Constituent quark model study of the meson spectra

J. Vijande, F. Fernández, A. Valcarce
Nuclear Physics Group,
University of Salamanca,
Plaza de la Merced s/n, E-37008 Salamanca, Spain

Abstract

The $q\bar{q}$ spectrum is studied in a generalized constituent quark model constrained in the study of the NN phenomenology and the baryon spectrum. An overall good fit to the available experimental data is obtained. A detailed analysis of all sectors from the light-pseudoscalar and vector mesons to bottomonium is performed paying special attention to the existence and nature of some non well-established states. These results should serve as a complementary tool in distinguishing conventional quark model mesons from glueballs, hybrids or multiquark states.

Keywords:
nonrelativistic quark models, meson spectrum, scalar mesons

Pacs:
12.39.Jh, 14.40.-n

I. INTRODUCTION

Meson spectroscopy is an extremely broad subject covering from the few hundred MeV masses of the light pseudoscalar mesons to the 10 GeV scale of the $b\bar{b}$ system. Such a wide energy region allows to address perturbative and nonperturbative phenomena of the underlying theory. The continuously increasing huge amount of data and its apparent simplicity made mesons ideal systems to learn about the properties of QCD. As an exact solution of the theory seems not attainable at present and the big effort done in developing its lattice approximation is still not able to produce, without guidance, realistic results, phenomenological models are the most important theoretical tool to study the meson properties. Last years are being extremely exciting due to the new data obtained at the B factories, reopening the interest of classifying the meson experimental data as $q\bar{q}$ states according to $SU(N)$ irreducible representations. The recently measured $D^*_{sJ}(2317)$ and $X(3872)$ states seem to be hardly accommodated in a pure $q\bar{q}$ description. The $D^*_{sJ}(2317)$ presents a mass much lower than the prediction of potential models, whereas the $X(3872)$ is too light to be a $2P$ charmonium state and too heavy for a $1D$ state. Such discrepancies could be related with the structure of some of the light scalar mesons still not well established theoretically. To be able to understand the nature of new resonances it is important that we have a template against which to compare observed states with theoretical predictions.

Much has been learned during the last years about the structure and properties of QCD. The study of charmonium and bottomonium made clear that heavy-quark systems are properly described by nonrelativistic potential models reflecting the dynamics expected from QCD [1]. The *a priori* complicated light-meson sector was quite surprisingly well reproduced on its bulk properties by means of a universal one-gluon exchange plus a linear confining potential [2]. However, the dynamics of the light-quark sector is expected to be dominated by the nonperturbative spontaneous breaking of chiral symmetry, basic property of the QCD Lagrangian not satisfied by the first constituent quark model approaches to the light-meson sector. Chiral symmetry breaking suggests to divide the quarks into two different sectors: light quarks (u, d, and s) where the chiral symmetry is spontaneously broken, and heavy quarks (c and b) where the symmetry is explicitly broken. The origin of the constituent quark mass can be traced back to the spontaneous breaking of chiral symmetry and consequently constituent quarks should interact through the exchange of Goldstone bosons [3], these exchanges being essential to obtain a correct description of the NN phenomenology and the light baryon spectrum. Therefore, for the light sector hadrons can be described as systems of confined constituent quarks (antiquarks) interacting through gluons and boson exchanges, whereas for the heavy sector hadrons are systems of confined current quarks interacting through gluon exchanges. Concerning the other basic property of the theory, confinement, little analytical progress has been made. Lattice calculations indicate that the linear confining potential at short distances is screened due to pair creation making it flat at large distances [4].

A preliminary study of the light-meson spectra and their strong decays has already been done with an interacting potential constrained on the study of the NN system and the nonstrange baryon spectrum [5]. Our purpose is to generalize this work obtaining a quark-quark interaction that allows for a complete study of the meson spectra, from the light to the heavy sector. To find new physics, it is important that we test the quark model against known states to understand its strengths and weakness. To this end we shall begin in the next section discussing the theoretical ingredients of the constituent quark model. In Sec. III we perform a detail comparison of the predictions of our model with experiment from the light pseudoscalar and vector mesons to bottomonium. This will allow us to identify discrepancies between the quark model predictions and experiment that may signal physics beyond conventional hadron spectroscopy. We shall go over these puzzles to decide whether the discrepancy is most likely a problem with the model or the experiment, or whether it most likely signals some new physics. In particular the possible presence of four quark systems in the scalar meson sector will be addressed. Finally, in Sec. IV we summarize our most important findings.

II. SU(3) CONSTITUENT QUARK MODEL

Since the origin of the quark model hadrons have been considered to be built by constituent (massive) quarks. Nowadays it is widely recognized that the constituent quark mass appears because of the spontaneous breaking of the original $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale. The picture of the QCD vacuum as a dilute medium of instantons [6] explains nicely such a symmetry breaking, which is the most important nonperturbative phenomenon for hadron structure at low energies. Quarks interact with fermionic zero modes of the individual instantons in the medium and therefore the propagator of a light quark gets modified and quarks acquire a momentum dependent mass which drops to zero for momenta higher than the inverse of the average instanton size $\bar{\rho}$. The momentum dependent quark mass acts as a natural cutoff of the theory. In the domain of momenta $k < 1/\overline{\rho}$, a simple Lagrangian invariant under the chiral transformation can be derived as [6]

$$L = \overline{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - M U^{\gamma_5})\psi \tag{1}$$

being $U^{\gamma_5} = \exp(i\pi^a \lambda^a \gamma_5 / f_\pi)$. π^a denotes the pseudoscalar fields $(\vec{\pi}, K_i, \eta_8)$ with i=1,...,4, and M is the constituent quark mass. An expression of the constituent quark mass can be obtained from the theory, but it also can be parametrized as $M = m_q F(q^2)$ with

$$F(q^2) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{\frac{1}{2}} \tag{2}$$

where Λ determines the scale at which chiral symmetry is broken. Once a constituent quark mass is generated such particles have to interact through Goldstone modes. Whereas the Lagrangian $\overline{\psi}(i\gamma^\mu \partial_\mu - M)\psi$ is not invariant under chiral rotations, that of Eq. (1) is invariant since the rotation of the quark fields can be compensated renaming the bosons fields. U^{γ_5} can be expanded in terms of boson fields as,

$$U^{\gamma_5} = 1 + \frac{\mathrm{i}}{f_\pi}\gamma^5 \lambda^a \pi^a - \frac{1}{2f_\pi^2}\pi^a \pi^a + ... \tag{3}$$

The first term generates the constituent quark mass and the second one gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which can be simulated by means of a scalar exchange potential. Inserting Eqs. (2) and (3) in Eq. (1), one obtains the simplest Lagrangian invariant under the chiral transformation $SU(3)_L \otimes SU(3)_R$ with a scale dependent constituent quark mass, containing $SU(3)$ scalar and pseudoscalar potentials. The nonrelativistic reduction of this Lagrangian has been performed for the study of nuclear forces and will not be repeated here, although the interested reader can follow the particular steps in several theoretical works [7,8]. The different terms of the potential contain central and tensor or central and spin-orbit contributions that will be grouped for consistency. Therefore, the chiral part of the quark-quark interaction can be resumed as follows,

$$V_{qq}(\vec{r}_{ij}) \;=\; V_{qq}^{C}(\vec{r}_{ij}) \;+\; V_{qq}^{T}(\vec{r}_{ij}) \;+\; V_{qq}^{SO}(\vec{r}_{ij})\,, \tag{4}$$

where C stands for central, T for tensor, and SO for spin-orbit potentials. The central part presents four different contributions,

$$V_{qq}^{C}(\vec{r}_{ij}) \;=\; V_{\pi}^{C}(\vec{r}_{ij}) \;+\; V_{\sigma}^{C}(\vec{r}_{ij}) \;+\; V_{K}^{C}(\vec{r}_{ij}) \;+\; V_{\eta}^{C}(\vec{r}_{ij})\,, \tag{5}$$

being each interaction given by,

$$\begin{aligned}
V_{\pi}^{C}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_im_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2-m_\pi^2}m_\pi\left[Y(m_\pi\, r_{ij})-\frac{\Lambda_\pi^3}{m_\pi^3}Y(\Lambda_\pi\, r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\sum_{a=1}^{3}(\lambda_i^a\cdot\lambda_j^a)\,,\\
V_{\sigma}^{C}(\vec{r}_{ij}) &= -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2-m_\sigma^2}m_\sigma\left[Y(m_\sigma\, r_{ij})-\frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma\, r_{ij})\right]\,,\\
V_{K}^{C}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_im_j}\frac{\Lambda_K^2}{\Lambda_K^2-m_K^2}m_K\left[Y(m_K\, r_{ij})-\frac{\Lambda_K^3}{m_K^3}Y(\Lambda_K\, r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\sum_{a=4}^{7}(\lambda_i^a\cdot\lambda_j^a)\,,\\
V_{\eta}^{C}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_im_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2-m_\eta^2}m_\eta\left[Y(m_\eta\, r_{ij})-\frac{\Lambda_\eta^3}{m_\eta^3}Y(\Lambda_\eta\, r_{ij})\right](\vec{\sigma}_i\cdot\vec{\sigma}_j)\left[cos\theta_P(\lambda_i^8\cdot\lambda_j^8)-sin\theta_P\right]\,,
\end{aligned} \tag{6}$$

the angle θ_P appears as a consequence of considering the physical η instead the octet one. $g_{ch}=m_q/f_\pi$, the $\lambda's$ are the $SU(3)$ flavor Gell-Mann matrices. m_i is the quark mass and m_π, m_K and m_η are the masses of the $SU(3)$ Goldstone bosons, taken to be their experimental values. m_σ is determined through the PCAC relation $m_\sigma^2 \sim m_\pi^2 + 4\,m_{u,d}^2$ [9]. Finally, $Y(x)$ is the standard Yukawa function defined by $Y(x)=e^{-x}/x$.

There are three different contributions to the tensor potential,

$$V_{qq}^{T}(\vec{r}_{ij}) \;=\; V_{\pi}^{T}(\vec{r}_{ij}) \;+\; V_{K}^{T}(\vec{r}_{ij}) \;+\; V_{\eta}^{T}(\vec{r}_{ij})\,, \tag{7}$$

each term given by,

$$\begin{aligned}
V_{\pi}^{T}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12m_im_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2-m_\pi^2}m_\pi\left[H(m_\pi\, r_{ij})-\frac{\Lambda_\pi^3}{m_\pi^3}H(\Lambda_\pi\, r_{ij})\right]S_{ij}\sum_{a=1}^{3}(\lambda_i^a\cdot\lambda_j^a)\,,\\
V_{K}^{T}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12m_im_j}\frac{\Lambda_K^2}{\Lambda_K^2-m_K^2}m_K\left[H(m_K\, r_{ij})-\frac{\Lambda_K^3}{m_K^3}H(\Lambda_K\, r_{ij})\right]S_{ij}\sum_{a=4}^{7}(\lambda_i^a\cdot\lambda_j^a)\,,\\
V_{\eta}^{T}(\vec{r}_{ij}) &= \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12m_im_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2-m_\eta^2}m_\eta\left[H(m_\eta\, r_{ij})-\frac{\Lambda_\eta^3}{m_\eta^3}H(\Lambda_\eta\, r_{ij})\right]S_{ij}\left[cos\theta_P(\lambda_i^8\cdot\lambda_j^8)-sin\theta_P\right]\,,
\end{aligned} \tag{8}$$

being $S_{ij} = 3\,(\vec{\sigma}_i\cdot\hat{r}_{ij})(\vec{\sigma}_j\cdot\hat{r}_{ij}) - \vec{\sigma}_i\cdot\vec{\sigma}_j$ the quark tensor operator and $H(x) = (1+3/x+3/x^2)\,Y(x)$.

Finally, the spin-orbit potential only presents a contribution coming form the scalar part of the interaction,

$$V^{SO}_{qq}(\vec{r}_{ij}) = V^{SO}_{\sigma}(\vec{r}_{ij}) = -\frac{g^2_{ch}}{4\pi}\frac{\Lambda^2_\sigma}{\Lambda^2_\sigma - m^2_\sigma}\frac{m^3_\sigma}{2m_i m_j}\left[G(m_\sigma\, r_{ij}) - \frac{\Lambda^3_\sigma}{m^3_\sigma}G(\Lambda_\sigma\, r_{ij})\right]\vec{L}\cdot\vec{S} \tag{9}$$

where $G(x) = (1+1/x)\,Y(x)/x$. The chiral coupling constant g_{ch} is determined from the πNN coupling constant through

$$\frac{g^2_{ch}}{4\pi} = \left(\frac{3}{5}\right)^2\frac{g^2_{\pi NN}}{4\pi}\frac{m^2_{u,d}}{m^2_N} \tag{10}$$

what assumes that flavor $SU(3)$ is an exact symmetry only broken by the different mass of the strange quark.

