Faddeev study of heavy baryon spectroscopy

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Abstract

We investigate the structure of heavy baryons containing a charm or a bottom quark. We employ a constituent quark model successful in the description of the baryon-baryon interaction which is consistent with the light baryon spectra. We solve exactly the three-quark problem by means of the Faddeev method in momentum space. Heavy baryon spectrum shows a manifest compromise between perturbative and nonperturbative contributions. The flavor dependence of the one-gluon exchange is analyzed. We assign quantum numbers to some already observed resonances and we predict the first radial and orbital excitations of all states with $J = 1/2$ or $3/2$. We combine our results with heavy quark symmetry and lowest-order $SU(3)$ symmetry breaking to predict the masses and quantum numbers of six still non-measured ground-state beauty baryons.

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I. INTRODUCTION

Since the hypothesis at BNL [1] and posterior confirmation at Fermilab [2] of the existence of charmed baryons, an increasing interest on heavy baryon spectroscopy arose. It became evident that baryons containing heavy flavors c or b could play an important role in our understanding of QCD. In the early 90's the first beauty baryon, Λ_b, was discovered at the CERN e^+e^- collider LEP [3]. Since then, several new hadrons containing a single charm or bottom quark have been identified [4], they are resumed in Table I. Recent years have been specially fruitful in discovering heavy baryon states [5–9], and new states will be soon reported [10]. In many cases signals for the production of charmed and beauty baryons have been observed, the definitive measurement of the mass awaiting confirmation. This is, for example, the case of Ξ_b and Ω_b baryons already produced in $Z^0 \to b\bar{b}$ decays [11]. Thus, there are more states still to be found [12].

While the mass of these particles is usually measured as part of the discovery process, other quantum numbers such as the spin or parity often prove more elusive. For heavy baryons, no spin or parity quantum numbers of a given state have been measured directly. These properties can only be extracted by studying angular distributions of the particle decays, but these are available only for the lightest and most abundant species. For excited heavy baryons the data set are typically one order of magnitude smaller than for heavy mesons. Besides, knowledge of orbitally excited states is very much limited. Therefore, a powerful guideline for assigning quantum numbers to new states or even to indicate new states to look for is required by experiment.

All these reasons make heavy baryon spectroscopy an extremely rich and interesting subject. On the one hand it represents a three-body problem that may be solved exactly within the Faddeev formalism. On the other hand, it allows to study how the interaction between quarks evolves from light to heavy systems. Understanding baryon spectroscopy is expected to improve our knowledge about basic properties of QCD. In particular, heavy baryons provide with an excellent and perhaps even dramatic way of testing the approximate flavor independence of confinement forces. Besides, heavy baryons provide an excellent laboratory to study the dynamics of a light diquark in the environment of a heavy quark, allowing the predictions of different theoretical approaches to be tested.

After the discovery of the first charmed baryons, several theoretical works [13–15] based on potential models developed for the light baryon or meson spectra started analyzing properties of the observed and expected states. Later on, charmed and beauty baryons were studied with different Bhaduri-like potentials [16], calculating the ground state by a Faddeev method in configuration space and the excited states by diagonalization in a harmonic oscillator basis up to eight quanta. The charmed baryon spectrum has also been recently analyzed by means of a relativistically covariant quark model based on the Bethe-Salpeter equation in the instantaneous approximation [17]. Finally, the ground states of charmed and beauty baryons have been calculated by means of lattice techniques [18].

Nowadays, we have to our disposal *realistic* quark models accounting for most part of the one- and two-body low-energy hadron phenomenology. Among the several quark models proposed in the literature [19], either they were designed to study the baryon-baryon interaction [20–24] or the baryon spectra [25–30]. To our knowledge, the ambitious project of a simultaneous description of the baryon-baryon interaction and the baryon (and meson) spectra has only been undertaken by the constituent quark model of Ref. [19]. It was originally designed to study the nonstrange sector and it has been recently generalized to all flavors [31]. The success in describing the properties of the strange and non-strange one and two-hadron systems encourages its use as a guideline in order to assign parity and spin quantum numbers to already determined heavy baryon states as well as to predict still non-observed resonances.

This is why in this work we pursue the study of heavy baryons containing a charm or a bottom quark making use of the constituent quark model of Ref. [31]. Consistency with the baryon-baryon interaction and the light-baryon spectrum will be required. For the first time a Faddeev calculation in momentum space is done for the ground and excited states of baryons made of a heavy c or b quark. This is also the first time we study systems made of three distinguishable particles. The paper is organized as follows, we will start in the next section resuming the basic properties of the constituent quark model and describing the Faddeev method in momentum space for three distinguishable particles. Section III will be devoted to present and discuss the results in comparison to other models in the literature. Finally, in Sec. IV we will summarize our conclusions.

II. WORKING FRAMEWORK

A. Constituent quark model

Let us first outline the basic ingredients of the constituent quark model of Ref. [31]. Since the origin of the quark model hadrons have been considered to be built by constituent (massive) quarks. Nowadays it is widely recognized that the constituent quark mass appears because of the spontaneous breaking of the original chiral symmetry of the QCD Lagrangian, which gives rise to boson-exchange interactions between quarks (V_χ).

QCD perturbative effects are taken into account through the one-gluon-exchange (OGE) potential [32]. The δ-function, arising as a consequence of the nonrelativistic reduction of the one-gluon exchange diagram between point-like particles, has to be regularized in order to perform exact calculations. This regularization, controlled by a parameter r_0, has to be flavor dependent [33]. As this will be a central issue of our discussion, we will make explicit the expressions concerning the OGE. It reads,

$$V_{OGE}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s \, \vec{\lambda^c}_i \cdot \vec{\lambda^c}_j \left\{ \frac{1}{r_{ij}} - \frac{1}{6m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \, \frac{e^{-r_{ij}/r_0}}{r_{ij}\, r_0^2} \right\} , \tag{1}$$

where λ^c are the $SU(3)$ color matrices, α_s is the quark-gluon coupling constant, and r_0 is a flavor-dependent parameter to be determined from data. The scale-dependent strong coupling constant [34] is given by [31],

$$\alpha_s(\mu) = \frac{\alpha_0}{ln\left[(\mu^2+\mu_0^2)/\gamma_0^2\right]}, \tag{2}$$

where μ is the reduced mass of the interacting qq pair and $\alpha_0 = 2.118$, $\mu_0 = 36.976$ MeV and $\gamma_0 = 0.113$ fm^{-1}.

Finally, any model imitating QCD should incorporate confinement (CON). Lattice calculations in the quenched approximation derived, for heavy quarks, a confining interaction linearly dependent on the interquark distance. The consideration of sea quarks apart from valence quarks (unquenched approximation) suggests a screening effect on the potential when increasing the interquark distance [35].

