i}\epsilon^{\mu\nu\alpha\beta}k'_{\alpha}k_{\beta}\right) \tag{16}$$

where k, k' represent the momenta of the final charged lepton and antineutrino respectively. We use the convention $\epsilon^{0123}=-1$. Using Lorentz covariance one can write

$$\mathcal{L}^{\mu\nu} = A(q^2)\,g^{\mu\nu} + B(q^2)\,\frac{q^{\mu}q^{\nu}}{q^2} \tag{17}$$

where neglecting neutrino masses

$$\begin{aligned} A(q^2) &= -\frac{I(q^2)}{6}\left(2q^2-m_l^2-\frac{m_l^4}{q^2}\right) \\ B(q^2) &= \frac{I(q^2)}{3}\left(q^2+m_l^2-2\frac{m_l^4}{q^2}\right) \end{aligned} \tag{18}$$

with

$$I(q^2) = \frac{\pi}{2q^2}(q^2-m_l^2) \tag{19}$$

Note that for a light lepton $l=e,\mu$ we can neglect terms in m_l^2/q^2 over most of the q^2 interval and thus use $B(q^2)\approx -A(q^2)$.

The hadron tensor is given by

$$\begin{aligned} \mathcal{H}_{\mu\nu}(p,p') = \frac{1}{2S+1}\sum_{r,r'} &\left\langle B'(S'),r'\ \vec{p}\,'\left|\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\right|B(S),r\ \vec{p}\right\rangle \\ &\times\left\langle B'(S'),r'\ \vec{p}\,'\left|\overline{\Psi}^c(0)\gamma_\nu(I-\gamma_5)\Psi^b(0)\right|B(S),r\ \vec{p}\right\rangle^* \end{aligned} \tag{20}$$

In Ref. [14] it is shown how $1/2\to 1/2$ hadronic matrix elements are evaluated within our model. The extension to the $1/2\longleftrightarrow 3/2$ and $3/2\to 3/2$ cases is straightforward.

In Table II we compare our results for $\Xi\to\Xi$ transitions with the ones calculated in different models. Our central values have been obtained with the AL1 potential of Ref. [15], while the errors shown indicate the spread of the results when using four other interquark potentials, three more taken from Ref. [15] and another one from Ref. [25]. In all cases we have used a value $|V_{cb}|=0.0413$. Our results are in a global reasonable agreement with the ones in Ref. [16] where they use a relativistic quark model evaluated in the quark-diquark approximation[6]. In Ref. [19], and using HQET, results around a factor of 4 larger than ours are found[7]. We believe this factor of 4 discrepancy stems from the fact that in Ref. [19] the author approximates $\eta(\omega)$ by $\eta(1)$ in the calculation of the decay width[8]. In Ref. [21] they obtain results similar to the ones in the previous reference[9]. Finally in Ref. [17] they obtain in general much larger

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[6] Note the results we show under Ref. [16] are a factor of 2 smaller than the originally published. The reason beint that the authors of that reference also missed a normalization factor $1/\sqrt{2}$ for diquarks with two equal quarks [26]. An erratum has been sent.

[7] Note the results we show under Ref. [19] are a factor of 2 larger than the originally published. There is a factor $\sqrt{2}$ wrong in the normalization of matrix elements that affects the published results [27].

[8] If we take for instance the approximate expression in the Eq. (21), which is closer to the approximations used in Ref. [19], and make $\eta(\omega)=\eta(1)$ we get $\Gamma(\Xi_{bc}\to\Xi_{cc}) = 9.4\times 10^{-14}$ GeV in agreement with the result in Ref. [19]. On the other hand if we take the actual $\eta(\omega)$ values we get $\Gamma(\Xi_{bc}\to\Xi_{cc}) = 2.4\times 10^{-14}$ GeV, roughly a factor of 4 smaller and in agreement with our full calculation result.

