# Nature of the light scalar mesons 

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#### Abstract

Despite the apparent simplicity of meson spectroscopy, light scalar mesons cannot be accommodated in the usual $q \bar{q}$ structure. We study the description of the scalar mesons below 2 GeV in terms of the mixing of a chiral nonet of tetraquarks with conventional $q \bar{q}$ states. A strong diquark-antidiquark component is found for several states. The consideration of a glueball as dictated by quenched lattice QCD drives a coherent picture of the isoscalar mesons.


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## I. INTRODUCTION

Nearly all known mesons made of $u, d$, and $s$ quarks fit neatly into the multiplets expected in generic constituent quark models. The single striking exception are the scalar mesons, i.e., $J^{P C}=0^{++}$. Either they form an anomalously light nonet (their masses do not fit into the quark model predictions in its many variations) or a nonet and more scalar mesons appear in the energy region $1.2-1.5 \mathrm{GeV}$, they overpopulate the expected number of states [1]. The understanding of mesons with vacuum quantum numbers is a crucial problem in low-energy QCD since they can shed some light on the symmetry breaking mechanism in QCD and presumably also on confinement.

There are too many $0^{++}$mesons observed below 2 GeV to be explained as $q \bar{q}$ states. There have been reported by the Particle Data Group (PDG) [2] two isovectors: $a_{0}(980)$ and $a_{0}(1450)$; five isoscalars: $f_{0}(600), f_{0}(980), f_{0}(1370), f_{0}(1500)$ and $f_{0}(1710)$; and three $I=1 / 2$ states: $K_{0}^{*}(1430), K_{0}^{*}(1950)$ and recently $K_{0}^{*}(800)$. The latest K-matrix fit of the $p \bar{p}$ annihilation data of Crystal Barrel Collaboration [3] gave rather definite information on the isoscalar resonances $f_{0}(600), f_{0}(980), f_{0}(1370)^{1}, f_{0}(1500), f_{0}(1710)^{2}$, and a broad state $f_{0}(1200-1600)$. Such state is introduced to explain a bump observed simultaneously in the channels $\pi \pi, K \bar{K}, \eta \eta$ and $\eta \eta^{\prime}$. Its large width, of the order of $1200 \pm 400 \mathrm{MeV}$, does not allow to determine reliably its mass. Finally, the recent analyses by BES of the $J / \Psi \rightarrow \phi \pi^{+} \pi^{-}$ and $J / \Psi \rightarrow \phi K^{+} K^{-}$data [4] require a state $f_{0}(1790)$ distinct from the $f_{0}(1710)$. The experimental situation of the scalar mesons is resumed in Table I.

In contrast to the vector and tensor mesons, scalar resonances are difficult to resolve. Their large decay widths cause a strong overlap between resonances and background, which is also obscured by the presence of several decay channels open up within a short mass interval. In addition, the $K \bar{K}$ and $\eta \eta$ thresholds produce sharp cusps in the energy dependence of the resonant amplitude. Furthermore, theoretically one expects non $-q \bar{q}$ scalar objects, like glueballs and multiquark states in the mass range below 2 GeV . On the one hand, multiquarks have been justified to coexist with $q \bar{q}$ states in the energy region around 1 GeV because they can couple to $0^{++}$, avoiding penalty due to orbital excitation [5]. On the other hand, lattice QCD in the quenched approximation predicts the existence of a scalar glueball with a mass around 1.6 GeV [6].

While the low-energy hadron phenomenology has been successfully understood in terms of constituent quark models, the scalar mesons are still puzzling. Without exhausting the list of problems on the scalar sector we recall that the quark structure as probed by the electromagnetic interaction is definitively not consistent with a naive $q \bar{q}$ composition [7]. The interest and rather complicated situation of the scalar mesons claims for a comprehensive study where other possible components are considered. States of the quark model are most easily identified with the hadrons we observe experimentally when unquenching (virtual hadron loop contributions) is not important. Thus, for example, $\phi$ is dominantly an $s \bar{s}$ state,

[^0]what follows from its decay to $K \bar{K}$. Only a small fraction of the Fock space of the physical $\phi$ is $K \bar{K}$. This is in part due to the $P$-wave nature of the $K \bar{K}$ component. In contrast, scalar mesons are strongly affected by their coupling to open hadron channels. The fact that the $f_{0}(980)$ and the $a_{0}(980)$ couple to both $\pi \pi / \pi \eta$ and $K \bar{K}$ channels means that virtual hadron loop contributions are important. As a consequence, conventional $q \bar{q}$ states are expected to mix with four-quark ( $q q \bar{q} \bar{q})$ states to yield physical mesons. A color singlet four-quark state can be obtained in two different coupling schemes, $[(q q)(\bar{q} \bar{q})]$ or $[(q \bar{q})(q \bar{q})]$. While in the first case the singlet color states are obtained from the $\left[(q \otimes q)_{6}(\bar{q} \otimes \bar{q})_{\overline{6}}\right]$ (the subindex standing for the color state) and $\left[(q \otimes q)_{\overline{3}}(\bar{q} \otimes \bar{q})_{3}\right]$ couplings, with a non-intuitive physical interpretation, in the second case the total singlet color states are driven by $\left[(q \otimes \bar{q})_{1}(q \otimes \bar{q})_{1}\right] \equiv(M M)$ and $\left[(q \otimes \bar{q})_{8}(q \otimes \bar{q})_{8}\right] \equiv(Q Q)$, where the physical interpretation is made evident. The first component will correspond to a molecule of mesons, while the second one is a compact component that is not factorizable into singlet color meson-meson channels.

In this work we will address the study of hadrons with zero baryon number described as clusters of constituent (massive) quarks confined by a realistic interaction. We make use of a standard constituent quark model applied to the study of the nonstrange baryon spectra and the baryon-baryon interaction [8]. This model has been recently generalized to all flavor sectors giving a reasonable description of the meson spectra except for the scalar mesons [9]. The model is strongly constrained by its application to other hadron sectors, representing in this way an advance with respect to similar recent studies based on models explicitly designed to study the scalar mesons [10]. The paper is organized as follows. In the next section we will resume the most relevant aspect of the constituent quark model used. Sect. III will be devoted to present and discuss the results. Firstly we will analyze the results based on a naive $q \bar{q}$ scheme, then we will consider the mixing to four-quark components and finally we will include a scalar glueball. In Sect. IV we will conclude summarizing our main findings.