Once perturbative (one-gluon exchange) and nonperturbative (confinement and chiral symmetry breaking) aspects of QCD have been considered, one ends up with a quark-quark interaction of the form

$$V_{q_iq_j} = \left\{ \begin{array}{l} q_iq_j = nn/sn \Rightarrow V_{CON} + V_{OGE} + V_\chi \\ q_iq_j = cn/cs/bn/bs/cc/bb \Rightarrow V_{CON} + V_{OGE} \end{array} \right. . \tag{3}$$

Notice that for the particular case of heavy quarks (c or b) chiral symmetry is explicitly broken and therefore boson exchanges do not contribute. For the explicit expressions of the interacting potential and a more detailed discussion of the model we refer the reader to Refs. [31,36]. For the sake of completeness we resume the parameters of the model in Table II.

We have not considered the noncentral contributions arising from the different terms of the interacting potential. Experimentally, there is no evidence for important effects of the noncentral terms on the light baryon spectra. This is clearly observed in the almost degeneracy of the nucleon ground states with $J^P = 1/2^-$ and $J^P = 3/2^-$, or their first excited states with the nucleon ground state with $J^P = 5/2^-$. The same is observed around the whole baryon spectra except for the particular problem of the relative large separation between the $\Lambda(1405)$, $J^P = 1/2^-$, and the $\Lambda(1520)$, $J^P = 3/2^-$, related to the vicinity of the $N\overline{K}$ threshold [37].

Theoretically, the spin-orbit force generated by the OGE has been justified to cancel with the Thomas precession term obtained from the confining potential [38]. This is not however the case for the two-baryon system where, by means of an explicit model for confinement, it has been demonstrated that the strong cancellation in the baryon spectra translates into a constructive effect for the two-baryon system [39]. One should notice that the scalar boson-exchange potential also presents a spin-orbit contribution with the same properties as before, it cancels the OGE spin-orbit force in the baryon spectra while it adds to the OGE contribution for the nucleon-nucleon $P-$waves and cancels for $D-$waves [40], as it is observed experimentally. Such a different behavior in the one- and two-baryon systems is due to the absence of a direct term in the OGE spin-orbit force (due to the color of the gluon only quark-exchange diagrams are allowed), while the spin-orbit contribution of the confining interaction in Ref. [39] and that of the scalar boson-exchange potential in Ref. [40] are dominated by a direct term, without quark exchanges. Regarding the tensor terms of the meson-exchange potentials, they have been explicitly evaluated in the light-baryon case (in a model with stronger meson-exchange potentials) finding contributions not bigger that 25 MeV [41]. This is due to the fact that the tensor terms give their most important contributions at intermediate distances (of the order of $1-2$ fm), due to the direct term in the quark-quark potential. The regularization of the boson-exchange potentials below the chiral symmetry breaking scale suppresses their contributions for the very small distances involved in the one-baryon problem. This allows to neglect the noncentral terms of the interacting potential that would provide with a fine tune of the final results and would make very much involved and time-consuming the solution of the three-body problem by means of the Faddeev method in momentum space we pretend to use. The same result has been obtained in Ref. [17] in the study of the noncentral terms of instanton induced interactions for charmed baryons.

The noncentral terms are relativistic corrections whose effect is known to decrease for heavy baryons [17,42], either due to the absence of the interaction (boson exchanges) or to the fact that they are $1/m^2$ corrections (OGE). In particular, the noncentral terms have also been explicitly evaluated in the past by the present authors, demonstrating that their contribution is already very small in the light baryon sector. The tensor potential of the one-pion exchange potential has been explicitly evaluated in Ref. [43], obtaining contributions smaller than 20 MeV. Spin-orbit terms have been calculated in Ref. [44] by means of a relativistic treatment of the spin variables, obtaining corrections one order of magnitude smaller than those due to the use of relativistic momentum variables.

B. Three-body equations

After partial-wave decomposition, the Faddeev equations are integral equations in two continuous variables as shown in Ref. [36]. They can be transformed into integral equations in a single continuous variable by expanding the two-body t-matrices in terms of Legendre polynomials as shown in Eqs. (32)−(36) of Ref. [45]. One obtains the final set of equations

$$\psi_{i;LST}^{n\ell_i\lambda_iS_iT_i}(q_i) = \sum_{j\neq i}\sum_{m\ell_j\lambda_jS_jT_j}\int_0^\infty q_j^2dq_j\, K_{ij;LST}^{n\ell_i\lambda_iS_iT_im\ell_j\lambda_jS_jT_j}(q_i,q_j;E) \times\psi_{j;LST}^{m\ell_j\lambda_jS_jT_j}(q_j), \tag{4}$$

with

$$K_{ij;LST}^{n\ell_i\lambda_iS_iT_im\ell_j\lambda_jS_jT_j}(q_i,q_j;E) = \frac{1}{2} < S_iT_i|S_jT_j >_{ST}\ \sum_r \tau_{i;nr}^{\ell_iS_iT_i}(E-q_i^2/2\nu_i) \times\int_{-1}^1 d\cos\theta\, \frac{P_r(x_i')P_m(x_j)}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}A_L^{\ell_i\lambda_i\ell_j\lambda_j}(p_i'q_ip_jq_j). \tag{5}$$

$P_r(x_i')$ and $P_m(x_j)$ are Legendre polynimials, $x_i'=(p_i'-b)/(p_i'+b)$, $x_j=(p_j-b)/(p_j+b)$, and b a scale parameter. $\tau_{i;nr}^{\ell_iS_iT_i}(E-q_i^2/2\nu_i)$ are the coefficients of the expansion of the two-body $t-$matrices in terms of Legendre polynomials defined by Eq. (34) of Ref. [45]. S_i and T_i are the spin and isospin of the pair jk while S and T are the total spin and isospin. ℓ_i is the orbital angular momentum of the pair jk, λ_i is the orbital angular momentum of particle i with respect to the pair jk, and L is the total orbital angular momentum. The reduced masses η_i and ν_i, the spin-isospin recoupling coefficients $< S_iT_i|S_jT_j >_{ST}$, and the orbital angular momentum recoupling coefficients $A_L^{\ell_i\lambda_i\ell_j\lambda_j}(p_i'q_ip_jq_j)$, including the momentum variables p_i' and p_j, are defined by Eqs. (8)−(13) of Ref. [36].

The integral equations (4) couple the amplitude ψ_i to the amplitudes ψ_j and ψ_k. In the cases when the three particles are identical or when two are identical and one is different the equations can be reduced to integral equations involving just one of the three amplitudes as it has been shown in Ref. [36]. In contrast, when the three particles are different, by substituting the equation for ψ_i into the corresponding equations for ψ_j and ψ_k, one obtains at best integral equations that involve two independent amplitudes which means that in that case the numerical calculations are more time consuming.

