

Light Flavour and Heavy Quark Spin Symmetry in Heavy Meson Molecules

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We propose an effective field theory incorporating light SU(3)-flavour and heavy quark spin symmetry to describe charmed meson-antimeson bound states. At lowest order the effective field theory entails a remarkable simplification: it only involves contact range interactions among the heavy meson and antimeson fields. We show that the isospin violating decays of the $X(3872)$ can be used to constraint the interaction between the D and a \bar{D}^* mesons in the isovector channel. As a consequence, we can rule out the existence of a isovector partner of the $X(3872)$. If we additionally assume that the $X(3915)$ and $Y(4140)$ are $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ molecular states, we can determine the full spectrum of molecular states with isospin $I = 0, \frac{1}{2}$ and 1.

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I. INTRODUCTION

The $X(3872)$ resonance [1] has opened new perspectives in hadron spectroscopy. The $X(3872)$, even though it clearly contains a $c\bar{c}$ pair, does not fit well within the standard charmonium spectrum. Disentangling its nature requires a more exotic explanation. Among the theoretical proposals available, the interpretation of the $X(3872)$ as a hadronic molecule [2–6] is especially promising and has attracted the attention of the community. The motivation behind the molecular picture, in which the $X(3872)$ is a bound state of a charmed meson and antimeson, is the striking closeness of this resonance to the $D^0\bar{D}^{0*}$ threshold, $m_{X(3872)} = 3871.68 \pm 0.17$ MeV, to be compared with $m_{D^0} + m_{D^{0*}} = 3871.84 \pm 0.20$ MeV [7]. However, the likelihood of the molecular hypothesis depends on the J^{PC} quantum numbers of the $X(3872)$, which have not been experimentally determined yet. They are either 1^{++} or 2^{-+} [8–10] (see also the interesting analysis of Ref. [11]), of which only 1^{++} is compatible with a low-lying S-wave bound state.

The discovery of the $X(3872)$ has been followed by the experimental observation of a series of hidden charm resonances above the open-charm threshold, the so-called XYZ states. Some of the XYZ states fit well within the charmonium spectrum (most notably the $Z(3940)$ [12]), but others do not and may require, just like the $X(3872)$, non-conventional explanations. A few might be molecular: several authors [13–15] have proposed the $X(3915)$ [16] and $Y(4140)$ [17] to be $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ bound states. The $Y(4260)$ [18] might even have a three body structure ($J/\Psi K\bar{K}$) [19]. The $Y(4660)$ [20] and $X(4630)$ [21] have been theorized to be a $f_0(980)\Psi'$ molecule [22, 23] (they may be the same state [23]). More recently, the Belle collaboration has observed two hidden bottom resonances, the $Z_b(10610)$ and $Z_b(10650)$ [24, 25],

located just a few MeV away from the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds. Hence, the Z_b 's may also have a molecular nature [26–28].

Heavy meson-antimeson molecules are very interesting objects from the theoretical point of view. As far as they are not tightly bound, the meson and antimeson will preserve their individuality and will not probe the specific details of the short range interaction responsible of their binding. Thus, there exists a scale separation and molecular states may be amenable to an effective field theory (EFT) treatment in which they can be described as mesons interacting via contact interactions and pion exchanges (e.g. X-EFT [29], Heavy Meson Molecule EFT [30]). In addition, the heavy-light quark content of heavy meson molecules implies a high degree of symmetry. They are subjected to heavy quark spin symmetry (HQSS) [31–34], which imposes interesting constraints in the heavy meson-antimeson interaction [35]. As a consequence, HQSS can be used to predict the existence of so far unobserved molecular states [26, 28, 36]. On the other hand, we must not forget the light quark content of heavy meson molecules. If we consider $q = u, d, s$, we expect SU(3) flavour symmetry to hold: we can arrange molecular states within SU(3) multiplets. In this work we will explore the consequences of this symmetry in the spectrum of heavy meson molecules.

The effect of HQSS and SU(3) flavour symmetry is to generate relationships among the heavy meson-antimesons interactions in different channels. Four parameters are enough to describe the twenty-four possible S-wave molecules¹. That is, we need four data points to predict the full molecular spectrum. For this purpose we will assume the molecular nature of certain XYZ states

¹ This is the total dimension, without considering the spin-isospin third component multiplicities of the $H\bar{H}$ space ($H = D^+, D^0, D_s^0, D^{*+}, D^{*0}$ and D_s^{*0}) or states that are connected by a C-parity transformation, such as $D_s\bar{D}$ and $\bar{D}_s D$ that are not counted twice.

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such as the $X(3872)$, $X(3915)$ and $Y(4140)$. If the predictions turn out to be correct, this will serve as a confirmation that the states used as input are molecular. We notice that the $X(3915)$ could also be a charmonium state, as the decay properties are similar to a molecule's, with the most important decay channels being $J/\Psi \omega$ and $D\bar{D}$. However, according to Ref. [37], the charmonium hypothesis implies a $D\bar{D}$ partial decay width of about one hundred MeV at least, larger than the experimental total decay width (about 30 MeV). In contrast, the molecular picture predicts a partial decay width to $D\bar{D}$ of the order of tens of MeV [36]. We also warn that the $Y(4140)$ is far from being confirmed and might not even exist: after its discovery [17], it has not been observed in subsequent experiments [38, 39]. Finally, we notice that SU(2) isospin and SU(3) flavour symmetry are broken in the masses of the heavy mesons. From this we can derive interesting consequences. In particular, from the analysis of the isospin violating branching of the $X(3872)$ into $J/\Psi \omega$ and $J/\Psi \rho$, we will be able to determine the strength of the contact interaction in the isovector channel and to discard the existence of an isovector or $D_s \bar{D}_s^*$ counterpart of the $X(3872)$.

The article is structured as follows: in Sect. II we briefly review the effective field theory formalism that we use for the description of heavy meson molecules. In Sect. III we consider the isospin violating decay of the $X(3872)$ into $J/\Psi \rho$, which can be used to obtain information about the interaction of the $D\bar{D}^*$ in the isovector channel. In Sect. IV we calculate the location of the SU(3)-flavour and HQSS partners of the $X(3872)$, $X(3915)$ and $Y(4140)$. Finally, we present our conclusions in Sect. V.

II. THE EFT DESCRIPTION AT LOWEST ORDER

In this section we review the EFT we use for describing heavy meson molecules. The presentation is brief – we have already discussed this EFT in previous publications [30, 36] – and we concentrate on the ideas rather than the technicalities. The EFT description involves pions and heavy meson/antimeson fields and the local interactions among these degrees of freedom that are compatible with the known low energy symmetries, most notably HQSS and chiral symmetry. A remarkable simplification is that pion exchanges are weaker than naively expected [30] and only enter as a perturbation at subleading orders. A similar thing happens to coupled channel dynamics. Hence, at lowest or leading order (LO), the EFT consists on heavy meson and antimeson interacting through a contact range potential.

A. Overview of the EFT Formalism

EFTs are generic theoretical descriptions of low energy phenomena. They become an adequate tool for situations in which a more fundamental description in terms of the underlying high-energy dynamics is impractical for whatever reasons. Thus they are very useful for low-energy hadronic processes where quantum chromodynamics (QCD) is not solvable owing to asymptotic freedom and confinement.

The formulation of an EFT requires the identification of the degrees of freedom and symmetries that are important for the low energy dynamics. For heavy meson molecules, the degrees of freedom are the heavy meson and antimeson, plus the pion field. The relevant symmetries, which provide the connection to the underlying theory (QCD), are chiral symmetry and HQSS. It is important to notice that EFTs are only useful if there exists a separation of scales between low and high energy physics. If we name the characteristic low energy scale at which the EFT is expected to work as Q , and the high energy scale as Λ_0 , EFTs allows to construct the amplitudes as a power series expansion in terms of the small parameter $x_0 = Q/\Lambda_0$

$$\mathcal{A} = \sum_{\nu} \mathcal{A}^{(\nu)} = \sum_{\nu} x_0^{\nu} \hat{\mathcal{A}}^{(\nu)}, \quad (1)$$

meaning that if we truncate the expansion at $\nu = \nu_{\max}$, the *a priori* error of the calculation will be $x_0^{\nu_{\max}+1}$. The ordering principle behind the EFT expansion is power counting. It is related to the scaling properties of the diagrams contributing to the EFT amplitude \mathcal{A} under a rescaling of all the Q 's

$$\mathcal{A}^{(D)}(\lambda Q, \Lambda_0) = \lambda^{\nu_D} \mathcal{A}^{(D)}(Q, \Lambda_0), \quad (2)$$

where the superscript (D) indicates that we are considering a single diagram of order ν_D (equivalently, we say that D is of order Q^{ν_D}). At each order Q^{ν} in the EFT expansion there is only a finite number of diagrams involving the low energy fields and their symmetries.

B. The EFT Potential

Heavy mesons are non-relativistic and form bound states. The first property entails a simplification of the EFT description, while the second triggers a series of non-trivial changes in the power counting that we will comment later. The simplification is that the heavy meson-antimeson potential is a well-defined quantity. Instead of directly expanding the scattering amplitude within EFT, we can expand (and truncate) the heavy meson-antimeson potential

$$V = \sum_{\nu=0}^{\nu_{\max}} V^{(\nu)} + \mathcal{O}(x_0^{\nu_{\max}+1}). \quad (3)$$

Then we calculate wave functions and observables in the standard quantum mechanical fashion. For example, we can generate bound states by iterating the EFT potential in the Schrödinger / Lippmann-Schwinger equation ².