This interaction, arising as a consequence of the instanton induced chiral symmetry breaking, gives rise, among other effects, to vector-pseudoscalar meson mass splitting and it also generates flavor mixing for the η mesons. This is the very same effect obtained by other instanton induced approaches as those based on the 't Hooft Lagrangian [10,11]. In the heavy-quark sector chiral symmetry is explicitly broken and the Goldstone boson exchange interaction is not active in such a way that we cannot reproduce the hyperfine splittings for heavy mesons. This is a common feature of other instanton induced interactions [12] unless a phenomelogical parametrization is done [13]. Beyond the chiral symmetry breaking scale one expects the dynamics being governed by QCD perturbative effects. They mimic the gluon fluctuations around the instanton vacuum and are taken into account through the one-gluon-exchange (OGE) potential. Such a potential nicely describes the heavy-meson phenomenology. Following de Rújula *et al.* [14] the OGE is a standard color Fermi-Breit interaction given by the Lagrangian,

$$L = i\sqrt{4\pi}\alpha_s\overline{\psi}\gamma_\mu G^\mu\lambda^c\psi \tag{11}$$

where λ^c are the $SU(3)$ color matrices, G^μ is the gluon field and α_s is the quark-gluon coupling constant. The nonrelativistic reduction of the OGE diagram in QCD for point-like quarks presents a contact term that, when not treated perturbatively, leads to collapse [15]. This is why one maintains the structure of the OGE, but the δ function is regularized in a suitable way. This regularization is justified based on the finite size of the constituent quarks and should be therefore flavor dependent [16]. As a consequence, the central part of the OGE reads,

$$V^C_{OGE}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s\,\vec{\lambda^c}_i\cdot\vec{\lambda^c}_j\left\{\frac{1}{r_{ij}} - \frac{1}{6m_i m_j}\vec{\sigma}_i\cdot\vec{\sigma}_j\,\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}\,r^2_0(\mu)}\right\} \tag{12}$$

where $r_0(\mu) = \hat{r}_0/\mu$, scaling with the reduced mass as expected for a coulombic system. Let us note that the nonrelativistic reduction of the one-gluon exchange diagram in QCD yields several spin-independent contributions [14] that have not been considered. As the OGE is an effective interaction we have included only the relevant different structures obtained from the nonrelativistic reduction and neglected the other terms that are supposed to be included in the fitted parameters [2].

The noncentral terms of the OGE present a similar problem. For point-like quarks they contain an $1/r^3$ term that in spite of its strength has been usually treated perturbatively. Once again the finite size of the constituent quarks allows for a regularization and therefore for an exact treatment of these contributions, obtaining tensor and spin-orbit potentials of the form,

$$
\begin{aligned}
V_{OGE}^{T}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_i m_j}\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}\left(\frac{1}{r_{ij}^2} + \frac{1}{3r_g^2(\mu)} + \frac{1}{r_{ij}\, r_g(\mu)}\right)\right] S_{ij}\,, \\
V_{OGE}^{SO}(\vec{r}_{ij}) &= -\frac{1}{16}\frac{\alpha_s}{m_i^2 m_j^2}\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3}\left(1 + \frac{r_{ij}}{r_g(\mu)}\right)\right] \times \\
&\qquad \left[\left((m_i+m_j)^2 + 2m_i m_j\right)(\vec{S}_+ \cdot \vec{L}) + \left(m_j^2 - m_i^2\right)(\vec{S}_- \cdot \vec{L})\right]\,,
\end{aligned} \tag{13}
$$

where $\vec{S}_\pm = \vec{S}_i \pm \vec{S}_j$, and $r_g(\mu) = \hat{r}_g/\mu$ presents a similar behavior to the scaling of the central term. The wide energy covered to describe the light, strange and heavy mesons requires an effective scale-dependent strong coupling constant [17] that cannot be obtained from the usual one-loop expression of the running coupling constant because it diverges when $Q \to \Lambda_{QCD}$. The freezing of the strong coupling constant at low energies studied in several theoretical approaches [18,19] has been used in different phenomenological models [20]. The momentum-dependent quark-gluon coupling constant is frozen for each flavor sector. For this purpose one has to determine the typical momentum scale of each flavor sector that, as explained in Ref. [21], can be assimilated to the reduced mass of the system. As a consequence, we use an effective scale-dependent strong coupling constant given by

$$
\alpha_s(\mu) = \frac{\alpha_0}{ln\left(\frac{\mu^2+\mu_0^2}{\Lambda_0^2}\right)}, \tag{14}
$$

where μ is the reduced mass of the $q\bar{q}$ system and α_0, μ_0 and Λ_0 are determined as explained in Sec. III. This equation gives rise to $\alpha_s \sim 0.54$ for the light-quark sector, a value consistent with the one used in the study of the nonstrange hadron phenomenology [5,22], and it also has an appropriate high Q^2 behavior, $\alpha_s \sim 0.127$ at the Z_0 mass [23]. In Fig. 1 we compare our parametrization to the experimental data [24,25]. We also show for comparison the parametrization obtained in Ref. [18] from an analytical model of QCD.

When Goldstone-boson exchanges are considered together with the OGE, the possibility of double counting emerges. This question connects directly with the nature of the pion, studied for a long time concluding its dual character as $q\bar{q}$ pair and Goldstone boson [26,27]. In first approximation, it should be reasonable to construct a theory in which chiral symmetry is retained in the Goldstone mode but the internal structure of the pion is neglected. This would be essentially a long-wave length approximation [27]. This may be the reason why the OPE generates contributions that are not obtained from the OGE, while the ρ and ω meson exchanges give rise to contributions already generated by the OGE [28]. Explicit studies on the literature about the double counting problem concluded that while the pion can be safely exchanged together with the gluon, the vector and axial mesons cannot [29].

Finally, any model imitating QCD should incorporate another nonperturbative effect, confinement, that takes into account that the only observed hadrons are color singlets. It remains an unsolved problem to derive confinement from QCD in an analytic manner. The only indication we have on the nature of confinement is through lattice studies, showing that $q\bar{q}$ systems are well reproduced at short distances by a linear potential. Such potential can be physically interpreted in a picture in which the quark and the antiquark are linked with a one-dimensional color flux tube. The spontaneous creation of light-quark pairs may give rise to a breakup of the color flux tube. It has been proposed that this translates into a screened potential [4], in such a way that the potential does not rise continuously but it saturates at some interquark distance. Although string breaking has not been definitively confirmed through lattice calculations [30], a quite rapid crossover from a linear rising to a flat potential is well established in SU(2) Yang-Mills theories [31]. A screened potential simulating these results can be written as,

$$V_{CON}^{C}(\vec{r}_{ij}) = \{-a_c\,(1-e^{-\mu_c\,r_{ij}}) + \Delta\}(\vec{\lambda^c}_i \cdot \vec{\lambda^c}_j) \tag{15}$$

where Δ is a global constant fixing the origin of energies. At short distances this potential presents a linear behavior with an effective confinement strength $a = a_c\,\mu_c\,\vec{\lambda^c}_i \cdot \vec{\lambda^c}_j$, while it becomes constant at large distances. Such screened confining potentials provide with an explanation to the missing state problem in the baryon spectra [32] and also to the deviation of the Regge trajectories from the linear behavior for higher angular momentum states [33].

One important question which has not been properly answered is the covariance property of confinement. While the spin-orbit splittings in heavy-quark systems suggest a scalar confining potential [34], Ref. [35] showed that the Dirac structure of confinement is of vector nature in the heavy-quark limit of QCD. On the other hand, a significant mixture of scalar and vector confinement has been used to explain the decay widths of P-wave D mesons [36]. Nonetheless, analytic techniques [37] and numerical studies using lattice QCD [38] have shown that the confining forces are spin independent apart from the inevitable spin-orbit pseudoforce due to the Thomas precession [39]. Therefore, we will consider a confinement spin-orbit contribution as an arbitrary combination of scalar and vector terms,

$$\begin{aligned} V_{CON}^{SO}(\vec{r}_{ij}) = -(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\,\frac{a_c\,\mu_c\,e^{-\mu_c\,r_{ij}}}{4m_i^2m_j^2r_{ij}}\Big[\Big((m_i^2+m_j^2)(1-2\,a_s) \\ +4m_im_j(1-a_s)\Big)\,(\vec{S}_+\cdot\vec{L}) + (m_j^2-m_i^2)(1-2\,a_s)(\vec{S}_-\cdot\vec{L})\Big] \end{aligned} \tag{16}$$

where a_s would control the ratio between them.

Once perturbative (one-gluon exchange) and nonperturbative (confinement and chiral symmetry breaking) aspects of QCD have been considered, one ends up with a quark-quark interaction of the form (from now on we will refer to a light quark, u or d, as n, s will be used for the strange quark and Q for the heavy quarks c and b):

$$V_{q_iq_j} = \begin{cases} q_iq_j = nn \Rightarrow V_{CON} + V_{OGE} + V_\pi + V_\sigma + V_\eta \\ q_iq_j = ns \Rightarrow V_{CON} + V_{OGE} + V_\sigma + V_K + V_\eta \\ q_iq_j = ss \Rightarrow V_{CON} + V_{OGE} + V_\sigma + V_\eta \\ q_iq_j = nQ \Rightarrow V_{CON} + V_{OGE} \\ q_iq_j = QQ \Rightarrow V_{CON} + V_{OGE} \end{cases} \tag{17}$$

The corresponding $q\bar{q}$ potential is obtained from the qq one as detailed in Ref. [40]. In the case of $V_K(\vec{r}_{ij})$, where G parity is not well defined, the transformation is given by $\lambda_1^a \cdot \lambda_2^a \to \lambda_1^a \cdot (\lambda_2^a)^T$, which recovers the standard change of sign in the case of the pseudoscalar exchange between two nonstrange quarks.

Apart from models incorporating the one-gluon exchange potential for the short-range part of the interaction, one finds in the literature other attempts to study hadron phenomenology based on instanton induced forces. In Ref. [41] a flavor antisymmetric quark-quark instanton induced interaction was derived. It was used in a nonrelativistic framework for the study of the two-baryon system and the baryon spectrum, but it was never applied to study the meson spectrum. Such a model does not contain Goldstone boson exchanges, which are essential to make contact with the one and two baryon systems. The short-ranged instanton induced force was supplemented by a baryonic meson exchange potential to give a quantitative explanation of experimental data. Moreover, in Ref. [42] it was demonstrated that this instanton induced force reproduces the baryon spectrum as well as the one-gluon exchange. The other extensive work using instanton induced interactions is that of the group of Bonn [43,44]. The interaction is derived in Ref. [43] obtaining a sum of contact terms that are regularized (as we have done for the OGE potential) and whose flavor mixing matrix elements are fitted to experimental data. This interaction was supplemented by a phenomenological confining potential and applied in a Bethe-Salpeter framework to study the light meson and baryon spectra. This instanton induced force is only valid for light quarks in such a way that the extension of the model to study heavy flavors is done in a completely phenomenological way [13]. Therefore, these instanton induced models do not allow for the moment for a coherent study of the light and heavy flavors neither for a simultaneous description of the baryon-baryon phenomenology. These two approaches, the one based on the 't Hooft interaction and the one followed in the present work, clearly shares the most important features of the quark-quark interaction. The reason why we adopt the first scheme lies in the fact that it includes explicitly Goldstone boson exchanges between quarks which are essential to make contact with the one and two-baryon phenomenology.