## II. SU(3) CONSTITUENT QUARK MODEL

The model is based on the assumption that the constituent quark mass appears because of the spontaneous breaking of the original $S U(3)_{L} \otimes S U(3)_{R}$ chiral symmetry at some momentum scale, which is the most important nonperturbative phenomenon for hadron structure at low energies. In this domain of momenta quarks are quasiparticles with a constituent mass interacting through scalar and pseudoscalar boson-exchange potentials.

Beyond the chiral symmetry breaking scale one expects the dynamics being governed by QCD perturbative effects. They are taken into account through the one-gluon-exchange (OGE) potential. Following de Rújula et al. [11] the OGE is a standard color Fermi-Breit interaction.

Finally, any model imitating QCD should incorporate confinement. Lattice calculations in the quenched approximation derived, for heavy quarks, a confining interaction linearly dependent on the interquark distance. The consideration of sea quarks apart from valence quarks (unquenched approximation) suggest a screening effect on the potential when increasing the interquark distance [12]. Creation of light-quark pairs out of vacuum in between the quarks becomes energetically preferable resulting in a complete screening of quark color
charges at large distances. Although string breaking has not been definitively confirmed through lattice calculations [13], a quite rapid crossover from a linear rising to a flat potential is well established in $S U(2)$ Yang-Mills theories [14]. Such screened confining potentials provide with an explanation to the missing state problem in the baryon spectra [15], improve the description of the heavy meson spectra [16], and justify the deviation of the meson Regge trajectories from the linear behavior for higher angular momentum states [17].

Explicit expressions of the interacting potential derived from the nonrelativistic reduction of the lagrangian on the static approximation and a more detailed discussion of the model can be found in Ref. [9].

## III. RESULTS

In nonrelativistic quark models, gluon degrees of freedom are frozen and therefore the wave function of a zero baryon number $(\mathrm{B}=0)$ hadron may be written as

$$
\begin{equation*}
|\mathrm{B}=0\rangle=\Omega_{1}|q \bar{q}\rangle+\Omega_{2}|q q \bar{q} \bar{q}\rangle+\ldots \tag{1}
\end{equation*}
$$

where $q$ stands for quark degrees of freedom and, as mentioned above, the coefficients $\Omega_{i}$ will take into account the possibility that four-quark states could mix with $q \bar{q}$ states in the 1 GeV energy region. $|\mathrm{B}=0\rangle$ systems could then be described in terms of a hamiltonian

$$
H=H_{0}+H_{1} \quad \text { being } \quad H_{0}=\left(\begin{array}{cc}
H_{q \bar{q}} & 0  \tag{2}\\
0 & H_{q q \bar{q} \bar{q}}
\end{array}\right) \quad H_{1}=\left(\begin{array}{cc}
0 & V_{q \bar{q} \leftrightarrow q q \bar{q} \bar{q}} \\
V_{q \bar{q} \leftrightarrow q q \bar{q} \bar{q}} & 0
\end{array}\right)
$$

where, as will be shown below, the non-diagonal terms can be treated perturbatively, allowing to solve therefore the two- and four-body sectors separately.

## A. The $q \bar{q}$ sector

For the two-body problem, we have solved the Schrödinger equation using the Numerov algorithm [18]. As mentioned above the model provides with a correct description of the full meson spectra except for the scalar states [9]. In Table II we compare the masses of the scalar $q \bar{q}$ states to experimental data. In the last two columns we indicate the name and the mass of the hypothetical experimental state corresponding to the theoretical result.

Let us examine a possible correspondence between $q \bar{q}$ states and $0^{++}$experimental states. The isovector sector is spectroscopically simple. The $a_{0}(980)$ would correspond to the ${ }^{3} P_{0}$ member of the $1{ }^{3} P_{J}$ isovector multiplet. The $a_{0}(1450)$ should be considered as the scalar member of the $2^{3} P_{J}$ excited isovector multiplet. This reinforces the prediction of the quark model, the spin-orbit force making lighter the $J=0$ states with respect to the $J=2$. The assignment of the $a_{0}(1450)$ to the scalar member of the $1{ }^{3} P_{J}$ multiplet $[7,19]$ would contradict this idea, because the $a_{2}(1320)$ is a well established $q \bar{q}$ pair and much lower in energy than the $a_{0}(1450)$. The same behavior is evident in the $c \bar{c}$ and $b \bar{b}$ spectra, making impossible to describe the $a_{0}(1450)$ as a member of the $1^{3} P_{J}$ isovector multiplet without spoiling the description of heavy-quark multiplets [9]. However, there appear several problems with the
decay patterns. For example, the $a_{0}(980)$, considered as a pure light $q \bar{q}$ state, is known to underestimate the $\phi \rightarrow \gamma a_{0}$ (980) width by one or two orders of magnitude [20].

Concerning the $I=1 / 2$ sector, as a consequence of the larger mass of the strange quark, the quark model predicts a mass for the lowest $0^{++}$state 200 MeV greater than the $a_{0}(980)$ mass. Therefore, being the $a_{0}(980)$ the member of the lowest isovector scalar multiplet, the $K_{0}^{*}(800)$ cannot be explained as a $q \bar{q}$ pair.

The most obvious problems appear in the case of the isoscalar states. It is clear from Table II the absence of theoretical isoscalar $q \bar{q}$ states in three different energy regions. No states are predicted between $0.5-1.3 \mathrm{GeV}$, where the $f_{0}(980)$ resides, neither between $1.4-1.7 \mathrm{GeV}$, the energy range of the $f_{0}(1500)$, nor between $1.9-2.2 \mathrm{GeV}$, where the $f_{0}(2100)$ is placed.

Similar conclusions concerning the $f_{0}(980)$ and the $K_{0}^{*}(800)$ have been obtained using the extended Nambu-Jona-Lasinio model in an improved ladder approximation of the BetheSalpeter equation [21]. This indicates that relativistic corrections may not improve the situation. Therefore, a different structure rather than a naive $q \bar{q}$ pair seems to be needed. In particular, the $f_{0}(980)$ has been suggested as a possible four-quark state [22].

