Open-charm meson spectroscopy

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Abstract

We present a theoretical framework that accounts for the new D_J and D_{sJ} mesons measured in the open-charm sector. These resonances are properly described if considered as a mixture of conventional $P-$wave quark-antiquark states and four-quark components. The narrowest states are basically $P-$wave quark-antiquark mesons, while the dominantly four-quark states are shifted above the corresponding two-meson threshold, being broad resonances. We study the electromagnetic decay widths as basic tools to scrutiny their nature. The proposed explanation incorporates in a natural way the most recently discovered mesons in charmonium spectroscopy.

arXiv:hep-ph/0601143v1 18 Jan 2006

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I. INTRODUCTION

During the last few years, heavy meson spectroscopy is living a continuous excitation due to the discovery of several new charmed mesons. Two years ago BABAR Collaboration at the Standford Linear Accelerator Center (SLAC) reported the observation of a charm-strange state, the $D^*_{sJ}(2317)$ [1]. It was confirmed by CLEO Collaboration at the Cornell Electron Storage Ring [2] and also by Belle Collaboration at KEK [3]. Besides, BABAR had also pointed out to the existence of another charm-strange meson, the $D_{sJ}(2460)$ [1]. This resonance was measured by CLEO [2] and confirmed by Belle [3]. Belle results [3] are consistent with the spin-parity assignments of $J^P = 0^+$ for the $D^*_{sJ}(2317)$ and $J^P = 1^+$ for the $D_{sJ}(2460)$. Thus, these two states are definitively well established, confirmed independently by different experiments. They present unexpected properties, quite different from those predicted by quark potential models. If they would correspond to standard $P-$wave mesons made of a charm quark, c, and a strange antiquark, $\overline{s}$, their masses would be larger [4], around 2.48 GeV for the $D^*_{sJ}(2317)$ and 2.55 GeV for the $D_{sJ}(2460)$. They would be therefore above the DK and D^*K thresholds, respectively, being broad resonances. However the states observed by BABAR and CLEO are very narrow, $\Gamma < 4.6$ MeV for the $D^*_{sJ}(2317)$ and $\Gamma < 5.5$ MeV for the $D_{sJ}(2460)$.

The intriguing situation of the charm-strange mesons has been translated to the non-strange sector with the Belle observation [5] of a nonstrange broad scalar resonance, D^*_0, with a mass of $2308 \pm 17 \pm 15 \pm 28$ MeV/c^2 and a width $\Gamma = 276 \pm 21 \pm 18 \pm 60$ MeV. A state with similar properties has been suggested by FOCUS Collaboration at Fermilab [6] during the measurement of masses and widths of excited charm mesons D^*_2. They found significant evidence for a broad excess parametrized as an $S-$wave resonance. Without being able to clearly distinguish the origin of this broad excess, they conclude that their results are in agreement with the Belle results [5]. This state generates for the open-charm nonstrange mesons a very similar problem to the one arising in the strange sector with the $D^*_{sJ}(2317)$. If the $D^*_0(2308)$ would correspond to a standard $P-$wave meson made of a charm quark, c, and a light antiquark, $\overline{n}$, its mass would have to be larger, around 2.46 GeV. In this case, the quark potential models prediction and the measured resonance are both above the $D\pi$ threshold, the large width observed being expected although not its low mass.

The last step in this series of discoveries has been the observation of a new charm-strange meson, D_{sJ}, announced by SELEX Collaboration at Fermilab [7] with a mass of 2632.5 ± 1.7 MeV/c^2 and a small width, $\Gamma < 17$ MeV. However, up to now no other experiment has been able to confirm the existence of this resonance [8].

There have been many theoretical interpretations for the masses and widths of the new resonances, but most part of them have been devoted to explain the strange states. Ref. [9] made use of a unitarized meson model, the existence of a quasi-bound $c\overline{s}$ state due to the coupling with the nearby $S-$wave DK threshold. The coupling to the DK channel within a QCD string model an a chiral Lagrangian was used in Ref. [10]. Ref. [11] proposed a $c\overline{s}$ structure with modified non-central forces. Ref. [12] combined HQET with a chiral effective Lagrangian to interpret these states as the missing $j = 1/2$ member of the $c\overline{s}$ $L = 1$ ground state multiplet, being j the angular momentum of the strange quark. The smaller mass of the $D^*_{sJ}(2317)$ is attributed in Ref. [13] to a coupled channel effect. Finally more involved qualitative solutions like $DK-$molecules [14], $D_s\pi-$atoms [15], four-quark states [16, 17] and their combination with $q\overline{q}$ states [18] have been invoked to explain why the new states have a nature so different from canonical $q\overline{q}$ mesons. Although the nonstrange

partners of the D_{sJ}'s have received much less attention, as discussed above, they present similar spectroscopic properties that should be acknowledge altogether with those of the strange states in any reliable model.