At LO, which corresponds to $\nu = 0$, the heavy meson-antimeson potential is local

$$\langle \vec{p}' | V^{(0)} | \vec{p} \rangle = V^{(0)}(\vec{p}' - \vec{p}), \quad (4)$$

and receives the contribution of two diagrams, a four heavy meson vertex and the one pion exchange (OPE) potential. The LO potential reads (schematically)

$$V^{(0)}(\vec{q}) = C_0^{(0)} + \eta \frac{g^2}{2f_\pi^2} \frac{(\vec{a} \cdot \vec{q})(\vec{b} \cdot \vec{q})}{\vec{q}^2 + m_\pi^2}. \quad (5)$$

The contact range coupling $C_0^{(0)}$ is a free parameter. Its value can be determined from the location of a bound state. The OPE contribution depends on the pion decay constant $f_\pi \simeq 132$ MeV and the axial coupling $g \simeq 0.6$, which we have particularized for the charmed meson case. The sign η and the spin operators \vec{a} and \vec{b} depend on whether the incoming/outgoing heavy meson and antimeson are pseudoscalar or vector (the details can be consulted in Ref. [30]).

A problematic feature of the EFT potential is that it tends to a constant, non-zero value at large exchanged momentum. Thus, heavy meson-antimeson loops containing the EFT potential will be divergent. We solve this issue by renormalizing the EFT calculations. For that, we define a regularized potential V_Λ as follows

$$\langle \vec{p}' | V_\Lambda | \vec{p} \rangle = f\left(\frac{p}{\Lambda}\right) \langle \vec{p}' | V | \vec{p} \rangle f\left(\frac{p'}{\Lambda}\right), \quad (6)$$

where Λ is an ultraviolet cut-off, $p = |\vec{p}|$, $p' = |\vec{p}'|$ and $f(x)$ a regulator function obeying the conditions (i) $f(0) = 1$, (ii) $f(x) \rightarrow 0$ for $x \rightarrow \infty$. For concreteness we use the gaussian regulator function

$$f_{\text{Gauss}}(x) = e^{-x^2}, \quad (7)$$

though other choices of the regulator are equally valid ³. The cut-off dependence of the theory is absorbed in the counterterms, completing the renormalization process. We comment that there is no universal criterion for choosing the cut-off, but as a general rule it does not need to be much larger than the high energy scale Λ_0 [44]. Here we employ the window $\Lambda = 0.5 - 1.0$ GeV, which is of the order of Λ_0 .

² This is in fact the basis of the Weinberg counting as originally formulated in the context of two-nucleon scattering in EFT [40, 41]. However, the power counting we obtain is equivalent to the Kaplan, Savage and Wise proposal [42, 43], modulo the obvious difference that here we are dealing with heavy mesons instead of nucleons.

³ We notice that multipolar regulators are closer in spirit to the form factors that are usually employed in more phenomenological approaches. Yet in EFT treatments there is no privileged choice of a regulator function. Nonetheless we will briefly comment on the impact of using other regulators later on when we present our results.

C. Power Counting and Bound States

The existence of bound states implies changes in the power counting of the potential. This can be easily appreciated by considering the bound state equation

$$|\Psi_B\rangle = G_0(E)V|\Psi_B\rangle, \quad (8)$$

where $|\Psi_B\rangle$ is the wave function, $G_0(E) = 1/(E - H_0)$ the resolvent operator and V the non-relativistic potential. Then it is apparent from the EFT point of view that power counting requires the successive iterations of G_0V to be of the same order

$$\mathcal{O}(G_0V) = \mathcal{O}(G_0VG_0V). \quad (9)$$

If we take into account that $G_0(E)$ is of order Q , it is clear that the existence of a bound states requires that the EFT potential contains a contribution of order Q^{-1} . That is, the naive assignment of order Q^0 to the lowest order potential is incompatible with the existence of bound states. The solution is to promote at least one of the diagrams conforming the Q^0 potential to order Q^{-1} [42, 43, 45–47]. For heavy meson-antimeson molecules we usually move the contact range potential to Q^{-1} and left the OPE potential at Q^0 . The consequences of this promotion are that (i) we redefine LO as Q^{-1} and (ii) OPE is a subleading order effect, that is, a small perturbation over the LO results. As we are only considering the LO calculation, we are left with a very simple theory that only contains contact interactions. The exception is the isoscalar bottom sector, in which OPE is stronger than naively expected and is moved to order Q^{-1} together with the counterterm [30].

To end the discussion, we find it useful to write down the eigenvalue equation for the bound states:

$$\frac{1}{C_0(\Lambda)} = - \int \frac{d^3\vec{q}}{(2\pi)^3} f^2\left(\frac{q}{\Lambda}\right) \frac{2\mu}{q^2 + \gamma^2}, \quad (10)$$

which relates the position of the bound state with the strength of the contact interaction C_0 (i.e. we are assuming that this is the only operator that enters at LO). In the equation above $f(x)$ is the regulator function and $\gamma^2 = -2\mu E_B$, with $E_B < 0$ the bound state energy and μ the reduced mass of the two-body system. Provided the cut-off Λ and the wave number γ , we can determine the value of C_0 . Conversely, from C_0 and Λ , we can predict the location of a bound state. Further details on the eigenvalue equation above and on the wave functions we obtain from it can be found in Refs. [36, 48].

D. The Heavy Quark Spin Structure of the EFT Potential

According to the previous arguments, at lowest order in the EFT expansion the potential only contains

an energy- and momentum- independent S-wave contact range interaction (per channel). In a given isospin-strangeness sector, naively this translates into six counterterms, one for each of the possible S-wave configurations of a heavy meson molecule. However, HQSS reduces the number of independent counterterms to two, which we call C_a and C_b [35].

Owing to HQSS the LO potential mixes the different particle channels. The reason is that the P (\bar{P}) and P^* (\bar{P}^*) mesons (antimesons) can be transformed into each other by means of a flip of the spin of the heavy quark. Nevertheless the total angular momentum and parity of a heavy meson molecule is conserved. If we consider the following set of basis (with well-defined J^P)

$$\mathcal{B}(0^+) = \{|P\bar{P}\rangle, |P^*\bar{P}^*(0)\rangle\}, \quad (11)$$

$$\mathcal{B}(1^+) = \{|P\bar{P}^*\rangle, |P^*\bar{P}\rangle, |P^*\bar{P}^*(1)\rangle\}, \quad (12)$$

$$\mathcal{B}(2^+) = \{|P^*\bar{P}^*(2)\rangle\}, \quad (13)$$

the LO potential takes the form

$$V^{\text{LO}}(\vec{q}, 0^+) = \begin{pmatrix} C_a & \sqrt{3}C_b \\ \sqrt{3}C_b & C_a - 2C_b \end{pmatrix}, \quad (14)$$

$$V^{\text{LO}}(\vec{q}, 1^+) = \begin{pmatrix} C_a & -C_b & \sqrt{2}C_b \\ -C_b & C_a & \sqrt{2}C_b \\ \sqrt{2}C_b & \sqrt{2}C_b & C_a - C_b \end{pmatrix}, \quad (15)$$

$$V^{\text{LO}}(\vec{q}, 2^+) = C_a + C_b. \quad (16)$$

The notation $|P^{(*)}\bar{P}^{(*)}\rangle$ is used to indicate a system of a heavy meson/antimeson containing a heavy quark/antiquark, regardless of the light quark content. That is, the heavy meson $P^{(*)}$ is not necessarily the antiparticle of the heavy antimeson $\bar{P}^{(*)}$, as they could contain different types of light quarks. The number in parenthesis in the $|P^*\bar{P}^*(J)\rangle$ states is the total intrinsic spin J to which the two vector meson system couples.

In the specific case in which the heavy meson and antimeson are each other antiparticle, the C-parity is a well-defined (and conserved) quantum number. The $J^P = 0^+$ and 2^+ states are now $J^{PC} = 0^{++}$ and 2^{++} states, but neither the potentials nor the bases we use to express them change

$$\mathcal{B}(0^{++}) = \mathcal{B}(0^+), \quad (17)$$

$$\mathcal{B}(2^{++}) = \mathcal{B}(2^+), \quad (18)$$

$$V^{\text{LO}}(\vec{q}, 0^{++}) = V^{\text{LO}}(\vec{q}, 0^+), \quad (19)$$

$$V^{\text{LO}}(\vec{q}, 2^{++}) = V^{\text{LO}}(\vec{q}, 2^+). \quad (20)$$

In contrast, the 1^+ states can be further subdivided into 1^{++} and 1^{+-} states that do not mix under the effect of the LO potential. We can write a 1^{++} and 1^{+-} basis

$$\mathcal{B}(1^{+-}) = \left\{ \frac{1}{\sqrt{2}} (|P\bar{P}^*\rangle + |P^*\bar{P}\rangle), |P^*\bar{P}^*(1)\rangle \right\}, \quad (21)$$

$$\mathcal{B}(1^{++}) = \left\{ \frac{1}{\sqrt{2}} (|P\bar{P}^*\rangle - |P^*\bar{P}\rangle) \right\}, \quad (22)$$

for which the LO potential now reads

$$V^{\text{LO}}(\vec{q}, 1^{+-}) = \begin{pmatrix} C_a - C_b & 2C_b \\ 2C_b & C_a - C_b \end{pmatrix}, \quad (23)$$

$$V^{\text{LO}}(\vec{q}, 1^{++}) = C_a + C_b. \quad (24)$$

As can be seen, the 1^{++} heavy meson-antimeson molecule decouples from the two 1^{+-} components. In addition, the 1^{++} potential is identical to the 2^{++} one. A 1^{++} heavy meson-antimeson molecule implies a 2^{++} HQSS partner [36].