We will now describe the application of the Faddeev method to the case of three distinguishable particles. If one represents in Eq. (4) the integration over dq_j by a numerical quadrature [46], then for a given set of the conserved quantum numbers L, S, and T, Eq. (4) can be written in the matrix form

$$\psi_i = \sum_{j \neq i} B_{ij}(E)\psi_j, \tag{6}$$

where ψ_i is a vector whose elements correspond to the values of the indices n, ℓ_i, λ_i, S_i, T_i, and r, i.e.,

$$\psi_i \equiv \psi_{i;LST}^{n\ell_i\lambda_i S_i T_i}(q_r), \tag{7}$$

with q_r the abscisas of the integration quadrature. The matrix $B_{ij}(E)$ is given by

$$B_{ij}(E) \equiv q_s^2 w_s K_{ij;LST}^{n\ell_i\lambda_i S_i T_i m\ell_j\lambda_j S_j T_j}(q_r, q_s; E), \tag{8}$$

where the vertical direction is defined by the values of the indices n, ℓ_i, λ_i, S_i, T_i, and r while the horizontal direction is defined by the values of the indices m, ℓ_j, λ_j, S_j, T_j, and s. q_s and w_s are the abscisas and weights of the integration quadrature.

Substituting the Eq. (6) for $i = 1$ into the corresponding equations for $i = 2$ and $i = 3$ one obtains

$$[B_{21}(E)B_{12}(E) - 1]\psi_2 + [B_{21}(E)B_{13}(E) + B_{23}(E)]\psi_3 = 0, \tag{9}$$

$$[B_{31}(E)B_{12}(E) + B_{32}(E)]\psi_2 + [B_{31}(E)B_{13}(E) - 1]\psi_3 = 0, \tag{10}$$

so that the binding energies of the system are the zeroes of the Fredholm determinant

$$|M(E)| = 0, \tag{11}$$

where

$$M(E) = \begin{pmatrix} B_{21}(E)B_{12}(E) - 1 & B_{21}(E)B_{13}(E) + B_{23}(E) \\ B_{31}(E)B_{12}(E) + B_{32}(E) & B_{31}(E)B_{13}(E) - 1 \end{pmatrix}. \tag{12}$$

III. RESULTS AND DISCUSSION

The results we are going to present have been obtained by solving exactly the Schrödinger equation by the Faddeev method in momentum space as we have just described.

Our results are shown in Tables III and IV for charmed and beauty baryons, respectively. As we are asking for consistency with the light baryon spectra, let us note that the corresponding results for light baryons are given in Fig. 2 of Ref. [36]. Comparing the gross structure of the known experimental states and the theoretical spectrum we find a good overall agreement. One can observe that, in general, the supposed orbital excitations (negative parity states) are predicted lower than experiment, we will return to this point at the end of this section. Note that the radial excitation of $1/2^+$ baryons is always around 350 MeV above the ground state, both for charmed and beauty baryons. The only exception is the $\Xi_i(1/2^+)$ ($i = c$ or b) with an excitation energy around 80 MeV. This resonance is not indeed a radial excitation. The ground state corresponds to a us pair in a singlet spin state while the excited one corresponds to the same pair in a triplet spin state. These two levels are often denoted in the literature as $\Xi_i(1/2^+)$ and $\Xi'_i(1/2^+)$.

Let us first stress the flavor independence of the confining interaction. It was fixed in Ref. [36] to drive the nucleon Roper resonance to its correct position and this is the value, denoted as (A) in Table II, we will use all along this work except for the results shown in Fig. 3. Once confinement is fixed in the nucleon sector it provides with the needed strength for baryons of different flavor.

Another interesting feature arising from our results is the clear flavor dependence of the regularization of the delta function appearing in the OGE. In the first calculations of the heavy baryon spectra this regularization was done by calculating the $\delta-$function perturbatively in a properly cut Hilbert space [13]. When dealing with exact solutions, as those based on Faddeev equations, the delta function has to be regularized. This regularization has to be flavor dependent if one wants to obtain the experimentally observed spin splitting, i.e., mass difference between the ground $3/2^+$ and $1/2^+$ states. In the case of pairs of quarks of similar mass one can use for the regularization parameter r_0 a formula depending on the reduced mass of the system as has been proved in the past [16,31,36]. However, in the case of heavy baryons the masses of the interacting quarks may be quite different, such that a reduced mass based formula will give the same result for a light-charm than for a light-bottom pair. As the color-magnetic term of the one-gluon exchange interaction depends on the inverse of the product of the masses of the interacting quarks, such a potential will be strongly reduced for the heavier pair producing a too small spin-splitting. This is the reason why Refs. [16,47] predict a small spin-splitting for beauty baryons (see the AL1 results in Table V), because the parameters were adjusted on the charm sector and a reduced mass based formula was used to move to beauty baryons.

The interplay between the pseudoscalar and the one-gluon exchange interactions is a key problem for both the baryon spectra and the two-nucleon system [48]. This can be illustrated noting that while the $\Sigma_i(3/2^+) - \Lambda_i(1/2^+)$ mass difference varies slowly from the strange to the bottom sector, the $\Sigma_i(3/2^+) - \Sigma_i(1/2^+)$ mass difference varies very fast (see Table V). The former mass difference is given by the pseudoscalar and one-gluon exchange forces (see columns V_1 and V_3 of Table III in Ref. [36]), but the last one is only provided by the one-gluon exchange interaction between the light and the heavy or strange quark. This is very easy to understand when the wave function is explicitly constructed. In the case of Λ baryons the two light quarks are in a flavor antisymmetric spin 0 state, the pseudoscalar and the one-gluon exchange forces being both attractive. For Σ baryons they are in a flavor symmetric spin 1 state. The pseudoscalar force, being still attractive, is suppressed by one order of magnitude due to the expectation value of the $(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\lambda}_i \cdot \vec{\lambda}_j)$ operator [36] and the one-gluon exchange between the two light quarks becomes repulsive. Therefore, the attraction is provided by the interaction between the light and the heavy or strange quarks, which for heavy quarks c or b is given only by the one-gluon exchange potential. In Table VI we have calculated the mass of Σ_i and Λ_i ($i = s$ or b) baryons with and without the pseudoscalar contribution. As can be seen the effect of the pseudoscalar interaction between the two-light quarks is approximately the same independently of the third quark. As the mass difference between the $\Sigma_i(3/2^+)$ and $\Sigma_i(1/2^+)$ states decreases when increasing the mass of the baryon, being almost constant the effect of the one pion-exchange, the remaining mass difference has to be accounted for by the one-gluon exchange. When using an ad hoc combination of gluon and boson exchanges one may generate a too large $\Sigma_i(3/2^+) - \Lambda_i(1/2^+)$ ($i = c$ or b) mass difference and a too small $\Sigma_i(3/2^+) - \Sigma_i(1/2^+)$ mass difference (see the AL1$_\chi$ results in Table V). This rules out any ad hoc recipe for the relative strength of both potentials, in any manner consistent with experiment. An incorrect scaling of the regularization parameter or a wrong balance between pseudoscalar and one-gluon exchange contributions drive incorrect results. This reinforces the importance of constraining models for the baryon spectra in the widest possible set of experimental data.