The difficulties to identify the D_J and D_{sJ} states with conventional $c\bar{q}$ mesons are rather similar to those appearing in the light-scalar meson sector [19] and may be indicating that other configurations are playing a role, as could be for example four-quark contributions. $q\bar{q}$ states are more easily identified with physical hadrons when virtual quark loops are not important. This is the case of the pseudoscalar and vector mesons, mainly due to the $P-$wave nature of this hadronic dressing. On the contrary, in the scalar sector it is the $q\bar{q}$ pair the one in a $P-$wave state, whereas quark loops may be in a $S-$wave. In this case the intermediate hadronic states that are created may play a crucial role in the composition of the resonance, in other words unquenching may be important. The vicinity of these components to the lightest $q\bar{q}$ state implies that they have to be considered. This has been shown as a possible interpretation of the low-lying light-scalar mesons, where the coupling of the scalar $q\bar{q}$ nonet to the lightest $qq\bar{q}\bar{q}$ configurations allows for an almost one-to-one correspondence between theoretical states and experiment [20].

In this work we pretend to explore the same ideas for the understanding of the properties of the D_J and D_{sJ} meson states. For this purpose, in the next section we will present our calculating scheme and the basic ingredients of the constituent quark model used. Section III will be devoted to present and discuss our results in connection with those obtained for the light-scalar mesons. Finally, in Sec. IV we will resume the most important conclusions of our work.

II.   CALCULATING FRAMEWORK

In non-relativistic quark models gluon degrees of freedom are frozen and therefore the wave function of a zero baryon number (B=0) hadron may be written as

$$|\text{B} = 0\rangle = \Omega_1 |q\bar{q}\rangle + \Omega_2 |qq\bar{q}\bar{q}\rangle + ... \tag{1}$$

where q stands for quark degrees of freedom and the coefficients Ω_i take into account the mixing of four-quark and $q\bar{q}$ states. $|\text{B} = 0\rangle$ systems could then be described in terms of a hamiltonian

$$H = H_0 + H_1 \quad \text{being} \quad H_0 = \begin{pmatrix} H_{q\bar{q}} & 0 \\ 0 & H_{qq\bar{q}\bar{q}} \end{pmatrix} \quad H_1 = \begin{pmatrix} 0 & V_{q\bar{q}\leftrightarrow qq\bar{q}\bar{q}} \\ V_{q\bar{q}\leftrightarrow qq\bar{q}\bar{q}} & 0 \end{pmatrix}, \tag{2}$$

where H_0 is a constituent quark model hamiltonian described below and H_1, that takes into account the mixing between $q\bar{q}$ and $qq\bar{q}\bar{q}$ configurations, includes the annihilation operator of a quark-antiquark pair into the vacuum. This operator could be described using the $^3P_0$ model, however, since this model depends on the vertex parameter, we prefer in a first approximation to parametrize this coefficient by looking to the quark pair that is annihilated and not to the spectator quarks that will form the final $q\bar{q}$ state. Therefore we have taken $V_{q\bar{q}\leftrightarrow qq\bar{q}\bar{q}} = \gamma$. If this coupling is weak enough one can solve independently the eigenproblem for the hamiltonians $H_{q\bar{q}}$ and $H_{qq\bar{q}\bar{q}}$, treating H_1 perturbatively. To ensure that the perturbative treatment is justified, γ cannot take all possible values, being restricted to $|\gamma/(E^n_{JPC} - E^{n+1}_{JPC})|^2 \leq 1$. This restriction will limit the energy range of the mixed states once the unmixed energies are calculated. The two-body problem has been solved exactly by

means of the Numerov algorithm [21]. The four-body problem has been solved by means of a variational method using the most general combination of gaussians as trial wave functions [22, 23]. In particular, the so-called *mixed terms* (mixing the various Jacobi coordinates) that are known to have a great influence in the light quark case have been considered.

Although the constituent quark model used is described in Ref. [21], let us outline here its basic ingredients. Since the origin of the quark model hadrons have been considered to be built by constituent (massive) quarks. Nowadays it is widely recognized that the constituent quark mass of light quarks appears because of the spontaneous breaking of the original chiral symmetry of the QCD Lagrangian, what gives rise to boson-exchange interactions between quarks. The different terms of the potential contain central and tensor or central and spin-orbit contributions that will be grouped for consistency. Therefore, the chiral part of the quark-quark interaction can be resumed as follows,

$$V_\chi(\vec{r}_{ij}) = V_\chi^C(\vec{r}_{ij}) + V_\chi^T(\vec{r}_{ij}) + V_\chi^{SO}(\vec{r}_{ij}), \tag{3}$$

where C stands for central, T for tensor, and SO for spin-orbit potentials. The central part of the quark-quark meson-exchange potentials are given by:

$$V_\chi^C(\vec{r}_{ij}) = V_\pi^C(\vec{r}_{ij}) + V_\sigma^C(\vec{r}_{ij}) + V_K^C(\vec{r}_{ij}) + V_\eta^C(\vec{r}_{ij}), \tag{4}$$

each contribution given by,

$$V_\pi^C(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\pi^2}{12 m_i m_j} \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi \left[Y(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3} Y(\Lambda_\pi r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j) \sum_{a=1}^3 (\lambda_i^a \cdot \lambda_j^a),$$

$$V_\sigma^C(\vec{r}_{ij}) = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right], \tag{5}$$

$$V_K^C(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_K^2}{12 m_i m_j} \frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K \left[Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3} Y(\Lambda_K r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j) \sum_{a=4}^7 (\lambda_i^a \cdot \lambda_j^a),$$