III. RESULTS AND DISCUSSION

Let us first discuss how the parameters of the model are fixed. Most of them, as for example those of the Goldstone boson fields, are taken from calculations on two-baryon systems. Once the Goldstone boson exchange part of the interaction has been determined, the one-gluon exchange controls the hyperfine splittings. It is worth to notice the relevance of the OGE contribution for the description of the meson spectra, as has already been emphasized in Ref. [5]. The conclusion of this work, that the $\rho - \pi$ mass difference cannot be obtained from the Goldstone-boson exchanges alone is fully maintained. For the sake of completeness let us mentioned that in the present model this mass difference would be around 38 MeV if the OGE is not used. As it is also illustrated in Ref. [5] the dependence of the $\rho - \pi$ mass difference on the Goldstone-boson cut-off masses is small, in such a way that the spectra come never determined by the election of the cut-off masses. We have fixed the OGE parameters by a global fit to the hyperfine splittings well established by the Particle Data Group (PDG) [45] from the light to the heavy-quark sector. The strength of confinement determines the energy difference between any J^{PC} meson ground state and its radial excitations. We have fitted a_c and μ_c in order to reproduce two well measured energy differences: the ρ meson and its first radial excitation and the J/ψ and the $\psi(2S)$, obtaining values close to those inferred in Ref. [46] through the analysis of the screening of QCD suggested by unquenched lattice calculations in the heavy-quark sector. Finally, it only remains to fix the relative strength of scalar and vector confinement. For this purpose one has to look for some states where the spin-orbit contribution, being important, can be easily isolated from other effects as for example flavor mixing. This is the case of the isovector mesons $a_1(1260)$ and $a_2(1320)$. In Fig. 2 we have plotted their masses as a function of a_s, the relative strength of scalar and vector confinement. As can be seen, using a strict scalar confining potential, $a_s = 1$, one would obtain 1343 MeV and 1201 MeV, respectively, in complete disagreement with the ordering and magnitude of the experimental data. Introducing a mixture of vector confinement, a_s=0.777, the experimental order is recovered being now the masses 1205 MeV and 1327 MeV, respectively, both within the experimental error bars. This value also allows to obtain a good description of the experimental data in the $c\bar{c}$ and $b\bar{b}$ systems. The parameters of the model are resumed in Table I.

Acceptable results for the meson spectra have been provided by relativistic as well as nonrelativistic approaches [48]. In both cases a QCD-inspired interaction is used, the difference being in quarks masses and kinematics. In fact, several works in the literature [49] have studied the connections existing between relativistic, semirelativistic, and nonrelativistic potential models of quarkonium using an interaction composed of an attractive Coulomb potential and a confining power-law term. The spectra of these very different models become nearly similar provided specific relations exist between the dimensionless parameters peculiar to each model.

As a consequence we will solve the Schrödinger equation for the relative motion of the $q\bar{q}$ pair with the interacting potential of Eq. (17). There is a particularly simple and efficient method for integrating this type of second order differential equations, the commonly called Numerov method [50]. The noncentral potentials (tensor and spin-orbit) give their most important contribution for the diagonal terms. For example for the isovector states $J^{PC} = 1^{--}$ ($L = 0$ or 2, $S = 1$, $J = 1$) we would have the following matrix elements (in MeV):

$$\begin{pmatrix} <^3 S_1|H|^3S_1 >= 772 & <^3 S_1|H|^3D_1 >= 22 \\ <^3 D_1|H|^3S_1 >= 22 & <^3 D_1|H|^3D_1 >= 1518 \end{pmatrix} , \tag{18}$$

and for the isovector $J^{PC} = 2^{++}$ ($L = 1, 3$, $S = 1$, $J = 2$)

$$\begin{pmatrix} <^3 P_2|H|^3P_2 >= 1327 & <^3 P_2|H|^3F_2 >= 5 \\ <^3 F_2|H|^3P_2 >= 5 & <^3 F_2|H|^3F_2 >= 1797 \end{pmatrix} , \tag{19}$$

where the matrix elements are calculated with the wave functions solution of the single-channel Schrödinger equation for the total hamiltonian, including central and noncentral terms. The perturbative effect of the nondiagonal contributions of the noncentral potentials is observed. The importance of the diagonal contributions can be easily inferred from the energies obtained when the different interactions are connected. In table II we give the energies obtained when solving the Schrödinger equation for two different partial waves: one of the cases discussed above, the isovector $^3D_1$, and the isoscalar $^3P_0$ $(n\overline{n})$. One can see that the corrections induced by the diagonal contributions of the noncentral potentials are much important than the nondiagonal ones. This effect becomes clearly nonperturbative for the isoscalar $^3P_0$ state, that on the other hand does not admit coupling to partial waves with different orbital angular momentum. Therefore the noncentral potentials will be treated exactly in their diagonal contributions and by diagonalizing the corresponding matrix when nondiagonal contributions are present. The same reasoning as before applies in the case of the spin-orbit contribution proportional to S_- as will be discussed in Sect. III H.

In Tables III to XII we compare the masses obtained within the present model to experimental data [45]. Tables III, IV, and V do not include the scalar mesons which will be discussed separately in Sec. III L and are shown in Table XII. In all cases we show the name and mass of the experimental state and the prediction of our model together with the J^{PC} quantum numbers, or J^P if C parity is not well defined (in these cases we have explicitly indicated the spin). If there is an experimental meson without assignment of quantum numbers, those indicated by question marks, and we find a corresponding state in our model, we indicate between square brackets its quantum numbers. In the tables we also include in parenthesis the radial excitation and the orbital angular momentum corresponding to the state under consideration.

A final comment about the flavor mixing angles is in order. Our interacting hamiltonian needs as input the pseudoscalar octet-singlet mixing angle (see Eq. (6)). For this angle, values in the range of -10^o to -23^o have been obtained depending on the analysis performed [45,51–56]. We have taken an intermediate value of -15^o. The model provides a theoretical mixing $n\bar{n} \leftrightarrow s\bar{s}$ ($\omega \leftrightarrow \phi$, $\eta \leftrightarrow \eta'$, ...) as a consequence of the V_K potential. The mixing angle obtained is $\theta_V = 34.7^o$ for the $\omega \leftrightarrow \phi$ case, and $\theta_P = -21.7^o$ for the $\eta \leftrightarrow \eta'$. We want to emphasize the independence of our result on the input value for θ_P. For example if we had used a theoretical input of $\theta_P = -23^o$, our predicted pseudoscalar mixing angle would have been $\theta_P = -21.9^o$, almost the same as before. Our result is compatible with many others reported in the literature [52,54,56] and close to the θ_P^{lin} value given on the PDG [45] ($\theta_P^{lin} = -23^o$), as expected because we use a mixing formula based on linear and not quadratic masses. The mixing angle obtained for most meson nonets is approximately ideal, exceptions are the nonet of pseudoscalar and scalar mesons. The mixing angle, calculated for the ground state of each J^{PC} nonet, is assumed for the radial excitations with the same quantum numbers.

There are a number of states whose quantum numbers are not clearly determined or do not seem to present a clear correspondence with a $q\bar{q}$ state. Let us analyze in detail the results.

A. $I = 0$ $J^{PC} = 0^{-+}$ states

There is an overall good agreement between the constituent quark model results and the experimentally observed mesons. Recently there has been a modification in the experimental situation. In the 2002 PDG [71] there were two resonances in the 1.5 GeV energy region, $\eta(1295)$ and $\eta(1440)$. However, the 2004 PDG reports three states, $\eta(1295)$, $\eta(1405)$ and

$\eta(1475)$, all of them interpreted as pseudoscalar 0^{-+} states. This modification was mainly due to a recent experiment by the E852 Collaboration [72]. They found evidence for three pseudoscalar resonances: $\eta(1295)$, $\eta(1416)$ and $\eta(1485)$ in the analysis of the reaction $\pi^- p \to K^+ K^- \pi^0 n$. The first one decays exclusively into $a_0\pi^0$, the second into $a_0\pi^0$ and $K^*\bar{K}$ and the last one exclusively into $K^*\bar{K}$. We only obtain two 0^{-+} states in this energy region, one at 1290 MeV and the other at 1563 MeV, both being predominantly $n\bar{n}$. Based on our results one would identify the first state with the $\eta(1295)$. However, our model predicts only one state corresponding either to $\eta(1405)$ or $\eta(1475)$. Although the energy seems to favor an assignment of our second state with the $\eta(1475)$, its dominant $n\bar{n}$ component makes difficult to explain its experimental decay modes ($K^*\bar{K}$). However, the expectation of the $\eta(1475)$ being a dominantly $s\bar{s}$ $2\,^1S_0$ (sometimes the discussion is more clear using the spectroscopic notation $n\,^{2S+1}L_J$) state [73] as suggested by the observation of a large $K^*\bar{K}$ decay mode is highly dependent on the strong KK final state interaction [74]. The existence of the third resonance would therefore imply the presence of additional states beyond the two obtained in the quark model. However the experimental situation is not definitively settled. There are some speculations that one of these states could be somehow related to the $f_1(1420)$ [75]. To disentangle the flavor content of the $\eta(1405)$ and $\eta(1475)$ could be a very important experimental contribution feasible at CLEO, what would help to discriminate among different theoretical models.

To illustrate the uncertain situation with these resonances let us finally mention that the analysis of the $K\bar{K}\pi$ and $\eta\pi\pi$ channels in $\gamma\gamma$ collision performed in Ref. [76] observed the $\eta(1475)$ in $K\bar{K}\pi$, but not the $\eta(1405)$ in $\eta\pi\pi$. Since gluonium production is presumably suppressed in $\gamma\gamma$ collisions this result suggests that the $\eta(1405)$ may have a large glueball component [77]. This interpretation, however, is not favored by lattice calculations, which predict the 0^{-+} state above 2.5 GeV.

B. $I = 0$ $J^{PC} = 1^{++}$ states

The historically confused experimental status of light axial vectors has improved a lot with high statistic central production experiments on $\eta\pi\pi$, $K\,K\pi$ and 4π by WA102 [78] and $K\,K\pi$ by E690 [79]. In these experiments very clearly $f_1(1285)$ and $f_1(1420)$ states have been observed, but there is no evidence of $f_1(1510)$. Although the $f_1(1420)$ and the $f_1(1510)$ are well separated in mass and resolved in different experiments, there are no experiment or production reactions in which both states have been detected. All these observations express skepticism regarding the existence of the $f_1(1510)$ [80]. In view of their masses the obvious assumption is that the $f_1(1285)$ is the $1\,^3P_1$ $n\bar{n}$ state and the $f_1(1420)$ its $s\bar{s}$ partner. In Ref. [80] a significant singlet-octet mixing for the nonet of the $f_1(1285)$ and $f_1(1420)$ has been obtained from several independent analysis, $\theta \sim 50^o$. We obtain a value of 46.3^o in complete agreement with the former study. One should remember that the mixing in our model is fully determined by the structure of the potential.

C. $I = 0$ $J^{PC} = 2^{-+}$ states

Regarding the $I = 0$ 2^{-+} the situation seems to be clear. Experimentally there are two states: the $\eta_2(1645)$ with a mass of 1617±5 MeV and decaying into $a_2(1320)\pi$, and the $\eta_2(1870)$ with a mass of 1842±8 and decaying into $a_2(1320)\pi$ and 4π. We obtain three theoretical states in this energy region: the first one with a mass of 1600 MeV and pure light-quark content, a second with a mass of 1853 MeV and pure $s\bar{s}$ content and a third state with a mass of 1863 MeV and also pure light-quark content. The assignment of our first state to the $\eta_2(1645)$ seems clear. With respect to the $\eta_2(1870)$ its reported decay modes are not accessible in the ${}^3P_0$ model having a pure $s\bar{s}$ content. This enforces to assign the $\eta_2(1870)$ to the second excited state with light-quark content. Thus, it may exist an unobserved resonance close to the $\eta_2(1870)$ being a pure $s\bar{s}$ state.

D. $I = 0$ $J^{PC} = 1^{--}$ and $J^{PC} = 3^{--}$ states

In the PDG there are three isoscalar 1^{--} states in the 1.5 GeV energy region: $\omega(1420)$, $\omega(1650)$ and $\phi(1680)$. The first and third state seem to be fairly well established, however in the last two years there have been several modifications on the mass of the $\omega(1650)$. The 2002 PDG gave a mass of 1649 ± 24 MeV, the 2003 electronic version reported a mass between $1600 - 1800$ MeV, while the 2004 PDG quotes a value of 1670±30 MeV. This modification is based on the analysis of the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ [81] where two ω states are reported, the first one with a mass of 1490 ± 75 MeV decaying into $\pi^+\pi^-\pi^0$ and the second with a mass of 1790 ± 50 and decaying, with approximately the same probability, into 3π and $\omega\pi\pi$. While the first state clearly corresponds to the $\omega(1420)$, the second state was included by the electronic 2003 PDG as an experimental result for the $\omega(1650)$, increasing drastically the error bars. In the 2004 PDG this experiment has not been considered within the statistical fit, therefore reducing again the error bars. A similar situation may happen with the $\phi(1680)$. The FOCUS Collaboration at Fermilab [82] has reported a high statistic study of the diffractive photoproduction of K^+K^- confirming the existence of a clear enhancement of the cross section corresponding with a fitted mass of 1753 ± 4 MeV.