[9] We must say we believe the calculation in Ref. [21] is affected by the same normalization mistake made in the original calculation in Ref. [19] as they give $F_1+F_2+F_3=\eta$ instead of $F_1+F_2+F_3=\sqrt{2}\,\eta$. To our understanding their present results have to be multiplied by a factor of 2.

	This work	[16]†	[17]	[21]		This work	[16]†	[17]	[19]‡	[21]
$\Gamma(\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	$1.92^{+0.25}_{-0.05}$	1.63	28.5	8.99	$\Gamma(\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$2.57^{+0.26}_{-0.03}$	2.30	8.93	8.0	8.87
$\Gamma(\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	$1.06^{+0.13}_{-0.03}$	0.82	4.28		$\Gamma(\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$1.36^{+0.10}_{-0.03}$	0.88	7.76		
$\Gamma(\Xi_{bb} \to \Xi^*_{bc}\, l\bar{\nu}_l)$	$0.61^{+0.04}$	0.53	27.2	2.70	$\Gamma(\Xi_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	$0.75^{+0.06}$	0.72	14.1	2.4	2.66
$\Gamma(\Xi^*_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	$1.04^{+0.06}$	0.82	8.57		$\Gamma(\Xi'_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	$2.33^{+0.16}$	1.70	28.8		
$\Gamma(\Xi^*_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	$0.35^{+0.03}$	0.28	52.0		$\Gamma(\Xi^*_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$0.43^{+0.06}$	0.38	27.5		
$\Gamma(\Xi^*_{bb} \to \Xi^*_{bc}\, l\bar{\nu}_l)$	$2.09^{+0.16}$	1.92	12.9		$\Gamma(\Xi^*_{bc} \to \Xi^*_{cc}\, l\bar{\nu}_l)$	$2.63^{+0.40}$	2.69	17.2		

TABLE II: Decay widths in units of 10^{-14} GeV for doubly heavy Ξ baryon semileptonic decay. Our central results have been obtained with the AL1 potential of Ref. [15]. The errors show the spread of results when using four other interquark potentials taken from Refs. [15, 25]. We have used a value $|V_{cb}| = 0.0413$. l stands for a light charged lepton, $l = e,\ \mu$. For results with † and ‡ see text for details. The results of Ref. [21] are given as quoted in Ref. [16].

	This work	[16]†		This work	[16]†
$\Gamma(\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	$2.14^{+0.20}_{-0.02}$	1.70	$\Gamma(\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$2.59^{+0.20}$	2.48
$\Gamma(\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	$1.16^{+0.13}$	0.83	$\Gamma(\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$1.36^{+0.9}$	0.95
$\Gamma(\Omega_{bb} \to \Omega^*_{bc}\, l\bar{\nu}_l)$	$0.67^{+0.08}$	0.55	$\Gamma(\Omega_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	$0.76^{+0.13}$	0.74
$\Gamma(\Omega^*_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	$1.13^{+0.11}_{-0.08}$	0.85	$\Gamma(\Omega'_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	$2.36^{+0.33}$	1.83
$\Gamma(\Omega^*_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	$0.38^{+0.04}_{-0.02}$	0.29	$\Gamma(\Omega^*_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$0.44^{+0.06}$	0.40
$\Gamma(\Omega^*_{bb} \to \Omega^*_{bc}\, l\bar{\nu}_l)$	$2.29^{+0.31}_{-0.04}$	2.0	$\Gamma(\Omega^*_{bc} \to \Omega^*_{cc}\, l\bar{\nu}_l)$	$2.79^{+0.60}$	2.88

TABLE III: Same as Table II for doubly heavy Ω baryon semileptonic decay. Decay widths are given in units of 10^{-14} GeV.

results for all transitions. In this latter calculation the authors take the Bethe–Salpeter equation model to analyze the weak transition matrix elements between two heavy diquarks, and then use "superflavor" symmetry [8, 9, 10] to evaluate the transition matrix elements at the baryon level. The global results show a contradiction between the calculation by Guo *et al.* [17] in one hand, and ours and the one by Ebert *et al.* [16] on the other.

