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Heavy quark impact factor in k_T-factorization

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ABSTRACT: We present the calculation of the finite part of the heavy quark impact factor at next-to-leading logarithmic accuracy in a form suitable for phenomenological studies such as the calculation of the cross-section for single bottom quark production at the LHC within the k_T-factorization scheme.

Contents

1 Introduction

One of the key issues in QCD phenomenology at the Large Hadron Collider (LHC) is gauging the importance of small-x physics related effects on a number of physical observables and consequently, getting a definite answer on the validity and the applicability of the high energy resummation programme.

At very large center-of-mass energies, $\sqrt{s}$, or alternatively at very small-x, the appearance of large logarithms in energy ($\log s \sim \log 1/x$) can spoil the convergence in the perturbative calculation of scattering amplitudes. More concretely, terms of the form $(\alpha_{\mathrm{S}} \log 1/x)^n$, where α_{S} is the strong coupling constant, can be of order unity, for small enough x, and therefore need to be resummed to all orders. The Balitsky-Fadin-Kuraev-Lipatov (BFKL) framework enables the resummation of high center-of-mass energy logarithms at leading [1–3] (Lx) and next-to-leading [4, 5] logarithmic accuracy (NLx). At Lx, all the terms of the form $(\alpha_{\mathrm{S}} \log 1/x)^n$ are resummed whereas, at NLx one has also to resum terms in which the strong coupling lacks one power compared to the logarithm in energy, that is, terms that behave like $\alpha_{\mathrm{S}} (\alpha_{\mathrm{S}} \log 1/x)^n$.

In the last two decades or so and after the first small-x data from Deep Inelastic Scattering (DIS) collisions at HERA became available in the beginning of the 90's, the resummation of the high energy logarithms and its phenomenological relevance exhibited some major developments. To give but a sample of very important works in the field, focusing more on the phenomenological side, we would have to mention the formal study of the k_T-factorization scheme [6–8], the computation of the NLx BFKL kernel [4] and the collinear improvements to the NLx kernel [9–12]. For the study of scattering amplitudes within the BFKL formalism, necessary ingredients are the gluon Green's function which is obtained after solving the BFKL equation (see for example Refs. [13–23]), the gluon

Regge trajectory [24–33] and the impact factors [34–42], the latter being process-dependent objects. In a very general definition, the impact factors are the effective couplings of the scattering projectiles to whatever is exchanged in the t-channel for a process studied in the k_T-factorization scheme. One can claim that resumming small-x logarithms is finally well understood for a number of processes and observables at HERA and the LHC: perturbative evolution of parton distribution functions [43–49], photoproduction [6–8] and double-DIS [50–52] processes, hadroproduction of heavy quarks [53, 54], Drell-Yan [55], Higgs boson hadroproduction [56], Mueller-Navelet jets and forward jets in DIS [57–61].

The impact factors for gluons and massless quarks were calculated in Ref. [62] at NLx accuracy and in momentum space. This allows in principle for the calculation of various DIS and double-DIS processes with massless quarks and gluons in the initial state whereas the extension to the case of hadron-hadron collisions was also established [63–65].

What we are interested in though, starting from this work, is to set up a programme for phenomenological studies –within the k_T-factorization scheme– of processes involving massive quarks (mainly bottom quarks) at the LHC, given the excellent tagging capabilities of the ATLAS [66], CMS [67] and LHCb [68] detectors. For that to be possible we need the NLx impact factor for a massive quark which was first calculated by Ciafaloni and Rodrigo in Refs. [69, 70]. However, their final expression for the massive quark impact factor was written in the form that contains a sum of an infinite number of terms. To make their result directly applicable for phenomenological studies we recalculate the NLx heavy quark impact factor in a compact and resummed form, ready to be used for the convolution with the gluon Green's function in a numerically straightforward manner.

After this introduction, we proceed to Section 2 where we set up our notation and provide the necessary definitions. In Section 3, we present the two terms that contribute with massive corrections to the massless NLx quark impact factor and we calculate these terms in Sections 4 and 5. The full result in a closed resummed form appears in Section 6, in which we also offer a first numerical study of the behavior of the finite part of the result. Finally, we conclude in Section 7.

2 High energy factorization

In the high energy limit: $\Lambda_{QCD} \ll |t| \ll s$, the partonic cross-section of $2 \rightarrow 2$ processes factorizes into the impact factors $h_{\mathsf{a}}(\boldsymbol{k}_1)$ and $h_{\mathsf{b}}(\boldsymbol{k}_2)$ of the two colliding partons a and b, and the gluon Green's function $\mathcal{G}_\omega(\boldsymbol{k}_1, \boldsymbol{k}_2)$ (here in Mellin space) so that the differential cross-section can be written as

$$\frac{\mathrm{d}\sigma_{\mathsf{ab}}}{\mathrm{d}[\boldsymbol{k}_1]\,\mathrm{d}[\boldsymbol{k}_2]} = \int \frac{\mathrm{d}\omega}{2\pi i\,\omega}\, h_{\mathsf{a}}(\boldsymbol{k}_1)\,\mathcal{G}_\omega(\boldsymbol{k}_1, \boldsymbol{k}_2)\, h_{\mathsf{b}}(\boldsymbol{k}_2) \left(\frac{s}{s_0(\boldsymbol{k}_1, \boldsymbol{k}_2)}\right)^\omega , \qquad (2.1)$$

where ω is the dual variable to the rapidity Y, and $\mathrm{d}[\boldsymbol{k}] = \mathrm{d}^{2+2\varepsilon}\boldsymbol{k}/\pi^{1+\varepsilon}$ is the transverse space measure. The impact factor at the leading $\log x$ order (Lx) can be expressed by a very simple formula

