Dirac neutrinos from flavor symmetry

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We present a model where Majorana neutrino mass terms are forbidden by the flavor symmetry group $\Delta(27)$. Neutrinos are Dirac fermions and their masses arise in the same way as those of the charged fermions, due to very small Yukawa couplings. The model fits current neutrino oscillation data and correlates the octant of the atmospheric angle θ_{23} with the magnitude of the lightest neutrino mass, with maximal mixing excluded for any neutrino mass hierarchy.

Introduction

The historic observation of neutrino oscillations [1–4] implies that neutrinos are massive in contrast with the Standard Model (SM) prediction. Incorporating small masses requires an extension of the SM in which neutrinos are generally expected to be of Majorana type, hence violating lepton number symmetry [5] [1]. On the other hand in many schemes, such as for example the so-called seesaw mechanism lepton number violation is expected to account for the observed smallness of neutrino mass relative to that of charged fermions [5]. Yet, so far current neutrino oscillation experiments have been insensitive to the Majorana nature of neutrinos [7, 8] and, despite intense ongoing efforts it has not been confirmed through the observation of lepton number violation processes such as neutrinoless double beta decay ($0\nu\beta\beta$) [9]. Hence neutrinos could very well be Dirac fermions [10]. In short, the status of lepton and baryon number symmetries remains as one of the deepest unsolved mysteries of nature [11]. An equally puzzling challenge is associated to the origin of the peculiar flavor pattern of mixing angles indicated by global fits of neutrino oscillation experiments [12].

Here we suggest a possible interconnection between these puzzles, namely, that lepton number conservation can be an accidental consequence of the flavor symmetry that accounts for the neutrino mixing pattern.

Over the last decade non-Abelian discrete groups have been widely used as family symmetries because of their potential in restricting neutrino mixing patterns [13, 14]. As examples we mention the successful models based on the A_4 group predicting $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ [15, 16]. However the recent discovery of a large reactor angle, $\theta_{13} > 0$ [2–4], and a possible hint in favor of non-maximal atmospheric mixing present in recent oscillation fits θ_{23} [12] suggests the need for generalizing these models [17] and/or seeking for alternative schemes based upon different fla-

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[1] Recently it has been claimed that one can find models where lepton number is violated by four units, $\Delta L = 4$, even if neutrinos are of the Dirac type [6].

vor symmetries [18].

Here we present a flavor model for leptons using the non-Abelian group $\Delta(27)$ [19–22] that is able to provide automatic lepton number conservation in a way consistent with current global fits of neutrino oscillation data [12]. Recently other non-Abelian flavor symmetries have been used for pure Dirac neutrinos, see for instance [10, 23–25], however Majorana mass terms are forbidden by means of extra Abelian symmetries. Here we focus on the possibility that Majorana mass terms are not allowed from the flavor symmetry without requiring any extra additional Abelian symmetry. We note that since neutrinos are Dirac fermions $0\nu\beta\beta$ is exactly forbidden. In addition the model gives a correlation between neutrino oscillation parameters that excludes the maximal θ_{23} value.

Preliminaries

In order to generate Dirac neutrino mass terms we introduce singlet "right handed" (RH) neutrinos transforming under the flavor symmetry group $\mathcal{G}_F$ in such a way that their tensor product does not contain the trivial element of $\mathcal{G}_F$. This means that, even though lepton number conservation is not necessarily required *a priori*, Majorana mass terms are forbidden in the Lagrangian as a result of the flavor symmetry $\mathcal{G}_F$.

Although this may be achieved by using an Abelian symmetry $\mathbb{Z}_N \ \forall \ N \geq 3$ our focus relies on simple non-Abelian flavor symmetry groups. We assume that RH-neutrinos (N_R) transform as a 3-dimensional irreducible representation (irrep) of $\mathcal{G}_F$. Hence if N_R transforms as 3-dimensional irrep ($\mathbf{3}$) under $\mathcal{G}_F$, one finds that the non-Abelian symmetries which forbid a term like $N_R^T N_R$ are

- $\Delta(3N^2)$ for $N \geq 3$: these groups contain nine singlets and $(N^2 - 3)/3$ triplets for $N = 3\mathbb{Z}$. Otherwise, for $N \neq 3\mathbb{Z}$, they have three singlets and $(N^2 - 1)/3$ triplets.