In Table III we show results for $\Omega \to \Omega$ transitions. Again we get a global reasonable agreement with the calculation by Ebert *et al.* [16].

It is worthwhile to mention that for the case of baryons with a bc heavy quark content the actual physical states Ξ and Ω will be an admixture of Ξ_{bc}, Ξ'_{bc} and Ω_{bc}, Ω'_{bc} respectively. If we look for instance at our model predictions we see the widths are very different for transitions involving Ξ_{bc} or Ξ'_{bc}, and Ω_{bc} or Ω'_{bc}. Accurate measurements of decay widths could thus give information on the admixtures.

V. HQSS CONSTRAINTS ON SEMILEPTONIC DECAY WIDTHS

To the extent that one is close enough to the infinite heavy quark mass limit and near zero recoil we can combine the HQSS results in Eqs.(4-9) with Eq.(17), to approximate the tensor product $\mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu}$ by

$$B_{bc} \to B_{cc} \qquad \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \approx \eta^2 \frac{1}{9}\left\{A(q^2)\,(-26\,\omega + 20) + B(q^2)\left[26\,\frac{(v'\cdot q)(v\cdot q)}{q^2} + (5 - 13\,\omega)\right]\right\} \tag{21}$$

$$B'_{bc} \to B_{cc} \qquad \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \approx \eta^2 \frac{1}{9}\left\{A(q^2)\,(-6\,\omega - 12) + B(q^2)\left[6\,\frac{(v'\cdot q)(v\cdot q)}{q^2} - 3(1 + \omega)\right]\right\} \tag{22}$$

$$B_{bc} \to B^*_{cc} \qquad \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \;\approx\; \eta^2\frac{1+\omega}{9}\left\{-6A(q^2)+2B(q^2)\left[\frac{(v'\cdot q)^2}{q^2}-1\right]\right\} \tag{23}$$

$$B'_{bc} \to B^*_{cc} \qquad \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \;\approx\; \eta^2\frac{1+\omega}{3}\left\{-6A(q^2)+2B(q^2)\left[\frac{(v'\cdot q)^2}{q^2}-1\right]\right\} \tag{24}$$

$$B^*_{bc} \to B_{cc} \qquad \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \;\approx\; \eta^2\frac{1+\omega}{9}\left\{-3A(q^2)+B(q^2)\left[\frac{(v\cdot q)^2}{q^2}-1\right]\right\} \tag{25}$$

$$B^*_{bc} \to B^*_{cc} \qquad \mathcal{L}^{\mu\nu}\,\mathcal{H}_{\mu\nu} \;\approx\; \eta^2\frac{1}{9}\left\{-A(q^2)\,\omega\left(4+8\omega^2\right)+B(q^2)\left[-\,\omega\left(6+4\omega^2\right)+\frac{(v'\cdot q)(v\cdot q)}{q^2}\left(20+8\omega^2\right)\right]\right\} \tag{26}$$

and similar ones for $bb \to bc$ decays.

Working in the strict near zero recoil approximation, $\omega \approx 1$ or equivalently q^2 quite close to its maximum value $q^2_{\rm max}$, we can approximate

$$\frac{(v\cdot q)^2}{q^2} \approx \frac{(v'\cdot q)(v\cdot q)}{q^2} \approx \frac{(v'\cdot q)^2}{q^2} \approx 1, \tag{27}$$

and $A(q^2) \approx -B(q^2)$ near $q^2_{\rm max}$. In these circumstances, and using

$$m_{B_{bb}} \approx m_{B^*_{bb}}\ ;\ m_{B_{bc}} \approx m_{B'_{bc}} \approx m_{B^*_{bc}}\ ;\ m_{B_{cc}} \approx m_{B^*_{cc}}, \tag{28}$$