There is still a significant simplification in the EFT potential that we have not discussed yet. In the charm sector the EFT description of heavy meson molecules is only expected to be valid if the binding energy is smaller than a maximum value B_{max} of the order of 150 – 300 MeV (that is, we take $B_{\text{max}} = \gamma_{\text{max}}^2/2\mu$, with $\gamma_{\text{max}} = \Lambda_0 \sim 0.5 - 1$ GeV and $\mu = m_D/2$, where m_D is the D meson mass). This figure is similar to the mass gap among the heavy meson-antimeson particle channels:

$$\Delta(0^+) \equiv M(D^*\bar{D}^*) - M(D\bar{D}) \simeq 280 \text{ MeV}, \quad (25)$$

$$\Delta(1^+) \equiv M(D^*\bar{D}^*) - M(D\bar{D}^*) \simeq 140 \text{ MeV}, \quad (26)$$

where $\Delta(J^P)$ denotes the gap in the J^P channel. The prize of ignoring the coupled channel effects is a reduction of the expansion parameter of the theory from $x_0 = \sqrt{B/B_{\text{max}}}$ to $x'_0 = \sqrt{B/\Delta(J^P)}$, where B is the binding energy of the molecular state and we have included the square roots to translate the energy ratios into momentum ratios (we are using a non-relativistic EFT). As the sizes of B_{max} and $\Delta(J^P)$ are comparable, we can safely ignore the coupled channel dynamics without compromising the range of validity of the EFT description of the bound states. Moreover, explicit calculations carried out in Ref. [36, 48] for the isoscalar channels confirm that the size of the coupled channel effects is the expected one in EFT.

Thus, we can simplify the EFT potential in the 0^+ and 1^+ channels to

$$V_{P\bar{P}}^{\text{LO}}(\vec{q}, 0^+) = C_a, \quad (27)$$

$$V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 0^+) = C_a - 2C_b, \quad (28)$$

$$V_{P^*\bar{P}/P\bar{P}^*}^{\text{LO}}(\vec{q}, 1^+) = \begin{pmatrix} C_a & -C_b \\ -C_b & C_a \end{pmatrix}, \quad (29)$$

$$V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 1^+) = C_a - C_b, \quad (30)$$

where we only need to keep track of the coupled channel dynamics in the 1^+ $P^*\bar{P}/P\bar{P}^*$ case. If C-parity is a good quantum number, the $P^*\bar{P}$ and $P\bar{P}^*$ thresholds coincide and the 1^{+-} component separates from the 1^{++} one:

$$V_{P^*\bar{P}/P\bar{P}^*}^{\text{LO}}(\vec{q}, 1^{+-}) = C_a - C_b \quad (31)$$

$$V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 1^{++}) = C_a + C_b, \quad (32)$$

while there is no change in the other channels (except the

additional C-parity quantum number)

$$V_{P\bar{P}}^{\text{LO}}(\vec{q}, 0^{++}) = V_{P\bar{P}}^{\text{LO}}(\vec{q}, 0^+), \quad (33)$$

$$V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 0^{++}) = V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 0^+), \quad (34)$$

$$V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 1^{+-}) = V_{P^*\bar{P}^*}^{\text{LO}}(\vec{q}, 1^+). \quad (35)$$

Now the potentials in the 1^{+-} $P^*\bar{P}/P\bar{P}^*$ and $P^*\bar{P}^*$ channels coincide. That is, the 1^{+-} molecular states come in pairs, one state per each of the particle channels, as happens with the $Z_b(10610)$ and $Z_b(10650)$ in the bottom sector.

E. The SU(3)-Flavour Structure of the EFT Potential

SU(3)-flavour symmetry implies that we can organize the heavy meson-antimeson states into SU(3)-multiplets. As there is a light quark and antiquark (3 and $\bar{3}$ representations), heavy meson-antimeson molecules can be organized in a singlet and octet representation, $3 \otimes \bar{3} = 1 \oplus 8$. We can label the heavy meson with a flavour index, that is, we write P_a , \bar{P}^a , P_a^* and \bar{P}^{*a} with $a = 1, 2, 3$, where the quark content is

$$P_a, P_a^* = (Q\bar{u}, Q\bar{d}, Q\bar{s}) \quad \text{and} \quad \bar{P}^a, \bar{P}^{*a} = \begin{pmatrix} \bar{Q}u \\ \bar{Q}d \\ \bar{Q}s \end{pmatrix}, \quad (36)$$

and $Q = c, b$ represents the heavy quark. In this notation we can construct the singlet and octet representations as follows

$$|P\bar{P}, 1\rangle = \frac{1}{\sqrt{3}} |P_a\bar{P}^a\rangle, \quad (37)$$

$$|P\bar{P}, 8; j\rangle = \frac{1}{\sqrt{2}} (\lambda_j)_b^a |P_a\bar{P}^b\rangle, \quad (38)$$

plus the analogous expressions for $P\bar{P}^*$, $P^*\bar{P}$ and $P^*\bar{P}^*$, where the λ_j 's are the Gell-Mann matrices. SU(3) flavour symmetry implies that the heavy meson-antimeson interaction distinguishes the singlet and octet representations

$$\langle P\bar{P}, 1 | V^{\text{LO}} | P\bar{P}, 1 \rangle = C_a^{(1)}, \quad (39)$$

$$\langle P\bar{P}, 8; i | V^{\text{LO}} | P\bar{P}, 8; j \rangle = C_a^{(8)} \delta_{ij}, \quad (40)$$

and do not mix them

$$\langle P\bar{P}, 1 | V^{\text{LO}} | P\bar{P}, 8; j \rangle = 0, \quad (41)$$

where we have particularized for the case of a $P\bar{P}$ molecule with quantum numbers $J^{PC} = 0^{++}$ and ignored the particle coupled channel dynamics stemming from HQSS. The extension to other particle channels and J^{PC} quantum numbers is trivial and only entails the substitution of $C_a^{(\mu)}$ in the equation above by the adequate linear combination of $C_a^{(\mu)}$ and $C_b^{(\mu)}$, with $\mu = 1$ or 8 .

Flavour symmetry also conserves the strangeness S and the isospin I of the heavy meson-antimeson system. For the singlet representation we have $S = 0$ and $I = 0$.

For the octet, however, we can have different values of the quantum numbers S and I , and thus we may find more convenient to write them in the basis

$$|P\bar{P}, 8; S; I, M_I\rangle \quad (42)$$

instead of $|P\bar{P}, 8; j\rangle$, where the relation between the two basis can be readily obtained (the only technicality involved is the isospin phase convention for the light antiquarks). In this basis we have

$$\begin{aligned} \langle P\bar{P}, 8; S; I; M_I | V^{\text{LO}} | P\bar{P}, 8; S'; I'; M_I' \rangle \\ = C_a^{(8)} \delta_{SS'} \delta_{II'} \delta_{M_I M_I'}, \end{aligned} \quad (43)$$

plus the corresponding expressions for the particle channels ($P\bar{P}^*$, $P^*\bar{P}$ and $P^*\bar{P}^*$). The conservation of S , I and M_I is now evident.

However, flavour symmetry is broken at the level of particle masses. In general, this only represents a problem for isoscalar states with hidden strangeness ($I = 0$ and $S = 0$), where the threshold of the particle channel that contains the $s\bar{s}$ light quark pair is considerably above the $u\bar{u}$ and $d\bar{d}$ channels. For example, in the charm sector the energy gap between the $D_s\bar{D}_s$ and the $D^0\bar{D}^0/D^+\bar{D}^-$ thresholds is about 200 MeV, of the order of the maximum binding energy we expect to be able to describe within the EFT framework. In other cases, the particle channels within a given S and I channel have approximately the same masses. Isospin symmetry breaking is of the order of a few MeV and we will not take it into account unless the molecular state we are describing is very shallow, as in the $X(3872)$, a case we will consider in detail at the end of this section.