$$h^{(0)}(\boldsymbol{k}) = \sqrt{\frac{\pi}{N_c^2 - 1}}\, \frac{2C_F \alpha_{\mathrm{S}} N_\varepsilon}{\boldsymbol{k}^2\,\mu^{2\varepsilon}} , \quad N_\varepsilon = \frac{(4\pi)^{\varepsilon/2}}{\Gamma(1 - \varepsilon)} , \qquad (2.2)$$

and it is the same (up to a color factor) for quarks and gluons, where μ is the renormalization scale and ε is the dimensional regularization parameter. Using the expression for the leading order impact factor we define the constant

$$A_\varepsilon = \boldsymbol{k}^2 \, h^{(0)}(\boldsymbol{k}) \, \frac{\overline{\alpha}_S}{\Gamma(1-\varepsilon)\mu^{2\varepsilon}} \ , \tag{2.3}$$

which contains the dependence on the strong coupling and on color factors. The dimensionless strong coupling $\overline{\alpha}_S$ is expressed by using the gauge coupling parameter g and already introduced parameters:

$$\overline{\alpha}_S = \frac{\alpha_S N_c}{\pi} \ , \qquad \alpha_S = \frac{g^2 \Gamma(1-\varepsilon)\mu^{2\varepsilon}}{(4\pi)^{1+\varepsilon}} \ , \tag{2.4}$$

where N_c is the number of colors in QCD. Finally, the gluon Regge trajectory, $\omega^{(1)}(\boldsymbol{k})$, which accounts for the virtual correction to the BFKL kernel, has the simple form:

$$\omega^{(1)}(\boldsymbol{k}) = -\frac{g^2 N_c \boldsymbol{k}^2}{(4\pi)^{2+\varepsilon}} \int \frac{\mathrm{d}[\boldsymbol{p}]}{\boldsymbol{p}^2(\boldsymbol{k}-\boldsymbol{p})^2} = -\frac{\overline{\alpha}_S}{2\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} \left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^\varepsilon . \tag{2.5}$$

3 The integral representation of the impact factor

According to Ref. [69], the NLx result for the impact factor of a heavy quark can be written as the sum of three contributions:

$$h_{\mathsf{q}}^{(1)}(\boldsymbol{k}_2) = h_{\mathsf{q},m=0}^{(1)}(\boldsymbol{k}_2) + \int_0^1 \mathrm{d}z_1 \int \mathrm{d}[\boldsymbol{k}_1] \Delta F_{\mathsf{q}}(z_1, \boldsymbol{k}_1, \boldsymbol{k}_2)$$
$$+ \int \mathrm{d}[\boldsymbol{k}_1] \, \overline{\alpha}_S \, h_{\mathsf{q}}^{(0)}(\boldsymbol{k}_1) \, K_0(\boldsymbol{k}_1, \boldsymbol{k}_2) \, \log \frac{m}{k_1} \, \Theta_{m \, k_1} \ , \tag{3.1}$$

with the convention $\Theta_{m \, k_1} = \theta(m - k_1)$, where the θ-function is the well-known Heaviside step function and $k_1 = |\boldsymbol{k}_1|$. The first term on the right hand side of Eq.(3.1) is the NLx correction to the impact factor of a massless quark, which can be expressed by using the leading order impact factor $h^{(0)}(\boldsymbol{k})$ in Eq.(2.2) and the gluon Regge trajectory $\omega^{(1)}(\boldsymbol{k})$ in Eq.(2.5),

$$h_{\mathsf{q},m=0}^{(1)}(\boldsymbol{k}_2) = h^{(0)}(\boldsymbol{k}_2) \, \omega^{(1)}(\boldsymbol{k}_2) \left[b_0 + \frac{3}{2} - \varepsilon \left(\frac{1}{2} + \mathcal{K} \right) \right] \ , \tag{3.2}$$

with the beta function b_0 and $\mathcal{K}$ defined as

$$b_0 = \frac{11}{6} - \frac{n_f}{3N_c} \ , \qquad \mathcal{K} = \frac{67}{18} - \frac{\pi^2}{6} - \frac{5n_f}{9N_c} \ . \tag{3.3}$$

The second term on the right hand side of Eq.(3.1) is the NLx correction induced by the heavy quark mass m, with $\Delta F_{\mathsf{q}}(z_1, \boldsymbol{k}_1, \boldsymbol{k}_2)$ defined in Ref. [69]. The third term comes from the introduction of the mass scale to the leading order BFKL kernel $K_0(\boldsymbol{k}_1, \boldsymbol{k}_2)$ which is defined as

$$\overline{\alpha}_S \, K_0(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{\overline{\alpha}_S}{\boldsymbol{q}^2 \Gamma(1-\varepsilon)\mu^{2\varepsilon}} + 2\omega^{(1)}(\boldsymbol{k}_1)\delta[\boldsymbol{q}] \ , \qquad \delta[\boldsymbol{q}] = \pi^{1+\varepsilon}\delta^{2+2\varepsilon}(\boldsymbol{q}) \ , \tag{3.4}$$

with $\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2$. The first term on the right hand side of Eq.(3.4) is the real component of the BFKL kernel and the second one corresponds to the virtual corrections. In the following two Sections we reanalyze the second and third terms in the right hand side of Eq.(3.1).