- $\Sigma(3N^3)$ for $N \geq 3$: the set of groups with $N(N^2 + 8)/3$ conjugacy classes, $3N$ singlets and $N(N^2 - 1)/3$ triplets.

	$\overline{L}$	ℓ_{1R}	ℓ_{2R}	ℓ_{3R}	N_R	H
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
$\Delta(27)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{1''}$	$\mathbf{3}$	$\mathbf{3'}$

Table I: Matter assignments of the model.

- T_N for the N values given in [26] these groups have 3 singlets and $(N - 1)/3$ three-dimensional irreducible representations.

- $Z_9 \rtimes Z_3$.

In fact the mass term $N_R^T N_R$ is forbidden because the tensor product $\mathbf{3}_i \otimes \mathbf{3}_i$ (where $i = 1, ..., n_d$ and $n_d = (N^2 - 3)/3$ for $\Delta(3N^2)$ and $n_d = N(N^2 - 1)/3$ for $\Sigma(3N^3)$) does not contain a trivial 1-dimensional irrep 1^0 [26, 27].

The model

Searching for the the smallest realistic flavor symmetry group of the above class, i.e. used in the context of forbidding Majorana mass terms, we find that [2] it is $\Delta(27)$. The $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes \Delta(27)$ multiplet assignment is given in Table I,[3] where we have extended the SM by adding three right-handed neutrinos and two Higgs doublets apart from that of the Standard Model. The most general invariant Lagrangian for leptons is written as

$$\mathcal{L}_\ell = \sum_{i=1}^{3} Y_i^\ell \bar{L} \ell_{iR} H + Y^\nu \bar{L} N_R \tilde{H} + h.c., \quad (1)$$

where we use the compact notation $H = (H_1, H_2, H_3)$ and $\tilde{H} = (\tilde{H}_1, \tilde{H}_2, \tilde{H}_3)$ with $\tilde{H}_i \equiv i\sigma_2 H^*$. After electroweak symmetry breaking one gets the following patterns for the neutrino and charged lepton mass matri-

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[2] T_7 has the desired product and has indeed been used as a successful flavor symmetry, however not in the context of Dirac neutrinos [28–31].

[3] We denote, by convenience, $\mathbf{1} \equiv \mathbf{1}_{(0,0)}$, $\mathbf{1'} \equiv \mathbf{1}_{(1,0)}$, $\mathbf{1''} \equiv \mathbf{1}_{(2,0)}$, $\mathbf{3} \equiv \mathbf{3}_{(0,1)}$ and $\mathbf{3'} \equiv \mathbf{3}_{(0,2)}$, where the index notation is that used in [26, 27]

ces:

$$M_\nu = \begin{bmatrix} av_1 & bv_3 & cv_2 \\ cv_3 & av_2 & bv_1 \\ bv_2 & cv_1 & av_3 \end{bmatrix} \qquad (2)$$

$$M_\ell = \begin{bmatrix} Y_1^\ell v_1 & Y_2^\ell v_1 & Y_3^\ell v_1 \\ Y_1^\ell v_2 & \omega Y_2^\ell v_2 & \omega^2 Y_3^\ell v_2 \\ Y_1^\ell v_3 & \omega^2 Y_2^\ell v_3 & \omega Y_3^\ell v_3 \end{bmatrix}$$

where v_i are Higgs scalar vacuum expectation values (vevs), $\langle H \rangle = (\langle H_1 \rangle, \langle H_2 \rangle, \langle H_3 \rangle) = (v_1, v_2, v_3)$. The parameters $\{a, b, c, Y_i\}$ are real if CP invariance is assumed where the CP transformation is properly defined in [32–35]. One sees that in such minimal scenario the smallness of neutrino masses w.r.t. those of the charged leptons must arise due to very small Yukawa couplings [4]. The structure of M_ℓ and M_ν are well known in the literature [16, 19] and the alignment $\langle H \rangle = v(1, 1, 1)$ turns out to be natural in $\Delta(27)$ [19, 21].