## B. Including four-quark states

Let us examine if the naive $q \bar{q}$ picture supplemented by four-quark states could acknowledge for the experimental situation. The four-body problem has been solved by means of a variational method using as trial wave function the most general linear combination of gaussians [23]. In particular, the so-called "mixed terms" (mixing the various Jacobi coordinates) that are known to have a great influence in the light quark case have been considered. The method to solve the four-body problem has been tested in the case of a system whose quantum numbers can only be obtained by means of a multiquark state, the isospin two $X(1600)$. This state has been observed in the reaction $\gamma \gamma \rightarrow \rho \rho$ near threshold, reported with a mass of $1600 \pm 100 \mathrm{MeV}$ and quantum numbers $I^{G}\left(J^{P C}\right)=2^{+}\left(2^{++}\right)$[2]. It cannot be described as a $q \bar{q}$ state, being therefore an exotic meson that can be understood as four light quarks coupled to $I=2, S=2$ and $L=0$. The energy obtained for this configuration is 1500 MeV , in agreement with the experimental data, giving confidence to the results obtained in the four-quark calculation.

The masses and flavor dominant component obtained for the low-lying four-quark states are shown in Table II, compared to the possible experimental assignment. As can be seen, for the isoscalar mesons the four-quark states appear precisely in the energy region where no $q \bar{q}$ states are predicted. In spite of that, neither the $q \bar{q}$ nor the $q q \bar{q} \bar{q}$ configurations match the experimental data. In particular, the $f_{0}(980)$ cannot be understood as a $q \bar{q}$ pair without failing to describe the isovector sector. Nonetheless, it is worth to notice that the nearly mass degeneracy observed experimentally between the $a_{0}(980)$ and the $f_{0}(980)$ could be explained spectroscopically if the $a_{0}(980)$ is considered as a $q \bar{q}$ pair and the $f_{0}(980)$ as a four-quark state, although such assignment fails to explain the observed properties of these states.

The Hamiltonian $H_{1}$ in Eq. (2) describes the mixing between two- and four-body configurations. Its explicit expression would require the knowledge of the operator annihilating
a quark-antiquark pair into the vacuum. This could be done, for example, using a ${ }^{3} P_{0}$ model, but the result will always depend on the parametrization used to describe the vertex. For simplicity, we have parametrized this mixing by looking to the quark pair that it is annihilated, and not to the spectator quarks that will form the final $q \bar{q}$ state:

$$
\begin{align*}
\langle n n \bar{n} \bar{n}| V|n \bar{n}\rangle & =\langle n s \bar{n} \bar{s}| V|s \bar{s}\rangle=\langle n n \bar{n} \bar{s}| V|n \bar{s}\rangle=C_{n}  \tag{3}\\
\langle s s \bar{s} \bar{s}| V|s \bar{s}\rangle & =\langle n s \bar{n} \bar{s}| V|n \bar{n}\rangle=\langle n s \bar{s} \bar{s}| V|n \bar{s}\rangle=C_{s} .
\end{align*}
$$

The mixing parameters, $C_{s}$ and $C_{n}$, are chosen to drive the $f_{0}(980)$ state to its physical mass. A fine tune of the $q \bar{q}$ model has been done to maintain the description of the isovector sector. In particular, the mass obtained for the bare $1 P q \bar{q}$ state (that would represent the $a_{0}(980)$ as a two-quark state) is 1079 MeV . The final resulting values are $C_{n}=165 \mathrm{MeV}$ and $C_{s}=70 \mathrm{MeV}$. Taking into account the degeneracy observed in Table II between the $a_{0}(980)$ and the $f_{0}(980)$, this correction is consistent with the assumption that $H_{1}$ can be treated perturbatively.

The obtained mass and dominant flavor component for all the scalar mesons are given in Table III. The interpretation of the light scalar mesons in terms of two- and four-quark components allows for an almost one-to-one correspondence between theoretical states and experiment. A dominant tetraquark component is found for the $f_{0}(980), f_{0}(1500), a_{0}(1450)$ and the $K_{0}^{*}(1430)$, and an important tetraquark component is predicted for the $a_{0}(980)$, although it has a dominant $q \bar{q}$ structure (see Table IV). The four-quark structure of the $a_{0}(980)$ and $f_{0}(980)$ would avoid the underestimation of the partial width for the decays $\phi \rightarrow \gamma f_{0}(980)$ and $\phi \rightarrow \gamma a_{0}(980)$ obtained in the case of a pure $q \bar{q}$ component [20]. In the case of the $f_{0}(980)$ it would also make it compatible with the similar branching ratios observed for the $J / \psi \rightarrow f_{0}(980) \phi$ and $J / \psi \rightarrow f_{0}(980) \omega$ decays [22]. Concerning the $f_{0}(1370)$ (which has already been suggested as corresponding to two different states [24]) we obtain two states around this energy, the heavier one with a dominant nonstrange content which favors its assignment to the $f_{0}(1370)$; the other with a high $s \bar{s}$ content without experimental partner in the PDG. This state, which couples strongly to the $K \bar{K}$ channel, may correspond to the broad resonance $f_{0}(1200-1600)$ predicted by Anisovich and collaborators [3].

Our results support the conclusion of the non- $q \bar{q}$ structure of the $f_{0}(1500)$ [25]. The $f_{0}(1500)$ has not been measured either in $\gamma \gamma \rightarrow K_{S} K_{S}$ by L3 Collaboration [26] or in $\gamma \gamma \rightarrow \pi^{+} \pi^{-}$by ALEPH Collaboration [27], implying that this state, if quarkonium, should be dominantly $s \bar{s}$. However, the branching ratios for the decay into two pseudoscalar mesons are only compatible with an almost pure $n \bar{n}$ structure. One should notice that the experimental situation for the two-photon decay is not definitively settled, since no partial wave analysis has been performed either by L3 or ALEPH and the possible coupling of $f_{0}(1500)$ and $f_{0}(1370)$ has not been considered. Nonetheless, there are other experimental evidences supporting the non $-q \bar{q}$ structure of the $f_{0}(1500)$. For instance, in the central production experiments performed by the WA102 Collaboration [28] it has been discovered a kinematic filter able to discriminate between $P$-wave $q \bar{q}$ states and $S$-wave states like glueballs, $q q \bar{q} \bar{q}$ multiquarks or $K \bar{K}$ molecules [29]. Its essence is that the pattern of resonances depends on the vector difference of the transverse momentum recoil of the final state protons $\left(d P_{T}\right)$. When $d P_{T}$ is large $P$-wave $q \bar{q}$ states are prominent, whereas at small $d P_{T}$ they are suppressed. Following this discovery there has been an intensive experimental program that
has reported that the only isoscalar scalar states that appear in the low $d P_{T}$ regime are the $f_{0}(980), f_{0}(1500)$ and $f_{0}(1710)$, therefore confirming their non $-q \bar{q}$ structure. Another experimental evidence has also been obtained from the analysis of the data of the WA102 Collaboration [30]. The azimuthal dependences as a function of $J^{P C}$ and the momentum transferred at the proton vertices appear to divide the scalar mesons into two classes: the $f_{0}(980), f_{0}(1500)$ and $f_{0}(1710)$ which are all strongly peaked at small angles and the $f_{0}(1370)$ which peaks at large angles, what may indicate a different nature of these states.