4 The ΔF_{q} term

The second term in the right hand side of Eq.(3.1),

$$\Delta F_{\mathsf{q}}(\boldsymbol{k}_2) = \int_0^1 \mathrm{d}z_1 \int \mathrm{d}[\boldsymbol{k}_1] \Delta F_{\mathsf{q}}(z_1, \boldsymbol{k}_1, \boldsymbol{k}_2) \ , \tag{4.1}$$

receives contributions from virtual and real corrections. The explicit expression of the integrand of Eq.(4.1) is given in Ref. [69] in momentum space, after integration over $\boldsymbol{k}_1$. Note, however, that the remaining integrations cannot be performed directly in an straightforward way. Instead, it is easier to calculate the Mellin transform:

$$\Delta \tilde{F}_{\mathsf{q}}(\gamma) = \Gamma(1 + \varepsilon)\,(m^2)^{-\varepsilon} \int \mathrm{d}[\boldsymbol{k}_2] \left(\frac{\boldsymbol{k}_2^2}{m^2} \right)^{\gamma - 1} \Delta F_{\mathsf{q}}(\boldsymbol{k}_2) \ , \tag{4.2}$$

which leads to this expression

$$\begin{aligned}
\Delta \tilde{F}_{\mathsf{q}}(\gamma) = {}& A_\varepsilon \, (m^2)^\varepsilon \, \frac{\Gamma(\gamma + \varepsilon)\Gamma(1 - \gamma - 2\varepsilon)\Gamma^2(1 - \gamma - \varepsilon)}{8\Gamma(2 - 2\gamma - 2\varepsilon)} \\
& \times \left[\frac{1 + \varepsilon}{\gamma + 2\varepsilon} + \frac{2}{1 - 2\gamma - 4\varepsilon} \left(\frac{1}{1 - \gamma - 2\varepsilon} - \frac{1}{3 - 2\gamma - 2\varepsilon} \right) \right] \ .
\end{aligned} \tag{4.3}$$

Then, the function $\Delta F_{\mathsf{q}}(\boldsymbol{k}_2)$ in momentum space is recovered by computing the inverse Mellin transform:

$$\Delta F_{\mathsf{q}}(\boldsymbol{k}_2) = \frac{1}{m^2} \int_{1 - 2\varepsilon < \mathrm{Re}\,\gamma < 1 - \varepsilon} \frac{\mathrm{d}\gamma}{2\pi i} \left(\frac{\boldsymbol{k}_2^2}{m^2} \right)^{-\gamma - \varepsilon} \Delta \tilde{F}_{\mathsf{q}}(\gamma) \ . \tag{4.4}$$

This integral is a contour integral in the complex plane which is well defined when the integration contour is a straight line parallel to the imaginary axis and which intersects the real axis in the strip $1 - 2\varepsilon < \mathrm{Re}\,\gamma < 1 - \varepsilon$. To perform the integration in Eq.(4.4) we use Cauchy's residue theorem. The ratio $\boldsymbol{k}_2^2/m^2$ may, in principal, take any value between 0 and ∞. If $\boldsymbol{k}_2^2/m^2 < 1$, we deform the integration contour at $-i\infty$ and at $+i\infty$ to the right, such that the two ends meet at $+\infty$ of the real axis, whereas, if $\boldsymbol{k}_2^2/m^2 > 1$ we deform the integration contour at $-i\infty$ and at $+i\infty$ to the left, such that the two ends meet at $-\infty$ of the real axis. In both cases, we change the initial integration contour to a closed one which consists of the initial one and of a semi-circle on which the integrand is zero, allowing this way for the integration to be done by summing the residue contributions enclosed by each contour. It turns out though, that in order to obtain the result in a simple resummed form, we are forced to close the contour to the left, assuming that $\boldsymbol{k}_2^2/m^2 > 1$, in which case we denote the result by $\Delta F_{\mathsf{q}}^-(\boldsymbol{k}_2)$. We have checked that this closed resummed result is the correct result for all allowed values of the ratio $\boldsymbol{k}_2^2/m^2$ by comparing to the expression –denoted as $\Delta F_{\mathsf{q}}^+(\boldsymbol{k}_2)$– we get after closing the initial contour to the right. It is probably noteworthy to add that in the case of deforming the contour to the left, we were able to resum the residue contributions in a closed form both before and after expanding in ε whereas in the case of closing the contour to the right, the resummation of the residue contributions in a closed form is only possible before expanding in ε.

In detail, after deforming the integration contour as described above, we have:

$$\Delta F_{\mathsf{q}}^-(\boldsymbol{k}_2) = \frac{1}{m^2} \sum_{\gamma \leq 1 - 2\varepsilon} \operatorname{Res}\left[\left(\frac{\boldsymbol{k}_2^2}{m^2}\right)^{-\gamma-\varepsilon} \Delta\tilde{F}_{\mathsf{q}}(\gamma)\right] . \tag{4.5}$$

The distinct pole contributions that need to be accounted for come from the residues at $\gamma = 1 - 2\varepsilon$ (which provides the singular terms in ε), $\frac{1}{2} - 2\varepsilon$, $-\varepsilon$, -2ε and finally from the poles at $\gamma = -n - \varepsilon$ with $n \in \mathbb{N}$ and $n > 0$. To simplify the formalism, we factorize the leading order impact factor $h^{(0)}(\boldsymbol{k}_2)$ and the gluon Regge trajectory $\omega^{(1)}(\boldsymbol{k}_2)$ at each residue contribution and define

$$h_{\gamma_i}(\boldsymbol{k}_2) = \left(h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2)\right)^{-1} \frac{1}{m^2} \operatorname{Res}_{\{\gamma=\gamma_i\}}\left[\left(\frac{\boldsymbol{k}_2^2}{m^2}\right)^{-\gamma-\varepsilon} \Delta\tilde{F}_{\mathsf{q}}(\gamma)\right] . \tag{4.6}$$