In such a case M_ℓ can be written as $M_\ell = U_\omega \hat{Y}$ where $\hat{Y} = \mathrm{diag}(Y_1, Y_2, Y_3)$ and

$$U_\omega = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \qquad (3)$$

is the so–called "magic" matrix. However, given the structure of the neutrino mass matrix M_ν, the previous alignment $\langle H \rangle = v(1, 1, 1)$ cannot be assumed since then U_ω diagonalizes both $M_\nu M_\nu^\dagger$ and $M_\ell M_\ell^\dagger$. This results in a trivial lepton mixing matrix

$$U = U_\ell^\dagger U_\nu = U_\omega^\dagger U_\omega = \mathbb{I}. \qquad (4)$$

Moreover, when $v_1 = v_2 = v_3 = v$ and the couplings a, b and c are real the resulting neutrino masses are also not suitable to account for current neutrino oscillation data.

All of this can be avoided by deviating from the simplest vev alignment, i.e. we can fit the neutrino squared mass differences, as well as induce large lepton mixing angles by

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[4] Suppressed Yukawa coefficients can arise in extra dimension schemes, i.e. [24], as well as supersymmetric schemes, see for instance [23]

assuming that the vev alignment is generalized to

$$\langle H \rangle = \hat{v}(1 + \epsilon_1, 1 + \epsilon_2, 1)^T , \qquad (5)$$

where $|\langle H \rangle|^2 = v^2 = (246 \text{ GeV})^2$. The above vev configuration is a solution of the minimization conditions of the scalar potential provided it softly breaks the flavor symmetry, the deviation parameters $\epsilon_{1,2}$ being then associated to this soft breaking.

Taking into account Eq. (5) the mass matrices for the lepton sector are now given by

$$M_\nu = \hat{v} \begin{bmatrix} a(1 + \epsilon_1) & b & c(1 + \epsilon_2) \\ c & a(1 + \epsilon_2) & b(1 + \epsilon_1) \\ b(1 + \epsilon_2) & c(1 + \epsilon_1) & a \end{bmatrix} \qquad (6)$$

$$M_\ell = \hat{v} \begin{bmatrix} Y_1^\ell(1 + \epsilon_1) & Y_2^\ell(1 + \epsilon_1) & Y_3^\ell(1 + \epsilon_1) \\ Y_1^\ell(1 + \epsilon_2) & \omega Y_2^\ell(1 + \epsilon_2) & \omega^2 Y_3^\ell(1 + \epsilon_2) \\ Y_1^\ell & \omega^2 Y_2^\ell & \omega Y_3^\ell \end{bmatrix} .$$

Note that an immediate consequence of the generalized vev alignment is that the U_ω no longer diagonalizes the neutrino mass matrix nor that of the charged leptons, and therefore, as desired, the lepton mixing matrix is now non-trivial,

$$U = U_\ell^\dagger U_\nu \neq \mathbb{I} . \qquad (7)$$

Furthermore one can indeed fit all neutrino observables as we now show.