$$V_\eta^C(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\eta^2}{12 m_i m_j} \frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} m_\eta \left[Y(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3} Y(\Lambda_\eta r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j) \left[cos\theta_P (\lambda_i^8 \cdot \lambda_j^8) - sin\theta_P \right],$$

the angle θ_P appears as a consequence of considering the physical η instead the octet one. $g_{ch} = m_q/f_\pi$, the $\lambda's$ are the $SU(3)$ flavor Gell-Mann matrices and the σ's are the spin quark Pauli matrices. m_i is the quark mass and m_π, m_K and m_η are the masses of the $SU(3)$ Goldstone bosons, taken to be their experimental values. The Λ_i's are cutoff parameters. m_σ is determined through the PCAC relation $m_\sigma^2 \sim m_\pi^2 + 4 m_{u,d}^2$. Finally, $Y(x)$ is the standard Yukawa function defined by $Y(x) = e^{-x}/x$.

There are three different contributions to the tensor potential,

$$V_{qq}^T(\vec{r}_{ij}) = V_\pi^T(\vec{r}_{ij}) + V_K^T(\vec{r}_{ij}) + V_\eta^T(\vec{r}_{ij}), \tag{6}$$

each term given by,

$$V_\pi^T(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\pi^2}{12 m_i m_j} \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi \left[H(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3} H(\Lambda_\pi r_{ij}) \right] S_{ij} \sum_{a=1}^3 (\lambda_i^a \cdot \lambda_j^a),$$

$$V_K^T(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_K^2}{12 m_i m_j} \frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K \left[H(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3} H(\Lambda_K r_{ij}) \right] S_{ij} \sum_{a=4}^7 (\lambda_i^a \cdot \lambda_j^a), \tag{7}$$

$$V_\eta^T(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\eta^2}{12 m_i m_j} \frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} m_\eta \left[H(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3} H(\Lambda_\eta r_{ij}) \right] S_{ij} \left[cos\theta_P (\lambda_i^8 \cdot \lambda_j^8) - sin\theta_P \right],$$

being $S_{ij} = 3\,(\vec{\sigma}_i \cdot \hat{r}_{ij})(\vec{\sigma}_j \cdot \hat{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j$ the quark tensor operator and $H(x) = (1 + 3/x + 3/x^2)\,Y(x)$.

Finally, the spin-orbit potential only presents a contribution coming form the scalar part of the interaction,

$$V_{qq}^{SO}(\vec{r}_{ij}) = V_{\sigma}^{SO}(\vec{r}_{ij}) = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{\sigma}^2}{\Lambda_{\sigma}^2 - m_{\sigma}^2} \frac{m_{\sigma}^3}{2m_i m_j} \left[G(m_{\sigma}\,r_{ij}) - \frac{\Lambda_{\sigma}^3}{m_{\sigma}^3} G(\Lambda_{\sigma}\,r_{ij}) \right] \vec{L}\cdot\vec{S} \qquad (8)$$

where $G(x) = (1 + 1/x)\,Y(x)/x$.

QCD perturbative effects are taken into account through the one-gluon-exchange (OGE) potential [24]. The nonrelativistic reduction of the one-gluon-exchange diagram in QCD for point-like quarks presents a contact term that, when not treated perturbatively, leads to collapse [25]. This is why one maintains the structure of the OGE, but the δ function is regularized in a suitable way. This regularization, justified by the finite size of the systems studied, has to be flavor dependent [26]. As a consequence, the central part of the OGE reads,

$$V_{OGE}^C(\vec{r}_{ij}) = \frac{1}{4}\alpha_s \, \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \left\{ \frac{1}{r_{ij}} - \frac{1}{6m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}\, r_0^2(\mu)} \right\}, \qquad (9)$$

where λ^c are the $SU(3)$ color matrices, α_s is the quark-gluon coupling constant, and $r_0(\mu) = \hat{r}_0 \mu_{nn}/\mu_{ij}$, where μ_{ij} is the reduced mass of quarks ij (n stands for the light u and d quarks) and $\hat{r}_0$ is a parameter to be determined from the data.

The noncentral terms of the OGE present a similar problem. For point-like quarks they contain an $1/r^3$ term. Once again the finite size of the constituent quarks allows for a regularization, obtaining tensor and spin-orbit potentials of the form,

$$
\begin{aligned}
V_{OGE}^T(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_i m_j} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}} \left(\frac{1}{r_{ij}^2} + \frac{1}{3r_g^2(\mu)} + \frac{1}{r_{ij}\, r_g(\mu)} \right) \right] S_{ij}, \\
V_{OGE}^{SO}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_i^2 m_j^2} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3} \left(1 + \frac{r_{ij}}{r_g(\mu)} \right) \right] \times \qquad (10) \\
&\quad \left[\left((m_i + m_j)^2 + 2m_i m_j \right) (\vec{S}_+ \cdot \vec{L}) + \left(m_j^2 - m_i^2 \right) (\vec{S}_- \cdot \vec{L}) \right],
\end{aligned}
$$

where $\vec{S}_\pm = \vec{S}_i \pm \vec{S}_j$, and $r_g(\mu) = \hat{r}_g/\mu$ presents a similar behavior to the scaling of the central term.