Then

$$\Delta F_{\mathsf{q}}^-(\boldsymbol{k}_2) = h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2) \sum_{\gamma_i \leq 1 - 2\varepsilon} h_{\gamma_i}(\boldsymbol{k}_2) \tag{4.7}$$

and the contributions of the different residua at the poles located at $\gamma \leq 1 - 2\varepsilon$ are given by

$$\begin{aligned}
h_{1-2\varepsilon}(\boldsymbol{k}_2) &= -\frac{1 + 5\varepsilon - 2\varepsilon^2}{2(1 + 2\varepsilon)} - \log(4R) + \psi(1-\varepsilon) - \psi(1) - 2\psi(\varepsilon) + 2\psi(2\varepsilon) , \\
h_{1/2-2\varepsilon}(\boldsymbol{k}_2) &= \sqrt{R}\,\frac{(3 + 4\varepsilon)\,\Gamma(1 + 2\varepsilon)\,\pi\tan(\pi\varepsilon)}{4^{1+\varepsilon}(1 + \varepsilon)\,\Gamma^2(1 + \varepsilon)} , \\
h_{-2\varepsilon}(\boldsymbol{k}_2) &= R\,\frac{1 + \varepsilon}{1 + 2\varepsilon} , \\
h_{-n-\varepsilon}(\boldsymbol{k}_2) &= (-1)^{1+n}\,(4R)^{1+n-\varepsilon}\,\frac{\varepsilon\,\Gamma(1 + 2\varepsilon)\Gamma(1 + n)\,\Gamma(1 + n - \varepsilon)}{4\,\Gamma(1 - \varepsilon)\,\Gamma^2(1 + \varepsilon)\,\Gamma(2 + 2n)} \\
&\quad \times \left[\frac{1 + \varepsilon}{\varepsilon - n} + \frac{2}{1 + 2n - 2\varepsilon}\left(\frac{1}{1 + n - \varepsilon} - \frac{1}{3 + 2n}\right)\right] ,
\end{aligned} \tag{4.8}$$

where $R = \boldsymbol{k}_2^2/(4m^2)$. The residue at $\gamma = -\varepsilon$ is accounted for when $n = 0$ in the last expression in Eq.(4.8). To obtain a closed analytic expression we still need to resum $h_{-n-\varepsilon}(\boldsymbol{k}_2)$ for $n \geq 1$. The result is given by:

$$\begin{aligned}
\sum_{n=1}^{\infty} h_{-n-\varepsilon}(\boldsymbol{k}_2) = -(4R)^{2-\varepsilon}\,\frac{\varepsilon\,\Gamma(1 + 2\varepsilon)}{24\,\Gamma^2(1 + \varepsilon)} &\left[(1 + \varepsilon)\,{}_2F_1\!\left(1, 1 - \varepsilon; \frac{5}{2}; -R\right)\right. \\
&- \frac{4(1 - \varepsilon)}{3 - 2\varepsilon}\,{}_3F_2\!\left(1, \frac{3}{2} - \varepsilon, 2 - \varepsilon; \frac{5}{2}, \frac{5}{2} - \varepsilon; -R\right) \\
&+ \frac{2(1 - \varepsilon)}{5(3 - 2\varepsilon)}\,{}_3F_2\!\left(1, \frac{3}{2} - \varepsilon, 2 - \varepsilon; \frac{7}{2}, \frac{5}{2} - \varepsilon; -R\right) \\
&\left.+ \frac{2(1 - \varepsilon)}{2 - \varepsilon}\,{}_3F_2\!\left(1, 2 - \varepsilon, 2 - \varepsilon; \frac{5}{2}, 3 - \varepsilon; -R\right)\right] ,
\end{aligned} \tag{4.9}$$

where $_pF_q$ are generalized hypergeometric functions. By expanding in ε we obtain

$$\sum_{n=1}^{\infty} h_{-n-\varepsilon}(\boldsymbol{k}_2) = \varepsilon \left[1 + \frac{10}{3} R + \log(Z) \left((1+2R)\sqrt{\frac{1+R}{R}} + 2\log(Z) \right) \right.$$
$$\left. + 3\sqrt{R} \left(\text{Li}_2(Z) - \text{Li}_2(-Z) + \log(Z) \log\left(\frac{1-Z}{1+Z}\right) - \frac{\pi^2}{4} \right) \right] + \mathcal{O}(\varepsilon^2) , \qquad (4.10)$$

where Li_2 is the usual dilogarithm function, and $Z = (\sqrt{1+R} + \sqrt{R})^{-1}$. The final result is obtained by summing up the contributions of all the residua and reads

$$\Delta F_{\mathsf{q}}(\boldsymbol{k}_2) = h^{(0)}(\boldsymbol{k}_2) \, \omega^{(1)}(\boldsymbol{k}_2) \left\{ \frac{1}{\varepsilon} - \log(4R) - \frac{1}{2} \right.$$
$$+ \varepsilon \left[\frac{\pi^2}{6} - \frac{1}{2} + R \log(4R) + \log(Z) \left((1+2R)\sqrt{\frac{1+R}{R}} + 2\log(Z) \right) \right.$$
$$\left. \left. + 3\sqrt{R} \left(\text{Li}_2(Z) - \text{Li}_2(-Z) + \log(Z) \log\left(\frac{1-Z}{1+Z}\right) \right) \right] \right\} + \mathcal{O}(\varepsilon) . \qquad (4.11)$$

What we have achieved so far is a compact analytic expression for the second term in Eq.(3.1), that is, $\Delta F_{\mathsf{q}}(\boldsymbol{k}_2)$, for any value of R keeping the whole singularity structure. The double log singularities ($1/\varepsilon^2$ and $1/\varepsilon \, \log(R)$ terms) cancel [69] against the double log singularities from the remaining third term of Eq.(3.1).