Finally, we obtain a dominant $n \bar{n}$ state corresponding to the $f_{0}(1790)$. This energy region is expected to allocate the radial excitation of the $n \bar{n}_{2 P}$ state, $f_{0}(1370)$. We also find a candidate for the $f_{0}(2200)$, experimentally identified as an $s \bar{s}$ state [31], with an energy of 2212 MeV .

A crucial test of the quark structure of the scalar mesons would be the systematic study of two-photon decay widths of neutral scalars, which is still lacking. The two-photon decay is dominated by the $q \bar{q}$ component of the wave function (the four-quark component is known to give a small contribution [32]). In this case the scalar two-photon decay width can be related to the well-known experimental data of the $J=2$ multiplet member through [33]

$$
\begin{equation*}
\Gamma_{\gamma \gamma}\left(0^{++}\right)=k\left(\frac{m_{0}}{m_{2}}\right)^{3} \Gamma_{\gamma \gamma}\left(2^{++}\right) \tag{4}
\end{equation*}
$$

with obvious notation. The factor $k$ arises from spin multiplicities and the consideration of relativistic corrections to $O\left(v^{2} / c^{2}\right)$ in three different approaches leads to a value lower than the corresponding nonrelativistic one, $k \approx 2$ as compared to $k=15 / 4$ [34]. Data on the charmonium states $\chi_{c 2}$ and $\chi_{c 0}$ using the nonrelativistic value are in good agreement with Eq. (4). Using the results given in Table IV we obtain

$$
\begin{equation*}
\Gamma_{\gamma \gamma}\left[a_{0}(980)\right]=0.65 \pm 0.04 \mathrm{keV} \quad ; \quad \Gamma_{\gamma \gamma}\left[f_{0}(980)\right]=0.23 \pm 0.02 \mathrm{keV} \tag{5}
\end{equation*}
$$

that compares rather well with the experiment [2],

$$
\begin{equation*}
\Gamma_{\gamma \gamma}^{\mathrm{Exp}}\left[a_{0}(980)\right]=0.3 \pm 0.1 \mathrm{keV} ; \Gamma_{\gamma \gamma}^{\mathrm{Exp}}\left[f_{0}(980)\right]=0.39_{-0.13}^{+0.10} \mathrm{keV} . \tag{6}
\end{equation*}
$$

Let us note that the experimental width for the decay $a_{0}(980) \rightarrow \gamma \gamma$ requires the knowledge of the branching ratio of the decay $a_{0}(980) \rightarrow \eta \pi$, that has been taken to be $0.24 \pm 0.08 \mathrm{keV}$. Small variations on this number will induce large modifications on the $a_{0}(980)$ two-photon decay width. These results would be modified by the $K \bar{K}$ component of the four-quark wave function [35]. However, this component is rather small, less than $1 \%$, what would not modify the results significantly.

The final physical picture arising shows an involved structure for the flavor wave function of the light scalar mesons, in agreement with their complicated pattern decays. In Table IV we give a detailed description of the flavor wave function of some selected scalar states. Regarding the four-quark structure it presents at the same time compact and molecule components. For example the 72 \% of ( $n s \bar{n} \bar{s}$ ) of the $f_{0}(1500)$ can be decomposed in the following way $f_{0}(1500)=0.723|M M\rangle+0.691|Q Q\rangle$, where the meson-meson component involves the asymptotic states: $\eta^{\prime} \eta^{\prime}, \eta \eta, K \bar{K}, \eta \eta^{\prime}, \omega \phi$ and $K^{*} \bar{K}^{*}$, generating decays that will feed a wide range of physical states. The meson-meson component contains a weakly
bound $K \bar{K}$ molecule which may decay by annihilation through an intermediate $0^{++} q \bar{q}$ meson into two pions [36] as observed experimentally. The same reasoning could be applied to the $f_{0}(980)$ and $a_{0}(980)$ in such a way that they would be at the same time four-quark states and $K \bar{K}$ molecules as suggested by the nearness of the $K \bar{K}$ threshold [7]. Finally, let us stress the presence of an important diquark-antidiquark, $\left[(q \otimes q)_{\overline{3}}(\bar{q} \otimes \bar{q})_{3}\right]$, component in the four-quark wave function ( $57 \%$ for the case of the $f_{0}(1500)$ discussed above), where QCD predicts a strong attraction in the $S$-wave when in a flavor nonet [1,37]. The presence of a diquark-antidiquark component in the four-quark wave function of the scalar mesons has been recently recognized in a schematic calculation of the scalar mesons [38].

In the literature one finds alternative approaches to understand the rather complicated scenario of the scalar mesons. An earlier attempt to link the understanding of the $N N$ interactions with meson spectroscopy was done based on the Jülich potential model [39]. The structure of the scalar mesons $a_{0}(980)$ and $f_{0}(980)$ was investigated in the framework of a meson exchange model for $\pi \pi$ and $\pi \eta$ scattering. The $K \bar{K}$ interaction generated by the vector-meson exchange, which for isospin $I=0$ is strong enough to generate a bound state is much weaker for $I=1$, making a degeneracy of $a_{0}(980)$ and $f_{0}(980)$ impossible, as found in our model as $q \bar{q}$ pairs. Although both scalar mesons result from the coupling to the $K \bar{K}$ channel explaining in a natural way their similar properties, the underlying structure obtained was, however, quite different. Whereas the $f_{0}(980)$ appears to be a $K \bar{K}$ bound state the $a_{0}(980)$ was found to be a dynamically generated threshold effect. Similar conclusions have been obtained in a chiral unitary coupled channel approach, where the $f_{0}(600)$, the $a_{0}(980)$, and the $K_{0}^{*}(800)$ rise up as dynamically generated resonances, being poles in the $S$-wave meson-meson scattering amplitudes, while the $f_{0}(980)$ is a combination of a strong $S$-wave meson-meson unitarity effect and a preexisting singlet resonance [40].