Theoretically we find three 1^{--} states in this energy region: two almost pure light-quark states with masses of 1444 MeV and 1784 MeV, and an almost pure $s\bar{s}$ state with a mass of 1726 MeV. Based on the aforementioned discussion, the first two states may correspond to the $\omega(1420)$ and $\omega(1650)$, whereas the third one would correspond to the $\phi(1680)$. Our analysis also assigns the $\omega(1650)$ to a $3\,{}^3S_1$ state instead of a $1\,{}^3D_1$ wave [73], that would have a mass of 1475 MeV with a pure light-quark content, in complete disagreement with the experimental data.

For the 3^{--} case we obtain two almost degenerate states with completely different flavor content. Experimentally there is a clear evidence that the $\phi_3(1850)$ has a dominant $s\bar{s}$ flavor content, and therefore it would correspond to our $s\bar{s}$ state at 1875 MeV, predicting the existence of an unobserved $n\bar{n}$ 3^{--} state in the same energy region.

E. $I = 0$ $J^{PC} = 2^{++}$ states

Experimentally there is a proliferation of isoscalar 2^{++} states in an energy region that has been suggested as coexisting with 2^{++} glueballs. Theoretically the results of our model may confirm that the $f_2(1270)$ and $f_2'(1525)$ are the $n\bar{n}$ and $s\bar{s}$ members of the $1\,^3P_2$ $q\bar{q}$ flavor nonet. Although $n\bar{n} \leftrightarrow s\bar{s}$ mixing is present due to the kaon exchange, the $n\bar{n}$ content of the $f_2'(1525)$ is smaller than 0.1% in agreement with the experimental $f_2'(1525)\gamma\gamma$ coupling, which limits the $n\bar{n}$ content to a few percent [74]. Our model also finds that the $f_2(1565)$, observed in $p\bar{p}$ annihilation at rest, and the $f_2(1640)$, decaying into $\omega\omega$ and 4π, could be the same state, the $n\bar{n}$ member of the $2\,^3P_2$ $q\bar{q}$ flavor nonet as is suggested in the PDG. The $f_2(1950)$ would be its $s\bar{s}$ partner in agreement with the experimentally observed decays. Finally the $f_2(1810)$ corresponds to the $n\bar{n}$ member of the $1\,^3F_2$ nonet and the $f_2(1910)$ to the $n\bar{n}$ member of the $3\,^3P_2$ nonet, as expected from the decay patterns.

In this sector there is a meson, the $f_2(1430)$, that has no equivalent in our $q\bar{q}$ scheme. This state is not confirmed in the PDG and even recent measurements have suggested a different assignment of quantum numbers, being a 0^{++} state [83], what could make it compatible with the lightest scalar glueball [84]. It was seen together with another resonance, the $f_2(1480)$, in an experiment where the $f_2(1270)$ was not detected [85]. A full understanding of the nature of the $f_2(1430)$ will probably also require an explanation for the absence of the $f_2(1270)$ in the experimental data of Ref. [85].

F. $I = 0$ $J^{PC} = 1^{+-}$ states

While the $n\bar{n}$ $1\,^3P_1$ quark model prediction, 1257 MeV, differs slightly from its corresponding experimental state, $f_1(1285)$, the $n\bar{n}$ $1\,^1P_1$ is a little bit far from the expected corresponding experimental state, $h_1(1170)$. Annihilation contributions could improve the agreement with data, having in mind that they are expected to be negative and larger in the $S = 0$ light-quark sector while almost negligible for $S = 1$ states [2]. There are some estimations of the contribution of annihilation in the light-quark sector, but its dependence on fitted parameters prevents from making any definitive conclusion. The $h_1(1380)$ it is a convincing candidate for the $s\bar{s}$ partner of the $1\,^1P_1$ $h_1(1170)$. Although a first glance to the quark model result seems to indicate that the mass is too high, the most recent experimental measurement for this state reports a mass of 1440 ± 60 [86]. The last measured 1^{+-} state, the $h_1(1595)$, will correspond to the $n\bar{n}$ $2\,^1P_1$ state, its $s\bar{s}$ partner being an unobserved meson around 1973 MeV as also noticed from the analysis of strange decays in Ref. [74].

G. $I = 1$ mesons

The results shown in Table IV present an almost perfect parallelism between the theoretical predictions and the experimentally observed states. The only not well defined correspondence comes from the $\rho(1700)$, which is compatible with an S or D wave. Our calculation indicates a mass for this state a little bit higher than the one reported in the PDG and compatible with the observations of Ref. [87].

H. $I = 1/2$ strange mesons

From a theoretical perspective strange mesons exhibit explicit flavor and therefore they do not present the additional complications of annihilation mixing what makes the isoscalar mesons a priori much more complicated.

There is a good correspondence between the 0^- and 1^- quark model results and the experimental mesons. In particular, the $K(1630)$, whose quantum numbers have not been yet determined, would correspond to the $2\,{}^3S_1$ $n\bar{s}$ state. The only problem in this sector appears for the $K^*(1410)$, in clear disagreement with the predictions of the quark model. The $K^*(1410)$ could be the radial excitation of the $K^*(892)$, because it is the first 1^- vector resonance observed. However, the first $n\bar{s}$ radial excitation is predicted at 1620 MeV. This is reasonable taking into account that in the $S = 1$ light-quark sector the radial excitation has a mass around 700 MeV above the ground state, what confirms that the first radial excitation of the $K^*(892)$ should be around 1600 MeV. This is why we assign in Table V our 1620 MeV state to the $K(1630)$. It would be interesting for future experiments to check the quantum numbers of this state. With respect to the $K^*(1410)$ its assignment to the $2\,{}^3S_1$ state is not only excluded by its mass, but also by its decay modes. A possible interpretation of the low mass of this state due to the mixing with hybrids has been suggested in Ref. [74] and therefore seems to be excluded as a pure $q\bar{q}$ pair.

The 2^+ states are also in reasonable agreement with the theoretical predictions. This can be attributed to the understanding of the $(1P)\,2^{++}$ and $(1F)\,2^{++}$ isoscalar light-quark sector. The only difference is the mass of the strange quark in the interacting potential what makes clear the $n\bar{s}$ structure of $K_2^*(1430)$ and $K_2^*(1980)$.

Strange mesons are the lighter ones where a mixing of the 1^+ states with $S = 0$ and $S = 1$ can occur (also the 2^- states) due to the fact that C parity is not a good quantum number. Although there is not theoretical consensus about the origin of such a mixing, in our model it is induced by the spin-orbit contributions of the OGE and confinement potentials proportional to $\vec{S}_-$. As a consequence the physical states would be given by

$$
\begin{aligned}
\mid K_1^*\rangle &= \cos\theta \mid 1^+(S=0)\rangle - \sin\theta \mid 1^+(S=1)\rangle \\
\mid K_1\rangle &= \sin\theta \mid 1^+(S=0)\rangle + \cos\theta \mid 1^+(S=1)\rangle
\end{aligned} \tag{20}
$$

In the literature this mixing angle has been estimated by different methods. In Ref. [88] it was calculated from the ratio $B(\tau \to \nu K_1(1270))/B(\tau \to \nu K_1(1400))$, obtaining a value $\theta \sim +62^o$. In Ref. [74] an angle $\theta \sim +45^o$ was calculated from the pattern of the decay branching fractions of the two experimental states, $K_1(1270)$ and $K_1(1400)$. HQET [89] gives two possible mixing angles, $\theta \sim +35.3^o$ and $\theta \sim -54.7^o$. We obtain a mixing angle of $\theta \sim 55.7^o$ and therefore a mass for the physical states of 1352 MeV and 1414 MeV, improving the agreement with the experiment with respect to the unmixed masses. The mixing angle is close to the preferred value by the experimental decays and far from the results of Ref. [90] in a relativized quark model based only on OGE and confinement potentials. Their mixing angle $\theta \sim +5^o$ seems to be definitively excluded by the experimental decays.

The same mixing occurs for the $(2P)\,1^+$ states, giving two resonances around 1.85 GeV, one would correspond to the $K_1(1650)$ and the other to an unobserved state. Although this identification could seem inconsistent, the PDG states that the $K_1(1650)$ entry contains various peaks reported in partial-wave analysis in the 1600−1900 mass region. The election of a mass of 1650 MeV is done based on three measurements, one of 1650 ± 50 MeV and two around 1840 MeV. The same identification for $K_1(1650)$ together with the prediction of a $2P$ tensor state $K_2^*(1850)$ has been done in Ref. [74] from the analysis of experimental data on strong decays.

We have included in Table V a state given in the PDG as $K_2(1580)$ that has no equivalent in the $q\bar{q}$ spectrum. This state is clearly uncertain, it was reported in only one experimental work more than twenty years ago and has never been measured again.

I. D and D_s mesons

Recently, a clear evidence for the existence of new open-charm mesonic states has been reported by three different collaborations, two states with charm-strange quark content and other with charm-light quark content.

BaBar Collaboration reported a narrow state at 2316.8±0.5 MeV [62] called $D_{sJ}^*(2317)$. CLEO Collaboration [64,66,91] provided confirmation of the existence of this state and furthermore reported the observation of a new state called $D_{sJ}(2463)$. In return, BaBar experiment also confirmed the existence of this state [63]. Finally both discoveries have been confirmed by the Belle Collaboration, not only in the analysis of inclusive e^+e^- annihilation [67], but also in the exclusive B meson decays [68]. All the experimental observations are consistent with the assignment of P-wave states with spin-parity $J^P = 0^+$ for the $D_{sJ}^*(2317)$ and $J^P = 1^+$ for the $D_{sJ}(2463)$. Belle Collaboration [59] has also reported the existence of a $J^P = 0^+$ charm-light state with a mass of 2308±36 MeV.

There are two intriguing aspects of the new D mesons. First of all they have a mass significantly smaller than the predictions of most QCD inspired quark potential models regarding these mesons as P-wave bound states, $c\bar{s}$ and $c\bar{n}$ respectively. Secondly they do not show the decay patterns favored by theoretical expectations. This has triggered a variety of articles either supporting the $q\bar{q}$ interpretation or presenting alternative hypothesis of exotic states.

Our results are shown in Tables VI and VII. In Table VI we have only included those states reported by the PDG, while in Table VII we include all $1P$ excited states. As can be seen the low-lying 0^- and 1^- D and D_s states are perfectly reproduced. We obtain a theoretical state with the mass of the $D^*(2640)$ and spin-parity $J^P = 0^-$, and also a state with a similar mass to the $D_{sJ}(2573)$ and $J^P = 2^+$. However there is a discrepancy both with the $J^P = 1^+$ states and specially with the $J^P = 0^+$ states. In the case of the 1^+ states the same mixing as discussed in Eq. (20) between the $^1P_1$ and $^3P_1$ partial waves appears. As a consequence, for the charm-light sector we obtain two 1^+ states with masses of 2454 MeV and 2535 MeV, with a mixing angle of 43.5^o, and for the charm-strange case 2543 MeV and 2571 MeV, with a mixing angle of 58.4^o. Therefore, while we would find a candidate for the $D_{s1}(2536)$ and the $D_1(2420)$, the recently measured $D_{sJ}(2463)$ and its equivalent state in the charm-light sector (see Table VII) do not fit into the predictions of the nonrelativistic $q\bar{q}$ models [92].

The situation with the 0^+ states being in principle more discouraging is, however, similar to the problem observed for the light-scalar mesons (see Sec. III L), they hardly fit in a

$q\bar{q}$ scheme. As seen in Table VII the quark model predictions for the 0^+ D and D_s are around 150 MeV above the experimental results. In a pure $q\bar{q}$ scheme the 0^+ states are influenced by the noncentral terms of the interaction, in particular they strongly depend on the spin-orbit force and therefore on the scalar/vector rate in the potential, controlled by a_s. As this relation was fixed in the light-meson sector, where the potential has a much more involved structure (Goldstone boson exchanges), we wonder if the disagreement could be solved by modifying the scalar/vector rate of confinement. In Table XIII we present the results obtained by fixing a_s to reproduce the experimental mass of one of the new measured D_s mesons, the $D_{sJ}^*(2317)$. Apart from obtaining a completely unusual value for the scalar/vector rate, $a_s = 0.46$ [35,36], in doing this one observes how the situation of the charm-light sector does not improve, what makes evident the rather complicated situation appeared with the new measurements.

Let us finally note that in a $q\bar{q}$ scheme the 0^+ states are obtained through an orbital angular momentum excitation, what could explain the large masses predicted. The 0^+ quantum numbers may also be obtained in the absence of orbital excitation considering more complex structures [93] which can help to understand the experimental situation.

