Charmonium spectroscopy above thresholds

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We present a systematic and selfconsistent analysis of four-quark charmonium states and applied it to study compact four-quark systems and meson-meson molecules. Our results are robust and should serve to clarify the situation of charmonium spectroscopy above the threshold production of charmed mesons.

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Understanding of charmonium spectroscopy is challenging for experimentalists and theorists alike. Charmonium has been used as the test bed to demonstrate the color Fermi-Breit structure of quark atoms obeying the same principles as ordinary atoms [1]. Its nonrelativistic character ($v/c \approx 0.2 - 0.3$) gave rise to an amazing agreement between experiment and simple quark potential model predictions as $c\bar{c}$ states [2]. The opening of charmed meson thresholds was expected to modify the trend in the construction of quark-antiquark models. In the adiabatic approximation meson loops were absorbed into the static interquark potential. Thus, close to the threshold production of charmed mesons models required of an improved interaction [3]. The corrections introduced to the quark-antiquark spectra explained some deviations observed experimentally [4].

Since 2003, with the discovery of several states in the open charm sector, we have witnessed a growth of puzzling new mesons, being $D^*_{sJ}(2317)$, $D_{sJ}(2460)$ and $D^*_0(2308)$ the most prominent examples. Later, several new states have joined this exclusive group either in the open-charm sector, $D_{sJ}(2860)$ and $D_{sJ}(3040)$, or in the charmonium spectra, like the well established $X(3872)$ and $Y(4260)$, $Z(3930)$, $X(3940)$, $Y(3940)$, $X(4008)$, $X(4160)$, $X(4260)$, $Y(4350)$, and $Y(4660)$. In addition, the Belle Collaboration has reported the observation of similar states with non-zero electric charge: the $Z(4430)$, the $Z_1(4040)$ and the $Z_2(4240)$ that have not yet been confirmed by other experiments and remain somewhat controversial [5]. These new states do not fit, in general, the simple predictions of the quark-antiquark schemes and, moreover, they overpopulate the expected number of states in (simple) two-body theories. This situation is not uncommon in particle physics. For example, in the light scalar-isoscalar meson sector hadronic molecules seem to be needed to explain the experimental data [6–8]. Also, the study of the NN system above the pion production threshold required new degrees of freedom to be incorporated in the theory, either as pions or as excited states of the nucleon, i.e., the Δ [9, 10]. This discussion suggests that charmonium spectroscopy could be rather simple below the threshold production of charmed mesons but much more complex above it. In particular, the coupling to the closest $(c\bar{c})(n\bar{n})$ system, referred to as *unquenching the naive quark model* [11], could be an important spectroscopic ingredient. Besides, hidden-charm four-quark states could explain the overpopulation of quark-antiquark theoretical states. Thus, the new experimental discoveries are offering exciting new insights into the subtleties of the strong interaction.

In an attempt to disentangle the role played by multiquark configurations in the charmonium spectroscopy we have obtained an exact solution of the four-body problem based on an infinite expansion of the four-quark wave function in terms of hyperspherical harmonics [12]. The method is exact but is not completely adequate to study states that are close to, but below, the charmed meson production threshold. Such states are called *molecular*, in the sense that they can be exactly expanded in terms of a single singlet-singlet color vector. Close to a threshold, methods based on a series expansion fail to converge since arbitrary large number of terms are required to determine the wave function. From our analysis, we concluded that those four-quark states with two different asymptotic physical thresholds (as it is the case of the $c\bar{c}n\bar{n}$ system that may split either into a $(c\bar{c})(n\bar{n})$ or $(c\bar{n})(n\bar{c})$ two-meson states) can hardly present a bound state since the interaction between any pair of quarks contributes to the energy of one of the two physical thresholds. However, we observed that the root mean square radius of a few channels did not grow in the same manner as in those channels clearly converging to an unbound two-meson threshold. Instead, their radius remained stable and their energy did not cease slightly decreasing.

For this reason, we have used a different technique that we developed when studying baryon spectra with screened potentials and that showed to be very powerful close to a threshold [13]. In this case, the hyperspherical harmonic expansion of the wave function was computationally very expensive. Instead, we solved the Faddeev equations for negative energies using the Fredholm determinant method that permitted us to obtain robust predictions even for zero-energy bound states. For the charmonium the situation is similar but simpler. Similar because we are working on the region where methods based on infinite expansions are inefficient, but simpler since it is a two-body problem, the scattering of two mesons. Thus, we solve the Lippmann-Schwinger equation looking for attractive channels that may contain a meson-meson molecule. In order to account for all basis states we allow for the coupling to charmonium-light two-meson systems. With this method we circumvent the uncertainties associated to the slow convergence of the hyperspherical harmonic method for large grand angular momenta.