Let us therefore face the problem of the regularization parameter of the OGE, r_0. As mentioned in Sec. II this parameter must vary with the masses of the interacting quarks. The larger the system (the lighter the masses of the quarks involved) the larger the value of r_0 that can be used without risk of collapse. In Fig. 1 we plot the mass of the $\Xi(1/2^+)[nss]$, $\Xi_c(1/2^+)[nsc]$ and $\Xi_b(1/2^+)[nsb]$ ground states as a function of the regularization parameter of the one-gluon exchange potential for the ns subsystem, r_0^{ns}, for a fixed value of the other two, r_0^{nb} and r_0^{sb}. We observe how the variation of the energy is very similar in all three cases. It seems that the dynamics of any two-particle subsystem is not affected by the nature of the third particle. In particular, the unstable area for the one-gluon exchange interaction starts exactly at the same value. It is one of the Faddeev amplitudes that becomes unstable independently of the other two. Therefore the regularization parameter should depend on the interacting pair, independently of the baryon the pair belongs to. The values of r_0 reproducing the experimental data are quoted in Table VII. They obey a formula depending on the product of the masses of the interacting quarks that can be represented by

$$r_0^{q_i q_j} = A\mu \left(m_{q_i} m_{q_j}\right)^{-3/2} \tag{13}$$

being A a constant and μ the reduced mass of the interacting quarks. While working with almost equal or not much different masses this law can be easily replaced by a formula depending on some inverse power of the mass (or reduced mass) of the pair as obtained in Ref. [31], but this is not any more the case for quarks with very different masses, like those present in heavy baryons. This is one of the reasons why these systems constitute an excellent laboratory for testing low-energy QCD realizations. For the sake of completeness we have plotted in Fig. 2 the mass of the $\Xi_c(1/2^+)$ and $\Xi_b(1/2^+)$ states as a function of the r_0 values of the two subsystems with a heavy and a light quark. We observe how the value where the unstable region starts is greatly reduced in going from the nc (sc) to the nb (sb) subsystem, what should rule out a reduced mass dependent law that would imply a similar value of r_0 in both cases.

This behavior of the regularization parameter for large mass differences of the interacting quarks was not known, and therefore not taken into account in Ref. [49], predicting a very small $3/2^+ - 1/2^+$ spin splitting for double charmed baryons. This is exactly the same problem observed in the calculations of Refs. [16] and [47]. When the calculation of Ref.

[49] is repeated with the scaling law of the regularization parameter derived in this work, the spin splitting augments up to

$$\begin{aligned} \Xi_{cc}(3/2^+) - \Xi_{cc}(1/2^+) &= 66 \text{ MeV} \\ \Omega_{cc}(3/2^+) - \Omega_{cc}(1/2^+) &= 54 \text{ MeV}\,. \end{aligned} \tag{14}$$

Apart from the spin-splitting, the structure of the spectra is not significantly modified. In the case of the triply charmed baryons the results would remain the same as in Ref. [49].

A widely discussed issue on the light baryon spectra has been the so-called level ordering problem, the experimentally opposite order of the $N^*(1440)$ $J^P = 1/2^+$ and the $N^*(1535)$ $J^P = 1/2^-$ compared to the harmonic limit. Theoretically, this situation in the light baryon spectra has been cured by means of appropriate phenomenological interactions as it is the case of anharmonic terms [38], scalar three-body forces [27], or pseudoscalar interactions [29,50]. For heavy baryons these mechanisms are not expected to work or their strength is strongly diminished in such a way that the level ordering problem may not be present. In the case of the scalar three-body force of Ref. [27], a simultaneous exchange of a scalar particle among the three quarks, if existing in nature, should not be active in the presence of a heavy quark due to the explicit breaking of chiral symmetry. In the case of the chiral pseudoscalar interaction, its $(\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\lambda}_i \cdot \vec{\lambda}_j)$ structure gives attraction for symmetric spin-flavor pairs and repulsion for antisymmetric ones. The reduction of the strength of the pseudoscalar potential, acting only between the two light quarks, makes this level ordering inversion highly not probable for heavy baryons. This effect was already noticed in the strange baryon spectra. We observed how the one pion exchange is very much reduced in the case of $\Sigma(1/2^+)$ states (see Table III of Ref. [36]), its role being replaced by the kaon exchange between the light and *heavy* (strange in that case) quarks. Even then, the strength is not enough to reverse the ordering of the states (see Fig. 3 of Ref. [36]). When the $SU(3)$ symmetry is broken and one puts a heavy quark (c or b) the situation is clearly favored for having a first negative parity excited state below the Roper resonance. The presence of the heavy quark will also diminish the importance of relativistic effects, being responsible for most part of the level ordering problem [51]. We consider that the determination of the position of the first radial excitation of the Σ_c could help in understanding this problem.

Our results allow to predict quantum numbers to the experimentally observed resonances. These assignments are not mandatory. At this point we have to return to the observation that the orbital excitations are predicted lower than observed. This is a consequence of fitting the confinement strength in the light baryon sector such as to reproduce the nucleon Roper resonance instead of the negative parity states. If we modify the confinement strength to reproduce the negative parity states in the light baryon sector (this would only imply to loose the description of the nucleon Roper resonance that it is known to be highly sensitive to relativistic kinematics [51]) and recalculate the spectra with the new confining strength, denoted by (B) in Table II, we obtain the results shown in Fig. 3. Let us notice that all our previous discussion and the conclusions derived hold for the new confinement strength. In Fig. 3, column [A] represents the results of the smaller confinement strength and [B] those of the larger confinement strength in Table II. A much better agreement is observed with the model reproducing the orbital excitations of the light baryon sector. There is no experimental state that we do not predict and there is no low-lying theoretical resonance that has not been observed. The smaller confinement strength would give rise to several states still not observed. The recently discovered $\Sigma_c(2800)$ [5] would correspond to an orbital excitation with $J^P = 1/2^-$ or $3/2^-$ (they are degenerate in our model), any other correspondence being definitively excluded. For Λ_c baryons, the recently confirmed as a Λ_c state, $\Lambda_c(2880)$ [6], and the new state $\Lambda_c(2940)$ [6] may constitute the second orbital excitation of the Λ_c baryon. Finally, there is an state with a mass of 2765 MeV reported in Ref. [9] as a possible Λ_c or Σ_c state and also observed in Ref. [5]. While the first reference (and also the PDG) are not able to decide between a Λ_c or a Σ_c state, the second one prefers a Λ_c assignment. As seen in Fig. 3, this state may constitute the second member of the first orbital excitation of Σ_c states or the first radial excitation of Λ_c baryons. An experimental effort to confirm the existence of this state and its decay modes would help on the symbiotic process between experiment and theory to disentangle the details of the structure of heavy baryons.