In the literature the new $(0^+, 1^+)$ states have triggered other alternative explanations as for example that they could be the members of the $J^P = (0^+, 1^+)$ doublet predicted by the heavy quark effective theory (HQET) [94] or that these states together with the lowest $(0^-, 1^-)$ constitute the chiral doublet of the $(0^+, 1^+)$ HQET spin multiplet [95].

J. Charmonium and the new states

We present in Table VIII our results for charmonium. Confinement parameters were fitted to describe the energy difference between the $S = 1$ ground state, $J/\psi(1S)$, and its first radial excitation, $\psi(2S)$. One also observes a pretty good description of the first negative parity orbital excitation, $\psi(3770)$. The splitting among the χ_{cJ} states is correctly given both in order and magnitude. With respect to the $S = 0$ sector, as happened for the light-quark systems, the orbital excitation is expected to be 600 MeV above the ground state, in complete agreement with the mass of the $h_c(1P)$. This state, although not yet confirmed is a clear candidate for being the 1^{+-} resonance. There is also a set of L =even $S = 1$ states with an obvious correspondence to $q\bar{q}$ states although in some cases the S or $D-$wave identification is not unique, as seen on Table VIII for the $\psi(4415)$.

Recently the Belle Collaboration has reported the observation of a narrow peak that has been interpreted as the $2S$ singlet charmonium state, the $\eta_c(2S)$,

$$M[\eta_c(2S)] = \begin{cases} 3654 \pm 10 \text{ MeV Ref. [96]} \\ 3622 \pm 8 \text{ MeV Ref. [68]} \\ 3630 \pm 8 \text{ MeV Ref. [67]} \end{cases} .$$

These masses are larger than the experimental value quoted by the 2002 PDG: $M[\eta_c(2S)] = 3594 \pm 5$ MeV and it was pointed out that cannot be easily explained in the framework of constituent quark models. The reason for that stems on the $2S$ hyperfine splitting (HFS) that would be smaller than the predicted for the $1S$ ones. In fact the theoretical predicted $2S/1S$ HFS ratio is

$$R = \frac{\Delta M_{2S}^{HFS}}{\Delta M_{1S}^{HFS}} = \begin{cases} 0.84 \text{ Ref. [97]} \\ 0.67 \text{ Ref. [98]} \\ 0.60 \text{ Ref. [99]} \end{cases} ,$$

larger than the experimental value, R=0.273 if $M[\eta_c(2S)] = 3654$ MeV, R=0.547 for 3622 MeV, and R=0.479 for 3630 MeV. Our result is $M[\eta_c(2S)] = 3627$ MeV, within the error bar of the last two Belle measurements, the ones obtained with higher statistics. Moreover the ratio $2S/1S$ HFS is found to be 0.537, in agreement with these last two experimental data. The reason for this agreement can be found in the radial structure of the confining potential that also influences the HFS, the linear confinement being not enough flexible to accommodate both excitations [46]. A similar agreement has also been obtained by other theoretical models including the effect of the open thresholds [47] (note that the result of Ref. [2] was far from the available experimental data at that time).

Finally HFS is closely connected with the leptonic decay widths $V(\text{vector meson}) \to e^+e^-$. Although their absolute values depend on radiative and relativistic corrections, the ratios are a test of the wave functions at the origin, and still closely connected to HFS. We obtain $\Gamma_{e^+e^-}[\psi(2S)]/\Gamma_{e^+e^-}[J/\psi(1S)]$ =0.44, in good agreement with the experimental value 0.41±0.07, what gives us confidence about the correct description of the $\eta_c(2S)$.

The most recently discovered charmonium state is the $X(3872)$, reported by Belle [100] in the $J/\Psi\pi^+\pi^-$ invariant mass distribution of the $B^\pm \to K^\pm J/\psi\pi^+\pi^-$ reaction, and confirmed by the CDF collaboration at Fermilab [101]. Both experiments report a similar mass, $3872.0 \pm 0.6 \pm 0.5$ MeV in Ref. [100] and $3871.4 \pm 0.7 \pm 0.4$ MeV in Ref. [101], very close to the $D^0 D^{*0}$ threshold (3871.5 ± 0.5 MeV).

The proposed interpretations for the $X(3872)$ include 1^3D_3, 1^3D_2, 1^1D_2, 2^3P_1 and 2^1P_1 $c\bar{c}$ states. None of them comfortably fit the observed properties of this state and therefore a considerable experimental uncertainty still remains. The results of our model for the possible quantum numbers are shown in Table IX. Although the well-established 1^3D_1 is well reproduced by our model the predicted $1D$ candidates lie 70−80 MeV below its experimental mass whereas the $2P$ states are 40 MeV above. These results clearly show that the theoretical splitting in the ^3D_J multiplet is much smaller [∼25 MeV] than the one necessary to correctly described the $X(3872)$ and the $\Psi(3770)$ [∼100 MeV]. Similar results have been found in other potential models which suggests that the $X(3872)$ could present a more involved structure [102,103].

K. Bottomonium and open beauty states

A pretty good description of the experimental states shown in Tables X and XI is obtained. Some caution is necessary with respect of some of the experimental data reported in Table XI. The most important discrepancy observed between the data and our results arise from the $\eta_b(1S)$, all the other states being described with similar accuracy to any other spectroscopic model designed to study a particular sector [46]. However, the result for this state is based in only one experimental work where only one event has been observed [45], and therefore this data needs confirmation. The uncertainty on this data can be easily understood from the surprisingly large spin splitting that it would produce: $m_\Upsilon - m_{\eta_b} = 160$

MeV. Such an energy difference clearly spoils the evolution of the spin splittings with the mass of the quarks: $m_\rho - m_\pi = 630$ MeV, $m_{K^*} - m_K = 397$ MeV, $m_{D^*} - m_D = 146$ MeV, $m_{J/\Psi} - m_{\eta_c} = 117$ MeV, $m_{B^*} - m_B = 46$ MeV, $m_\Upsilon - m_{\eta_b} = 160$ MeV. The S or D assignment of 1^{--} high energy $b\bar{b}$ excitations is not conclusive. Let us mention again that with the noncentral terms of our interaction a correct description of the hyperfine splittings both in order and magnitude is obtained. For the open-beauty mesons (Table X) we find theoretical states corresponding to $B_J^*(5732)$ and $B_{sJ}^*(5850)$, in both cases being $J^P = 2^+$.

L. $J^{PC} = 0^{++}$ states

It is still not clear which are the members of the 0^{++} nonet corresponding to the $L = S = 1$ $n\bar{n}$ and $s\bar{s}$ multiplets. There are too many 0^{++} mesons observed in the region below 2 GeV to be explained as $q\bar{q}$ states. There have been reported in the PDG two isovectors: $a_0(980)$ and $a_0(1450)$; five isoscalars: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$; and three $I = 1/2$: $K_0^*(1430)$, $K_0^*(1950)$ and recently $\kappa(800)$. The quark model predicts the existence of one isovector, two isoscalars and two $I = 1/2$ states for each nonet. Our results are shown in Table XII. Using this table one can try to assign the physical states to 0^{++} nonet members.

Let us discuss each state separately. With respect to the isovector states, there is a candidate for the $a_0(980)$, the $^3P_0$ member of the lowest ^3P_J isovector multiplet. The other candidate, the $a_0(1450)$, is predicted to be the scalar member of the $2\,^3P_J$ excited isovector multiplet. This reinforces the predictions of the quark model, the spin-orbit force making lighter the $J = 0$ states with respect to the $J = 2$. The assignment of the $a_0(1450)$ as the scalar member of the lowest ^3P_J multiplet [104] would contradict this idea, because the $a_2(1320)$ is well established as a $q\bar{q}$ pair. The same behavior is evident in the $c\bar{c}$ and $b\bar{b}$ spectra, making impossible to describe the $a_0(1450)$ as a member of the lowest ^3P_J isovector multiplet without spoiling the description of heavy-quark multiplets. However, in spite of the correct description of the mass of the $a_0(980)$, the model predicts a pure light-quark content, what seems to contradict some experimental observations [105]. The $a_0(1450)$ is predicted to be also a pure light-quark structure obtaining a mass somewhat higher than the experiment.

In the case of the isoscalar states, we find a candidate for the $f_0(600)$ with a strangeness content around 10%. There are no $I = 0$ states with a mass close to 1 or 1.5 GeV, which would correspond to the $f_0(980)$ and the $f_0(1500)$, and they cannot be found for any combination of the model parameters. It seems that a different structure rather than a naive $q\bar{q}$ pair is needed. In particular, the $f_0(1500)$ is a clear candidate for the lightest glueball [106], while the $f_0(980)$ has been suggested as a possible four-quark state [107,108] what would make it compatible with the similar branching ratios observed for the $J/\psi \to f_0(980)\phi$ and $J/\psi \to f_0(980)\omega$ decays [105]. Concerning the $f_0(1370)$ (which may actually correspond to two different states [109]) we obtain two states around this energy, the heavier one with a dominant nonstrange content which favors its assignment to the $f_0(1370)$; the other with a high $s\bar{s}$ content without having an experimental partner. Let us however remember that in this energy region there is a state, the $f_2(1430)$, that does not fit into the isoscalar 2^{++} sector and has been recently suggested as a possible 0^{++} state [83]. Finally a dominant

$n\bar{n}$ state corresponding to the $f_0(1710)$ is obtained. Our results concluding that $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ are dominantly $n\bar{n}$, non $q\bar{q}$, and $n\bar{n}$ respectively differ from the conclusion of Refs. [106,110] obtaining that $f_0(1710)$ is dominantly $s\bar{s}$ and are also in contrast to the predictions of Ref. [111] which prefers to assign $f_0(1500)$ to an $s\bar{s}$ state and $f_0(1710)$ to a glueball. This makes clear the complicated situation in the scalar sector with several alternative interpretations of the observed states. The study of radiative transitions and two photon decay widths should help to understand the flavor mixing and the nature of the $I = 0$ scalar sector. We obtain two states around 1.9 GeV, a $2\,^3P_0$ state with a dominant $s\bar{s}$ content and a $4\,^3P_0$ with a dominant $n\bar{n}$ content. Being the $f_0(2020)$ an experimentally known $n\bar{n}$ meson, its identification with the $4\,^3P_0$ state is clear. Therefore, we find an unobserved $s\bar{s}$ scalar with a mass around 1.9 GeV as has also been suggested in Ref. [74]. Finally, although for consistency not given in the tables, we find a candidate for the $3\,^3P_0$ state $f_0(2200)$, experimentally identified as an $s\bar{s}$ state [112], with an energy of 2212 MeV.

Concerning the $I = 1/2$ sector, as a consequence of the larger mass of the strange quark as compared to the light ones, our model always predicts a mass for the lowest 0^{++} state 200 MeV greater than the $a_0(980)$ mass. Therefore, being the $a_0(980)$ the member of the lowest isovector scalar multiplet, the $\kappa(800)$ cannot be explained as a $q\bar{q}$ pair. We find a candidate for the $K_0^*(1430)$ although with a smaller mass.

Our results indicate that the light-scalar sector cannot be described in a pure $q\bar{q}$ scheme and more complicated structures or mixing with multiquark system seems to be needed. Concerning the $f_0(980)$ and the $\kappa(800)$ our conclusions are very similar to those obtained in Ref. [113] using the extended Nambu-Jona-Lasinio model in an improved ladder approximation of the Bethe-Salpeter equation. This seems to indicate that relativistic corrections would not improve the situation and the conclusions remain model independent.

One finds in the literature several alternative explanations to understand the rather complicated scenario of the scalar mesons. An earlier attempt to link the understanding of the NN interactions with meson spectroscopy was done based on the Jülich potential model [114]. The structure of the scalar mesons $a_0(980)$ and $f_0(980)$ was investigated in the framework of a meson exchange model for $\pi\pi$ and $\pi\eta$ scattering. The $K\bar{K}$ interaction generated by the vector-meson exchange, which for isospin $I = 0$ is strong enough to generate a bound state is much weaker for $I = 1$, making a degeneracy of $a_0(980)$ and $f_0(980)$ impossible, as found in our model. Although both scalar mesons result from the coupling to the $K\bar{K}$ channel explaining in a natural way their similar properties, the underlying structure obtained was, however, quite different. Whereas the $f_0(980)$ appears to be a $K\bar{K}$ bound state the $a_0(980)$ was found to be a dynamically generated threshold effect.