In a different fashion within the quark model the same problem was illustrated in Ref. [41]. The bare mass used for the $n \bar{n}$ pair is much larger than the $a_{0}(980)$ and $f_{0}(980)$ experimental masses. It is the effect of the two-pseudoscalar meson thresholds the responsible for the substantial shift to a lower mass than what is naively expected from the $q \bar{q}$ component alone. This gives rise to an important $K \bar{K}$ and $\pi \eta^{\prime}$ components in the $a_{0}(980)$ and $K \bar{K}, \eta \eta$, $\eta^{\prime} \eta^{\prime}$ and $\eta \eta^{\prime}$ in the $f_{0}(980)$. In particular for the $a_{0}(980)$ they obtain the $K \bar{K}$ component to be dominant near the peak, being about $4-5$ times larger than the $q \bar{q}$ component. A similar conclusion, that the description of the $a_{0}(980)$ and $f_{0}(980)$ requires from more complex structures, is also obtained from our analysis.

## C. Including the light scalar glueball

Due to its non-abelian character QCD predicts the existence of isoscalar mesons containing only gluons, the glueballs $(G)$. Therefore one should finally wonder whether there is some place for them in the meson spectra. There are several alternatives in the literature concerning these states, although there seems to exist a consensus that the lowest glueball should be the scalar one [42]. Some of these approaches predict the existence of a low-lying glueball below 1 GeV [43], while others seem to prefer an scenario where the glueball appears in a higher energy region, around $1.4-1.8 \mathrm{GeV}[12,44-46]$. In our description of the scalar isoscalar states the second prescription is preferred. As can be seen in Table III, there is
only one experimental state without theoretical partner, being its energy around 1.5 GeV . Lattice QCD in the quenched approximation predicts the existence of a scalar glueball with a mass around 1.6 GeV [12]. Besides, recent lattice studies confirm that there is indeed significant mixing between $G$ and the nearest $q \bar{q}$ states, together with associated mass shifts in the $J^{P C}=0^{++}$sector [45]. For pure gluonium one expects coupling of similar strengths to $s \bar{s}$ and $n \bar{n}$ mesons since gluons are flavor blind. Lattice calculations derived a $G-q \bar{q}$ mixing nearly flavor blind with a maximum deviation of $20 \%$ [46] and suggested that the preferred glueball mass falls into the range

$$
\begin{equation*}
\frac{M_{s \bar{s}}+M_{n \bar{n}}}{2}>M_{G}>M_{n \bar{n}} \tag{7}
\end{equation*}
$$

where $M_{n \bar{n}}$ and $M_{s \bar{s}}$ are the bare quarkonium masses (before mixing). All these results obtained from lattice QCD indicate than glueballs and their flavor-mixing are a controlling feature of the meson spectroscopy in the $1.3-1.7 \mathrm{GeV}$ mass region.

Therefore, based on intuition from lattice QCD [12,45-48] we have investigated the mixing between a $J^{P C}=0^{++}$glueball and the $q \bar{q}$ nonet in its vicinity. We have included in our calculation a scalar glueball satisfying the Eq. (7) and with the mixing to the $2 P n \bar{n}$ and $s \bar{s}$ pairs predicted by lattice QCD [46]. In particular, within the model used the bare $n \bar{n}$ state has a mass of 1503 MeV , and the bare $s \bar{s}$ state 1850 MeV , giving a mass for the scalar glueball between 1503 and 1676.5 MeV . We have used $M_{G}=1643 \mathrm{MeV}$. Besides as suggested in Ref. [46] we take $\langle G| V|n \bar{n}\rangle=\sqrt{2} r\langle G| V|s \bar{s}\rangle$ and $\langle G| V|s \bar{s}\rangle=64 \mathrm{MeV}$ and $r \approx 1.2$ ( $r=1$ for exact $S U(3)$ symmetry, exact flavor blind). The results are resumed in Table V noting a correspondence between theoretical predictions and experiment. This assignment suggests that there are four isoscalar mesons that are not dominantly $q \bar{q}$ states, they are the $f_{0}(980)$ (dominantly a $n n \bar{n} \bar{n}$ state), the $f_{0}(1500)$ (dominantly a $n s \bar{n} \bar{s}$ state), the $f_{0}(1710)$ (dominantly a glueball) and the $f_{0}(2020)$ (dominantly a $s s \bar{s} \bar{s}$ state). This is clearly seen in Fig. 1 where we have constructed the two Regge trajectories associated to the isoscalar mesons. As it is observed the masses of the $f_{0}(600), f_{0}(1200-1600), f_{0}(1370), f_{0}(1790)$, $f_{0}(2100), f_{0}(2200)$ fit nicely in one of the two Regge trajectories, while those corresponding to the $f_{0}(980), f_{0}(1500), f_{0}(1710), f_{0}(2020)$ do not fit for any integer value. The exception would be the $f_{0}(2020)$ that it is the orthogonal state to the $f_{0}(2100)$ having almost $50 \%$ of four-quark component.

The glueball component is shared between the three neighboring states: $20 \%$ for the $f_{0}(1370), 2 \%$ for the $f_{0}(1500)$ and $76 \%$ for the $f_{0}(1710)$. Our results assigning the larger glueball component to the $f_{0}(1710)$ are on the line with Refs. [45,46] and differ from those of Refs. [25,49] concluding that the $f_{0}(1710)$ is dominantly $s \bar{s}$. One should notice that none of these studies consider the recent result of the BES Collaboration suggesting the existence of a scalar state close to the $f_{0}(1710)$ that could be the $q \bar{q}$ dominant one. It should be also mentioned that most studies in the literature [25,45,46,49] do only address a particular set of scalar states that they consider could be identified with the glueball. This makes clear the complicated situation in the scalar sector with several alternative interpretations.