The strong coupling constant, taken to be constant for each flavor sector, has to be scale-dependent when describing different flavor sectors [27]. Such an effective scale dependence has been related to the typical momentum scale of each flavor sector assimilated to the reduced mass of the system [28]. This has been found to be relevant for the study of the meson spectra and parametrized in Ref. [21] as

$$\alpha_s(\mu) = \frac{\alpha_0}{ln\left[(\mu^2 + \mu_0^2)/\gamma_0^2 \right]}, \qquad (11)$$

where μ is the reduced mass of the interacting qq pair and α_0, μ_0, and γ_0 are fitted parameters.

Finally, any model imitating QCD should incorporate confinement. Lattice calculations in the quenched approximation derived, for heavy quarks, a confining interaction linearly dependent on the interquark distance. The consideration of sea quarks apart from valence

quarks (unquenched approximation) suggests a screening effect on the potential when increasing the interquark distance [29]. Creation of light-quark pairs out of vacuum in between the quarks becomes energetically preferable resulting in a complete screening of quark color charges at large distances. String breaking has been definitively confirmed through lattice calculations [30] in coincidence with the quite rapid crossover from a linear rising to a flat potential well established in $SU(2)$ Yang-Mills theories [31]. The central part of a screened potential simulating these results can be written as,

$$V_{CON}^C(\vec{r}_{ij}) = -a_c\,(1 - e^{-\mu_c\,r_{ij}})(\vec{\lambda^c}_i \cdot \vec{\lambda^c}_j)\;. \tag{12}$$

At short distances it presents a linear behavior with an effective confinement strength $a = a_c\,\mu_c\,\vec{\lambda^c}_i \cdot \vec{\lambda^c}_j$, while it becomes constant at large distances. Screened confining potentials have been analyzed in the literature providing an explanation to the missing state problem in the baryon spectra [32], improving the description of the heavy-meson spectra [33], and justifying the deviation of the meson Regge trajectories from the linear behavior for higher angular momentum states [34].

Confinement also presents an spin-orbit contribution taken to be an arbitrary combination of scalar and vector terms of the form

$$\begin{aligned}
V_{CON}^{SO}(\vec{r}_{ij}) = & -(\vec{\lambda}^c_i \cdot \vec{\lambda}^c_j)\frac{a_c\,\mu_c\,e^{-\mu_c\,r_{ij}}}{4m_i^2m_j^2r_{ij}}\Big[\big((m_i^2 + m_j^2)(1 - 2\,a_s) \\
& + 4m_im_j(1 - a_s)\big)\,(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(1 - 2\,a_s)(\vec{S}_- \cdot \vec{L})\Big]
\end{aligned} \tag{13}$$

where a_s would control the ratio between them.

Once perturbative (one-gluon exchange) and nonperturbative (confinement and chiral symmetry breaking) aspects of QCD have been considered, one ends up with a quark-quark interaction of the form

$$V_{q_iq_j} = \begin{cases} q_iq_j = nn/sn \Rightarrow V_{CON} + V_{OGE} + V_\chi \\ q_iq_j = cn/cs \Rightarrow V_{CON} + V_{OGE} \end{cases}. \tag{14}$$

Note that for the particular case of heavy quarks, chiral symmetry is explicitly broken and therefore Goldstone boson exchanges do not contribute. The model parameters and a more detailed discussion of the model can be found in Refs. [20, 21, 23].

III.   RESULTS AND DISCUSSION

A thoroughly study of the full meson spectra has been presented in Ref. [21]. The results for the open-charm mesons are resumed in Table I. It can be seen how the open-charm states are easily identified with standard $c\bar{q}$ mesons except for the cases of the $D_{sJ}^*(2317)$, the $D_{sJ}(2460)$, and the $D_0^*(2308)$. We will also comment on the recent measurement of the $D_1^0(2430)$. This behavior is shared by almost all quark potential model calculations [4]. Although the situation from lattice QCD is far from being definitively established, similar problems are observed. Lattice NRQCD in the quenched approximation predicts for the $D_{sJ}^*(2317)$ a mass of 2.44 GeV [36], while using relativistic charm quarks the mass obtained is 2.47 GeV [37]. Unquenched lattice QCD calculations of $c\bar{s}$ states do not find a window for the $D_{sJ}^*(2317)$ [19], supporting the difficulty of a $P-$wave $c\bar{s}$ interpretation. The quenched lattice QCD calculation of the spectrum of orbitally excited D_s mesons of Ref. [38] concludes

that, although the results obtained are consistent with a $c\bar{s}$ configuration, the statistical and systematical uncertainties are too large to exclude the exotic states based on potential quark models [4]. The same situation may be drawn from heavy quark symmetry arguments. One finds that the scalar $c\bar{s}$ state belongs to the $j = 1/2$ doublet, but since the $j = 3/2$ doublet is identified with the narrow $D_{s2}(2573)$ and $D_{s1}(2536)$ (with total widths of 15^{+5}_{-4} MeV and < 2.3 MeV, respectively) the scalar state is expected to have a much larger width than the one measured for the $D^*_{sJ}(2317)$ [39].

Thus, one could be tempted to interpret these states as four-quark resonances within the quark model. The results obtained for the $cn\bar{s}\bar{n}$ and $cn\bar{n}\bar{n}$ configurations with the same interacting potential of Sec. II are shown in Table II. The $I = 1$ and $I = 0$ states are far above the corresponding strong decay threshold and therefore should be broad, what rules out a pure four-quark interpretation of the new open-charm mesons.