Let us finally note that heavy quark symmetry (HQS) and chiral symmetry can be combined together in order to describe the soft hadronic interactions of hadrons containing a heavy quark [52]. In the limit of the heavy quark mass $m_Q \to \infty$ HQS predicts that all states in the **6** $SU(3)$ representation (those where the light degrees of freedom are in a $s = 1$ state) would be degenerate. If one considers HQS and lowest order $SU(3)$ breaking [53] one obtains an equal spacing rule similar to the one that arises in the decuplet of uncharmed $J^P = 3/2^+$ baryons. The equal spacing rule obtained for charmed baryons is

$$\begin{aligned} &\Xi'_c(1/2^+) - \Sigma_c(1/2^+) = \Omega_c(1/2^+) - \Xi'_c(1/2^+) = \\ &= \Xi_c(3/2^+) - \Sigma_c(3/2^+) = \Omega_c(3/2^+) - \Xi_c(3/2^+) \end{aligned} \tag{15}$$

These relations are satisfied by experimental data as seen in the second column of Table VIII, the first three spacings being of the order of 123 MeV. This prediction is clearly sustained by our model giving rise to a spacing of 127 MeV (third column of Table VIII) and also by the results of OGE based models (fourth column of Table VIII). Lattice calculations based on HQS fulfill exactly this equal spacing rule (fifth column of Table VIII). However, the relativistically covariant quark model of Ref. [17] strongly violates this rule, contradicting the expectations of HQS. All these results allow to confirm the prediction of a $\Omega_c(3/2^+)$ state at around 2770 MeV.

This equal spacing rule should also held in the beauty baryon sector, giving rise to the relations

$$\begin{aligned} &\Xi'_b(1/2^+) - \Sigma_b(1/2^+) = \Omega_b(1/2^+) - \Xi'_b(1/2^+) = \\ &= \Xi_b(3/2^+) - \Sigma_b(3/2^+) = \Omega_b(3/2^+) - \Xi_b(3/2^+) \,. \end{aligned} \tag{16}$$

Once again this equal spacing rule is strictly fulfilled by our model, generating an spacing of 124 MeV in all four cases. If we now make use of the existing indications of experimental data for $\Sigma_b(1/2^+)$ and $\Sigma_b(3/2^+)$, then we can predict the existence of the following states: a $\Xi'_b(1/2^+)$ with a mass of 5920 MeV, a $\Omega_b(1/2^+)$ with a mass of 6044 MeV, a $\Xi_b(3/2^+)$ with a mass of 5976 MeV, and finally a $\Omega_b(3/2^+)$ with a mass of 6100 MeV.

IV. SUMMARY

We have used a constituent quark model incorporating the basic properties of QCD to study the heavy baryon spectra. Consistency with the baryon-baryon interaction and the light baryon spectra is asked for. The model takes into account the most important QCD nonperturbative effects: chiral symmetry breaking and confinement as dictated by unquenched lattice QCD. It also considers QCD perturbative effects trough a flavor dependent one-gluon exchange potential.

The three-body problem has been exactly solved by means of the Faddeev method in momentum space. For the first time we have studied baryons made of three different quarks, what makes the calculation time consuming, but providing an excellent test of our numerical method. We have found that the key interplay between pseudoscalar and one-gluon exchange forces, already observed for the light baryons, constitutes a basic ingredient for the description of heavy baryons. The final spectra results from a subtle but physically meaningful balance between different spin-dependent forces. The baryon spectra make manifest the presence of two different sources of spin-dependent forces that can be very well mimic by the operatorial dependence generated by the pseudoscalar and one-gluon exchange potentials.

While the flavor dependence of the regularized one-gluon exchange potential for equal mass quarks can be nicely described by the inverse of the reduced mass of the system, we have found that this is not the case for interacting quarks with large mass differences. It is instead a mass dependence considering explicitly the masses of the two quarks that provides with a nice agreement with data.

Heavy baryons constitute an extremely interesting problem joining the dynamics of light-light and heavy-light subsystems in an amazing manner. This is due to the remnant effect of pseudoscalar forces in the two-light quark subsystem. Models based only on boson exchanges cannot explain in any manner the dynamics of heavy baryons, but it becomes also difficult for models based only on gluon exchanges, if consistency between light and heavy baryons is asked for. One-gluon exchange models would reduce the problem to a two-body problem controlled by the dynamics of the heaviest subsystem, and we find evidences in the spectra for contributions of both subsystems.

Our results contain the equal mass spacing rules obtained for heavy baryons by means of heavy quark symmetry and lowest order $SU(3)$ chiral symmetry breaking. The obtained spacing is almost the same for charmed and beauty baryons. Making use of the available experimental data we can predict the existence of six new beauty baryons, their masses and quantum numbers, as well as we can confirm the prediction of a charmed baryon made some time ago. We also make predictions for the orbital and radial excitations of all quantum numbers that, if confirmed, would definitively prove the flavor independence of the confining interaction.

Finally, although we do not believe that explanations based on constituent quark models may rule out or contradict other alternative ones, one should acknowledge the capability of constituent quark models for a coherent understanding of the low-energy phenomena of the baryon spectroscopy and the baryon-baryon interaction in a simple framework based on the contribution of pseudoscalar, scalar and one-gluon-exchange forces between quarks.

V. ACKNOWLEDGMENTS

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TABLES

TABLE I. Experimentally known charmed and beauty baryons. The J and P quantum numbers have not been measured and we have quoted, when existing, the values given by the PDG that correspond to the quark model expectations. We have omitted the very small error bars in the mass. We have quoted in the last column the quark content of the different symbols used to denote the baryons, n stands for a light quark u or d. We have also included two charmed baryons recently reported in Refs. [5] and [6], but not appearing for the moment in the PDG. Finally, we also quote two beauty baryons suggested in Ref. [7]. For completeness, in the bottom part of the table we have included two double charmed baryons that will appear along the text.