## IV. CONCLUSIONS

In summary, the spectroscopy and many of the decay properties of the scalar mesons can be understood under the hypothesis of the mixing between two- and four-quark components. If one considers the existence of a scalar glueball within the intuition of lattice QCD in the quenched approximation, the description is improved. Our results suggest the existence of the state recently reported by BES Collaboration, $f_{0}(1790)$, as the radial excitation of the $f_{0}(1370)$ and also show evidence for the presence of a new scalar state around the $f_{0}(1370)$ already suggested by Ref. [24] and definitively predicted by the K-matrix analysis of the $p \bar{p}$ annihilation data of the Crystal Barrel Collaboration, $f_{0}(1200-1600)$. Our description of the $f_{0}(980), f_{0}(1500)$ and $f_{0}(1710)$ as non- $q \bar{q}$ states is in agreement with several experimental evidences, in particular those obtained by the WA102 Collaboration. The final flavor structure of the light scalar mesons becomes rather involved allowing to deep the understanding of their complicated decay patterns. A strong diquark-antidiquark component is found. Some problems remain open as it could be the high strange content predicted for the $f_{0}(2020)$. The study of radiative transitions and two-photon decay widths should help to clarify the flavor mixing and the nature of the $I=0$ scalar sector.

Our work presents an interpretation of the scalar mesons in a model constrained by the description of other hadron sectors. It drives to a final scenario that it is compatible with some other models in the literature and it differs from the results of others. The final answer could only be obtained from precise experimental data that would allow to discriminate between the predictions of different theoretical models. The set of data is so huge, and sometimes so poor, that one always may find a positive or negative interpretation of some of them.

## V. ACKNOWLEDGMENTS

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## REFERENCES

[1] R.L. Jaffe, Phys. Rep. 409, 1 (2005).
[2] S. Eidelman et al., Phys. Lett. B 592, 1 (2004).
[3] V.V. Anisovich and A.V. Sarantsev, Eur. Phys. J. A 16, 229 (2003). V.V. Anisovich, Yu.D. Prokoshkin, and A.V. Sarantsev, Phys. Lett. B 389, 388 (1996). V.V. Anisovich, Phys. Usp. 47, 45 (2004). V.V. Anisovich, AIP Conf. Proc. 619, 197 (2002). V.V. Anisovich, AIP Conf. Proc. 717, 441 (2004).
[4] M. Ablikim et al. (BES Collaboration), Phys. Lett. B 607, 243 (2005). L. Dong (BES Collaboration), hep-ex/0410040.
[5] R.L. Jaffe, Phys. Rev. D 15, 267 (1977).
[6] G.S. Bali et al., Phys. Lett. B 309, 378 (1993). J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. 75, 4563 (1995). C.J. Morningstar and M. Peardon, Phys. Rev. D 56, 4043 (1997). D. Weingarten, Nucl. Phys. B (Proc. Supp.) 53, 2321 (1997).
[7] F.E. Close and N.A. Törnqvist, J. Phys. G 28, R249 (2002).
[8] A. Valcarce, H. Garcilazo, F. Fernández, and P. Gonzalez, Rep. Prog. Phys. 68, 965 (2005).
[9] J. Vijande, F. Fernández, and A. Valcarce, J. Phys. G 31, 481 (2005).
[10] M. Napsuciale and S. Rodríguez, Phys. Rev. D 70, 094043 (2004). S. Rodríguez and M. Napsuciale, Phys. Rev. D 71, 074008 (2005).
[11] A. de Rújula, H. Georgi, and S.L. Glashow, Phys. Rev. D 12, 147 (1975).
[12] G.S. Bali, Phys. Rep. 343, 1 (2001) and references therein.
[13] G.S. Bali et al., Phys. Rev. D 62, 054503 (2000).
[14] P.W. Stephenson, Nucl. Phys. B 550, 427 (1999). F. Knechtli and R. Sommer, Nucl. Phys. B 590, 309 (2000). O. Philipsen and H. Wittig, Phys. Lett. B 451, 146 (1999). P. de Forcrand and O. Philipsen, Phys. Lett. B 475, 280 (2000).
[15] J. Vijande, P. González, H. Garcilazo, and A. Valcarce, Phys. Rev. D 69, 074019 (2004).
[16] P. González, A. Valcarce, H. Garcilazo, and J. Vijande, Phys. Rev. D 68, 034007 (2003).
[17] M.M. Brisudová, L. Burakovsky, and T. Goldman, Phys. Rev. D 61, 054013 (2000).
[18] S.E. Koonin and D.C. Meredith, Computational Physics, (Addison-Wesley, New York, 1990).
[19] P. Geiger and N. Isgur, Phys. Rev. Lett. 67, 1066 (1991).
[20] T. Barnes, AIP Conf. Proc. 619, 447 (2002). N.N. Achasov, AIP Conf. Proc. 619, 112 (2002). F.E. Close, Int. J. Mod. Phys. A 17, 3239 (2002).
[21] T. Umekawa, K. Naito, M. Oka, and M. Takizawa, Phys. Rev. C 70, 055205 (2004).
[22] N.N. Achasov, Nucl. Phys. A 675, 279c (2000); hep-ph/9803292.
[23] Y. Suzuki and K. Varga, Lect. Notes Phys. M54, 1 (1998).
[24] D. Black, A.H. Fariborz, and J. Schechter, Phys. Rev. D 61, 074001 (2000).
[25] C. Amsler, Phys. Lett. B 541, 22 (2002).
[26] M. Acciarri et al. (L3 Collaboration), Phys. Lett. B 501, 173 (2001).
[27] R. Barate et al. (ALEPH Collaboration), Phys. Lett. B 472, 189 (2000).
[28] D. Barberis et al. (WA102 Collaboration), Phys. Lett. B 397, 339 (1997); ibid 462, 462 (1999).
[29] F.E. Close and A. Kirk, Phys. Lett. B 397, 333 (1997).
[30] F.E. Close, Int. J. Mod. Phys. A 17, 3239 (2002).
[31] A.V. Anisovich et al., Nucl. Phys. A 662, 319 (2000).
[32] N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Phys. Lett. B 108, 134 (1982).
[33] K.A. Tumanov, Zh. Eksp. Teor. Fiz. 25, 385 (1953). A.I. Alekseev, Zh. Eksp. Teor. Fiz. 34, 1195 (1958).
[34] L. Bergström, G. Hulth, and H. Snellmann, Z. Phys. C 16, 263 (1983). Z.P. Li, F.E. Close, and T. Barnes, Phys. Rev. D 43, 2161 (1991).
[35] T. Barnes, Phys. Lett. B 165, 434 (1985).
[36] J. Weinstein and N. Isgur, Phys. Rev. D 41, 2236 (1990).
[37] R.L. Jaffe and F.E. Low, Phys. Rev. D 19, 2105 (1979).
[38] L. Maiani, F. Piccinini, A.D. Polosa, and V. Riquer, Phys. Rev. Lett. 93, 212002 (2004).
[39] G. Janssen, B.C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690 (1995).
[40] J.A. Oller and E. Oset, Phys. Rev. D 60, 074023 (1999). J.A. Oller, E. Oset and J.R. Peláez, Phys. Rev. Lett. 80, 3452 (1998).
[41] N.A. Tornqvist, Z. Phys. C 68, 647 (1995).
[42] G.B. West, Phys. Rev. Lett. 77, 2622 (1996).
[43] V. Vento, hep-ph/0401218 (2004). T. Barnes, F.E. Close, and F. de Viron, Nucl. Phys. B 224, 241 (1983). C. DeTar and J. Donoghue, Ann. Rev. Nucl. Part. Sci. 33, 325 (1983). P. Minkowski and W. Ochs, Eur. Phys. J. C 9, 283 (1999).
[44] N. Isgur and J. Paton, Phys. Lett. B 124, 247 (1983). N. Isgur and J. Paton, Phys. Rev. D 31, 2910 (1985). M.J. Teper, Phys. Rev. D 59, 014512 (1999).
[45] C. McNeile and C. Michael, Phys. Rev. D 63, 114503 (2001).
[46] W. Lee and D. Weingarten, Phys. Rev. D 61, 014015 (2000) and references therein.
[47] C. Amsler and F.E. Close, Phys. Lett. B 353, 385 (1995).
[48] F.E. Close, Act. Phys. Pol. B 31, 2557 (2000).
[49] C. Amsler and F.E. Close, Phys. Rev. D 53, 295 (1996). F.E. Close and A. Kirk, Eur. Phys. J. C 21, 531 (2001).
[50] F.E. Close and Q. Zhao, Phys. Rev. D 71, 094022 (2005).