5 The $K_0(\boldsymbol{k}_1, \boldsymbol{k}_2)$ related term

Let us now turn to the final ingredient in order to have the full NLx heavy quark impact factor with mass corrections. For the real emission part of the BFKL kernel, $K_0(\boldsymbol{k}_1, \boldsymbol{k}_2)$ (see Eq.(3.4)), we define the integral

$$I_m(\boldsymbol{k}_2) = \int \mathrm{d}[\boldsymbol{k}_1] \frac{\overline{\alpha}_{\mathrm{S}} h_{\mathsf{q}}^{(0)}(\boldsymbol{k}_1)}{\boldsymbol{q}^2 \Gamma(1-\varepsilon)\mu^{2\varepsilon}} \log \frac{m}{k_1} \Theta_{m\, k_1} . \qquad (5.1)$$

We use the following integral representation [69]:

$$\log \frac{a}{b} \Theta_{ab} = \lim_{\alpha \to 0^+} \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}\lambda}{2\pi i} \frac{1}{(\lambda+\alpha)^2} \left(\frac{a}{b}\right)^\lambda \equiv \int \mathrm{d}[\lambda] \left(\frac{a}{b}\right)^\lambda , \qquad (5.2)$$

valid for $a, b > 0$, which allows us to write

$$I_m(\boldsymbol{k}_2) = \frac{A_\varepsilon}{2} \int \mathrm{d}[\lambda] \, (m^2)^\lambda \int \frac{\mathrm{d}[\boldsymbol{k}_1]}{\boldsymbol{q}^2 \, (\boldsymbol{k}_1^2)^{1+\lambda}}$$
$$= \frac{A_\varepsilon}{2} \int \mathrm{d}[\lambda] \frac{\Gamma(1+\lambda-\varepsilon)\Gamma(\varepsilon)\Gamma(\varepsilon-\lambda)}{\Gamma(1+\lambda)\Gamma(2\varepsilon-\lambda)} (m^2)^\lambda (\boldsymbol{k}_2^2)^{-1-\lambda+\varepsilon} , \qquad (5.3)$$

or more explicitly

$$I_m(\boldsymbol{k}_2) = \frac{A_\varepsilon}{2} \lim_{\alpha \to 0^+} \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \frac{1}{(\lambda+\alpha)^2} \frac{\Gamma(1+\lambda-\varepsilon)\Gamma(\varepsilon)\Gamma(\varepsilon-\lambda)}{\Gamma(1+\lambda)\Gamma(2\varepsilon-\lambda)} (m^2)^\lambda (\boldsymbol{k}_2^2)^{-1-\lambda+\varepsilon} .$$
$$(5.4)$$

The integrand in Eq.(5.4) vanishes for $|\lambda| \to \infty$ in all directions apart from the real axis. As was the case in the previous Section, this is a contour integral and therefore we define

$$d_{\lambda_i}(\boldsymbol{k}_2) = \frac{A_\varepsilon}{2} \left(h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2) \right)^{-1}$$

$$\times \lim_{\alpha \to 0^+} \mathrm{Res}_{\{\lambda=\lambda_i\}} \left[\frac{1}{(\lambda+\alpha)^2} \frac{\Gamma(1+\lambda-\varepsilon)\Gamma(\varepsilon)\Gamma(\varepsilon-\lambda)}{\Gamma(1+\lambda)\Gamma(2\varepsilon-\lambda)} (m^2)^\lambda (\boldsymbol{k}_2^2)^{-1-\lambda+\varepsilon} \right] .(5.5)$$

For $m^2/\boldsymbol{k}_2^2 < 1$ we close the contour at infinity to the right of the complex plane, and evaluate the residua of the poles enclosed by the deformed contour. The first pole for $\lambda > 0$ is located at $\lambda = \varepsilon$ which gives the leading contribution including a singular term in ε. All remaining poles are located at $\lambda = n + \varepsilon$ with $n \in \mathbb{N}$ and give contributions of order $(m^2/\boldsymbol{k}_2^2)^n$. The complete expression for the residua of these poles, including the one at $n = 0$, reads:

$$d_{n+\varepsilon}(\boldsymbol{k}_2) = \frac{\Gamma(1+2\varepsilon)}{\Gamma(1-\varepsilon)\,\Gamma(1+\varepsilon)} \frac{(-1)^n\,(4R)^{-n-\varepsilon}}{(\varepsilon+n)^3\,\Gamma(\varepsilon-n)\,\Gamma(\varepsilon+n)} . \tag{5.6}$$

Keeping apart the contribution of the first pole at $n = 0$, we resum the series of residua at $\lambda = n + \varepsilon$ with $n \geq 1$ and we obtain

$$\sum_{n=1}^{\infty} d_{n+\varepsilon}(\boldsymbol{k}_2) = \frac{\varepsilon\,(1-\varepsilon)\,\Gamma(1+2\varepsilon)}{\Gamma(1-\varepsilon)\,\Gamma^3(2+\varepsilon)}$$

$$\times \frac{1}{(4R)^{1+\varepsilon}} \, {}_4F_3\left(1, 2-\varepsilon, 1+\varepsilon, 1+\varepsilon; 2+\varepsilon, 2+\varepsilon, 2+\varepsilon; \frac{1}{4R} \right). \tag{5.7}$$

By expanding in ε the contribution of the first pole of Eq.(5.6) at $n = 0$ and the result in Eq.(5.7) and after summing them up we get:

$$I_m^+(\boldsymbol{k}_2) = -h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2) \sum_{\lambda_i>0} d_{\lambda_i}(\boldsymbol{k}_2)$$

$$= h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2) \left[-\frac{1}{\varepsilon} + \log(4R) - \varepsilon \left(\frac{1}{2}\log^2(4R) + \mathrm{Li}_2\left(\frac{1}{4R} \right) \right) \right] + \mathcal{O}(\varepsilon)(5.8)$$