When we consider the system of two mesons M_1 and $\overline{M}_2$ ($M_i = D, D^*$) in a relative $S-$state interacting through a potential V that contains a tensor force then, in general, there is a coupling to the $M_1\overline{M}_2$ $D-$wave and the Lippmann-Schwinger equation of the system is

$$t_{ji}^{\ell s\ell'' s''}(p,p'';E) = V_{ji}^{\ell s\ell'' s''}(p,p'') + \sum_{\ell' s'}\int_0^\infty p'^2 dp' \times V_{ji}^{\ell s\ell' s'}(p,p')\frac{1}{E-p'^2/2\mu+i\epsilon}t_{ji}^{\ell' s'\ell'' s''}(p',p'';E), \quad (1)$$

where t is the two-body amplitude, j, i, and E are the angular momentum, isospin and energy of the system, and ℓs, $\ell' s'$, $\ell'' s''$ are the initial, intermediate, and final orbital angular momentum and spin; p and μ are the relative momentum and reduced mass of the two-body system, respectively. In the case of a two D meson system that can couple to a charmonium-light two-meson state, for example when $D\overline{D}^*$ is coupled to $J/\Psi\omega$, the Lippmann-Schwinger equation for $D\overline{D}^*$ scattering becomes

$$t_{\alpha\beta;ji}^{\ell_\alpha s_\alpha \ell_\beta s_\beta}(p_\alpha,p_\beta;E) = V_{\alpha\beta;ji}^{\ell_\alpha s_\alpha \ell_\beta s_\beta}(p_\alpha,p_\beta) + \sum_{\gamma=D\overline{D}^*,J/\Psi\omega}\sum_{\ell_\gamma=0,2}\int_0^\infty p_\gamma^2 dp_\gamma V_{\alpha\gamma;ji}^{\ell_\alpha s_\alpha \ell_\gamma s_\gamma}(p_\alpha,p_\gamma) \times G_\gamma(E;p_\gamma)\, t_{\gamma\beta;ji}^{\ell_\gamma s_\gamma \ell_\beta s_\beta}(p_\gamma,p_\beta;E), \quad (2)$$

with $\alpha,\beta = D\overline{D}^*, J/\Psi\omega$. For bound-state $E<0$ that the singularity of the propagator is never reached, we can neglect $i\epsilon$ in the denominator. By changing variables,

$$p'(p_\gamma) = b\frac{1+x'}{1-x'}, \quad (3)$$

where b is a scale parameter, and the same for $p(p_\alpha)$ and p"(p_β). Replacing the integral from -1 to 1 by a Gauss-Legendre quadrature, we obtain a set of linear equations. If a bound state exists at an energy E_B, the matrix determinant is zero. We took the scale parameter b of Eqs. (1) and (2) to be $b = 3$ fm^{-1} and used a Gauss-Legendre quadrature with $N = 20$ points.

We have consistently used the same interacting Hamiltonian to study the two- and four-quark systems to guarantee that thresholds and possible bound states are eigenstates of the same Hamiltonian. Such interaction contains a universal one-gluon exchange, confinement, and a chiral potential between light quarks [14]. We have solved the coupled channel problem of the $D\overline{D}$, $D\overline{D}^*$, and $D^*\overline{D}^*$. In all cases we have included the coupling to the relevant $(c\bar{c})(n\bar{n})$ channel (from now on denoted as $J/\Psi\omega$ channels).

As we study systems with well-defined $C-$parity, let us comment on the $D\overline{D}^*$ system. Since neither $D\overline{D}^*$ nor $\overline{D}D^*$ are eigenstates of $C-$parity, it is necessary to construct the proper linear combinations. Taking into account that $C(D) = \overline{D}$ and $C(D^*) = -\overline{D}^*$ (with a relative minus sin between them), it can be found that [17]:

$$D_1 = \frac{1}{\sqrt{2}}\left(D\overline{D}^* + \overline{D}D^*\right) \quad (4)$$