## TABLES

TABLE I. Experimentally reported light scalar mesons. I stands for the isospin. We denote by a star those states listed by the PDG [2], and by a dagger states reported in Refs. [3] and [4].

| $I$ | State | Mass | Status |
| :---: | :---: | :---: | :---: |
| 1 | $a_{0}(980)$ | $984.7 \pm 1.2$ | $*$ |
|  | $a_{0}(1450)$ | $1474 \pm 19$ | $*$ |
|  | $f_{0}(600)$ | $400-1200$ | $*$ |
|  | $f_{0}(980)$ | $980 \pm 10$ | $*$ |
|  | $f_{0}(1200-1600)$ | $1400 \pm 200$ | $\dagger$ |
|  | $f_{0}(1370)$ | $1200-1500$ | $*$ |
|  | $f_{0}(1500)$ | $1507 \pm 5$ | $*$ |
|  | $f_{0}(1710)$ | $1714 \pm 5$ | $*$ |
|  | $f_{0}(1790)$ | $1790_{-30}^{+40}$ | $\dagger$ |
|  | $f_{0}(2020)$ | $1992 \pm 16$ | $*$ |
|  | $f_{0}(2100)$ | $2103 \pm 17$ | $*$ |
|  | $f_{0}(2200)$ | $2197 \pm 17$ | $*$ |
|  | $K_{0}^{*}(800)$ | $\approx 800$ | $*$ |
|  | $K_{0}^{*}(1430)$ | $1412 \pm 6$ | $*$ |
|  | $K_{0}^{*}(1950)$ | $1945 \pm 22$ | $*$ |
|  |  |  |  |

TABLE II. Mass (QM), in MeV, and flavor dominant component (Flavor) of the light scalar mesons considered as $q \bar{q}$ or $q q \bar{q} \bar{q}$ states. $I$ stands for the isospin. $n L$ denotes the radial excitation and the orbital angular momentum corresponding to the state under consideration.

| I | $q \bar{q}$ |  |  | $q q \bar{q} \bar{q}$ |  | Experiment |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $n L$ | QM | Flavor | QM | Flavor | State | Mass |
| 1 | $1 P$ | 984 | $(n \bar{n})$ | 1308 | ( $n n \bar{n} \bar{n}$ ) | $a_{0}(980)$ | $984.7 \pm 1.2$ |
|  | $2 P$ | 1587 | $(n \bar{n})$ | 1522 | $(n s \bar{n} \bar{s})$ | $a_{0}(1450)$ | $1474 \pm 19$ |
| 0 | $1 P$ | 413 | $(n \bar{n})$ | - | - | $f_{0}(600)$ | 400-1200 |
|  | - | - | - | 949 | $(n n \bar{n} \bar{n})$ | $f_{0}(980)$ | $980 \pm 10$ |
|  | $1 P$ | 1340 | $(s \bar{s})$ | - | - | $f_{0}(1200-1600)$ | $1400 \pm 200$ |
|  | $2 P$ | 1395 | $(n \bar{n})$ | - | - | $f_{0}(1370)$ | 1200-1500 |
|  | - | - | - | 1525 | $(n s \bar{n} \bar{s})$ | $f_{0}(1500)$ | $1507 \pm 5$ |
|  | $3 P$ | 1754 | $(n \bar{n})$ | - | - | $f_{0}(1710)$ | $1714 \pm 5$ |
|  | - | - | - | - | - | $f_{0}(1790)$ | $1790_{-30}^{+40}$ |
|  | $2 P$ | 1894 | ( $s \bar{s}$ ) | - | - | $f_{0}(2020)$ | $1992 \pm 16$ |
|  | - | - | - | 1915 | $(s s \bar{s} \bar{s})$ | $f_{0}(2100)$ | $2103 \pm 17$ |
|  | $3 P$ | 2212 | $(s \bar{s})$ | - | - | $f_{0}(2200)$ | $2197 \pm 17$ |
| 1/2 | $1 P$ | 1213 | $(n \bar{s})$ | - | - | $K_{0}^{*}(800)$ | $\approx 800$ |
|  | $2 P$ | 1768 | $(n \bar{s})$ | 1295 | $(n n \bar{n} \bar{s})$ | $K_{0}^{*}(1430)$ | $1412 \pm 6$ |
|  | $3 P$ | 2046 | $(n \bar{s})$ | 1802 | $(n s \bar{s} \bar{s})$ | $K_{0}^{*}(1950)$ | $1945 \pm 22$ |

TABLE III. Mass (QM), in MeV, and flavor dominant component (Flavor) of the light isoscalar, isovector and $\mathrm{I}=1 / 2$ mesons, mixing two- and four-quark states as explained in the text.