Results

Here we consider deviations of the alignment $v(1, 1, 1)$ of the order $\mathcal{O}(\lambda_C)$ where $\lambda_C \sim 0.2$ is the Cabibbo angle. More precisely, using Eqs. (5) and (6) we have scanned over values for the small parameters $\epsilon_{1,2}$ within the range $|\epsilon_{1,2}| \leq 0.3$ and selected those solutions which satisfy the global fits for the mixing angles at 3σ [12]

$$0.017 < \sin^2 \theta_{13} < 0.033$$

$$0.36(0.37) < \sin^2 \theta_{23} < 0.68(0.67) \text{ NH(IH)}$$

$$0.27 < \sin^2 \theta_{12} < 0.37,$$

as well as the neutrino squared mass differences

$$\Delta m_{21}^2 = (7.12 - 8.20) \times 10^{-5} eV^2,$$

$$|\Delta m_{31}^2| = \left\{ \begin{array}{l} (2.31 - 2.74) \text{ for NH} \\ (0.21 - 2.64) \text{ for IH} \end{array} \right\} \times 10^{-3} eV^2 .$$
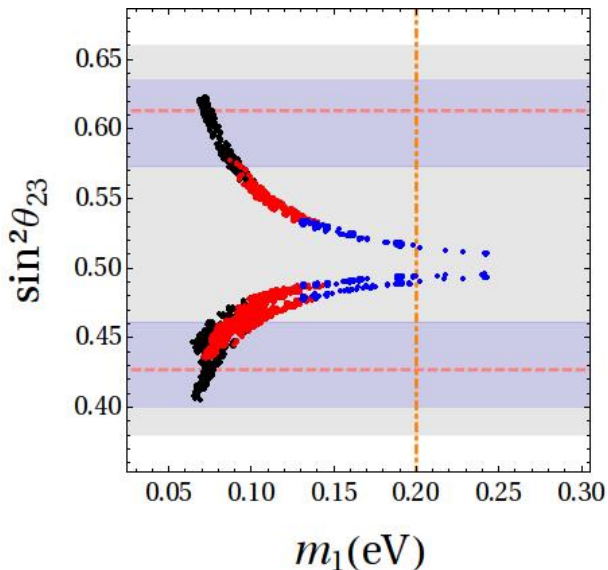
Figure 1: Correlation between the atmospheric angle and the lightest neutrino mass for the NH case. The horizontal dotted lines represent the best fit values, the (dark) blue and (light) gray horizontal bands are the 1σ and 2σ allowed ranges, respectively. The blue (light gray), red (gray) and black points are model expectations corresponding to vev deviations of 10%, 20% and 30% respectively (see text for more details). The vertical dot-dashed line indicates KATRIN's sensitivity [36].

Figure 2: Same as Figure 1 for the IH case. Note that in this case a 30% vev deviation is not enough to reach the best fit value of θ_{23}.

We have found a correlation between the atmospheric angle and the lightest neutrino mass for both the normal mass hierarchy (NH) and the inverted mass hierarchy (IH) cases. This is shown in Figures 1 and 2 for the NH and IH cases, respectively. In both figures the dotted horizontal lines represent the best fit values, while the (dark) blue and (light) gray horizontal bands are the 1σ and 2σ bands obtained in Ref. [12], respectively. For the NH case the global oscillation fit finds also a local minimum in the first octant of θ_{23} [12].

In order to explore the sensitivity of the observables with respect to the values of the vev deviation parameters, $\epsilon_{1,2}$, we consider the following cases, $|\epsilon_{1,2}| \lesssim 0.1$, $|\epsilon_{1,2}| \lesssim 0.2$ and $|\epsilon_{1,2}| \lesssim 0.3$ where each one represents deviations of 10%, 20% and 30% respectively. As we mentioned above, the free parameters
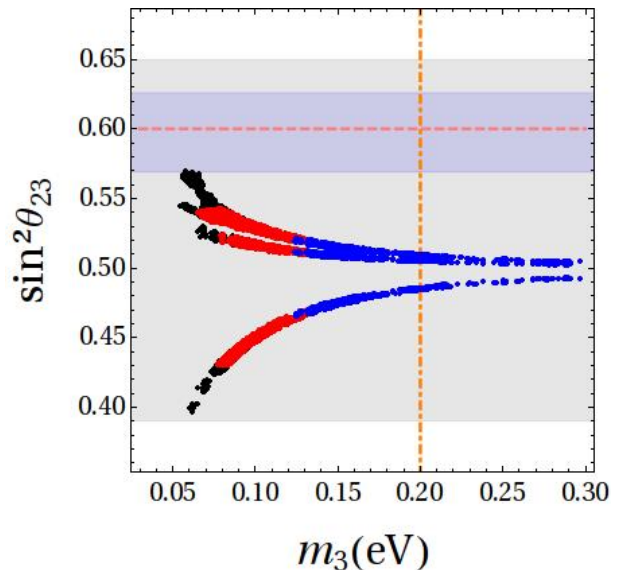
$\epsilon_{1,2}$ are associated to the $\Delta(27)$-soft breaking terms in the scalar potential and they are allowed to deviate at most at the order of the Cabibbo angle, $\epsilon_{1,2} \sim \mathcal{O}(\lambda_C)$.