State	J^P	Mass (MeV)	Status	Quark content
Λ_c	$1/2^+$	2285	$****$	udc
$\Lambda_c(2593)$	$1/2^-$	2594	$***$	
$\Lambda_c(2625)$	$3/2^-$	2627	$***$	
Λ_c or $\Sigma_c(2765)$	$?^?$	2765	$*$	
$\Lambda_c(2880)$	$?^?$	2881	$**$	
$\Lambda_c(2940)$	$?^?$	2940	Ref. [6]	
$\Sigma_c(2455)$	$1/2^+$	2452	$****$	nnc
$\Sigma_c(2520)$	$3/2^+$	2518	$***$	
$\Sigma_c(2800)$	$3/2^-$	2800	Ref. [5]	
Ξ_c	$1/2^+$	2469	$***$	nsc
Ξ'_c	$1/2^+$	2577	$***$	
$\Xi_c(2645)$	$3/2^+$	2646	$***$	
$\Xi_c(2790)$	$1/2^-$	2790	$***$	
$\Xi_c(2815)$	$3/2^-$	2816	$***$	
Ω_c	$1/2^+$	2698	$***$	ssc
Λ_b	$1/2^+$	5625	$***$	udb
$\Sigma_b(5796)$	$1/2^+$	5796	Ref. [7]	nnb
$\Sigma_b(5852)$	$3/2^+$	5852	Ref. [7]	
Ξ_b	$1/2^+$	?	$*$	nsb
Ω_b				ssb
Ξ_{cc}	$?^?$	3519	$*$	ucc
Ω_{cc}				scc

TABLE II. Quark-model parameters. (A) and (B) stand for two different confinement strengths and charm quark masses used along this work.

Quark masses	$m_u = m_d$ (MeV)	313
	m_s (MeV)	500
	m_c (MeV)	(A) 1650 / (B) 1740
	m_b (MeV)	5024
Boson exchanges	m_π (fm^{-1})	0.70
	m_σ (fm^{-1})	3.42
	m_η (fm^{-1})	2.77
	m_K (fm^{-1})	2.51
	$\Lambda_\pi = \Lambda_\sigma$ (fm^{-1})	4.20
	$\Lambda_\eta = \Lambda_K$ (fm^{-1})	5.20
	$g^2_{ch}/(4\pi)$	0.54
	$\theta_P(^o)$	-15
Confinement	a_c (MeV)	(A) 230 / (B) 340
	μ_c (fm^{-1})	0.70
OGE	r_0 (fm)	see text

TABLE III. Masses, in MeV, for baryons with a charm quark with the set (A) of parameters compared to experiment. Note that we have included a possible assignment of the state with a mass of 2765 MeV as a Λ_c or Σ_c state.

	$J^P = 1/2^+$		$J^P = 3/2^+$		$J^P = 1/2^-$		$J^P = 3/2^-$	
	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.
Λ_c	2285	2292	2940[a]	2906	2593	2559	2625	2559
	2765[b]	2669	–	3061	2880b	2779	2880[b]	2779
Σ_c	2455	2448	2520	2505	2765[b]	2706	2765[b]	2706
	–	2793	–	2825	2800[c]	2791	2800[c]	2791
Ξ_c	2468	2496	2645	2633	2790	2749	2815	2749
	2576	2574	–	2951	–	2829	–	2829
Ω_c	2697	2701	–	2759	–	2959	–	2959
	–	3044	–	3080	–	3029	–	3029

[a]Ref. [6]
[b]Ref. [9]
[c]Ref. [5]

TABLE IV. Masses, in MeV, for baryons with a bottom quark compared to experiment.

	$J^P = 1/2^+$		$J^P = 3/2^+$		$J^P = 1/2^-$		$J^P = 3/2^-$	
	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.
Λ_b	5624	5624	–	6246	–	5890	–	5890
	–	5996	–	6406	–	6132	–	6132
Σ_b	5796[a]	5789	5852[a]	5844	–	6039	–	6039
	–	6127	–	6158	–	6142	–	6142
Ξ_b	–	5825	–	5967	–	6076	–	6076
	–	5913	–	6275	–	6157	–	6157
Ω_b	–	6037	–	6090	–	6278	–	6278
	–	6367	–	6398	–	6373	–	6373

[a] Ref. [7]

TABLE V. Mass difference (in MeV) between Σ_i and Λ_i states for different flavor sectors.

Mass difference	Exp.	This work	AL1$_\chi$ [47]	AL1 [16]
$\Sigma(3/2^+) - \Lambda(1/2^+)$	269	260	–	–
$\Sigma(3/2^+) - \Sigma(1/2^+)$	195	169	–	–
$\Sigma_c(3/2^+) - \Lambda_c(1/2^+)$	235	223	395	253
$\Sigma_c(3/2^+) - \Sigma_c(1/2^+)$	65	57	78	79
$\Sigma_b(3/2^+) - \Lambda_b(1/2^+)$	228	220	394	239
$\Sigma_b(3/2^+) - \Sigma_b(1/2^+)$	56	55	28	31

TABLE VI. Masses, in MeV, of different beauty baryons with two-light quarks with (Full) and without ($V_\pi = 0$) the contribution of the one-pion exchange potential. The same results have been extracted from Table III of Ref. [36] for strange baryons. ΔE stands for the difference between both results.

State	Full	$V_\pi = 0$	ΔE
$\Sigma_b(1/2^+)$	5789	5802	-13
$\Sigma_b(3/2^+)$	5844	5854	-10
$\Lambda_b(1/2^+)$	5624	5804	-180
$\Lambda_b(3/2^+)$	6246	6246	< 1
State	$V_{CON} + V_{OGE} + V_\pi$	$V_{CON} + V_{OGE}$	ΔE
$\Sigma(1/2^+)$	1408	1417	-9
$\Sigma(3/2^+)$	1454	1462	-8
$\Lambda(1/2^+)$	1225	1405	-180

TABLE VII. $r_0^{q_i q_j}$ in fm.

(q_i, q_j)	$r_0^{q_i q_j}$
(n, n)	0.35
(s, n)	0.2845
(n, c)	0.0347
(s, c)	0.0285
(n, b)	0.0074
(s, b)	0.0060

TABLE VIII. Equal spacing rule of Eq. (15) for different models in the literature. Masses are in MeV.

Mass difference	Exp.	This work	Ref. [16]	Ref. [17]	Ref. [18]
$\Xi'_c(1/2^+) - \Sigma_c(1/2^+)$	121	126	112	136	110
$\Omega_c(1/2^+) - \Xi'_c(1/2^+)$	121	127	108	93	110
$\Xi_c(3/2^+) - \Sigma_c(3/2^+)$	125	128	112	112	110
$\Omega_c(3/2^+) - \Xi_c(3/2^+)$	–	126	103	70	110

FIGURES

FIG. 1. (a) $\Xi_b(1/2^+)[nsb]$, (b) $\Xi_c(1/2^+)[nsc]$, and (c) $\Xi(1/2^+)[nss]$ ground state masses as a function of the regularization parameter of the light-strange subsystem, r_0^{ns}.