and

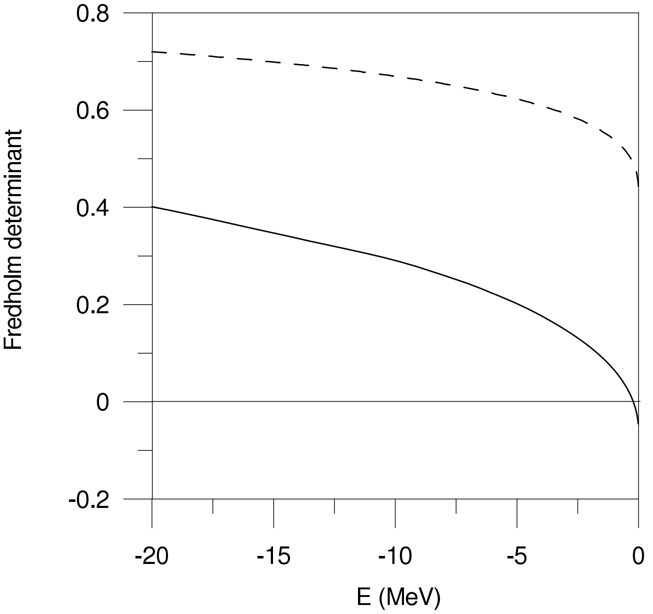
$$D_2 = \frac{1}{\sqrt{2}}\left(D\overline{D}^* - \overline{D}D^*\right) \quad (5)$$

are the eigenstates corresponding to $C=-1$ and $C=+1$, respectively. This does not depend on the quantum numbers of the system because D and D^* are not a particle-antiparticle pair. Once the $C-$parity of a $D\overline{D}^*$ state is fixed, its isospin is also determined. In particular, for a $D\overline{D}^*$ $S-$wave state, positive $C-$parity requires isospin 0, while negative $C-$parity implies isospin 1.

Table I and Figs. 1 and 2 summarize our results. We have specified the quantum numbers and plotted the Fredholm determinant of the attractive channels. The rest, not shown on the table, are either repulsive or have zero probability to contain a bound state or a resonance.

Let us remark that, of all possible channels, only a few are attractive. Of the systems made of a particle and its corresponding antiparticle, the $J^{PC}(I) = 0^{++}(0)$ channel is always attractive. In general, the coupling to the $\eta_c\eta$ channel reduces the attraction, but there is still enough attraction to expect a resonance close and above the threshold. This channel is much more attractive for the $D^*\overline{D}^*$ system than for $D\overline{D}$, thus, in the latter one could expect a wider resonance. It is easy to explain the reason for such a close-to-bind situation with these quantum numbers. They can be reached from a two-meson system without explicit orbital angular momentum, while through a simple $c\bar{c}$ pair it needs a unit of orbital angular momentum. Similar arguments were used to explain the proliferation of light scalar-isoscalar mesons [6–8]. The most attractive channel in the $D\overline{D}^*$ case is the $J^{PC}(I) = 1^{++}(0)$ and can be explained as before, except the unity of intrinsic spin due to the D^* meson. A simple calculation of the $D\overline{D}^*$ system (Eq. (1)) indicates that the $J^{PC}(I) = 1^{++}(0)$ and $1^{+-}(1)$ are degenerate. It is the coupling to the $J/\Psi\omega$ (Eq. (2)) that breaks the degeneracy to make the $1^{++}(0)$ more attractive. The isospin 1 channel becomes repulsive due to the coupling to the lightest channel that includes a

FIG. 1: Fredholm determinant for the $J^{PC}(I) = 1^{++}(0)$ $D\overline{D}^*$ system. Solid (dashed) line: results with (without) coupling to the $J/\Psi\omega$ channel.



pion. Then, the existence of meson-meson molecules in the isospin one $D\overline{D}^*$ channels can be discarded. Using the coupling to the $J/\Psi\omega$, not present in the calculations at the hadronic level of [15, 16], we obtain a binding energy for the $J^{PC}(I) = 1^{++}(0)$ in the range $0 - 1$ MeV, in good agreement with the experimental measurements of $X(3872)$ (see Fig. 1). This result supports the analysis of the Belle data on $B \to K + J/\Psi\pi^+\pi^-$ and

FIG. 2: Fredholm determinant of the most attractive $J^{PC}(I)$ channels for the $D\overline{D}$ and $D^*\overline{D}^*$ systems.