We need to consider in addition the case $\boldsymbol{k}_2^2/m^2 < 1$. Now, we close the integration contour to the left, to $-\infty$, enclosing this way the poles located at $\lambda = -\alpha$ with $\alpha \to 0^+$ and at $\lambda = -n + \varepsilon$, with $n \in \mathbb{N}$ and $n \geq 1$, with

$$I_m^-(\boldsymbol{k}_2) = h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2) \sum_{\lambda_i<0} d_{\lambda_i}(\boldsymbol{k}_2) . \tag{5.9}$$

These residua provide contributions of the order $(\boldsymbol{k}_2^2/m^2)^n$, their actual values are:

$$d_{-\alpha}(\boldsymbol{k}_2)|_{\alpha\to0^+} = 2\log(4R) + 2\left[\psi(1) - \psi(1-\varepsilon) + \psi(\varepsilon) - \psi(2\varepsilon) \right] , \tag{5.10}$$

$$d_{-n+\varepsilon}(\boldsymbol{k}_2) = \frac{\Gamma(1+2\varepsilon)}{\Gamma(1-\varepsilon)\,\Gamma(1+\varepsilon)} \frac{(-1)^n\,(4R)^{n-\varepsilon}}{(\varepsilon-n)^3\,\Gamma(\varepsilon-n)\,\Gamma(\varepsilon+n)} . \tag{5.11}$$

As before, we resum all the residua at $\gamma = -n + \varepsilon$ before we expand in ε. The result is:

$$\sum_{n=1}^{\infty} d_{-n+\varepsilon}(\boldsymbol{k}_2) = -\frac{\Gamma(1+2\varepsilon)\,\sin(\pi\varepsilon)}{\pi\,(1-\varepsilon)^2\,\Gamma(1+\varepsilon)^2}$$
$$\times\,(4R)^{1-\varepsilon}\,{}_4F_3\left(1, 1-\varepsilon, 1-\varepsilon, 1-\varepsilon; 2-\varepsilon, 2-\varepsilon, 1+\varepsilon; 4R\right)\ . \quad (5.12)$$

After summing up the contributions from Eq.(5.10) and Eq.(5.12), expanding in ε, and including the virtual term of the BFKL kernel $K_0(\boldsymbol{k}_1, \boldsymbol{k}_2)$, we obtain:

$$I_m^-(\boldsymbol{k}_2) - h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2)\,\log\left(\frac{\boldsymbol{k}_2^2}{m^2}\right)$$
$$= h^{(0)}(\boldsymbol{k}_2)\,\omega^{(1)}(\boldsymbol{k}_2)\left(-\frac{1}{\varepsilon} + \log(4R) - \varepsilon\,\mathrm{Li}_2(4R)\right) + \mathcal{O}(\varepsilon)\ . \quad (5.13)$$

6 The analytic result for the impact factor and a first numerical study

The final expression for the next-to-leading order correction $h_{\mathsf{q}}^{(1)}(\boldsymbol{k}_2)$ to the impact factor is obtained by using Eq.(3.2), Eq.(4.11), Eq.(5.8) and Eq.(5.13) into Eq.(3.1). Collecting all the contributions, the impact factor of a heavy quark at NLx accuracy reads

$$h_{\mathsf{q}}(\boldsymbol{k}_2) = h^{(0)}(\boldsymbol{k}_2) + h_{\mathsf{q}}^{(1)}(\boldsymbol{k}_2) \quad (6.1)$$

and can be expressed in terms of a singular and a finite contribution

$$h_{\mathsf{q}}(\boldsymbol{k}_2) = h_{\mathsf{q}}^{(1)}(\boldsymbol{k}_2)|_{\mathrm{sing}} + h_{\mathsf{q}}(\boldsymbol{k}_2)|_{\mathrm{finite}}\ . \quad (6.2)$$

The singular term $h_{\mathsf{q}}^{(1)}(\boldsymbol{k}_2)|_{\mathrm{sing}}$ reads [69]

$$h_{\mathsf{q}}^{(1)}(\boldsymbol{k}_2)|_{\mathrm{sing}} = h^{(0)}(\boldsymbol{k}_2)\left(\frac{3}{2}\,\omega^{(1)}(\boldsymbol{k}_2) - \frac{1}{2}\,\omega^{(1)}(m)\,\Theta_{k_2\,m} - \frac{1}{2}\,\omega^{(1)}(\boldsymbol{k}_2)\,\Theta_{m\,k_2}\right)\ , \quad (6.3)$$

whereas the finite contribution, which is the main result of this paper, is given by

$$\begin{aligned}
h_{\mathsf{q}}(\boldsymbol{k}_2)|_{\mathrm{finite}} = h^{(0)}(\boldsymbol{k}_2, \alpha_{\mathrm{S}}(\boldsymbol{k}_2))&\left\{1 + \frac{\alpha_{\mathrm{S}}\,N_c}{2\pi}\left[\mathcal{K} - \frac{\pi^2}{6} + 1 - R\,\log(4R)\right.\right.\\
&- \log(Z)\left((1+2R)\,\sqrt{\frac{1+R}{R}} + 2\log(Z)\right)\\
&- 3\,\sqrt{R}\left(\mathrm{Li}_2(Z) - \mathrm{Li}_2(-Z) + \log(Z)\,\log\left(\frac{1-Z}{1+Z}\right)\right)\\
&\left.\left.+ \mathrm{Li}_2(4R)\,\Theta_{m\,k_2} + \left(\frac{1}{2}\log(4R) + \frac{1}{2}\log^2(4R) + \mathrm{Li}_2\left(\frac{1}{4R}\right)\right)\Theta_{k_2\,m}\right]\right\}\ .
\end{aligned} \quad (6.4)$$