HQSS predicts that the different decay widths are in the relative ratios

$$\begin{array}{ccccccccccc} \Gamma(B_{bc}\to B_{cc}) & : & \Gamma(B'_{bc}\to B_{cc}) & : & \Gamma(B_{bc}\to B^*_{cc}) & : & \Gamma(B'_{bc}\to B^*_{cc}) & : & \Gamma(B^*_{bc}\to B_{cc}) & : & \Gamma(B^*_{bc}\to B^*_{cc}) \\ 4 & : & 3 & : & 2 & : & 6 & : & 1 & : & 5 \end{array} \tag{29}$$

$$\begin{array}{ccccccccccc} \Gamma(B_{bb}\to B_{bc}) & : & \Gamma(B_{bb}\to B'_{bc}) & : & \Gamma(B_{bb}\to B^*_{bc}) & : & \Gamma(B^*_{bb}\to B'_{bc}) & : & \Gamma(B^*_{bb}\to B_{bc}) & : & \Gamma(B^*_{bc}\to B^*_{bc}) \\ 4 & : & 3 & : & 2 & : & 3 & : & 1 & : & 5 \end{array} \tag{30}$$

In Table IV we show the above ratios obtained in different models. Our results and the ones by Ebert *et al.*[16] are in reasonable agreement with the HQSS predictions in this strict near zero recoil approximation. On the other hand the results by Guo *et al.*[17] deviate heavily form the above predictions. This disagreement does not improve much by using a different decay width to normalize the ratios

	$\Gamma(B_{bc}\to B_{cc})$	:	$\Gamma(B'_{bc}\to B_{cc})$	:	$\Gamma(B_{bc}\to B^*_{cc})$	:	$\Gamma(B'_{bc}\to B^*_{cc})$	:	$\Gamma(B^*_{bc}\to B_{cc})$	:	$\Gamma(B^*_{bc}\to B^*_{cc})$
HQSS	4	:	3	:	2	:	6	:	1	:	5
This work Ξ	6.04	:	3.20	:	1.75	:	5.48	:	1	:	6.18
This work Ω	5.88	:	3.08	:	1.73	:	5.36	:	1	:	6.33
[16] Ξ	6.12	:	2.35	:	1.91	:	4.53	:	1	:	7.16
[16] Ω	6.19	:	2.38	:	1.85	:	4.58	:	1	:	7.20
[17] Ξ	0.32	:	0.28	:	0.53	:	1.05	:	1	:	0.63

	$\Gamma(B_{bb}\to B_{bc})$	:	$\Gamma(B_{bb}\to B'_{bc})$	:	$\Gamma(B_{bb}\to B^*_{bc})$	:	$\Gamma(B^*_{bb}\to B'_{bc})$	:	$\Gamma(B^*_{bb}\to B_{bc})$	:	$\Gamma(B^*_{bb}\to B^*_{bc})$
HQSS	4	:	3	:	2	:	3	:	1	:	5
This work Ξ	5.56	:	3.07	:	1.75	:	3.01	:	1	:	6.04
This work Ω	5.71	:	3.09	:	1.77	:	3.01	:	1	:	6.11
[16] Ξ	5.93	:	2.98	:	1.91	:	2.96	:	1	:	6.96
[16] Ω	5.96	:	2.91	:	1.93	:	2.98	:	1	:	7.00
[17] Ξ	0.55	:	0.08	:	0.52	:	0.16	:	1	:	0.24

TABLE IV: Decay width ratios for semileptonic $bc \to cc$ and $bb \to bc$ decay of doubly heavy Ξ and Ω baryons compared to the HQSS predictions in the strict near zero recoil approximation. Our results have been obtained with the AL1 potential of Ref. [15]. l stands for a light charged lepton, $l = e,\, \mu$.