For the treatment of SU(3) breaking we define

$$|P\bar{P}, 0\rangle = \frac{1}{\sqrt{2}} (|P_1\bar{P}^1\rangle + |P_2\bar{P}^2\rangle), \quad (44)$$

$$|P_s\bar{P}_s\rangle = |P_3\bar{P}^3\rangle, \quad (45)$$

that is, we distinguish between the SU(2) isoscalar ($I = 0$) state and the hidden strangeness state, and the indices 1, 2 and 3 refer to the representation of Eq. (36). In this basis we can rewrite the singlet and octet isoscalar representations as

$$|P\bar{P}, 1\rangle = \frac{1}{\sqrt{3}} \left[\sqrt{2} |P\bar{P}[0]\rangle + |P_s\bar{P}_s\rangle \right], \quad (46)$$

$$|P\bar{P}, 8; 0; 0, 0\rangle = \frac{1}{\sqrt{3}} \left[|P\bar{P}[0]\rangle - \sqrt{2} |P_s\bar{P}_s\rangle \right]. \quad (47)$$

By inverting these relations, we find that in the basis

$$\mathcal{B} = \{ |P\bar{P}[0]\rangle, |P_s\bar{P}_s\rangle \}, \quad (48)$$

the flavour symmetric interaction can be written as

$$V^{\text{LO}} = \begin{pmatrix} \frac{2}{3}C_a^{(1)} + \frac{1}{3}C_a^{(8)} & \frac{\sqrt{2}}{3} [C_a^{(1)} - C_a^{(8)}] \\ \frac{\sqrt{2}}{3} [C_a^{(1)} - C_a^{(8)}] & \frac{1}{3}C_a^{(1)} + \frac{2}{3}C_a^{(8)} \end{pmatrix}, \quad (49)$$

where flavour symmetry is broken owing to the energy gap among the $|P\bar{P}[0]\rangle$ and $|P_s\bar{P}_s\rangle$ thresholds. As commented in the previous paragraph, the energy gap is large

enough as to justify the treatment of the isoscalar and hidden strangeness channels as uncoupled.

In terms of notation we find that working in the SU(2) isospin basis is easier than in the SU(3) one. If we define the SU(2) isovector state (with $M_I = 0$) as

$$|P\bar{P}[1]\rangle = \frac{1}{\sqrt{2}} (|P_1\bar{P}^1\rangle - |P_2\bar{P}^2\rangle), \quad (50)$$

we can rewrite the EFT potential in the SU(2) isospin basis as

$$\langle P\bar{P}[0]|V^{\text{LO}}|P\bar{P}[0]\rangle = C_{0a}, \quad (51)$$

$$\langle P\bar{P}[1]|V^{\text{LO}}|P\bar{P}[1]\rangle = C_{1a}, \quad (52)$$

plus the corresponding expressions for the other particle ($P\bar{P}^*$, $P^*\bar{P}$ and $P^*\bar{P}^*$) / spin / C-parity combinations, which require the inclusion of the new counterterms C_{0b} and C_{1b} .

The counterterms C_{0a} and C_{1a} introduced in Eqs. (51) and (52) are related to the ones in the original SU(3)-multiplet basis by

$$C_{0a} = \frac{2}{3}C_a^{(1)} + \frac{1}{3}C_a^{(8)}, \quad (53)$$

$$C_{1a} = C_a^{(8)}. \quad (54)$$

In the SU(2) basis, the interaction in the $|P_s\bar{P}_s\rangle$ channel reads

$$\langle P_s\bar{P}_s|V^{\text{LO}}|P_s\bar{P}_s\rangle = \frac{1}{2} (C_{0a} + C_{1a}), \quad (55)$$

that is, the average of the isoscalar and isovector contact range potentials.

Finally, we consider the $X(3872)$, where isospin breaking is important as this bound state is especially shallow. The energy gap between the $D^0\bar{D}^{*0}$ and D^+D^{*-} is 8 MeV, which is smaller than the binding energy of the $X(3872)$ (about 4 MeV in the isospin symmetric limit, or, equivalently, almost at the $D^0\bar{D}^{*0}$ threshold). What we do then is to treat the neutral ($D^0\bar{D}^{*0}$) and charged (D^+D^{*-}) channels independently, which are related to the isoscalar and isovector channels by

$$|D\bar{D}^*[0]\rangle = \frac{1}{\sqrt{2}} [|D^0\bar{D}^{*0}\rangle + |D^+D^{*-}\rangle], \quad (56)$$

$$|D\bar{D}^*[1]\rangle = \frac{1}{\sqrt{2}} [|D^0\bar{D}^{*0}\rangle - |D^+D^{*-}\rangle], \quad (57)$$

and assume that isospin symmetry is only broken at the level of the masses. If we consider the basis of physical states

$$\mathcal{B}_{X(3872)} = \left\{ \frac{1}{\sqrt{2}} (|D^0\bar{D}^{*0}\rangle - |D^{*0}\bar{D}^0\rangle), \right. \\ \left. \frac{1}{\sqrt{2}} (|D^+D^{*-}\rangle - |D^{*+}D^-\rangle) \right\}, \quad (58)$$

we find that we can express the EFT potential as

$$V^{\text{LO}} = \frac{1}{2} \begin{pmatrix} C_0 + C_1 & C_0 - C_1 \\ C_0 - C_1 & C_0 + C_1 \end{pmatrix}, \quad (59)$$

where C_0 and C_1 stand for

$$C_0 = C_{0a} + C_{0b}, \quad C_1 = C_{1a} + C_{1b}, \quad (60)$$

that is, the linear combination of isoscalar and isovector counterterms corresponding to the 1^{++} channel.

III. ISOSPIN SYMMETRY VIOLATION IN THE $X(3872)$

The Belle collaboration reported for the first time the decays of the $X(3872)$ into the (isoscalar) $J/\Psi\pi^+\pi^-\pi^0$ and the (isovector) $J/\Psi\pi^+\pi^-$ channels [8]. The non-negligible size of the latter one hints to the existence of isospin breaking terms in the dynamics that govern these decays. The size of the violation is remarkable, as indicated by the branching ratio [49]

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \rightarrow J/\Psi\pi^+\pi^-\pi^0)}{\Gamma(X(3872) \rightarrow J/\Psi\pi^+\pi^-)} = 0.8 \pm 0.3, \quad (61)$$

where the central value is even smaller than one. The most natural explanation for the large ratio is the isospin breaking generated by the mass difference of the neutral ($D^0\bar{D}^{*0}$) and charged (D^+D^{*-}) channels in the $X(3872)$ [50, 51], which would not have a definite isospin. In this picture, at short $D\bar{D}^*$ distances, the $X(3872)$ would be a linear combination of $I = 0$ and $I = 1$ components. The $J/\Psi 3\pi$ and $J/\Psi 2\pi$ decays would proceed through the isospin invariant coupling of these final states to the appropriated short distance $X(3872)$ isospin wave function components [51].

We expect the 2π and 3π decays to happen via an intermediate ρ and ω meson. In this regard it is interesting to notice the careful analysis of Hanhart et al. [11], in which the branching ratio \mathcal{B}_X is translated into the more convenient ratio

$$R_X = \frac{\mathcal{M}(X \rightarrow J/\Psi\rho)}{\mathcal{M}(X \rightarrow J/\Psi\omega)} = 0.26_{-0.05}^{+0.08}, \quad (62)$$

that, instead of the decay widths, involves the amplitude of the $X(3872)$ to decay into J/Ψ and the ρ or ω mesons. We stress that the amplitude ratio R_X is equivalent to the experimental branching ratio \mathcal{B}_X , provided we assume the quantum numbers of the $X(3872)$ to be $J^{PC} = 1^{++}$ (see Ref. [11] for further details). In particular the experimental errors in \mathcal{B}_X are completely accounted for by the errors in R_X . In Ref. [11], the transition amplitudes for the decays of the $X(3872)$ into J/Ψ and a vector meson are parametrized as

$$\mathcal{M}(X \rightarrow J/\Psi V) = g_{X(V)} f_X(p), \quad (63)$$

where $V = \omega, \rho$ and f_X is a Blatt-Weisskopf barrier factor that depends on the momentum p of the J/Ψ in the X rest frame. For the 1^{++} assignment, we have $f_X(p) = 1$ yielding the value quoted in Eq. (62) for R_X . Other J^{PC} assignments of the $X(3872)$, in particular $J^{PC} = 2^{-+}$,

will imply a different form for the decay amplitude and consequently a change in the value of R_X [11].