As in Ref. [69], we have absorbed the singularities proportional to the beta function b_0 into the running of the strong coupling $\alpha_{\mathrm{S}}(\boldsymbol{k}_2)$ [71, 72]. Our final result in Eq.(6.4) is valid in any kinematical regime, and provides a compact expression for the heavy quark impact factor which is suitable for phenomenological studies. We have checked that by

expanding Eq.(6.4) for either $\boldsymbol{k}_2^2/m^2 < 1$ or $m^2/\boldsymbol{k}_2^2 < 1$ we reproduce the results presented in Ref. [69][1]. In particular, the massless limit of the finite contribution to the impact factor in Eq.(6.4) is given by

$$h_{\mathsf{q}}(\boldsymbol{k}_2, m = 0)|_{\text{finite}} = h^{(0)}(\boldsymbol{k}_2, \alpha_{\text{S}}(\boldsymbol{k}_2)) \left\{1 + \frac{\alpha_{\text{S}}\, N_c}{2\pi} \left[\mathcal{K} - \frac{\pi^2}{6} - \frac{3}{2}\right]\right\} . \qquad (6.5)$$

With the result from Eq.(6.4) at hand, we proceed to a first numerical study of the magnitude of the mass corrections to the impact factor at NLx accuracy. As was stated previously, we have adopted the running coupling scheme as described in Refs. [71, 72] and with $n_f = 5$ flavors. At Lx accuracy, we use a fixed value for the strong coupling constant, namely, $\overline{\alpha}_{\text{S}} = 0.2$.
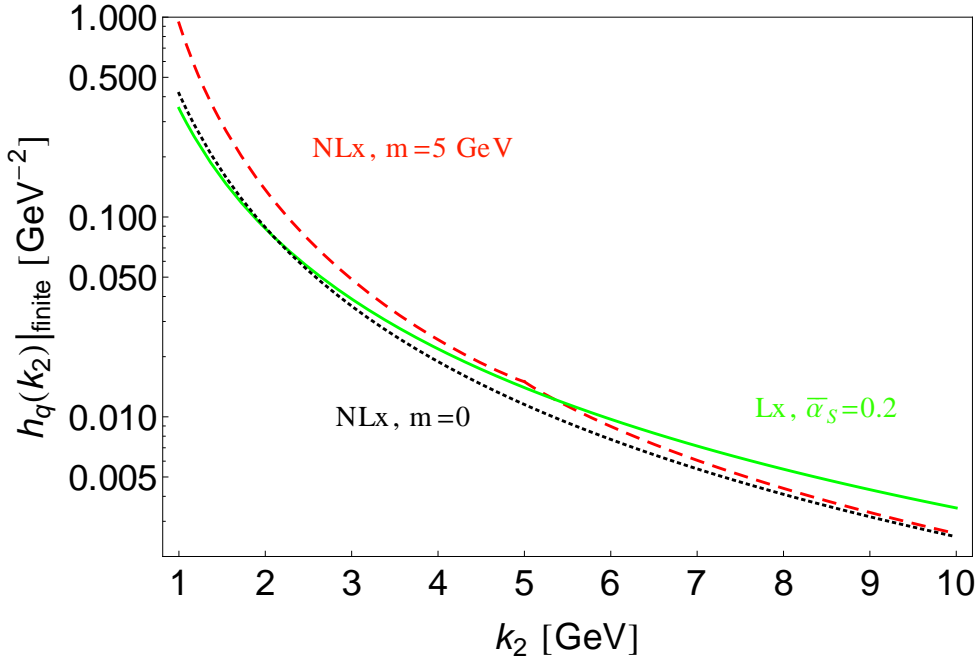

Figure 1. Finite part of the quark impact factor: Lx in green (solid line), massless NLx in black (dotted line) and NLx for quark mass $m = 5$ GeV in red (dashed line).

In Fig. 1 we plot the Lx as well as the NLx quark impact factor, the latter for two quark mass choices, $m = 0$ and $m = 5$ GeV. We see that the NLx correction to the leading order impact factor for massless quark is positive and moderate only for small $\boldsymbol{k}_2$ where the behavior is dominated by the running of the strong coupling constant, whereas, for most of the range of the plot the correction is negative. For a non-zero quark mass the overall correction is positive and large in the region $\boldsymbol{k}_2^2/m^2 < 1$. They turn to negative closely after $\boldsymbol{k}_2^2/m^2 = 1$ and for larger $\boldsymbol{k}_2^2/m^2$, they follow the NLx massless curve as expected. To get a better quantitative picture of the behavior of the NLx corrections in the massless