As outlined above, for $P-$wave mesons the hadronic dressing is in a $S-$wave, thus physical states may correspond to a mixing of two- and four-body configurations, Eq. (1). In the isoscalar sector, the $cn\bar{s}\bar{n}$ and $c\bar{s}$ states get mixed, as it happens with $cn\bar{n}\bar{n}$ and $c\bar{n}$ for the $I = 1/2$ case. Let us notice that the interacting potential contains terms mixing $P-$wave $S = 0$ and $S = 1$ states [21], arising from the different spin-orbit contributions for systems made of quarks of different mass. The mixing angle comes determined by the structure of the interaction. The parameter γ has been fixed to reproduce the mass of the $D^*_{sJ}(2317)$ meson, being $\gamma = 240$ MeV. Using this value one has $|\gamma/(E^n_{JPC} - E^{n+1}_{JPC})|^2 \approx 0.25 \ll 1$, a ratio consistent with the assumption that the hamiltonian H_1, Eq. (2), can be treated perturbatively. This perturbative condition together with $|\gamma/E_{q\bar{q}}| \approx |\gamma/E_{qq\bar{q}\bar{q}}| \approx 0.1 \ll 1$ also ensures that the mixing with higher states can be neglected.

The results obtained are shown in Table III. Let us first analyze the nonstrange sector. The $^3P_0$ $c\bar{n}$ pair and the $cn\bar{n}\bar{n}$ have a mass of 2465 MeV and 2505 MeV, respectively. Once the mixing is considered one obtains a state at 2241 MeV with 46% of four-quark component and 53% of $c\bar{n}$ pair. The lowest state, representing the $D^*_0(2308)$, is above the isospin preserving threshold $D\pi$, being broad as observed experimentally. The mixed configuration compares much better with the experimental data than the pure $c\bar{n}$ state. The orthogonal state appears higher in energy, at 2713 MeV, with and important four-quark component. A similar process would modify the state representing the $D^0_1(2430)$, but in this case one would need the mass of the $I = 1/2$ $J^P = 1^+$ four-quark state. The huge basis generated for such quantum numbers makes unfeasible its calculation. A correct description of this state would require a mass of 2.9 GeV for the four quark state mentioned above.

Concerning the strange sector, the $D^*_{sJ}(2317)$ and the $D_{sJ}(2460)$ are dominantly $c\bar{s}$ $J = 0^+$ and $J = 1^+$ states, respectively, with almost 30% of four-quark component. Without being dominant, it is fundamental to shift the mass of the unmixed states to the experimental values below the DK and D^*K thresholds. Being both states below their isospin-preserving two-meson threshold, the only allowed strong decays to $D^*_s\pi$ would violate isospin and are expected to have small widths. This width has been estimated assuming either a $q\bar{q}$ structure [12, 40], a four-quark state [41] or vector meson dominance [42] obtaining in all cases a width of the order of 10 keV. The second isoscalar $J^P = 1^+$ state, with an energy of 2555 MeV and 98% of $c\bar{s}$ component, corresponds to the $D_{s1}(2536)$. Regarding the $D^*_{sJ}(2317)$, it has been argued that a possible DK molecule would be preferred with respect to an $I = 0$ $cn\bar{s}\bar{n}$ tetraquark, what would anticipate an $I = 1$ $cn\bar{s}\bar{n}$ partner nearby in mass [14]. Our results confirm the last argument, the vicinity of the isoscalar and isovector tetraquarks (see Table II), however, the restricted coupling to the $c\bar{s}$ system allowed only for the $I = 0$ four-quark

states opens the possibility of a mixed nature for the $D_{sJ}^*(2317)$, the $I = 1$ tetraquark partner remaining much higher in energy. The $I = 1$ $J = 0^+$ and $J = 1^+$ four-quark states appear above 2700 MeV and cannot be shifted to lower energies.

Our results do not show any state around 2600 MeV, the mass region compatible with the $D_{sJ}^+(2632)$ measured by SELEX. Its two possible theoretical partners, the 2^3S_1 state (suggested in Ref. [9]) or the 1^3D_1 state, lie above 2700 MeV (2764 and 2873 MeV, respectively). This state has been proposed as the $cs\bar{s}\bar{s}$ partner of the $D_{sJ}^*(2317)$ [17]. Naively, the $D_{sJ}^*(2317)$ sets a scale of 2320 MeV for the $cn\bar{s}\bar{n}$ sector, and an augment of 150 MeV for each strange quark could accommodate a $cs\bar{s}\bar{s}$ system near the mass of the $D_{sJ}^+(2632)$. However, the mass of the $D_{sJ}^*(2317)$ is obtained through the coupling to the $c\bar{s}$ quark pair, being the mass of the tetraquark configuration above 2700 MeV, disregarding a possible $cs\bar{s}\bar{s}$ interpretation of the $D_{sJ}^+(2632)$. It has also been discarded as the first radial excitation of the $D_s^*(2112)$ [43]. A careful analysis of several theoretical interpretations has been done in Ref. [44], the surprising properties of this state not fitting in any of the scenarios considered. Several experiments have failed trying to confirm the existence of this state [8]. The confirmation or refutation of the $D_{sJ}^+(2632)$ is clearly an important priority for meson spectroscopy.