If we assume that the $X(3872)$ is a $D\bar{D}^*$ molecule with quantum numbers 1^{++} , we can calculate $g_{X(\omega)}$ and $g_{X(\rho)}$ in terms of the analogous g_ω and g_ρ couplings for the free $D\bar{D}^*$ meson pair, which are given by

$$\mathcal{M}(D\bar{D}^*(IS) \rightarrow J/\Psi \omega) = g_\omega, \quad (64)$$

$$\mathcal{M}(D\bar{D}^*(IV) \rightarrow J/\Psi \rho) = g_\rho, \quad (65)$$

where IS and IV indicate the isoscalar and isovector configurations respectively. When the mesons are bound in the $X(3872)$, we can define the isoscalar and isovector wave functions as the brackets

$$\langle X|D\bar{D}^*(IS) \vec{q} \rangle = \Psi_{X(IS)}(\vec{q}), \quad (66)$$

$$\langle X|D\bar{D}^*(IV) \vec{q} \rangle = \Psi_{X(IV)}(\vec{q}), \quad (67)$$

where we have explicitly indicated the relative momentum \vec{q} of the meson-antimeson pair. Now, we can relate the g_ω and g_ρ couplings with the $g_{X(\omega)}$ and $g_{X(\rho)}$ ones by noticing that we can rewrite the decay amplitude $\mathcal{M}(A \rightarrow B)$ as the bracket $\langle B|A \rangle$. Then, we insert the two-body identities

$$\mathbf{1}_{IS} = \int \frac{d^3 \vec{q}}{(2\pi)^3} |D\bar{D}^*(IS) \vec{q} \rangle \langle D\bar{D}^*(IS) \vec{q} |, \quad (68)$$

$$\mathbf{1}_{IV} = \int \frac{d^3 \vec{q}}{(2\pi)^3} |D\bar{D}^*(IV) \vec{q} \rangle \langle D\bar{D}^*(IV) \vec{q} |, \quad (69)$$

for the isoscalar and isovector channels within the brackets and end up with

$$\mathcal{M}(X \rightarrow J/\Psi \omega) \equiv g_{X(\omega)} = g_\omega \hat{\Psi}_{X(IS)}, \quad (70)$$

$$\mathcal{M}(X \rightarrow J/\Psi \rho) \equiv g_{X(\rho)} = g_\rho \hat{\Psi}_{X(IV)}, \quad (71)$$

where $\hat{\Psi}_{X(IS)}$ and $\hat{\Psi}_{X(IV)}$ are defined as

$$\hat{\Psi}_{X(IS)} = \int \frac{d^3 \vec{q}}{(2\pi)^3} \Psi_{X(IS)}(\vec{q}), \quad (72)$$

$$\hat{\Psi}_{X(IV)} = \int \frac{d^3 \vec{q}}{(2\pi)^3} \Psi_{X(IV)}(\vec{q}), \quad (73)$$

that is, they are the coordinate space wave functions at the origin (more properly, around the origin, as the regulator smears the wave functions)⁴. Putting all the pieces together, we find that the ratio R_X can be expressed as

$$R_X = \frac{g_{X(\rho)}}{g_{X(\omega)}} = \frac{g_\rho}{g_\omega} \frac{\hat{\Psi}_{X(IV)}}{\hat{\Psi}_{X(IS)}}, \quad (74)$$

However, we are not working in the isospin basis, but rather in the particle basis. If we denote the neutral and

charged channels by the subscripts 0 and 1, we have that

$$\hat{\Psi}_{X0} = \frac{1}{\sqrt{2}} \left(\hat{\Psi}_{X(IS)} + \hat{\Psi}_{X(IV)} \right), \quad (75)$$

$$\hat{\Psi}_{X1} = \frac{1}{\sqrt{2}} \left(\hat{\Psi}_{X(IS)} - \hat{\Psi}_{X(IV)} \right), \quad (76)$$

from which we can rewrite R_X as

$$R_X = \frac{g_\rho}{g_\omega} \frac{\hat{\Psi}_{X0} - \hat{\Psi}_{X1}}{\hat{\Psi}_{X0} + \hat{\Psi}_{X1}}. \quad (77)$$

At this point we notice that if we know R_X , the binding energy of the $X(3872)$ and the g_ρ/g_ω ratio, we can determine the contact range potential that binds the $X(3872)$ (which in turn determines whether the $X(3872)$ has an isovector partner). Of these quantities the only unknown is the g_ρ/g_ω ratio, which can be determined from the SU(3) relation

$$g_\rho - g_\omega = -\sqrt{2} g_\phi, \quad (78)$$

and the OZI rule, that implies

$$g_\rho, g_\omega \gg g_\phi, \quad (79)$$

which amounts to ignoring the decay of the $X(3872)$ into $J/\Psi \phi$. Thus, we can conclude that $g_\rho/g_\omega \simeq 1$ and that the R_X ratio simplifies to⁵

$$R_X = \frac{\hat{\Psi}_{X0} - \hat{\Psi}_{X1}}{\hat{\Psi}_{X0} + \hat{\Psi}_{X1}}. \quad (80)$$

The next step is to solve the bound state equation for the $X(3872)$. We consider that the $X(3872)$ is a $D\bar{D}^*$ molecule with quantum numbers $J^{PC} = 1^{++}$, where we can distinguish between the neutral ($D^0 \bar{D}^{*0}$) and charged ($D^+ D^{*-}$) components of the wave function. That is, there are two channels in the bound state equation. We regularize the LO potential of Eq. (59) with a Gaussian regulator function and take a cut-off $\Lambda = 0.5 - 1.0$ GeV. The couplings C_0 and C_1 (or equivalently $C_{0a} + C_{0b}$ and $C_{1a} + C_{1b}$) are determined by the condition of reproducing the location of the $X(3872)$ state and the ρ/ω ratio R_X , from which we obtain the values

$$C_0 = -1.693_{-0.023}^{+0.036} \text{ fm}^2 \quad (-0.731_{-0.005}^{+0.008} \text{ fm}^2), \quad (81)$$

$$C_1 = -0.08_{-0.41}^{+0.42} \text{ fm}^2 \quad (-0.373_{-0.090}^{+0.089} \text{ fm}^2), \quad (82)$$

for $\Lambda = 0.5$ GeV (1 GeV). The couplings indicate that the strength of the interaction in the isovector channel

⁴ Alternatively, we can employ the coupling versus wave-function language of Ref. [51] to arrive at the same result.

⁵ Of course, there are deviations from the result above coming from the approximate nature of the OZI rule and to a lesser extent to SU(3) breaking effects. However, estimating the size of these deviations is not straightforward and therefore we have not attempted to systematically include this error source in the calculations to come. In this regard we simply note that owing to the large relative error in R_X (20-30%) smaller relative deviations from $g_\rho/g_\omega \simeq 1$ (e.g. 10-20%), for which our results turn out to be little sensitive, will be inconsequential as the two errors are independent and thus are added quadratically.

is weaker than in the isoscalar one. In particular, the isovector coupling C_1 is not strong enough to generate a second bound state, the (mostly) isovector partner of the $X(3872)$.

We find it worth commenting the comparison of our results for the counterterms with the hidden gauge model of Gamermann et al. [50, 51], in which the value of the counterterms are determined from meson exchange saturation. While the hidden gauge model predicts $C_1 = 0$ (as the contribution from the ρ and ω mesons cancel out) and $R_X = 0.136$ (for a sharp cut-off regulator and $\Lambda = 653$ MeV, see Ref. [51] for details⁶), we obtain a small (yet important) contribution to C_1 . As can be seen, this small contribution is necessary to fine tune the isospin violating branching ratio of the $X(3872)$ decays to its exact value.

IV. THE SU(3) AND HQSS PARTNERS OF THE $X(3872)$

If we are able to determine the value of the counterterms of the LO EFT, we can calculate the location of the molecular partners of the $X(3872)$. There are four parameters. Isoscalar molecules ($I = 0, S = 0$) without hidden strangeness can be described with C_{0a} and C_{0b} . Isospinor ($I = 1/2, S = \pm 1$) and isovector ($I = 1, S = 0$) states are in turn determined by C_{1a} and C_{1b} . Finally, for molecular states with hidden strangeness, the contact range interactions are the average of the isoscalar and isovector ones.

We fix two of the counterterms from the location of the $X(3872)$ resonance and its isospin breaking branching ratio, as explained in the previous section. The remaining two require the identification of two partners of the $X(3872)$. We have chosen the $X(3915)$ [16] as a 0^{++} isoscalar $D^*\bar{D}^*$ molecule and the $Y(4140)$ [17] as a 0^{++} $D_s^*\bar{D}_s^*$ molecule, guided by its apparently dominant decay into $J/\Psi\phi$. We notice that these identifications were proposed for the first time in Refs. [13–15].

The 0^{++} assignment for the quantum numbers of the $X(3915)$ and $Y(4140)$ deserve some discussion. On the one hand, the 2^{++} option is excluded: HQSS fixes the location of the isoscalar 2^{++} $D^*\bar{D}^*$ partner of the $X(3872)$ in the vicinity of 4012 MeV, far away from the 3915 MeV region. If we additionally consider the isospin breaking decays of the $X(3872)$, we can determine that the isovector 2^{++} $D^*\bar{D}^*$ and the isoscalar 2^{++} $D_s^*\bar{D}_s^*$ molecules do not exist. On the other the 1^{+-} option can also be discarded from the decays of the $X(3915)$ and $Y(4140)$ into $J/\Psi\omega$ and $J/\Psi\phi$, requiring a positive C-parity state. Thus we are only left with $J^{PC} = 0^{++}$.

However, the choice of the $X(3915)$ and $Y(4140)$ states as input is not entirely free of problems. The first is the binding energy of the $X(3915)$, approximately 100 MeV. Around this binding there is power counting transition – one pion exchange changes from perturbative to non-perturbative – though the critical binding energy that marks this transition is not known exactly and can happen at larger bindings than expected [30]. In this regard the investigations of Ref. [36] shows (by performing the explicit calculations) that we are still in the perturbative regime for the $X(3915)$ and we can employ a contact theory to describe it at LO. The $Y(4140)$, with a binding energy of 80 MeV, is not affected by this issue because the $D_s^*\bar{D}_s^*$ system cannot exchange a single pion. The second problem is the debatable experimental status of the $Y(4140)$. After its first and only observation by the CDF Collaboration [17], subsequent experiments have failed to find it [38, 39]. This means that the consequences derived from the assumption that the $Y(4140)$ exists and that it has a molecular structure should be taken with a grain of salt.