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[1] There were two typos in Ref. [69] which were taken into account when we made the comparison.
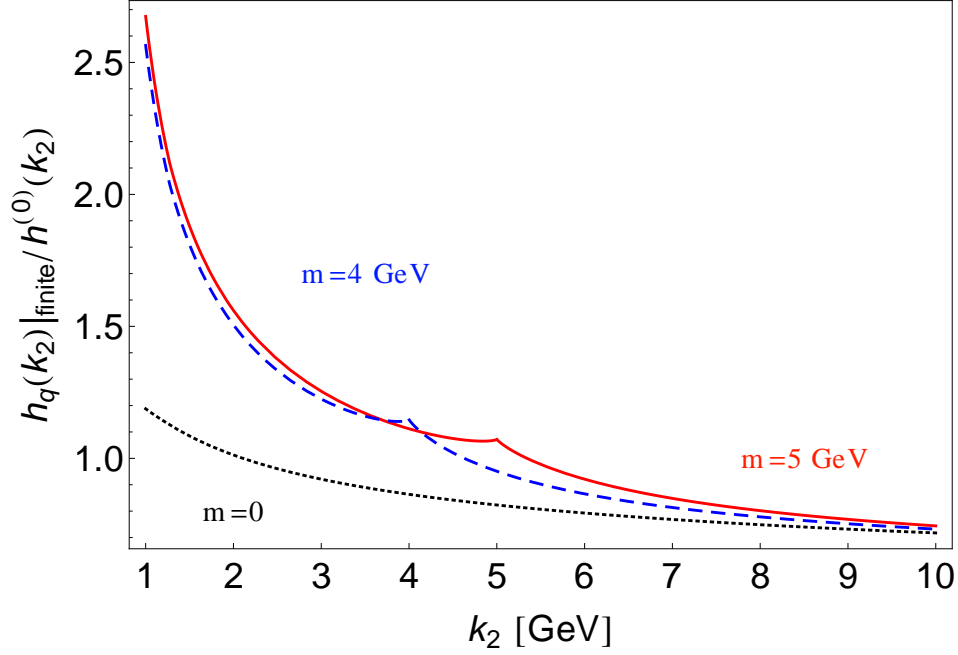
Figure 2. The ratio of the NLx impact factor to the Lx impact factor for different quark masses: $m = 0$ GeV (massless) in black (dotted line), $m = 4$ GeV in blue (dashed line) and $m = 5$ GeV in red (solid line).
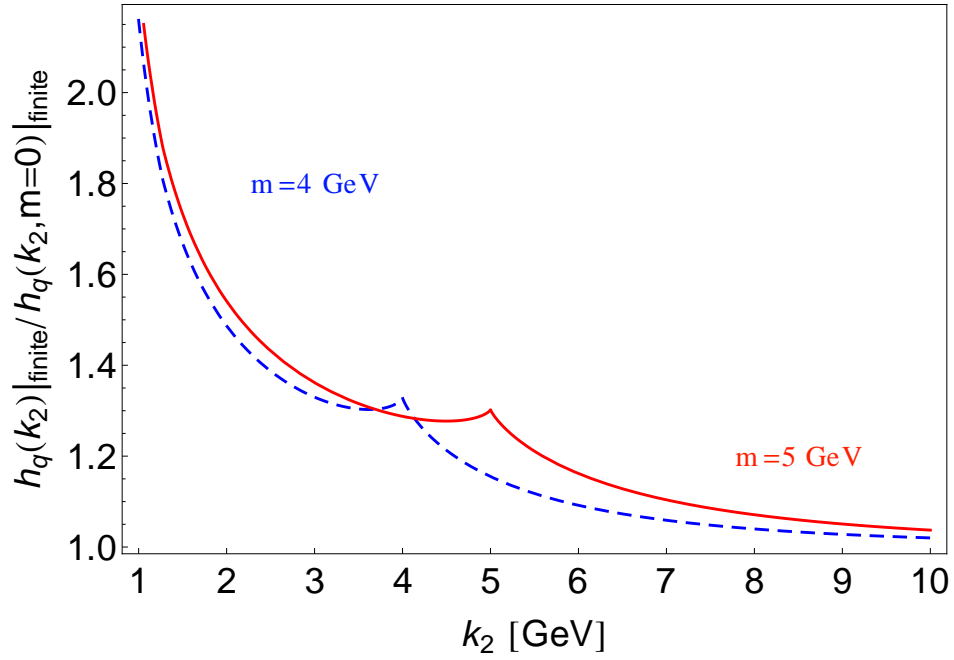
Figure 3. Ratio of the massive NLx impact factor over the massless NLx factor for two different mass choices: $m = 4$ GeV in blue (dashed line) and $m = 5$ GeV in red (solid line).

and massive case, it is useful to study the ratios of the impact factors at Lx and NLx accuracy. In Fig. 2 we can see that the relative size of the full NLx corrections in the

range $\boldsymbol{k}_2^2/m^2 < 10$ GeV, vary from more than $+100\%$ at very small $\boldsymbol{k}_2$ down to some -20% for larger $\boldsymbol{k}_2$, for similar mass choices $m = 4$ GeV and $m = 5$ GeV. In Fig. 3 the ratio between the finite parts of the NLx massive and massless quark impact factor is plotted. The corrections induced purely by a non-zero mass are of the order of a 100% in the small $\boldsymbol{k}_2^2/m^2$ limit and decrease as $\boldsymbol{k}_2$ is getting larger. The cusps in the curves are solely an artefact of the choice of the factorization scale, for details we refer the reader to Section 3 of Ref. [69]. As expected, in the limit $\boldsymbol{k}_2 \to \infty$, the massless and massive NLx impact factors coincide such that their ratio approaches 1.

7 Conclusions

In this work, we re-calculated the heavy quark impact factor at next-to-leading logarithmic accuracy and we obtained a closed analytic result for its finite part, suitable for an immediate numerical implementation. We performed a first comparative numerical study on the massless and massive NLx impact factor and found out that switching on a non-zero quark mass has as effect to amplify the magnitude of the overall corrections in the $\boldsymbol{k}_2^2/m^2 < 1$ region, while keeping them positive. We consider the re-calculated finite part of the heavy quark impact factor presented here as the first step toward new LHC phenomenological studies within the k_T-factorization scheme of processes with heavy quarks in the final state. Our next immediate project is to study the cross section for single bottom quark forward production at the LHC.

Acknowledgments

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