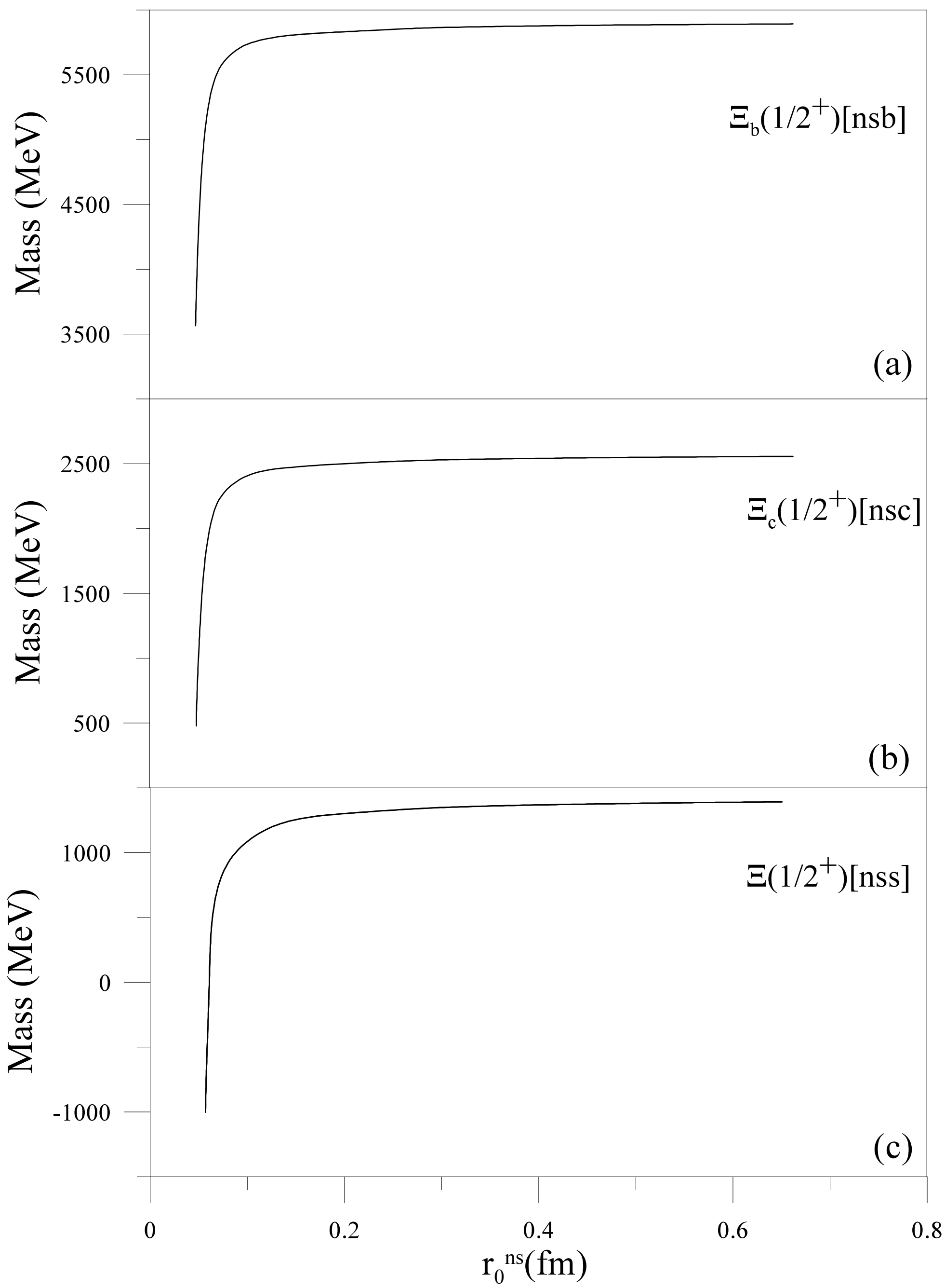


Figure 1

FIG. 2. (a) $\Xi_b(1/2^+)[nsb]$ ground state mass as a function of the regularization parameters r_0^{nb} and r_0^{sb}. (b) $\Xi_c(1/2^+)[nsc]$ ground state mass as a function of the regularization parameters r_0^{nc} and r_0^{sc}.