The structure of the $D_{sJ}^*(2317)$ and the $D_{sJ}(2460)$ mesons could be scrutiny, apart from their masses, also through the study of their electromagnetic decay widths. Using the standard formalism described, for example, in Ref. [40], the $E1$ radiative transitions for $c\bar{s} \to c\bar{s} + \gamma$ are given by:

$$\Gamma[c\bar{s} \to c\bar{s} + \gamma] = \frac{4}{27}\alpha \langle e_Q \rangle^2 \omega^3 (2J_f + 1)\left|\left\langle {}^{2S+1}S_{J'} \left| r \right| {}^{2S+1}P_J \right\rangle\right|^2 S_{if} \tag{15}$$

where S_{if} is a statistical factor with $S_{if} = 1$ for the transitions between spin-triplet states and $S_{if} = 3$ for the transition between spin-singlet states, $\langle e_Q \rangle$ is an effective quark charge given by

$$\langle e_Q \rangle = \frac{m_s e_c - m_c e_{\bar{s}}}{m_c + m_s} \tag{16}$$

where e_i is the charge of the quark(antiquark) i in units of $|e|$, α is the fine structure constant and ω is the photon energy. Once the mixing between two- and four-quark components has been included there would also be a contribution to the decay width coming from $cn\bar{s}\bar{n} \to c\bar{s} + \gamma$, that goes necessarily through the annihilation of a color singlet $n\bar{n}$ pair with photon quantum numbers within the four-quark wave function. The contribution to the electromagnetic decay width arising from the four-quark component gets suppressed. This can be illustrated by the analysis of the $M1$ electromagnetic decay width for $cn\bar{s}\bar{n}(1^+) \to cn\bar{s}\bar{n}(0^+) + \gamma$, that is given by

$$\Gamma[cn\bar{s}\bar{n}(1^+) \to cn\bar{s}\bar{n}(0^+) + \gamma] = \frac{4}{3}\alpha\omega^3(2J_f + 1)\left|\left\langle {}^1S_0 \left| \mu_z \right| {}^3S_1 \right\rangle\right|^2 \tag{17}$$

where

$$\mu_z = \sum_{i=1}^{4} \frac{e_i}{2m_i}\sigma_{iz} . \tag{18}$$

From the above equation one obtains $\Gamma[cn\bar{s}\bar{n}(1^+) \to cn\bar{s}\bar{n}(0^+) + \gamma] \approx 4$ eV, that will contribute to the $\Gamma[D_{sJ}(2460) \to D_{sJ}^*(2317) + \gamma]$ decay width, once the configuration probabilities given in Table III are considered, in the order of 0.25 eV. The value obtained in Ref.

[40] for this decay, assuming a pure $c\bar{s}$ configuration, is one order of magnitude larger confirming the smallness of the four to four-quark contribution. In Ref. [45] this decay width was estimated assuming a molecular structure, obtaining a larger value of the order of 17 keV. This makes evident the important difference between the assumed molecular structure of Ref. [45] and the full multiquark configuration considered in this work. In the case of the four-quark configuration the orthogonality between the several components of the wave functions diminishes the decay widths, an effect that does not seem to be present within the molecular interpretation. The same reasoning applies to the $\Gamma[cn\bar{s}\bar{n} \to c\bar{s} + \gamma]$ decay width, being suppressed due to the reduced singlet-singlet component with photon quantum numbers within the four-quark wave function. Similar conclusions were obtained in Ref. [18]. As a consequence, the presence of a four-quark component diminishes the decay widths, making them different from those predicted for a pure $c\bar{s}$ state [14, 40, 42].

We compare in Table IV our results for the radiative transitions of the $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ with different theoretical approaches and the experimental limits reported by CLEO and Belle. The main difference is noticed in the suppression predicted for the $D_{sJ}(2460) \to D^{*+}_s\gamma$ decay as compared to the $D_{sJ}(2460) \to D^+_s\gamma$. A ratio $D_{sJ}(2460) \to D^+_s\gamma / D_{sJ}(2460) \to D^{*+}_s\gamma \approx 1 - 2$ has been obtained assuming a $q\bar{q}$ structure for both states [12, 40] (what seems incompatible with their properties). Heavy-hadron chiral perturbation theory calculations find a similar ratio [46]. We find a larger value, $D_{sJ}(2460) \to D^+_s\gamma / D_{sJ}(2460) \to D^{*+}_s\gamma \approx 100$, due to the small 1^3P_1 $c\bar{s}$ probability of the $D_{sJ}(2460)$. A similar enhancement has been obtained in Ref. [47] (see penultimate column of Table IV) in the framework of light-cone QCD sum rules in contrast to a previous calculation of the same authors using vector meson dominance [42]. As a consequence, the radiative transitions are an important diagnostic tool to understand the nature of these states. In view of the different predictions of the electromagnetic decay widths, a precise measurement of this decay would allow to distinguish not only between $q\bar{q}$ and non$-q\bar{q}$ states, but also between pure molecular and four-quark interpretations.

Let us finally mention that the difficulties encountered for the interpretation of the new open-charm states as two-quark systems do not appear for the case of the recent charmed and B_c states measured at different facilities. They do nicely fit into the predictions of the model used, see Table V, giving confidence to the results obtained in the present work.