In a different fashion within the quark model the same problem was illustrated in Ref. [115]. The bare mass used for the $n\bar{n}$ pair is much larger than the $a_0(980)$ and $f_0(980)$ experimental masses. It is the effect of the two-pseudoscalar meson thresholds the responsible for the substantial shift to a lower mass than what is naively expected from the $q\bar{q}$ component alone. This gives rise to an important $K\bar{K}$ and $\pi\eta'$ components in the $a_0(980)$ and $K\bar{K}$, $\eta\eta$, $\eta'\eta'$ and $\eta\eta'$ in the $f_0(980)$. In particular for the $a_0(980)$ they obtain the $K\bar{K}$ component to be dominant near the peak, being about $4 - 5$ times larger than the $q\bar{q}$ component. A similar conclusion, that the description of the $a_0(980)$ and $f_0(980)$ requires from more complex structures, is also obtained from our analysis. The absence of the $f_0(1500)$ in our $q\bar{q}$ scheme make also contact with the indication that this state could correspond to the lightest scalar glueball [116].

A similar problem as the one observed with the light scalars appeared in the open charm sector. As we have already discussed, the two recently measured 0^+ states do not fit into a $q\bar{q}$ description. Possible alternatives to understand their masses, as for example being DK molecules [117] or tetraquarks [107], have been suggested. Using the same interacting hamiltonian presented in this work an estimation of the lowest scalar open charm tetraquark has been done in Ref. [108]. The variational estimation was performed under the assumption that internal orbital angular momentum does not give an important contribution, what has been strictly tested in the case of heavy-light tetraquarks [118], but it is well known to influence light-quark systems. Having in mind this precaution a mass of 2389 MeV was obtained for the light open-charm $[(ns)(\bar{n}\bar{c})]$ tetraquark that could very well correspond to the $D^*_{sJ}(2317)$. A consistent calculation of four light-quark structures seems to be advocated for a full understanding of the scalar sector.

IV. SUMMARY

We have performed an exhaustive study of the meson spectra from the light $n\bar{n}$ states to the $b\bar{b}$ mesons within the same model. The quark-quark interaction takes into account QCD perturbative effects by means of the one-gluon-exchange potential and the most important nonperturbative effects through the hypothesis of a screened confinement and the spontaneous breaking of chiral symmetry. The model incorporates in a natural way the $n\bar{n} \leftrightarrow s\bar{s}$ isoscalar mixing due to the V_K potential, and the 1^+ $S=0$ and $S=1$ mixing caused by the spin-orbit force. An arbitrary rate of scalar/vector confinement has been used, finding evidence of a strong scalar component. Annihilation effects have not been taken into account although their contribution seems to be important for the description of the 1^{+-} light isoscalar ground state. We have obtained a reasonable description of most part of the well established $q\bar{q}$ states for all flavors. The success of the model allowed us to make predictions with respect to those states whose quantum numbers, existence or nature is under debate.

In the light-isoscalar sector our results support the speculation pointed out by the PDG about the possible nonexistence of the $f_1(1420)$. We do not find a theoretical 2^{++} state corresponding to the $f_2(1430)$, in agreement with recent experiments that opened the possibility of a different assignment of quantum numbers for this meson being a 0^{++} state. Our model predicts a scalar meson in this energy region without an experimental partner. Besides, we have only found one $q\bar{q}$ state in the 1.6 GeV region, which seems to indicate that either the $f_2(1565)$ and the $f_2(1640)$ are the same state as suggested by the PDG, or that the $f_2(1565)$ goes beyond the naive $q\bar{q}$ structure. In the light-scalar sector it seems very difficult to accommodate the $f_0(980)$, the $\kappa(800)$ and the $f_0(1500)$ in a $q\bar{q}$ scheme. In the light-strange sector we do not find a $q\bar{q}$ state to be identified with the $K^*(1410)$, favoring its possible hybrid structure. The same situation occurs for the $K_2(1580)$, although in this case the poor experimental data do not assure its existence.

Concerning the flavor content of the $\eta(1440)$ our model indicates that it is a dominant $n\bar{n}$ state, with a probability of 70.3 %. The $\phi_3(1875)$ is compatible with an $s\bar{s}$ content but there should exist an unobserved $n\bar{n}$ state in the same energy region. We have also found evidence of the existence of a 1^{+-} light-isoscalar meson with a dominant $s\bar{s}$ content and a mass around 1.97 GeV, and an isoscalar $s\bar{s}$ 2^{+-} state at 1.85 GeV. Finally, our model assigns the quantum numbers 1^- to $K(1630)$.

For the heavy-quark sector the experimental situation is changing very fast. New experiments and reanalysis of old data are being done and new states being discovered. Some of them fit nicely in a $q\bar{q}$ scheme, but others are impossible to accommodate. In the open charm sector, the $D^*(2640)$ and $D_{sJ}(2573)$ are compatible with $c\bar{n}$ and $c\bar{s}$ mesons with quantum numbers 0^- and 2^+, respectively. However, the recently discovered 0^+ states, the $D^*_{sJ}(2317)$ and $D_J(2308)$, seem to have a completely different structure. We have argued a possible explanation to describe their low masses based in a tetraquark structure that may have positive parity without orbital excitation. Finally, there is an obvious identification of a $q\bar{q}$ state for the $h_c(1P)$ with quantum numbers 1^{+-}.

We consider that this type of study based on models whose parameters are constrained in the description of other low-energy systems should be a complementary useful tool to deepen the understanding of the meson spectra. The next step in this effort to a comprehensive description of the new data concerning the meson spectra should be the analysis of the electroweak and strong decays of mesons that will be the subject of a future publication [119].

V. ACKNOWLEDGMENTS

This work has been partially funded by Ministerio de Ciencia y Tecnología under Contract No. BFM2001-3563, by Junta de Castilla y León under Contract No. SA-104/04.

REFERENCES

[1] E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975).
[2] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[3] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).
[4] G.S. Bali, Phys. Rep. **343**, 1 (2001).
[5] L.A. Blanco, F. Fernández, and A. Valcarce, Phys. Rev. C **59**, 428 (1999); R. Bonnaz, L.A. Blanco, B. Silvestre-Brac, F. Fernández, and A. Valcarce, Nucl. Phys. A **683**, 425 (2001).
[6] D.I. Diakonov and V.Yu. Petrov, Nucl. Phys. B **245**, 259 (1984); *ibid* **272**, 457 (1986).
[7] S. Moszkowski, *Nuclear Physics* (New York: Gordon and Breach, 1968) p. 3.
[8] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[9] M.D. Scadron, Phys. Rev. D **26**, 239 (1982).
[10] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
[11] C.R. Münz, J. Resag, B.C. Metsch, and H.R. Petry, Nucl. Phys. A **578**, 418 (1994).
[12] T. Schäfer and E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[13] D. Merten, R. Ricken, M. Koll, B.C. Metsch, and H.R. Petry, Eur. Phys. J. A **13**, 477 (2002).
[14] A. de Rújula, H. Georgi, and S.L. Glashow, Phys. Rev. D **12**, 147 (1975).
[15] R.K. Bhaduri, L.E. Cohler, and Y. Nogami, Phys. Rev. Lett. **44**, 1369 (1980).
[16] J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983).
[17] S. Titard and F.J. Yndurain, Phys. Rev. D **51**, 6348 (1995); A.M. Badalian and V.L. Morgunov, *ibid* **60**, 116008 (1999); A.M. Badalian and B.L.G. Bakker, *ibid* **62**, 094031 (2000).
[18] D.V. Shirkov and I.L. Solovtsov, Phys. Rev. Lett. **79**, 1209 (1997).
[19] A.M. Badalian and Yu.A. Simonov, Yad. Fiz. **60**, 714 (1997).
[20] A.C. Mattingly and P.M. Stevenson, Phys. Rev. D **49**, 437 (1994); A.M. Badalian and D.S. Kuzmenko, Phys. Rev. D **65**, 016004 (2001).
[21] F. Halzen, C. Olson, M.G. Olsson and M.L. Stong, Phys. Rev. D **47**, 3013 (1993).
[22] B. Juliá-Díaz, J. Haidenbauer, A. Valcarce, and F. Fernández, Phys. Rev. C **65**, 034001 (2002); H. Garcilazo, A. Valcarce, and F. Fernández, Phys. Rev. C **64**, 058201 (2001); D.R. Entem, F. Fernández, and A. Valcarce, Phys. Rev. C **62**, 034002 (2000).
[23] C.T.H. Davies *et al.*, Phys. Rev. D **56**, 2755 (1997).
[24] S. Kluth, hep-ex/0309070.
[25] L3 Collaboration, P. Achard *et al.*, Phys. Lett. B **536**, 217 (2002).
[26] W. Weise, Int. Rev. Nucl. Phys. **1**, 58 (1984), p. 137.
[27] A.W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984), p. 55.
[28] F. Fernández, A. Valcarce, P. González, and V. Vento, Phys. Lett. B **287**, 35 (1992).
[29] K. Yazaki, Prog. Part. Nucl. Phys. **24**, 353 (1990).
[30] G.S. Bali *et al.*, Phys. Rev. D **62**, 054503 (2000).
[31] P.W. Stephenson, Nucl. Phys. B **550**, 427 (1999). F. Knechtli and R. Sommer, Nucl. Phys. B **590**, 309 (2000). O. Philipsen and H. Wittig, Phys. Lett. B **451**, 146 (1999). P. de Forcrand and O. Philipsen, Phys. Lett. B **475**, 280 (2000).
[32] J. Vijande, P. González, H. Garcilazo and A. Valcarce, Phys. Rev. D **69**, 074019 (2004).
[33] M.M. Brisudová, L. Burakovsky, and T. Goldman, Phys. Rev. D **61**, 054013 (2000).

[34] W. Lucha, F.F. Schöberl, and D. Gromes, Phys. Rep. **200**, 127 (1991).
[35] A.P. Szczepaniak and E.S. Swanson, Phys. Rev. D **55**, 3987 (1997).
[36] J. Sugiyama, S. Mashita, M. Ishida, and M. Oka, hep-ph/0306111.
[37] N. Brambilla and A. Vairo, Nucl. Phys. B (Proc. Supp.) **64**, 418 (1998), and references therein.
[38] G.S. Bali, A. Wachter, and K. Schilling, Phys. Rev. D **56**, 2556 (1997), and references therein.
[39] N. Isgur, Phys. Rev. D **62**, 014025 (2000); nucl-th/0007008.
[40] J. Burger, R. Müller, K. Tragl, and H.M. Hofmann, Nucl. Phys. A **493**, 427 (1989).
[41] M. Oka and S. Takeuchi, Phys. Rev. Lett. **63**, 1780 (1989).
[42] E.V. Shuryak and J.L. Rosner, Phys. Lett. B **218**, 72 (1989).
[43] W.H. Blask, U. Bohn, M.G. Huber, B.C. Metsch, and H.R. Petry, Z. Phys. A **337**, 327 (1990).
[44] M. Koll, R. Ricken, D. Merten, B.C. Metsch, and H.R. Petry, Eur. Phys. J. A **9**, 73 (2000).
[45] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
[46] P. González, A. Valcarce, H. Garcilazo, and J. Vijande, Phys. Rev. D **68**, 034007 (2003), and references therein.
[47] E.J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D **69**, 094019 (2004).
[48] W. Lucha, F.F. Schröberl, and D. Gromes, Phys. Rep. **200**, 127 (1991).
[49] C. Semay and B. Silvestre-Brac, Phys. Rev. D **46**, 5177 (1992), and references therein.
[50] S.E. Koonin and D.C. Meredith, *Computational Physics*, (Addison-Wesley, New York, 1990).
[51] KLOE Collaboration, A. Aloisio *et al.*, Phys. Lett. B **541**, 45 (2002).
[52] Crystal Barrel Collaboration, C. Amsler *et al.*, Phys. Lett. B **294**, 451 (1992).
[53] F.J. Gilman and R. Kauffman, Phys. Rev. D **36**, 2761 (1987).
[54] Th. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999).
[55] A. Bramon, R. Escribano, and M.D. Scadron, Eur. Phys. J. C **7**, 271 (1999).
[56] L. Burakovsky and T. Goldman, Phys. Lett. B **427**,361 (1998).
[57] FOCUS Collaboration, F.L. Fabri *et al.*, hep-ex/0011044.
[58] FOCUS Collaboration, talk presented by R. K. Kutschke at the 5th Workshop on Heavy Flavours at Fixed Target, Rio de Janeiro, Brazil (2000).
[59] Belle Collaboration, K. Abe *et al.*, Phys. Rev. D **69**, 112002 (2004).
[60] CLEO Collaboration, S. Anderson *et al.*, Nucl. Phys. A **663**, 647 (2000).
[61] DELPHI Collaboration, talk presented by D. Bloch at the EPS International Conference on High Energy Physics, Budapest, Hungary (2001).
[62] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **90**, 242001 (2003).
[63] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. D **69**, 031101 (2004).
[64] CLEO Collaboration, D. Besson *et al.*, Phys. Rev. D **68**, 032002 (2003).
[65] CLEO Collaboration, talk presented by H. Vogel at the International Europhysics Conference on High Energy Physics, Aachen, Germany (2003).
[66] CLEO Collaboration, S. Stone and J. Urheim, hep-ph/0308166.
[67] Belle Collaboration, Y. Mikami *et al.*, Phys. Rev. Lett. **92**, 012002 (2004).
[68] Belle Collaboration, P. Krokovny *et al.*, Phys. Rev. Lett. **91**, 262002 (2003).