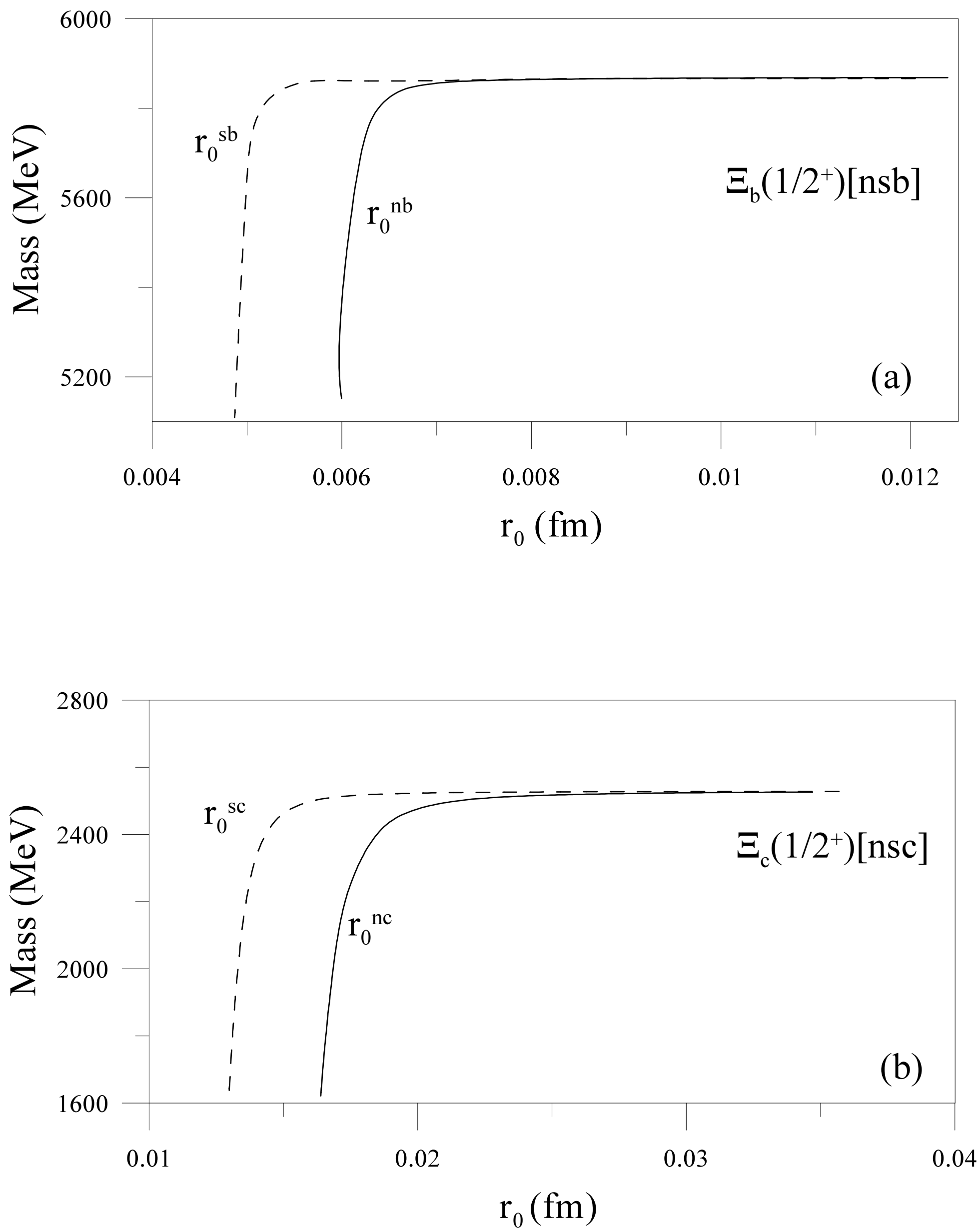


Figure 2

FIG. 3. (a) Spectra of Λ_c for two different confinement strengths compared to experiment. (b) Same as (a) for Σ_c states.

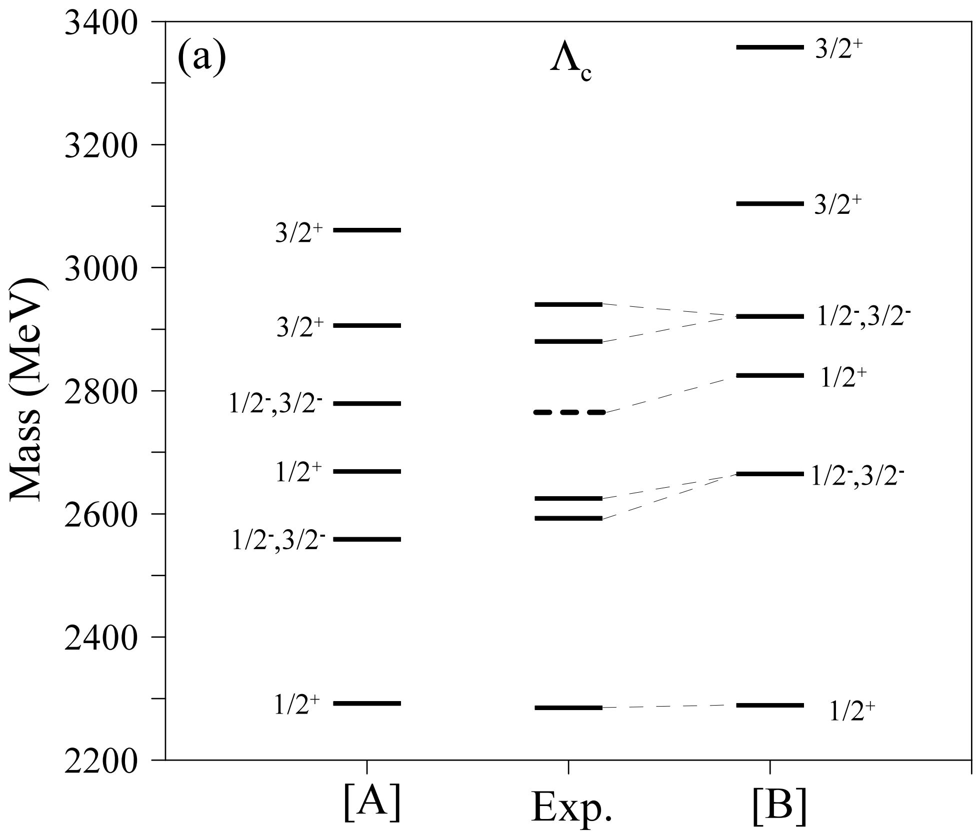
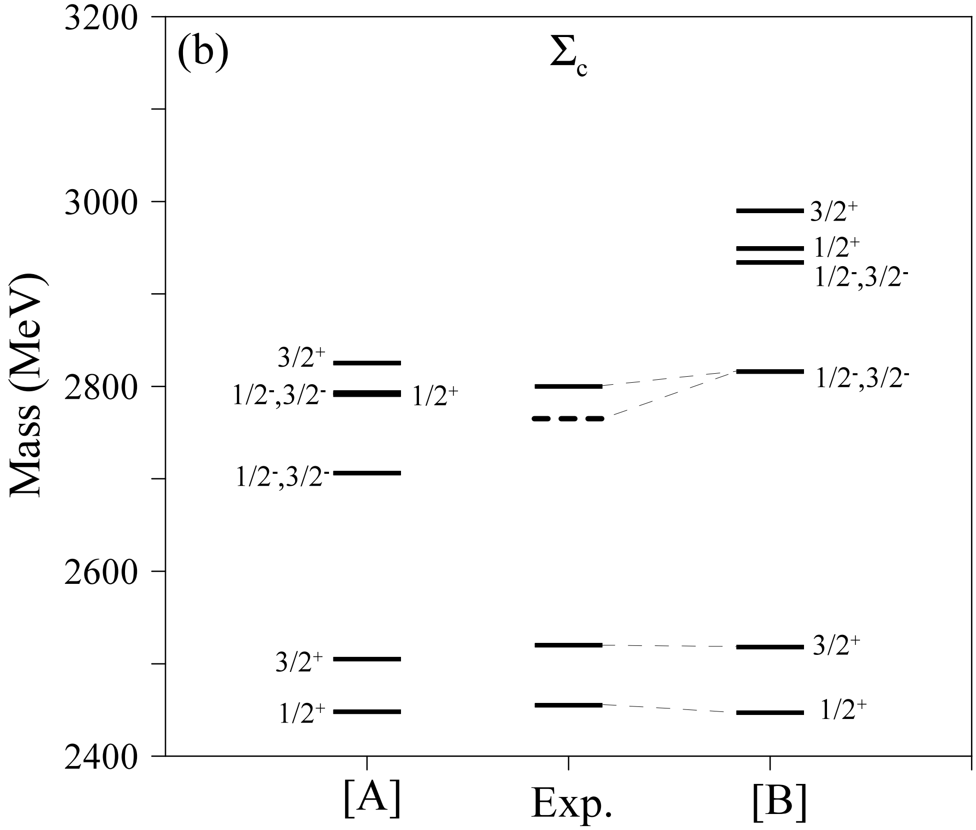


Figure 3



3200
3000
2800
2600
2400
Mass (MeV)
(b)
Σc
3/2+
1/2-,3/2-
1/2+
1/2-,3/2-
3/2+
1/2+
3/2+
1/2+
1/2-,3/2-
1/2-,3/2-
3/2+
1/2+
[A]
Exp.
[B]

Figure 3