| $\mathrm{I}=0$ |  |  | $\mathrm{I}=1$ |  |  | $\mathrm{I}=1 / 2$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| QM | Flavor | Mass | QM | Flavor | Mass | QM | Flavor | Mass |
| 568 | $(n \bar{n})$ | 400-1200 | 985 | $(n \bar{n})$ | $984.7 \pm 1.2$ | 1113 | $(n \bar{s})$ | $\approx 800$ |
| 999 | $(n n \bar{n} \bar{n})$ | $980 \pm 10$ | $\begin{aligned} & 1381 \\ & 1530 \end{aligned}$ | $\left.\begin{array}{r} (n n \bar{n} \bar{n}) \\ (n s \bar{n} \bar{s}) \end{array}\right\}$ | $1474 \pm 19$ | 1440 | $(n n \bar{n} \bar{s})$ | $1412 \pm 6$ |
| 1301 | $(s \bar{s})$ | $1400 \pm 200$ | 1640 | $(n \bar{n})$ | - |  |  |  |
| 1465 | $(n \bar{n})$ | 1200-1500 | 1868 | $(n \bar{n})$ | - |  |  |  |
| 1614 | $(n s \bar{n} \bar{s})$ | $1507 \pm 5$ |  |  |  | 1784 1831 2060 | $\left.\begin{array}{c}(n \bar{s}) \\ (n s \bar{s} \bar{s}) \\ (n \bar{s})\end{array}\right\}$ | $1945 \pm 20$ |
| - | - | $1714 \pm 5$ |  |  |  |  |  |  |
| 1782 | $(n \bar{n})$ | $1790_{-30}^{+40}$ |  |  |  |  |  |  |
| 1900 | $(s \bar{s})$ | $1992 \pm 16$ |  |  |  |  |  |  |
| 1944 | $(s s \bar{s} \bar{s})$ | $2103 \pm 17$ |  |  |  |  |  |  |
| 2224 | $(s \bar{s})$ | $2197 \pm 17$ |  |  |  |  |  |  |

TABLE IV. Flavor wave function components of some of the light scalar mesons. Masses are given in MeV and the probabilities in $\%$.

| $\mathrm{I}=0$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| QM | 568 | 999 | 1301 | 1465 | 1614 |
| $\mathrm{P}(n n \bar{n} \bar{n})$ | 15 | 82 | $\sim 1$ | $\sim 1$ | <1 |
| $\mathrm{P}(n s \bar{n} \bar{s})$ | $\sim 1$ | $\sim 1$ | 23 | $\sim 1$ | 72 |
| $\mathrm{P}\left(n \bar{n}_{1 P}\right)$ | 77 | 9 | 14 | $\sim 1$ | <1 |
| $\mathrm{P}\left(s \bar{s}_{1 P}\right)$ | 7 | 7 | 58 | 4 | 22 |
| $\mathrm{P}\left(n \bar{n}_{2 P}\right)$ | $<1$ | $\sim 1$ | $\sim 1$ | 84 | 5 |
| $\mathrm{P}\left(s \bar{s}_{2 P}\right)$ | $<1$ | $<1$ | <1 | 9 | <1 |
| $\mathrm{P}(s s \bar{s} \bar{s})$ | $<1$ | $<1$ | $\sim 1$ | $<1$ | $\sim 1$ |


| $\mathrm{I}=1$ |  |  | $\mathrm{I}=1 / 2$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| QM | 985 | 1381 | 1530 | QM | 1113 | 1440 |
| $\mathrm{P}(n n \bar{n} \bar{n})$ | 21 | 75 | $\sim 1$ | $\mathrm{P}(n n \bar{n} \bar{s})$ | 46 | 53 |
| $\mathrm{P}(n s \bar{n} \bar{s})$ | $\sim 1$ | 4 | 90 | $\mathrm{P}\left(n \bar{s}_{1 P}\right)$ | 53 | 44 |
| $\mathrm{P}\left(n \bar{n}_{1 P}\right)$ | 78 | 18 | 3 | $\mathrm{P}\left(n \bar{s}_{2 P}\right)$ | $<1$ | $<1$ |
| $\mathrm{P}\left(n \bar{n}_{2 P}\right)$ | $<1$ | 3 | 6 | $\mathrm{P}(n s \bar{s} \bar{s})$ | $<1$ | $\sim 1$ |

TABLE V. Mass, in MeV, and flavor dominant component of the light isoscalar mesons. The mixing with the glueball component is included as explained in the text.

| State | Exp. | Mass | Flavor |
| :---: | :---: | :---: | :---: |
| $f_{0}(600)$ | $400-1200$ | 568 | $\left(n \bar{n}_{1 P}\right)$ |
| $f_{0}(980)$ | $980 \pm 10$ | 999 | $(n n \bar{n} \bar{n})$ |
| $f_{0}(1200-1600)$ | $1400 \pm 200$ | 1299 | $\left(s \bar{s}_{1 P}\right)$ |
| $f_{0}(1370)$ | $1200-1500$ | 1406 | $\left(n \bar{n}_{2 P}\right)$ |
| $f_{0}(1500)$ | $1507 \pm 5$ | 1611 | $(n s \bar{n} \bar{s})$ |
| $f_{0}(1710)$ | $1714 \pm 5$ | 1704 | $($ glueball $)$ |
| $f_{0}(1790)$ | $1790_{-30}^{+40}$ | 1782 | $\left(n \bar{n}_{3 P}\right)$ |
| $f_{0}(2020)$ | $1992 \pm 16$ | 1902 | $(s s \bar{s} \bar{s})$ |
| $f_{0}(2100)$ | $2103 \pm 17$ | 1946 | $\left(s \bar{s}_{2 P}\right)$ |
| $f_{0}(2200)$ | $2197 \pm 17$ | 2224 | $\left(s \bar{s}_{3 P}\right)$ |

## FIGURES

FIG. 1. Regge trajectories for the isoscalar mesons. The squares represent the results of Table V. The lower solid line corresponds to $n \bar{n}$ systems and the upper line to $s \bar{s}$ systems. The dashed lines correspond to the mass of those states with a high non $-q \bar{q}$ component.


Fig. 1


[^0]:    ${ }^{1}$ Quoted as $f_{0}(1300)$ in the original work.
    ${ }^{2}$ Quoted as $f_{0}(1750)$ in the original work.