Now we fix the counterterms to the location of the $X(3915)$ and $Y(4140)$. For convenience we define first the linear combinations of counterterms:

$$C_2 = C_{0a} - 2C_{0b}, \quad (83)$$

$$C_3 = \frac{1}{2} ((C_{0a} - 2C_{0b}) + (C_{1a} - 2C_{1b})), \quad (84)$$

where C_2 / C_3 is a convenient way to write the LO potential of the $X(3915) / Y(4140)$ channel. We obtain

$$C_2 = -6.710 \text{ fm}^2 \quad (-1.611 \text{ fm}^2), \quad (85)$$

$$C_3 = -5.915 \text{ fm}^2 \quad (-1.459 \text{ fm}^2), \quad (86)$$

for a gaussian regulator $\Lambda = 0.5$ GeV (1 GeV). We can transform the values of the combinations above (and the corresponding ones for C_0 and C_1) to the standard counterterm representation, yielding

$$C_{0a} = -3.366_{-0.015}^{+0.024} \text{ fm}^2 \quad (-1.024_{-0.003}^{+0.005} \text{ fm}^2), \quad (87)$$

$$C_{0b} = +1.673_{-0.008}^{+0.012} \text{ fm}^2 \quad (+0.293_{-0.002}^{+0.004} \text{ fm}^2), \quad (88)$$

$$C_{1a} = -1.76_{-0.29}^{+0.29} \text{ fm}^2 \quad (-0.684_{-0.063}^{+0.064} \text{ fm}^2), \quad (89)$$

$$C_{1b} = +1.68_{-0.15}^{+0.15} \text{ fm}^2 \quad (+0.311_{-0.033}^{+0.033} \text{ fm}^2), \quad (90)$$

for $\Lambda = 0.5$ GeV (1 GeV), where the error comes from the uncertainty in R_X (a negligible effect in the isoscalar channels, but important in the isovector ones).

We notice that there are two additional error sources: the violations of HQSS due to the finite charm quark mass and the breaking of light flavour symmetry. The first of these effects can be taken into account by noticing that the EFT potential has a relative uncertainty of the order of

$$V_{(m_Q=m_c)}^{\text{LO}} = V_{(m_Q \rightarrow \infty)}^{\text{LO}} \left(1 \pm \frac{\Lambda_{QCD}}{m_c}\right), \quad (91)$$

with respect to the exact heavy quark limit. Taking a value of around 1.5 GeV for the charm quark mass and

⁶ Note that, when $C_1 = 0$, once the regulator/renormalization procedure and the cut-off is fixed, R_X is totally determined by the mass of the $X(3872)$.

$\Lambda_{\text{QCD}} \sim 200$ MeV, we should expect a 15% violation of HQSS for the LO contact range potentials ⁷.

The second error source affects molecules containing strange quarks ⁸, where we expect the contact range potential to deviate slightly from the pure SU(3) prediction, that is

$$V_{(s\text{-quark})}^{\text{LO}} = V_{\text{SU}(3)}^{\text{LO}} (1 \pm \delta_{\text{SU}(3)}). \quad (92)$$

In the expression above $\delta_{\text{SU}(3)}$ is the relative size of the SU(3)-breaking effects, which can be estimated from the ratio of the kaon and pion decay constants $f_K/f_\pi \sim 1.2$, yielding $\delta_{\text{SU}(3)} = 0.2$. This uncertainty also affects the determination of the C_3 counterterm from the mass of the $Y(4140)$ state. Thus we can assume a 20% relative error in the value we give for this parameter in Eq. (86). In turn, this will translate into an additional error in the isovector counterterms C_{1a} and C_{1b} , see Eq. (84) for details. Actually, their errors are fully anticorrelated to ensure that the linear combination $C_1 = C_{1a} + C_{1b}$ is free from the SU(3) uncertainties of C_3 . For simplicity and due to the exploratory nature of the present work we have neglected these correlations. Instead we have substituted the correlated error in C_{1a} and C_{1b} by an uncorrelated error in C_{1a} ⁹, a choice that overestimates the size of the errors in the molecular masses.

At this point we find it worth mentioning that the three error sources we have considered – the experimental error in R_X plus the breaking of HQSS and SU(3) light flavour symmetry – are independent: we can compute the total error by adding the partial errors in quadratures.

Finally, we emphasize that the choice of a regulator is inessential in EFT calculations. It does not matter which regulator we have chosen, as far as the cut-off window is sensible enough. We have explicitly tested this assumption by calculating the full molecular spectrum with other regulators. In general we find small changes in the central location of the states that are compatible with the cut-off uncertainty we already find in Tables I-IV, i.e. around 10 MeV for the most tightly bound cases. For instance, the location of the 1^{+-} isoscalar molecule we predict in Table I changes from 3955 MeV (3958 MeV) for a gaussian regulator to 3954 MeV (3957 MeV) with a sharp cut-off, 3965 MeV (3964 MeV) with a monopolar

regulator and 3960 MeV (3961 MeV) with a dipolar regulator, in all cases with a cut-off $\Lambda = 0.5$ GeV (1.0 GeV) ¹⁰.

A. The SU(2) Isoscalar ($I = 0$) Partners

We begin with the SU(2) isoscalar sector, in which we ignore the hidden strange components. The states are determined by the counterterms C_{0a} and C_{0b} . We do not take into account particle coupled channel effects as they are subleading, as explicitly checked in Ref. [36]. There is one exception though, the 1^{++} and 2^{++} channels, where the mass gap between the neutral and charged channels (8 and 6 MeV in each case) is similar in size to the binding energy in the isospin symmetric limit (4 and 5 MeV). This suggests that we may treat the neutral and charged channels as explicit degrees of freedom of the theory. We note that the inclusion of isospin violation is the only difference with respect to the previous analysis of Ref. [36]. The spectrum of molecular states is presented in Table I. As can be seen, isospin violation is a small perturbation over the former predictions of Ref. [36] (though it is still crucial to describe isospin violating decays properly).

We, however, would like to make a few remarks. The first one concerns the 2^{++} state. The central values of the counterterms as given in Eqs. (87)–(90) predict that the 2^{++} state lies very close (less than 1 MeV) to the lowest energy threshold, i.e. $D^{*0}\bar{D}^{*0}$. When we decrease the strength of the potential to account for the uncertainties of our approach, the pole reaches the neutral threshold and then bounces back into the second Riemann sheet. That is, the state becomes virtual (instead of bound). In any case, the existence of the pole will strongly influence the amplitude at threshold.

The second comment is about the 0^{++} $D\bar{D}$ channel. As can be seen in the Table I, this state is bound by about 20-25 MeV. For simplicity, we have used in this channel the isospin symmetric limit. Yet the $D^0\bar{D}^0 - D^+\bar{D}^-$ threshold gap is around 9 MeV, and it might make sense the explicit consideration of isospin breaking in this channel. However, as in the $X(3872)$ and $X(4012)$ cases, the effect is rather small, justifying the validity of the isospin symmetric limit for the spectroscopy problem.

Finally, there is a remark concerning the possible effect of hidden strange channels in the dynamics of the $X(3915)$ state. As we will see, there is a 0^{++} $D_s\bar{D}_s$ state at 3925 MeV (see Table IV) that in principle can mix with the $X(3915)$. However, explicit calculations show that the influence of the hidden strange channel is numerically marginal: it moves the position of the predicted

⁷ Actually, the 15% violation represents the full expected deviation from the heavy quark limit in the charm sector. Even though heavy quark symmetry involves heavy flavour symmetry as well as HQSS, only the later one is relevant for our purposes. Thus, when we are talking about HQSS violations, it is merely language abuse for heavy quark symmetry violations in general.

⁸ We do not consider isospin breaking effects for the potential, as their size will be negligible.

⁹ Note that we can rewrite C_3 as $\frac{1}{2}C_2 - C_1 + \frac{3}{2}C_{1a}$. The isoscalar part of the interaction C_2 and the isovector C_1 are fixed by the $X(3872)$ and $X(3915)$ inputs, meaning that all the error can be transferred to C_{1a} . As a matter of fact the decomposition is not unique: we could have transferred the error to C_{1b} instead.

¹⁰ Actually, one should take into account the optimal cut-off window depends on the choice of a regulator. In particular, for the monopolar regulator we should use larger cut-offs than for the sharp cut-off and gaussian cases. However, for the purposes of the current discussion we can ignore this effect and take the same cut-off value for all the regulators we have considered.

molecular states by a small fraction of a MeV (usually $\Delta M \sim 0.1 - 0.2$ MeV), a tiny effect compared to other error sources. The reason lies in the transition potential between the isoscalar 0^{++} $D^* \bar{D}^*$ and $D_s \bar{D}_s$ channels:

$$\langle D^* \bar{D}^* | V^{\text{LO}}(0^{++}) | D_s \bar{D}_s \rangle = \sqrt{\frac{3}{2}} (C_{0b} - C_{1b}), \quad (93)$$

which turns out to be numerically quite small, since we accidentally find $C_{0b} \simeq C_{1b}$, see Eqs. (87)–(90).