[69] CLEO Collaboration, S.E. Csorna *et al.*, hep-ex/0207060.
[70] E791 Collaboration, E.M. Aitala *et al.*, Phys. Rev. Lett. **89**, 121801 (2002).
[71] K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[72] E852 Collaboration, G.S. Adams *et al.*, Phys. Lett. B **516**, 264 (2001).
[73] V.V. Anisovich, AIP Conf. Proc. **717**, 441 (2004).
[74] T. Barnes, N. Black, and P.R. Page, Phys. Rev. D **68**, 054014 (2003).
[75] S. Godfrey and J. Napolitano, Rev. Mod. Phys. **71**, 1411 (1999).
[76] L3 Collaboration, M. Acciarri *et al*, Phys. Lett. B **501**, 1 (2001).
[77] D.M. Li, H. Yu and S.S. Fang, Eur. Phys. J. C **28**, 335 (2003).
[78] WA102 Collaboration, B. Barberis *et al.*, Phys. Lett. B **413**, 225 (1997); *ibid* **440**, 225 (1998).
[79] E690 Collaboration, M. Sosa *et al.*, Phys. Rev. Lett. **83**, 913 (1999).
[80] F.E. Close and A. Kirk, Z. Phys. C **76**, 469 (1997).
[81] M.N. Achasov *et al.*, Phys. Rev. D **66**, 032001 (2002).
[82] FOCUS Collaboration, J.M. Link *et al.*, Phys. Lett. B **545**, 50 (2002).
[83] V.V. Vladimirsky *et al.*, Phys. Atom. Nucl. **64**, 1895 (2001).
[84] C.J. Morningstar and M. Peardon, Phys. Rev. D **60**, 034509 (1999).
[85] AFS Collaboration, T. Akesson *et al.*, Nucl. Phys. B **264**, 154 (1986).
[86] A. Abele *et al.*, Phys. Lett. B **415**, 280 (1997).
[87] Crystal Barrel Collaboration, A. Abele *et al.*, Phys. Lett. B **391**, 191 (1997).
[88] J. Rosner, Com. Nucl. Part. Phys. **16**, 109 (1986).
[89] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989).
[90] S. Godfrey and R. Kokoski, Phys. Rev. D **43**, 1679 (1991).
[91] CLEO Collaboration, D. Besson *et al.*, hep-ex/0305017.
[92] W. Lucha and F.F. Schöberl, Mod. Phys. Lett. A **18**, 2837 (2003), and references therein.
[93] R.J. Jaffe, Phys. Rev. D **15**, 267 (1977).
[94] A. Deandrea, G. Nardulli, and A.D. Polosa, Phys. Rev. D **68**, 097501 (2003).
[95] W.A. Bardeen, E.J. Eichten, and C.T. Hill, Phys. Rev. D **68**, 054024 (2003).
[96] Belle Collaboration, S.-K. Choi *et al.*, Phys. Rev. Lett. **89**, 102001 (2002).
[97] D. Ebert, R.N. Faustov, and V.O. Galkin, Phys. Rev. D **62**, 034014 (2000).
[98] E.J. Eichten and C. Quigg, Phys. Rev. D **49**, 5845 (1994).
[99] R. Barbieri, R. Gatto, R. Kögerler, and Z. Kunszt, Phys. Lett. B **57**, 455 (1975).
[100] Belle Collaboration, S.-K.Choi *et al.*, Phys. Rev. Lett. **91**, 262001 (2003).
[101] CDF Collaboration, talk presented by G. Bauer at the Second International Workshop on Heavy Quarkonium, Fermilab, USA (2003).
[102] T. Barnes and S. Godfrey, Phys. Rev. D **69**, 054008 (2004).
[103] J. Vijande, talk presented at the MESON2004 Conference, Cracow, Poland (2004).
[104] P. Geiger and N. Isgur, Phys. Rev. Lett. **67**, 1066 (1991); F.E. Close and N.A. Törnqvist, J. Phys. G **28**, R249 (2002).
[105] N.N. Achasov, Nucl. Phys. A **675**, 279c (2000).
[106] C. Amsler, Phys. Lett. B **541**, 22 (2002).
[107] K. Terasaki, AIP Conf. Proc. **717**, 556 (2004).
[108] J. Vijande, F. Fernández and A. Valcarce, AIP Conf. Proc. **717**, 352 (2004).
[109] D. Black, A.H. Fariborz, and J. Schechter, Phys. Rev. D **61**, 074001 (2000).

[110] C. Amsler and F.E. Close, Phys. Rev. D **53**, 295 (1996).
[111] W. Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (2000) and references therein.
[112] A.V. Anisovich *et al.*, Nucl. Phys. A **662**, 319 (2000).
[113] T. Umekawa, hep-ph/0306040.
[114] G. Janssen, B.C. Pearce, K. Holinde and J. Speth, Phys. Rev. D **52**, 2690 (1995).
[115] N.A. Tornqvist, Z. Phys. C **68**, 647 (1995).
[116] M. Boglione and M.R. Pennington, Phys Rev Lett **79**, 1998 (1997).
[117] T. Barnes, F.E. Close, and H.J. Lipkin, Phys. Rev. D **68**, 054006 (2003).
[118] J. Vijande, F. Fernández, A. Valcarce, and B. Silvestre-Brac, Eur. Phys. J. A **19**, 383 (2004).
[119] J. Vijande, F. Fernández, and A. Valcarce, work in progress.

TABLES

TABLE I. Quark model parameters.

Quark masses	$m_u = m_d$ (MeV)	313
	m_s (MeV)	555
	m_c (MeV)	1752
	m_b (MeV)	5100
Goldstone bosons	m_π (fm^{-1})	0.70
	m_σ (fm^{-1})	3.42
	m_η (fm^{-1})	2.77
	m_K (fm^{-1})	2.51
	$\Lambda_\pi = \Lambda_\sigma$ (fm^{-1})	4.20
	$\Lambda_\eta = \Lambda_K$ (fm^{-1})	5.20
	$g_{ch}^2/(4\pi)$	0.54
	$\theta_P(^o)$	-15
Confinement	a_c (MeV)	430
	μ_c (fm^{-1})	0.70
	Δ (MeV)	181.10
	a_s	0.777
OGE	α_0	2.118
	Λ_0 (fm^{-1})	0.113
	μ_0 (MeV)	36.976
	$\hat{r}_0$ (MeV fm)	28.170
	$\hat{r}_g$ (MeV fm)	34.500

TABLE II. Energies (in MeV) obtained when solving the Schrödinger equation for two different partial waves when the different noncentral terms are switched on.

Potential	$^3D_1$ (I=1)	$^3P_0(n\overline{n})$ (I=0)
$V_{qq}^C(\vec{r}_{ij}) + V_{OGE}^C(\vec{r}_{ij}) + V_{CON}^C(\vec{r}_{ij})$	1602	1261
$+V_{qq}^T(\vec{r}_{ij})$	1598	1008
$+V_{OGE}^{SO}(\vec{r}_{ij})$	1474	225
$+V_{CON}^{SO}(\vec{r}_{ij})$	1508	335
$+V_{\sigma}^{SO}(\vec{r}_{ij})$	1518	500

TABLE III. Masses, in MeV, of $I = 0$ light-quark mesons up to 2 GeV. QM denotes the results of the present model and Flavor stands for the dominant component of the flavor wave function. Experimental data, PDG, are taken from Ref. [45]. In the second column we denote by a question mark those states whose existence is not clear. In the third column, if there are several candidates for an experimental state, we underline our preferred assignment. See text for details.

$(nL)\, J^{PC}$	State	QM	Flavor	PDG
$(1S)\, 0^{-+}$	η	572	$(n\bar{n})$	547.75±0.12
$(1S)\, 0^{-+}$	$\eta'(958)$	956	$(s\bar{s})$	957.8±0.1
$(2S)\, 0^{-+}$	$\eta(1295)$	1290	$(n\bar{n})$	1294±4
$(2S)\, 0^{-+}$	$\eta(1760)$	1795	$(s\bar{s})$	1760±11
$(3S)\, 0^{-+}$	$\begin{bmatrix}\eta(1405)\\ \eta(1475)\end{bmatrix}$	1563	$(n\bar{n})$	$\begin{bmatrix}1410.3 \pm 2.6\\ 1476 \pm 4\end{bmatrix}$
$(1D)\, 2^{-+}$	$\eta_2(1645)$	1600	$(n\bar{n})$	1617±5
$\begin{bmatrix}(1D)\, 2^{-+}\\ (2D)\, 2^{-+}\end{bmatrix}$	$\eta_2(1870)$	$\begin{bmatrix}1853\\ \underline{1863}\end{bmatrix}$	$\begin{bmatrix}(s\bar{s})\\ (n\bar{n})\end{bmatrix}$	1842±8
$(1S)\, 1^{--}$	$\omega(782)$	691	$(n\bar{n})$	782.54±0.11
$(1S)\, 1^{--}$	$\phi(1020)$	1020	$(s\bar{s})$	1019.46±0.02
$(2S)\, 1^{--}$	$\omega(1420)$	1444	$(n\bar{n})$	1400−1450
$(2S)\, 1^{--}$	$\phi(1680)$	1726	$(s\bar{s})$	1680±20
$(3S)\, 1^{--}$	$\omega(1650)$	1784	$(n\bar{n})$	1670±30
$(1D)\, 3^{--}$	$\omega_3(1670)$	1631	$(n\bar{n})$	1667±4
$\begin{bmatrix}(1D)\, 3^{--}\\ (2D)\, 3^{--}\end{bmatrix}$	$\phi_3(1850)$	$\begin{bmatrix}\underline{1875}\\ 1876\end{bmatrix}$	$\begin{bmatrix}\underline{(s\bar{s})}\\ (n\bar{n})\end{bmatrix}$	1854±7
$(1P)\, 1^{+-}$	$h_1(1170)$	1257	$(n\bar{n})$	1170±20
$(1P)\, 1^{+-}$	$h_1(1380)$	1511	$(s\bar{s})$	1386±19
$(2P)\, 1^{+-}$	$h_1(1595)$	1700	$(n\bar{n})$	1594^{+18}_{-62}
$(2P)\, 1^{+-}$		1973	$(s\bar{s})$	
$(1P)\, 2^{++}$	$f_2(1270)$	1311	$(n\bar{n})$	1275.4±1.2
$(1P)\, 2^{++}$	$f_2'(1525)$	1556	$(s\bar{s})$	1525±5
$(-)\, 2^{++}$	$f_2(1430)$	−	−	≈1430
$(2P)\, 2^{++}$	$\begin{bmatrix}f_2(1565)?\\ f_2(1640)\end{bmatrix}$	1725	$(n\bar{n})$	$\begin{bmatrix}1546 \pm 12\\ 1638 \pm 6\end{bmatrix}$
$(1F)\, 2^{++}$	$f_2(1810)$	1789	$(n\bar{n})$	1815±12
$(3P)\, 2^{++}$	$f_2(1910)$	1906	$(n\bar{n})$	1915±7
$(2P)\, 2^{++}$	$f_2(1950)$	1999	$(s\bar{s})$	1934±12
$(1P)\, 1^{++}$	$f_1(1285)$	1271	$(n\bar{n})$	1281.8±0.6
$(1P)\, 1^{++}$	$\begin{bmatrix}f_1(1420)?\\ f_1(1510)\end{bmatrix}$	1508	$(s\bar{s})$	$\begin{bmatrix}1426.3 \pm 1.1\\ 1518 \pm 5\end{bmatrix}$

TABLE IV. Masses, in MeV, of $I = 1$ light-quark mesons up to 2 GeV. QM denotes the results of the present model. Experimental data (PDG) are taken from Ref. [45]. We denote by a dagger those PDG states whose masses are not explicitly given. We use for them the more recent experimental data.