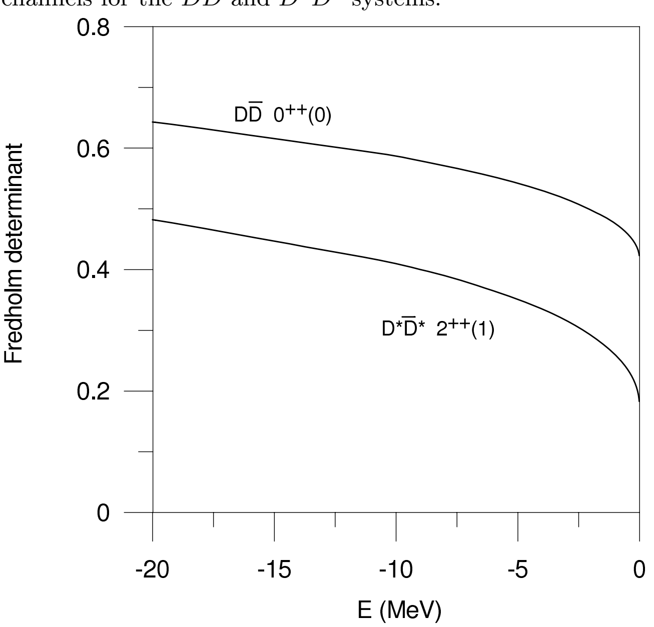


TABLE I: Attractive channels for the two $D-$mesons system.

System	$J^{PC}(I)$
$D\overline{D}$	$0^{++}(0)$
$D\overline{D}^*$	$1^{++}(0)$
$D^*\overline{D}^*$	$0^{++}(0)$
$D^*\overline{D}^*$	$2^{++}(0)$
$D^*\overline{D}^*$	$2^{++}(1)$

$B \to K + D^0\overline{D}^0\pi^0$ that favors the $X(3872)$ being a bound state whose mass is below the $D^0\overline{D}^0$ threshold [17]. The existence of a bound state in the $1^{++}(0)$ $D\overline{D}^*$ channel would not show up in the $D\overline{D}$ system because of quantum number conservation.

Finally, we have found that the $J^{PC}(I) = 2^{++}(0, 1)$ $D^*\overline{D}^*$ (see Fig. 2) are also attractive due to the coupling to the $J/\Psi\omega$ and $J/\Psi\rho$ channels, respectively. This would give rise to new states around 4 GeV/c^2 and one experimental candidate could be the $Y(4008)$. In this case, such a resonance would also appear in the $D\overline{D}$ system for large relative orbital angular momentum, $L = 2$. A similar behavior can be observed in resonances predicted for the $\Delta\Delta$ system [18].

In all cases, being loosely bound states whose masses are close to the sum of their constituent meson masses, their decay and production properties must be quite different from conventional $q\bar{q}$ mesons. Our calculation does not exclude a possible mixture of standard charmonium states in the channels where we have found attractive molecular systems. This admixture could explain some properties of the $X(3872)$ [19, 20]. We would like to emphasize the similarity of our results to those of Ref. [21] in spite of our different approach. Our treatment is general, dealing simultaneously with the two- and four-body problems and using an interaction containing gluon and quark exchanges instead of the simple two-body one-pion exchange potential of Ref. [21]. Nevertheless, we also concluded that the lighter meson-meson molecules are in the vector-vector and pseudoscalar-vector two-meson channels. Finally, let us remark that our approach could also be applied to the the $c\bar{c}s\bar{s}$ sector.

To summarize, we have performed the first systematic analysis of four-quark hidden-charm states as compact states or meson-meson molecules. For the first time we have performed a consistent study of all quantum numbers within the same model. Our predictions robustly show that no deeply bound states can be expected for this system. Only a few channels can be expected to present observable resonances or slightly bound states. Among them, we have found that the $D\overline{D}^*$ system must show a bound state slightly below the threshold for charmed mesons production with quantum numbers $J^{PC}(I) = 1^{++}(0)$, that could correspond to the widely discussed $X(3872)$. Of the systems made of a particle and its corresponding antiparticle, $D\overline{D}$ and $D^*\overline{D}^*$, the $J^{PC}(I) = 0^{++}(0)$ is attractive. It would be the only candidate to accommodate a wide resonance for the $D\overline{D}$ system. For the $D^*\overline{D}^*$ the attraction is stronger and structures may be observed close and above the charmed meson production threshold. Also, we have shown that the $J^{PC}(I) = 2^{++}(0,1)$ $D^*\overline{D}^*$ channels are attractive due to the coupling to the $J/\Psi\omega$ and $J/\Psi\rho$ channels. Due to heavy quark symmetry, replacing the charm quarks by bottom quarks decreases the kinetic energy without significantly changing the potential energy. In consequence, four-quark bottomonium mesons must also exist and have larger binding energies. An experimental effort in this direction will confirm or rule out the theoretical expectations. If the scenario presented here turns out to be correct, it will open a new interesting spectroscopic area.

When this work was finished we learned that particular studies of some of the new charmonium states coincide with our theoretical predictions about the more attractive quantum numbers [22].

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