The solutions in blue (light gray) correspond to deviations up to 10%, those in red (gray) up to 20% and those in black up to 30%. Figure 2 for the IH case shows that a 30% vev deviation is not enough to reach the best fit value for θ_{23}, so that larger deviations would be required in order to accomplish it.

In the near future the KATRIN experiment could discover a neutrino mass in the degenerate region, going from $m_\beta \sim 0.3$ eV at 3σ significance to $m_\beta = 0.35$ eV at 5σ significance [36]. If a neutrino mass is not seen in tritium β decays this will set an upper bound of 0.2 eV for neutrino mass and such a bound is depicted in each figure with the dot-dashed vertical line.

It is important to note that the atmospheric angle deviates significantly from the maximal value as the vev deviations increase.

Before concluding we mention that the model leads to contributions to flavor changing neutral current (FCNC) processes in the lepton sector, such as $\mu \to e\gamma$. However, we have checked a few representative points with normal neutrino mass hierarchy, and found that there is sufficient freedom in parameter

space to satisfy the current MEG bound for such a process [37]. Indeed, Table II gives the expected $\mu \to e\gamma$ branching ratios such points are all consistent with current bounds. Considering that these points are located in different parameter regions, we believe that a detailed analysis will give similar results, though a complete study is beyond the scope of this paper and will be considered elsewere. Note that the model does not lead to FCNC in the quark sector as its symmetry affects only the lepton sector. A model upgrading the flavour symmetry to both sectors is being developed and will be presented in a future publication, including a detailed phenomenological study.

Cases	$\mathrm{Br}^{\mathrm{th}}(\mu \to e\gamma)$	m_{ν_1} (eV)	$\sin^2 \theta_{23}$
i)	1.98×10^{-14}	0.2399	0.4956
ii)	1.74×10^{-14}	0.0930	0.4615
iii)	1.65×10^{-14}	0.0762	0.6107

Table II: Theoretical branching ratios for the process $\mu \to e\gamma$ for three different cases corresponding to three different sets of (ϵ_1,ϵ_2), m_{ν_1}, and $\sin^2 \theta_{23}$.

Summary

We have presented a model based on $\Delta(27)$ flavor symmetry. We showed that having RH neutrinos and LH leptons transforming as 3 dimensional irreps under $\Delta(27)$ forbids Majorana mass terms so that neutrinos are *naturally* Dirac-type, just as all other Standard Model fermions [10]. There is accidental lepton number conservation in the model caused by gauge symmetry, as in the SM, and it is present before and after EWSB. Furthermore, due to the particle content of the model, we find that all higher order Weinberg-type operators $LHLH(H^\dagger H)^n$ for $n = 0, 1, 2, ...$ that might yield a Majorana mass are not allowed by the symmetry $\Delta(27)$ and there are neither scalar singlets nor triplets to realize any diagram (operator) in [38–40]. This scenario is able to fit the current data in the lepton sector and establishes a correlation between the octant of the atmospheric angle θ_{23} and the magnitude of the lightest neutrino mass eigenvalue which may be probed by coming experiments.

Acknowledgments

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