We can relax the strict near near zero recoil approximation to obtain more accurate predictions based on HQSS in the following way. For the actual doubly heavy baryon masses $\omega_{max} \approx 1.22$ (1.08) for $bc \to cc$ ($bb \to bc$) transitions while the different differential decay widths $d\Gamma/d\omega$ show a maximum at around $\omega \approx 1.05$ (1.01). We can thus still use $\omega \approx 1$ and $A(q^2) \approx -B(q^2)$. On the other hand the quantities $(v \cdot q)^2/q^2$, $(v' \cdot q)^2/q^2$, $(v \cdot q)(v' \cdot q)/q^2$, that are all equal to 1 near zero recoil, can deviate rapidly from 1 because of the q^2 factor in the denominator. What is true, in and around the maximum of the differential decay width, is that we can reasonable approximate

$$\begin{aligned} \frac{(v\cdot q)^2}{q^2} &\approx \frac{(v'\cdot q)(v\cdot q)}{q^2} \\ \frac{(v'\cdot q)^2}{q^2} &\approx \frac{(v'\cdot q)(v\cdot q)}{q^2} \end{aligned} \tag{31}$$

With the above consideration we can still predict approximate ratios between different decay widths that one expects to be satisfied to an accuracy of $20 \sim 30\%$. We have chosen to define those ratios so that they are all equal to one,

$$\frac{\Gamma(B'_{bc} \to B^*_{cc}\, l\bar\nu_l)}{3\,\Gamma(B_{bc} \to B^*_{cc}\, l\bar\nu_l)} \approx \frac{\Gamma(B^*_{bb} \to B'_{bc}\, l\bar\nu_l)}{3\,\Gamma(B^*_{bb} \to B_{bc}\, l\bar\nu_l)} \approx 1 \tag{32}$$

$$\frac{\Gamma(B_{bc} \to B^*_{cc}\, l\bar\nu_l)}{\frac{2}{3}\,\Gamma(B'_{bc} \to B_{cc}\, l\bar\nu_l)} \approx \frac{\Gamma(B_{bb} \to B^*_{bc}\, l\bar\nu_l)}{\frac{2}{3}\,\Gamma(B_{bb} \to B'_{bc}\, l\bar\nu_l)} \approx 1 \tag{33}$$

$$\frac{\Gamma(B^*_{bc} \to B_{cc}\, l\bar\nu_l)}{\frac{1}{3}\,\Gamma(B'_{bc} \to B_{cc}\, l\bar\nu_l)} \approx \frac{\Gamma(B^*_{bb} \to B_{bc}\, l\bar\nu_l)}{\frac{1}{3}\,\Gamma(B_{bb} \to B'_{bc}\, l\bar\nu_l)} \approx 1 \tag{34}$$

$$\frac{\Gamma(B^*_{bc} \to B^*_{cc}\, l\bar\nu_l)}{\Gamma(B_{bc} \to B_{cc}\, l\bar\nu_l) + \frac{1}{2}\,\Gamma(B_{bc} \to B^*_{cc}\, l\bar\nu_l)} \approx \frac{\Gamma(B^*_{bb} \to B^*_{bc}\, l\bar\nu_l)}{\Gamma(B_{bb} \to B_{bc}\, l\bar\nu_l) + \frac{1}{2}\,\Gamma(B_{bb} \to B^*_{bc}\, l\bar\nu_l)} \approx 1 \tag{35}$$

Note, we consider as independent the phase-space integrals of $\eta^2(\omega)A(q^2)$ and $\eta^2(\omega)A(q^2)\frac{(v'\cdot q)(v\cdot q)}{q^2}$.

In Tables V,VI we show the above ratios evaluated in different quark model approaches. Once again calculations in this work and the ones in Ref.[16] are compatible, within the expected accuracy, with the approximate ratios obtained using HQSS results. On the other hand the deviations found in the results by Guo *et al.* are, in most cases, too large.