B. The Isospinor ($I = \frac{1}{2}$) Partners

The isospinor molecules are different in the sense that they have not well-defined C-parity, as they are not bound states of a heavy meson and its antimeson. In general this poses no problem (the formalism is identical to the one in the previous case) except for the $1^+ D_s \bar{D}^*$ and $D \bar{D}_s^*$ molecules. The $D_s \bar{D}^*$ and $D \bar{D}_s^*$ thresholds are separated by only 2 MeV and require a coupled channel treatment. The LO potential reads

$$V^{\text{LO}} = \begin{pmatrix} C_{1a} & -C_{1b} \\ -C_{1b} & C_{1a} \end{pmatrix}, \quad (94)$$

where channel 1 (2) corresponds to $D_s \bar{D}^*$ ($D^* \bar{D}$). We find that this coupled channel potential only leads to one bound state (of a maximum of two). We list this and the other isospinor molecules in Table II, where we have considered only the strangeness one states. The spectrum is identical for the strangeness minus one sector. We also notice that the error bands are bigger as they include the additional SU(3) breaking effects at the level of the contact range interaction.

There is a total of four states, one of them – the $0^+ D_s \bar{D}$ molecule – almost at threshold. Actually, there exists a small violation of the third component of isospin, because of the different masses of the D^- (D^{*-}) and \bar{D}^0 (\bar{D}^{*0}) mesons. In general the isospin splitting is smaller than the other errors in the calculations and we ignore it. The exception is the $0^+ D_s \bar{D}$ molecule: in Table II we only report the $I_3 = -1/2$ component of the molecule, which is being formed by the $D_s^+ D^-$ interaction¹¹. Similarly to what happens with the isoscalar 2^{++} state, with the central values of the counterterms in Eq. (87)–(90) we predict a 0^+ state that lies very close to threshold. When we decrease the strength of the potential, the molecule approaches the threshold and finally becomes a virtual state, at least within our simple scheme¹². In the real

world, it might well happen that in this situation the state could become a narrow resonance, placed very close to threshold, and decaying into the $D_s^+ \bar{D}^-$ pair instead of a becoming a virtual state. In any case, we predict the existence of some structure close to threshold. The upper errors for the mass of this state, as quoted in Table II, just account for the distance between its central mass value and the $D_s^+ \bar{D}^-$ threshold.

C. The Isovector ($I = 1$) Partners

The spectrum of the isovector molecules is similar to the isospinor one. The reason is that the LO potential is identical in both cases, with the exception of the 1^{++} and 2^{++} molecules owing to isospin violation, as we commented previously. We find four molecular states of a possible total of six that we list in Table III. The location of the states is similar to the isoscalar sector, only that they are a bit less bound. The two missing states would correspond to the isovector partners of the $X(3872)$ and $X(4012)$.

We also mention that there is a complication with the isovector 0^{++} $D \bar{D}$ molecule, which lies very close to the $D \bar{D}$ threshold. This implies that explicit isospin breaking should be taken into account. However, we notice that this is only necessary for the $I_3 = 0$ component of the isospin triplet, where the neutral ($D^0 \bar{D}^0$) and charged ($D^+ D^-$) channels are to be found. If we consider instead the $I_3 = \pm 1$ states (corresponding to the particle channels $D^+ \bar{D}^0$ and $D^0 D^-$), there is no mixing with the isoscalar components and no need for coupled channel dynamics. Thus, for simplicity, we have decided to report only the $I_3 = +1$ state in Table III.

D. The Hidden Strange Partners

Finally, we consider the molecules with hidden strangeness. In this sector the strength of the LO potential is the arithmetic mean of the isoscalar and isovector one. This means that if there is a bound state in the isoscalar and isovector sector, we can be confident about the existence of a heavy meson molecule containing a $s \bar{s}$ quark-antiquark pair. Conversely, if we consider the $X(3872)$ and $X(4012)$ molecules, the fact that they have no isovector partners is a strong hint that there will be no hidden strange partner either. The four states we obtain are listed in Table IV.

V. CONCLUSIONS

In this work we have shown how the heavy quark spin and light SU(3) flavour symmetries constrain the charmed meson-antimeson interaction ($H \bar{H} \rightarrow H \bar{H}$, being $H = D^+, D^0, D_s^0, D^{*+}, D^{*0}$ and D_s^{*0}). This has been done within the EFT framework, where the heavy meson

¹¹ The dynamics of the other state member of the isospin doublet is similar, being its mass just shifted by about 5 MeV. This shift is due to the mass difference between the $D_s^+ D^-$ and the $D_s^+ \bar{D}^0$ pairs.

¹² A resonance state in one channel is usually associated to a barrier in coordinate space which is not reproduced by a constant (in energy) potential. The situation may be different when we have coupled channels [52], in which case one of the channels can decay into the other.

J^{PC}	$H\bar{H}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^{++}	$D\bar{D}$	1S_0	C_{0a}	3709_{-10}^{+9}	3715_{-15}^{+12}	—	3734.5^*
1^{++}	$D^*\bar{D}$	3S_1	Eqs. (59) and (60)	Input	Input	3871.6	$3871.8/3879.9$
1^{+-}	$D^*\bar{D}$	3S_1	$C_{0a} - C_{0b}$	3815_{-17}^{+16}	3821_{-26}^{+23}	—	3875.9^*
0^{++}	$D^*\bar{D}^*$	1S_0	$C_{0a} - 2C_{0b}$	Input	Input	3917	4017.3^*
1^{+-}	$D^*\bar{D}^*$	3S_1	$C_{0a} - C_{0b}$	3955_{-17}^{+16}	3958_{-27}^{+24}	3942	4017.3^*
2^{++}	$D^*\bar{D}^*$	5S_2	Eqs. (59) and (60)	$4013_{-9}^{\dagger\dagger}$	$4013_{-12}^{\dagger\dagger}$	—	$4014.0/4020.6$

TABLE I. Predicted masses (in MeV) of the SU(2) isoscalar HQSS partners of the $X(3872)$ resonance for two different values of the Gaussian cutoff. We use as input 3871.6 MeV, 3917.4 MeV and 4140 MeV for the $X(3872)$, $X(3915)$ and $Y(4140)$, respectively. In the last column, we also give the thresholds for the different charge combination channels ($P^0\bar{P}^0/P^+P^-$) or the threshold value that results when we neglect isospin breaking effects and we employ charge averaged masses. In this latter case, we mark it with an asterisk (*). Errors in the predicted masses are obtained by adding in quadratures the uncertainties stemming from the two sources of systematic errors discussed at the end in Subsect. IV: errors in our determination of the LO counterterms in Eqs. (87)–(90) and variations obtained by modifying the strength of the contact interaction in each channel by $\pm 15\%$, which corresponds to the expected violations of HQSS for the charm quark mass. $\dagger\dagger$: the upper error of the 2^{++} state mass deserves a detailed discussion that can be found in the text (Subsect. IV A).

J^P	$H\bar{H}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^+	$D_s^+\bar{D}^-$	1S_0	C_{1a}	$3835.8_{-7.3}^{+2.3}$	$3837.7_{-8.1}^{+0.4}$	—	3838.1
1^+	$D_s\bar{D}^*, D_s^*\bar{D}$	3S_1	Eq. (94)	3949_{-21}^{+20}	3957_{-32}^{+22}	—	$3977.15^\dagger, 3979.55^\dagger$
0^+	$D_s^*\bar{D}^*$	1S_0	$C_{1a} - 2C_{1b}$	4056_{-35}^{+31}	4061_{-54}^{+45}	—	4120.9^\dagger
1^+	$D_s^*\bar{D}^*$	3S_1	$C_{1a} - C_{1b}$	4091_{-22}^{+19}	4097_{-33}^{+24}	—	4120.9^\dagger
2^+	$D_s^*\bar{D}^*$	5S_2	$C_{1a} + C_{1b}$	—	—	—	—

TABLE II. Predicted masses (in MeV) of the isospin ($I = \frac{1}{2}$) HQSS partners of the $X(3872)$ resonance, for two different values of the Gaussian cutoff. The meaning of the quoted errors in the table is the same as in Table I. We also give the relevant thresholds (in MeV) for each channel. We use isospin third component averaged masses for those cases marked with a \dagger symbol. Note that as we decrease the strength of the potential, the $D_s^+\bar{D}^-$ state becomes virtual (see discussion in Subsect. IV B).

interactions can be easily arranged from more to less relevant thanks to power counting, the ordering principle behind EFT. The bottom line of the EFT approach is that contact interactions (i.e. four meson vertices) dominate the low energy dynamics of heavy meson molecules. In turn, SU(3) flavour symmetry and HQSS reduce the number of contact interactions from twenty four to only four. The approach we advocate is actually an extension of the ideas of Ref. [36] for the SU(2) isoscalar channel to the isospinor, isovector and hidden strange sectors.