$(nL)\,J^{PC}$	State	QM	PDG
$(1S)\,0^{-+}$	π	139	139
$(2S)\,0^{-+}$	$\pi(1300)$	1288	1300±100
$(3S)\,0^{-+}$	$\pi(1800)$	1720	1812±14
$(1D)\,2^{-+}$	$\pi_2(1670)$	1600	1672.4±3.2
$(1S)\,1^{--}$	$\rho(770)$	772	775.8±0.5
$(2S)\,1^{--}$	$\rho(1450)$	1478	1465±25
$\begin{bmatrix}(3S)\,1^{--}\\(2D)\,1^{--}\end{bmatrix}$	$\rho(1700)$	$\begin{bmatrix}1802\\1826\end{bmatrix}$	1720±20
$(4S)\,1^{--}$	$\rho(1900)$	1927	$1911\pm5^\dagger$
$(1D)\,3^{--}$	$\rho_3(1690)$	1636	1691±5
$(2D)\,3^{--}$	$\rho_3(1990)$	1878	$1981\pm14^\dagger$
$(1P)\,1^{+-}$	$b_1(1235)$	1234	1229.5±3.2
$(1P)\,1^{++}$	$a_1(1260)$	1205	1230±40
$(2P)\,1^{++}$	$a_1(1640)$	1677	1647±22
$(1P)\,2^{++}$	$a_2(1320)$	1327	1318.3±0.6
$(2P)\,2^{++}$	$a_2(1700)$	1732	1732±16

TABLE V. Masses, in MeV, of the light-strange mesons up to 2 GeV. QM denotes the results of the present model. Experimental data (PDG) are taken from Ref. [45]. We denote by a dagger those PDG states whose masses are not explicitly given. We take for them the more recent experimental data. The spin is indicated because C parity is not well defined. In those cases where the PDG does not give the J^P quantum numbers and we find a candidate for the state, we quote between square brackets our predictions. QM(mixed) denotes the mass of the states after being mixed according to Eq. (20).

$(nL)\, J^P$	Spin	State	QM	QM(mixed)	PDG
$(1S)\, 0^-$	0	K	496		495
$(2S)\, 0^-$	0	$K(1460)$	1472		$\approx 1460^\dagger$
$(3S)\, 0^-$	0	$K(1830)$	1899		$\approx 1830^\dagger$
$(1S)\, 1^-$	1	$K^*(892)$	910		891.7±0.3
$(-)\, 1^-$	$-$	$K^*(1410)$	$-$		1414±15
$(1D)\, 1^-$	1	$K^*(1680)$	1698		1717±27
$(2S)\, ?^?\, [1^-]$	1	$K(1630)$	1620		1629±7
$(1P)\, 1^+$	1	$K_1(1270)$	1372	1352	1273±7
$(1P)\, 1^+$	0	$K_1(1400)$	1394	1414	1402±7
$(2P)\, 1^+$	1	$K_1(1650)$	1841	1836	1650±50
$(2P)\, 1^+$	0		1850	1856	
$(1P)\, 2^+$	1	$K_2^*(1430)$	1450		1425.6±1.5
$(1F)\, 2^+$	1	$K_2^*(1980)$	1968		1973±26
$(-)\, 2^-$	$-$	$K_2(1580)$	$-$		$\approx 1580^\dagger$
$(1D)\, 2^-$	1	$K_2(1770)$	1741	1709	1773±8
$(1D)\, 2^-$	0	$K_2(1820)$	1747	1779	1816±13
$(1D)\, 3^-$	1	$K_3^*(1780)$	1766		1776±7

TABLE VI. Same as Table V for D and D_s mesons.

Meson	$(nL)\,J^P$	Spin	State	QM	QM(mixed)	PDG
	$(1S)\,0^-$	0	D	1883		1867.7±0.5
	$(1S)\,1^-$	1	D^*	2010		2008.9±0.5
D	$(1P)\,1^+$	0	$D_1(2420)$	2492	2454	2425±4
	$(1P)\,2^+$	1	$D_2^*(2460)$	2502		2459±4
	$(2S)\,?^?\,[0^-]$	0	$D^*(2640)$	2642		2637±7
	$(1S)\,0^-$	0	D_s	1981		1968.5±0.6
	$(1S)\,1^-$	1	D_s^*	2112		2112.4±0.7
D_s	$(1P)\,0^+$	1	$D_{sJ}^*(2317)$	2469		2317.4±0.9
	$(1P)\,1^+$	0	$D_{sJ}^*(2460)$	2550	2543	2459.3±1.3
	$(1P)\,1^+$	1	$D_{s1}(2536)$	2563	2571	2535.3±0.6
	$(1P)\,?^?\,[2^+]$	1	$D_{sJ}(2573)$	2585		2572.4±1.5

TABLE VII. Masses, in MeV, of the first positive parity D and D_s mesons compared to recently measured experimental data. QM denotes the results of the present model. QM(mixed) indicates the mass of the states after being mixed according to Eq. (20).

Meson	$(nL)\ J^P$	$(1P)\ 0^+$	$(1P)\ 1^+$	$(1P)\ 1^+$	$(1P)\ 2^+$
	PDG [45]			2425±4	2459±4
	FOCUS [57,58]	~ 2420			2468±2
	Belle [59]	2308±36	2427±36	2421±2	2461±4
D	CLEO [60]		2461±51		
	DELPHI [61]		2470±58		
	QM	2436	2496	2492	2502
	QM(mixed)		2535	2454	
	PDG [45]	2317.4±0.9	2459.3±1.3	2535.3±0.6	2572.4±1.5
	FOCUS [57,58]			2535.1±0.3	2567.3±1.4
	BaBar [62,63]	2317.3±0.9	2458.0±1.4		
D_s	CLEO [64–66]	2318.1±1.2	2463.1±2.0		
	Belle [67,68]	2319.8±2.1	2456.5±1.8		
	QM	2469	2563	2550	2585
	QM(mixed)		2571	2543	

TABLE VIII. Same as Table III for charmonium.

$(nL)\, J^{PC}$	State	QM	PDG
$(1S)\, 0^{-+}$	$\eta_c(1S)$	2990	2979.6±1.2
$(1S)\, 1^{--}$	$J/\psi(1S)$	3097	3096.916±0.011
$(1P)\, 0^{++}$	$\chi_{c0}(1P)$	3436	3415.19±0.34
$(1P)\, 1^{++}$	$\chi_{c1}(1P)$	3494	3510.59±0.10
$(1P)\, 2^{++}$	$\chi_{c2}(1P)$	3526	3556.26±0.11
$(1P)\, ?^{??}\, [1^{+-}]$	$h_c(1P)$	3507	3526.21±0.25
$(2S)\, 0^{-+}$	$\eta_c(2S)$	3627	3654±10
$(2S)\, 1^{--}$	$\psi(2S)$	3685	3686.093±0.034
$(1D)\, 1^{--}$	$\psi(3770)$	3775	3770.0±2.4
$(1D)\, 2^{--}$	$\psi(3836)$	3790	3836±13
$(3S)\, 1^{--}$	$\psi(4040)$	4050	4040±10
$(2D)\, 1^{--}$	$\psi(4160)$	4103	4159±20
$\begin{bmatrix} (4S)\, 1^{--} \\ (3D)\, 1^{--} \end{bmatrix}$	$\psi(4415)$	$\begin{bmatrix} 4307 \\ 4341 \end{bmatrix}$	4415±6

TABLE IX. Masses predicted by our model for the $q\overline{q}$ states compatible with the $X(3872)$

1^3D_3	1^3D_2	1^1D_2	2^3P_1	2^1P_1
3802	3790	3793	3913	3924

TABLE X. Same as Table V for B, B_s and B_c mesons.

	$(nL)\, J^P$	Spin	State	QM	PDG
B	$(1S)\, 0^-$	0	B	5281	5279.2±0.5
	$(1S)\, 1^-$	1	B^*	5321	5325.0±0.6
	$(1P)\, ?^?\, [2^+]$	1	$B_J^*(5732)$	5790	5698±8
B_s	$(1S)\, 0^-$	0	B_s	5355	5369.6±2.4
	$(1S)\, 1^-$	1	B_s^*	5400	5416.6±3.5
	$(1P)\, ?^?\, [2^+]$	1	$B_{sJ}^*(5850)$	5855	5853±15
B_c	$(1S)\, 0^-$	0	B_c	6277	6400±410

TABLE XI. Same as Table III for bottomonium. We denote by an asterisk a experimental state recently reported in Ref. [69].

$(nL)\,J^{PC}$	State	QM	PDG
$(1S)\,0^{-+}$	$\eta_b(1S)$	9454	9300±28
$(1S)\,1^{--}$	$\Upsilon(1S)$	9505	9460.30±0.26
$(1P)\,0^{++}$	$\chi_{b0}(1P)$	9855	9859.9±1.0
$(1P)\,1^{++}$	$\chi_{b1}(1P)$	9875	9892.7±0.6
$(1P)\,2^{++}$	$\chi_{b2}(1P)$	9887	9912.6±0.5
$(2S)\,1^{--}$	$\Upsilon(2S)$	10013	10023.26±0.31
$(1D)\,2^{--}$	$\Upsilon(1D_2)^*$	10119	10162.2±1.6*
$(2P)\,0^{++}$	$\chi_{b0}(2P)$	10212	10232.1±0.6
$(2P)\,1^{++}$	$\chi_{b1}(2P)$	10227	10255.2±0.5
$(2P)\,2^{++}$	$\chi_{b2}(2P)$	10237	10268.5±0.4
$(3S)\,1^{--}$	$\Upsilon(3S)$	10335	10355.2±3.5
$(4S)\,1^{--}$	$\Upsilon(4S)$	10577	10580.0±3.5
$(5S)\,1^{--}$	$\Upsilon(10860)$	10770	10865±8
$(4D)\,1^{--}$		10803	
$(6S)\,1^{--}$	$\Upsilon(11020)$	10927	11019±8
$(5D)\,1^{--}$		10953	

TABLE XII. Same as Table III for the light-scalar mesons. We have included the $\kappa(800)$, whose isospin is not well determined being preferred $I=1/2$ [70].

	$(nL)\,J^{PC}$	State	QM	Flavor	PDG
$I=1$	$(1P)\,0^{++}$	$a_0(980)$	984	$(n\bar{n})$	984.7±1.2
	$(2P)\,0^{++}$	$a_0(1450)$	1587	$(n\bar{n})$	1474±19
$I=0$	$(1P)\,0^{++}$	$f_0(600)$	413	$(n\bar{n})$	400−1200
	$(-)\,0^{++}$	$f_0(980)$	−	−	980±10
	$(2P)\,0^{++}$	$f_0(1370)$	1395	$(n\bar{n})$	1200−1500
	$(1P)\,[0^{++}]$	−	1340	$(s\bar{s})$	−
	$(-)\,0^{++}$	$f_0(1500)$	−	−	1507±5
	$(3P)\,0^{++}$	$f_0(1710)$	1754	$(n\bar{n})$	1714±5
	$(4P)\,0^{++}$	$f_0(2020)$	1880	$(n\bar{n})$	1992±16
	$(2P)\,0^{++}$		1894	$(s\bar{s})$	
$I=1/2$	$(-)\,0^{+}$	$\kappa(800)$	−	−	≈ 800
	$(1P)\,0^{+}$	$K_0^*(1430)$	1213	$(n\bar{s})$	1412±6
	$(2P)\,0^{+}$	$K_0^*(1950)$	1768	$(n\bar{s})$	1945±22

TABLE XIII. Masses, in MeV, of the first positive parity D and D_s mesons with $a_s = 0.46$. They are already mixed according to Eq. (20).

Meson	(nL) J^P	State	QM(mixed)
	$(1P)$ 0^+	$D^*_{sJ}(2317)$	2317
D_s	$(1P)$ 1^+	$D_{sJ}(2463)$	2482
	$(1P)$ 1^+	$D_{s1}(2536)$	2574
	$(1P)$ 2^+	$D_{sJ}(2573)$	2633
	$(1P)$ 0^+	2308±36	2134
D	$(1P)$ 1^+	$D_1(2420)$	2354
	$(1P)$ 1^+	2427±36	2524
	$(1P)$ 2^+	$D^*_2(2460)$	2588

FIGURES

FIG. 1. Effective scale-dependent strong coupling constant α_s given in Eq. (14) as a function of momentum. We plot by the solid line our parametrization. Dots and triangles are the experimental results of Refs. [24] and [25], respectively. For comparison we plot by a dashed line the parametrization obtained in Ref. [18] using $\Lambda = 0.2$ GeV.

FIG. 2. Masses of the a_J triplet members as a function of the scalar/vector rate confinement, a_s. The solid line denotes the a_0 mass, the dashed that of a_1 and the dashed-dotted stands for the mass of the a_2 meson. A vertical solid line indicates the value chosen for a_s.