The interpretation we have just presented of the positive parity open-charmed mesons as a mixture of two- and four-quark states has also been used to account for the experimentally observed light-scalar mesons within the same constituent quark model [20], what gives us confidence on the mechanism proposed. Nonetheless, one should not forget that in the literature there is a wide variety of interpretations for the open-charm and also for the light-scalar mesons. As previously mentioned, they used to deal with one of both problems and even then with a particular set of states, being the most common ones the strange open-charm states and the isoscalar light mesons.

Regarding the light-scalar mesons, there have been reported by the PDG two isovectors, five isoscalar and three isodoublet states. The situation is far from being definitively settled, neither from the experimental nor from the theoretical point of view. Experimentally, let us only mention that the recent analysis by BES of the $J/\Psi \to \phi\pi^+\pi^-$ and $J/\Psi \to \phi K^+K^-$ data [53] requires a state $f_0(1790)$ distinct from the $f_0(1710)$. Recent reanalysis of the Crystal Barrel data [54] suggest the existence of a new state called $f_0(1200 - 1600)$. Theoretically, one would expect non$-q\bar{q}$ scalar objects in the mass range below 2 GeV. Multiquarks have been justified to coexist with $q\bar{q}$ states in the energy region around 1 GeV because they can

couple to 0^{++} without orbital excitation [55]. Lattice QCD in the quenched approximation predicts the existence of a scalar glueball with a mass around 1.6 GeV [56].

From this complicated scenario many different interpretations of the light-scalar mesons have arisen. The overpopulation of 0^{++} states below 2 GeV gave rise long ago to the speculation of the existence of four light-quark states. The most complete analysis was performed by Weinstein and Isgur [26, 57], concluding that, normally, $qq\bar{q}\bar{q}$ bound states do not exist, being the only exception the scalar sector where weakly bound states with a meson-meson molecule structure were found. Particular sets of states have been studied in the literature. There are several models analyzing the mixing between different configurations to yield the physical $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. Among them, Ref. [20] assigning the larger glueball component to the $f_0(1710)$ is on the line with Refs. [58] and differ from those of Refs. [59, 60] concluding that the $f_0(1710)$ is dominantly $q\bar{q}$. One should notice that in these studies only Refs. [20, 60] consider the recently reported $f_0(1790)$. In a different fashion within the quark model, the $a_0(980)$ and $f_0(980)$ mesons were analyzed in Ref. [61], being the effect of the two-pseudoscalar meson thresholds the responsible for the substantial shift to a lower mass than what is naively expected from the $q\bar{q}$ component alone. This gives rise to an important $K\overline{K}$ and $\pi\eta'$ components in the $a_0(980)$ and $K\overline{K}$, $\eta\eta$, $\eta'\eta'$ and $\eta\eta'$ in the $f_0(980)$.

The structure of the scalar mesons $a_0(980)$ and $f_0(980)$ have been also investigated in the framework of a meson exchange based on the Jülich potential model [62] for $\pi\pi$ and $\pi\eta$ scattering. Whereas the $f_0(980)$ appears to be a $K\overline{K}$ bound state the $a_0(980)$ was found to be a dynamically generated threshold effect. Similar conclusions have been obtained in a chiral unitary coupled channel approach, where the $f_0(600)$, the $a_0(980)$, and the $K_0^*(800)$ rise up as dynamically generated resonances, while the $f_0(980)$ is a combination of a strong $S-$wave meson-meson unitarity effect and a preexisting singlet resonance [63]. In Ref. [64] van Beveren *et al.* describe the light scalar mesons as resonances and bound states characterized by complex singularities of the scattering amplitude.

Finally, let us stress that Ref. [20] presents an interpretation of the scalar mesons in a model constrained by the description of other hadron sectors. The same mechanism has been applied here to disentangle the structure of the new D_s and D_{sJ} resonances. It drives to a final scenario that it is compatible with some other models in the literature and it differs from other results. Being the set of data so huge, and sometimes so poor, one always may find a positive or negative interpretation of some of them. Therefore, the final answer could only be obtained from precise experimental data that would allow to discriminate between the predictions of different theoretical models [65].

IV. SUMMARY

As a summary, we have obtained a rather satisfactory description of the positive parity open-charm mesons in terms of two- and four-quark configurations. The mixing between these two components is responsible for the unexpected low mass and widths of the $D_{sJ}^*(2317)$, $D_{sJ}(2460)$, and $D_0^*(2308)$. The same mechanism has been used to account for the spectroscopic properties of the light-scalar mesons. The obtained electromagnetic decay widths give hints that would help in distinguishing the nature of these states. We predict a ratio $D_{sJ}(2460) \to D_s^+\gamma/D_{sJ}(2460) \to D_s^{*+}\gamma$ much larger than the one obtained in a pure $q\bar{q}$ scheme. We did not find any theoretical partner for the recently measured $D_{sJ}^+(2632)$ whose existence awaits confirmation [8]. We encourage experimentalists on two different

directions: the measurement of the electromagnetic decay widths of the $D_{sJ}^*(2317)$ and the $D_{sJ}(2460)$, and the confirmation or refutation of the $D_{sJ}^+(2632)$ that would help to clarify the exciting situation of the open-charm mesons.