The four counterterms can be determined as follows: we fix three of them by identifying the $X(3915)$, $Y(4140)$ and $X(3872)$ resonances as molecular states. In particular we consider the $X(3915)$ and $Y(4140)$ to be a 0^{++} isoscalar $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ molecules respectively, while the $X(3872)$ is a 1^{++} isospin admixture $D\bar{D}^*$ state. Finally, the fourth counterterm is determined from the isospin breaking branching ratio of the $X(3872)$ into $J/\Psi 2\pi$ and $J/\Psi 3\pi$. We notice that the $X(3872)$ is weakly bound (~ 0.2 MeV) and lies extremely close to the $D^0\bar{D}^{0*}$ threshold. Its binding energy is much smaller than the mass splitting between the charged and neutral components of the $X(3872)$ (~ 8 MeV). Hence, we have taken into account these degrees of freedom explic-

itly. We work within a scheme where all interactions are isospin invariant, but where the $X(3872)$ does not have a well defined isospin¹³ as a consequence of the mass and kinetic terms of the $D\bar{D}^*$ Hamiltonian [50, 51]. The isospin breaking of the masses naturally explains the appearance of an isospin violation (and its quantity) in the decays of the $X(3872)$, as pointed out in Refs. [50, 51] for the first time. However, in the aforementioned works the experimental branching ratio [49]

$$\frac{\Gamma(X(3872) \rightarrow J/\Psi \pi^+ \pi^- \pi^0)}{\Gamma(X(3872) \rightarrow J/\Psi \pi^+ \pi^-)} = 0.8 \pm 0.3,$$

is not perfectly reproduced¹⁴. The reason is that the authors of Refs. [50, 51] assume that the isovector $D\bar{D}^*$ interaction vanishes. Here, we improve on that and take

¹³ The isospin mixing depends on the relative distance between the pseudoscalar and vector mesons.

¹⁴ Notice, however, that Refs. [50, 51] used an older experimental determination of the branching ratio ($1.0 \pm 0.3 \pm 0.4$) with much larger errors. It is only with the updated branching ratio that their explanation fails at the fine quantitative level.

J^{PC}	$\text{H}\bar{\text{H}}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^{++}	$D^+\bar{D}^0$	1S_0	C_{1a}	$3732.5^{+2.0}_{-6.9}$	$3734.3^{+0.2}_{-6.9}$	—	3734.5
1^{++}	$D^*\bar{D}$	3S_1	Eqs. (59) and (60)	—	—	—	—
1^{+-}	$D^*\bar{D}$	3S_1	$C_{1a} - C_{1b}$	3848^{+15}_{-17}	3857^{+15}_{-22}	—	3875.9*
0^{++}	$D^*\bar{D}^*$	1S_0	$C_{1a} - 2C_{1b}$	3953^{+24}_{-26}	3960^{+31}_{-37}	—	4017.3*
1^{+-}	$D^*\bar{D}^*$	3S_1	$C_{1a} - C_{1b}$	3988^{+15}_{-17}	3995^{+17}_{-23}	—	4017.3*
2^{++}	$D^*\bar{D}^*$	5S_2	Eqs. (59) and (60)	—	—	—	—

TABLE III. Predicted masses (in MeV) of the SU(2) isovector HQSS partners of the $X(3872)$ resonance for two different values of the Gaussian cutoff. The meaning of the quoted errors in the table is the same as in Table I. We also give the relevant thresholds (in MeV) for each channel. For the $D^*\bar{D}$ and $D^*\bar{D}^*$ cases, we give the threshold that would correspond to the zero isospin third component and calculated with charge averaged masses. Note that as we decrease the strength of the potential, the $D^+\bar{D}^0$ state becomes virtual. The upper errors for the mass of this state just account for the distance between its central mass value and the threshold.

J^{PC}	$\text{H}\bar{\text{H}}$	$^{2S+1}L_J$	V_C	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp [7]	Threshold [MeV]
0^{++}	$D_s\bar{D}_s$	1S_0	$\frac{1}{2}(C_{0a} + C_{1a})$	3924^{+10}_{-13}	3928^{+9}_{-19}	—	3937.0
1^{++}	$D_s^*\bar{D}_s$	3S_1	$\frac{1}{2}(C_{0a} + C_{1a} + C_{0b} + C_{1b})$	—	—	—	—
1^{+-}	$D_s^*\bar{D}_s$	3S_1	$\frac{1}{2}(C_{0a} + C_{1a} - C_{0b} - C_{1b})$	4035^{+23}_{-25}	4040^{+33}_{-39}	—	4080.8
0^{++}	$D_s^*\bar{D}_s^*$	1S_0	$\frac{1}{2}(C_{0a} + C_{1a} - 2C_{0b} - 2C_{1b})$	Input	Input	4140	4224.6
1^{+-}	$D_s^*\bar{D}_s^*$	3S_1	$\frac{1}{2}(C_{0a} + C_{1a} - C_{0b} - C_{1b})$	4177^{+23}_{-25}	4180^{+35}_{-40}	—	4224.6
2^{++}	$D_s^*\bar{D}_s^*$	5S_2	$\frac{1}{2}(C_{0a} + C_{1a} + C_{0b} + C_{1b})$	—	—	—	—

TABLE IV. Predicted masses (in MeV) of the hidden strange isoscalar HQSS partners of the $X(3872)$ resonance for two different values of the Gaussian cutoff. The meaning of the quoted errors is the same as in Table I. We also give the relevant thresholds (in MeV) for each channel.

advantage of the experimental ratio to constraint the non-vanishing interaction in the isovector channel.

Once we have fixed the counterterms, we have established the existence and the location of up to a total of 15 molecular partners of the $X(3915)$, $Y(4140)$ and $X(3872)$ states, see Tables I-IV. These predictions are subjected to a series of uncertainties, in particular the approximate nature of HQSS (especially in the charm sector). We have estimated the size of these corrections and concluded that the HQSS pattern of molecular states is stable, though the exact location of the states can change by a few tens of MeV in certain cases.

Actually, the family of $D^{(*)}\bar{D}^{(*)}$ states we theorize depends on the assumption that the $X(3872)$, $X(3915)$ and $Y(4140)$ resonances are molecular. While in the $X(3872)$ case the molecular interpretation is compelling and widely accepted, the $X(3915)$ and $Y(4140)$ states are merely compatible with it. Regarding the $X(3915)$, it is interesting to notice that the size of the decay width of this resonance is difficult to conciliate with the hypothesis that it is a charmonium [37]. This observation enhances the prospects that the $X(3915)$ may be a molecule after all. However, with the $Y(4140)$ we have a more serious problem: this resonance is far from being confirmed experimentally. Thus not all the states we predict are equally likely. Predictions derived from the $X(3872)$ should be regarded as more solid than those depending

on the $X(3915)$, which in turn are less speculative than the ones obtained from the $Y(4140)$. In this regard, as stressed in Ref. [36], the $2^{++} D^*\bar{D}^*$ isoscalar partner of the $X(3872)$ is still the most reliable prediction of the present work, followed by the other isoscalar states. If in the future we count with clearer molecular candidates than the $X(3915)$ and, particularly, the $Y(4140)$ resonances, they could be included in the current scheme instead of the later ones, helping to achieve more robust predictions. Conversely, the observation of any of the states predicted here can serve as proof of the molecular nature of the previous mentioned resonances.

Finally, we find it interesting to compare our results with those of the hidden gauge formalism, another theoretical approach for the study of hidden charm states. While the spectrum of the isoscalar molecules in the hidden gauge is similar to the one we obtain (with the notable exception of the 2^{++} state), fewer poles are reported in the $I = 1/2$, $I = 1$ and hidden strange sectors [53–55]. If we consider the case of two heavy pseudoscalar mesons, Gamermann et al. [53] predict an isoscalar $0^{++} D\bar{D}$ state in the vicinity of 3700 MeV. We can identify this state with the $X(3710) D\bar{D}$ molecule we obtain in the present work. However, they do not predict the existence of the isospinor, isovector and hidden strangeness partners of the $X(3710)$. The reason is that in Ref. [53] the interaction in the isovector chan-

nel is zero. The same comments apply to the extension of the hidden gauge formalism to pseudoscalar-vector molecules [54], where it is found a counterpart of the $1^{+-} D\bar{D}^*$ isoscalar molecule that we obtain at 3820 MeV at a slightly higher energy (3840 MeV) but no $I = 1/2$, $I = 1$ or hidden strangeness states. Last, in the case of two heavy vector mesons the hidden gauge predicts a series of isoscalar 0^{++} , 1^{+-} and $2^{++} D^*\bar{D}^*$ states, plus a few non-isoscalar ones [55]. In the isoscalar sector, the 0^{++} and $1^{+-} D^*\bar{D}^*$ resonances are located in the region around 3943 and 3945 MeV respectively, not very different to the masses we use (3917 and 3955 MeV). However, there is a striking difference in the mass of the $2^{++} D^*\bar{D}^*$ isoscalar state, which in Ref. [55] happens at 3922 MeV (instead of 4013 MeV). The reason for this value, which is incompatible with HQSS, is the remarkably strong vector-vector interaction that is obtained in the hidden gauge model. Probably as a result of this strong interaction, Ref. [55] also report the existence of broad 2^{++} isovector $D^*\bar{D}^*$

and hidden strange $D_s^*\bar{D}_s^*$ resonances, with widths above 100 MeV, and masses of around 3910 and 4160 MeV, respectively. These states are difficult to accommodate within our HQSS scheme.

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