	This work		[16]		[17]	
	Ξ	Ω	Ξ	Ω	Ξ	Ω
$\frac{\Gamma(B'_{bc}\to B^*_{cc}\, l\bar\nu_l)}{3\,\Gamma(B_{bc}\to B^*_{cc}\, l\bar\nu_l)}$	$1.04^{+0.03}_{-0.01}$	$1.04_{-0.03}$	0.79	0.82	0.68	—
$\frac{\Gamma(B_{bc}\to B^*_{cc}\, l\bar\nu_l)}{\frac{2}{3}\,\Gamma(B'_{bc}\to B_{cc}\, l\bar\nu_l)}$	$0.82^{+0.06}_{-0.01}$	$0.84^{+0.13}_{-0.01}$	1.22	1.17	2.72	—
$\frac{\Gamma(B^*_{bc}\to B_{cc}\, l\bar\nu_l)}{\frac{1}{3}\,\Gamma(B'_{bc}\to B_{cc}\, l\bar\nu_l)}$	$0.94^{+0.11}$	$0.97^{+0.10}_{-0.01}$	1.28	1.26	10.6	—
$\frac{\Gamma(B^*_{bc}\to B^*_{cc}\, l\bar\nu_l)}{\Gamma(B_{bc}\to B_{cc}\, l\bar\nu_l)+\frac{1}{2}\,\Gamma(B_{bc}\to B^*_{cc}\, l\bar\nu_l)}$	$0.89^{+0.11}$	$0.94^{+0.13}_{-0.01}$	1.01	1.01	1.08	—

TABLE V: Decay width ratios for semileptonic $bc \to cc$ decay of doubly heavy Ξ and Ω baryons. In all cases the approximate result obtained using HQSS is 1. Our central results have been obtained with the AL1 potential of Ref. [15]. The errors show the spread of results when using four other interquark potentials taken from Refs. [15, 25]. l stands for a light charged lepton, $l = e,\ \mu$.

VI. SUMMARY

We have checked the constraints imposed by HQSS on form factors and decay widths. To our knowledge those constraints have not been exploited before to check the consistency of different quark model calculations. We have shown that our calculation is consistent with HQSS. The ratios in Eqs.(29,30), obtained using HQSS with strict zero recoil approximation, and the approximate ratios in Eqs.(32-35), where we have relaxed that approximation, compare well with the results in our model and the one by Ebert *et al.* [16], but they are incompatible with the calculation in Ref. [17]. We think that although this is not enough guarantee for the predictions here and in Ref. [16] to be fully correct (in fact the few results in Refs. [19, 21] are not incompatible with HQSS constraints while they are a factor of four larger than ours), it certainly indicates problems either in the model or in the calculation performed in Ref. [17].

	This work		[16]		[17]	
	Ξ	Ω	Ξ	Ω	Ξ	Ω
$\dfrac{\Gamma(B_{bb}^{*}\to B_{bc}^{\prime}\, l\bar{\nu}_l)}{3\,\Gamma(B_{bb}^{*}\to B_{bc}\, l\bar{\nu}_l)}$	$1.00^{+0.01}_{-0.04}$	$1.00^{+0.03}_{-0.01}$	0.99	0.99	0.05	—
$\dfrac{\Gamma(B_{bb}\to B_{bc}^{*}\, l\bar{\nu}_l)}{\frac{2}{3}\,\Gamma(B_{bb}\to B_{bc}^{\prime}\, l\bar{\nu}_l)}$	$0.86^{+0.08}_{-0.06}$	$0.86^{+0.05}$	0.96	0.99	9.53	—
$\dfrac{\Gamma(B_{bb}^{*}\to B_{bc}\, l\bar{\nu}_l)}{\frac{1}{3}\,\Gamma(B_{bb}\to B_{bc}^{\prime}\, l\bar{\nu}_l)}$	$0.98^{+0.09}_{-0.03}$	$0.97^{+0.06}_{-0.14}$	1.01	1.03	36.4	—
$\dfrac{\Gamma(B_{bb}^{*}\to B_{bc}^{*}\, l\bar{\nu}_l)}{\Gamma(B_{bb}\to B_{bc}\, l\bar{\nu}_l)+\frac{1}{2}\,\Gamma(B_{bb}\to B_{bc}^{*}\, l\bar{\nu}_l)}$	$0.94^{+0.07}_{-0.06}$	$0.93^{+0.11}_{-0.10}$	1.01	1.01	0.31	—

TABLE VI: Same as Table V for semileptonic $bb \to bc$ decay of doubly heavy Ξ and Ω baryons.

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