V.   ACKNOWLEDGMENTS

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TABLE I: $c\bar{s}$ and $c\bar{n}$ masses (QM), in MeV. Experimental data (Exp.) are taken from Ref. [35], except for the state denoted by a dagger that has been taken from Ref. [5].

$nL\ J^P$	State	QM ($c\bar{s}$)	Exp.	State	QM ($c\bar{n}$)	Exp.
$1S\ 0^-$	D_s	1981	1968.5±0.6	D	1883	1867.7±0.5
$1S\ 1^-$	D_s^*	2112	2112.4±0.7	D^*	2010	2008.9±0.5
$1P\ 0^+$	$D_{sJ}^*(2317)$	2489	2317.4±0.9	$D_0^*(2308)$	2465	2308±17±15±28†
$1P\ 1^+$	$D_{sJ}(2460)$	2578	2459.3±1.3	$D_1(2420)$	2450	2422.2±1.8
$1P\ 1^+$	$D_{s1}(2536)$	2543	2535.3±0.6	$D_1^0(2430)$	2546	2427 ± 26 ± 25
$1P\ 2^+$	$D_{s2}(2573)$	2582	2572.4±1.5	$D_2^*(2460)$	2496	2459±4

TABLE II: $cn\bar{s}\bar{n}$ and $cn\bar{n}\bar{n}$ masses, in MeV.

$cn\bar{s}\bar{n}$				$cn\bar{n}\bar{n}$
$J^P = 0^+$		$J^P = 1^+$		$J^P = 0^+$
$I = 0$	$I = 1$	$I = 0$	$I = 1$	$I = 1/2$
2731	2699	2841	2793	2505

TABLE III: Probabilities (P), in %, of the wave function components and masses (QM), in MeV, of the open-charm mesons once the mixing between $q\bar{q}$ and $qq\bar{q}\bar{q}$ configurations is considered. Experimental data (Exp.) are taken from Ref. [35] except for the state denoted by a dagger that has been taken from Ref. [5].

$I = 0$						$I = 1/2$		
$J^P = 0^+$			$J^P = 1^+$			$J^P = 0^+$		
QM	2339	2847	QM	2421	2555	QM	2241	2713
Exp.	2317.4±0.9	–	Exp.	2459.3±1.3	2535.3±0.6	Exp.	2308±17±15±28†	–
P($cn\bar{s}\bar{n}$)	28	55	P($cn\bar{s}\bar{n}$)	25	~ 1	P($cn\bar{n}\bar{n}$)	46	49
P($c\bar{s}_{1^3P}$)	71	25	P($c\bar{s}_{1^1P}$)	74	~ 1	P($c\bar{n}_{1P}$)	53	46
P($c\bar{s}_{2^3P}$)	~ 1	20	P($c\bar{s}_{1^3P}$)	~ 1	98	P($c\bar{n}_{2P}$)	~ 1	5

TABLE IV: Comparison of 90% C.L. limits on radiative transitions obtained by CLEO [2] and Belle [3] with our results (QM) and those of two different quark models, Refs. [12, 40], based only on $q\bar{q}$ components. The BR's are with respect to the decay $D^*_{sJ}(2317) \to D^+_s \pi^0$ for the $D^*_{sJ}(2317)$ and with respect to the decay $D_{sJ}(2460) \to D^{*+}_s \pi^0$ for the $D_{sJ}(2460)$. We have assumed $\Gamma(D^*_{sJ}(2317) \to D^+_s \pi^0) \approx \Gamma(D_{sJ}(2460) \to D^{*+}_s \pi^0) \approx 10$ keV as explained in the text. We have also quoted results obtained by light-cone QCD sum rules [47] and vector meson dominance [42].

Transition	Quark models			Experiments		Other approaches	
	QM	Ref. [12]	Ref. [40]	CLEO [2]	Belle [3]	Ref. [47]	Ref. [42]
$D^*_{sJ}(2317) \to D^{*+}_s \gamma$	0.16	0.17	0.19	< 0.059	< 0.18	$0.4 - 0.6$	0.085
$D^*_{sJ}(2317) \to D^+_s \gamma$	0.0	0.0	0.0	< 0.052	< 0.05	0.0	0.0
$D_{sJ}(2460) \to D^{*+}_s \gamma$	0.006	0.47	0.55	< 0.16	< 0.31	$0.06 - 0.11$	0.15
$D_{sJ}(2460) \to D^+_s \gamma$	0.67	0.51	0.62	< 0.49	$0.55 \pm 0.13 \pm 0.08$	$1.9 - 2.9$	0.33

TABLE V: Masses (QM), in MeV, of the recently measured charmonium and B_c states obtained within the model of Ref. [21] used in this work.

Name	Mass	Ref.	$n^{2S+1}L_J$	QM
$X(3940)$	$3943 \pm 6 \pm 6$	[48]	2^1P_1	3923
−	−		2^3P_0	3878
$Y(3940)$	$3943 \pm 11 \pm 13$	[49]	2^3P_1	3915
$X'_{c2}(3940)$	$3931 \pm 4 \pm 2$	[50]	2^3P_2	3936
$Y(4260)$	$4260 \pm 8 \pm 2$	[51]	4^3S_1	4307
$B_c(6287)$	$6287 \pm 4.8 \pm 1.1$	[52]	1^1S_0	6277