

Non Perturbative Renormalization in Coordinate Space * †

V. Giménez ^a, L. Giusti ^b, S. Guerriero ^c, V. Lubicz ^c, G. Martinelli ^{d e}, S. Petrarca ^{d e}, J. Reyes ^{d e}, B. Taglienti ^e, E. Trevigne ^d

^aDep. de Física Teòrica and IFIC, Univ. de València, Dr. Moliner 50, E-46100, Burjassot, València, Spain.

^bTheory Division, CERN, CH-1211 Geneva 23, Switzerland

^cDip. di Fisica, Univ. di Roma Tre and INFN-Sezione di Roma III, Via della Vasca Navale 84, I-00146 Roma, Italy.

^dDip. di Fisica, Univ. di Roma “La Sapienza”, Piazzale A. Moro 2, I-00185 Roma, Italy.

^eINFN-Sezione di Roma I, Piazzale A. Moro 2, I-00185 Roma, Italy.

We present an exploratory study of a gauge-invariant non-perturbative renormalization technique. The renormalization conditions are imposed on correlation functions of composite operators in coordinate space on the lattice. Numerical results for bilinears obtained with overlap and $O(a)$ -improved Wilson fermions are presented. The measurement of the quark condensate is also discussed.

Renormalization constants (RC) of lattice operators are crucial inputs to obtain physical quantities from the matrix elements of these operators. We have presented in this conference a numerical study of a non-perturbative (NP) technique proposed in [1]. This method has the advantages of being gauge-invariant and free of contact terms, in addition with the very easy implementation on a simulation. The first two properties can be of vital interest to renormalize the operators which enter the $K \rightarrow \pi\pi$ decays.

A very exploratory analysis was already presented in the Lattice 2002 conference [2] for bilinears operators. The results presented there were not conclusive and claimed for a more detailed study, which is the purpose of this contribution. We have considered only bilinears operators to test the method, since they are the simplest operators for which there exist results from others NP techniques to compare with.

The method consists on imposing renormalization conditions on the bare green functions, $G_\Gamma(x) \equiv \langle O_\Gamma(x) O_\Gamma^\dagger(0) \rangle$, where $O_\Gamma(x)$ is a quark

bilinear operator, $O_\Gamma(x) = \bar{\psi}_1(x) \Gamma \psi_2(x)$. One can choose

$$Z_\Gamma^2(\mu = 1/x) G_\Gamma(x) = G_\Gamma(x)^{(free\ cont.)}, \quad (1)$$

in a window $1/a \gg 1/x \gg \Lambda_{QCD}$; we will call this definition as the X-space scheme. One can also obtain $Z_\Gamma(\mu = 1/x)$ directly in another continuum scheme, such \overline{MS} , by imposing,

$$Z_\Gamma^{\overline{MS}^2}(\mu = 1/x) G_\Gamma(x) = G_\Gamma(\mu, x)^{\overline{MS}} \quad (2)$$

We have analyzed results for $O(a)$ improved Wilson fermions at $\beta = 6.45$ ($a^{-1} = 3.87(19)$ GeV) on a $32^3 \times 70$ lattice and overlap fermions at $\beta = 6.00$ ($a^{-1} = 2.00(10)$ GeV) on a $16^3 \times 32$ volume. All the results presented here have been extrapolated to the chiral limit (further information can be found in [3]).

As an example of the general strategy adopted, we explain the analysis for the vector-vector correlator with Wilson fermions. In Fig. 1a) we show the dependence of the correlator $\langle O_V(x) O_V^\dagger(0) \rangle$ on x^2 . The naïve dimensional behaviour (x^2^{-3}) is well satisfied but the data points are quite dispersed (see also Fig. 1b)). To clarify the origin of these effects we show in Fig. 1c) the ratio between the free correlator on the lattice and

*Presented by J. Reyes at “Lattice 2003”, Tsukuba.

†This work has been supported in part by the EU IHP under HPRN-CT-2000-00145 Hadrons/LatticeQCD.

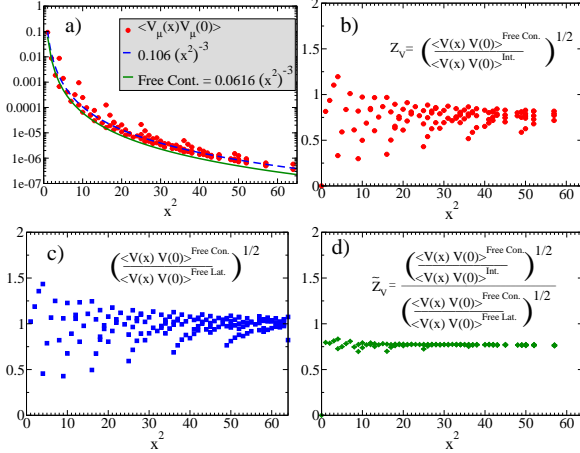


Figure 1.

in the continuum, both in infinite volume (note that in this case the expected value is 1 up to $O(a)$ effects). Comparing both figures we observe that the tree level calculation manifests the same pattern that the interacting case. These considerations together with similar analysis (see [3]), brought us to conclude that we are observing discretization effects, which we propose to reduce considering the following ratio:

$$Z_\Gamma^2(\mu = 1/x) = \frac{G_\Gamma(x)^{(F. con.)}}{G_\Gamma(x)^{(I.)}} \cdot \frac{G_\Gamma(x)^{(F. lat.)}}{G_\Gamma(x)^{(F. con.)}}, \quad (3)$$

where, I., F. mean interacting and free respectively. In this way we obtain the result plotted in Fig. 1d), which can be compared directly with Fig. 1b). It is clear that in this way a great part of $O(a)$ effects are canceled out in the ratio of eq. (3). The same procedure has been carried out for the other bilinears and for the overlap data.

As commented above one can also obtain Z_O directly in another continuum scheme ($\overline{\text{MS}}$), by imposing (2). Now one can evolve the RCs with the $\overline{\text{MS}}$ Renormalization Group (RG) factors, up to a fixed scale, say 2 GeV.

$$Z^{\overline{\text{MS}}}(2 \text{ GeV}) = \frac{Z(x)}{W(\mu, 2 \text{ GeV})R(\mu, 1/x)}, \quad (4)$$

where $R(\mu, 1/x)$ is the matching between the X-space scheme and $\overline{\text{MS}}$, which it has been calculated at NLO [3]. $W(\mu, 2 \text{ GeV})$ is the RGE factor. In principle μ is arbitrary as far as $\mu \sim 1/x$ in order to avoid large logarithmic contributions. This

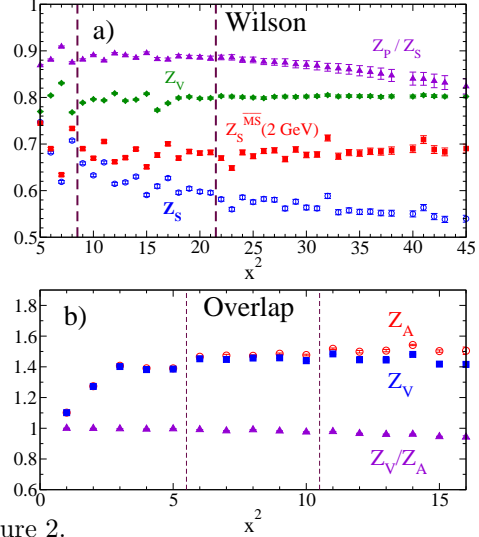


Figure 2.

scale dependence should cancel in a RGI quantity up to truncation of the perturbative series. In practice we have chosen $\mu = 1/x$ and $\mu = 2/x$ and included the difference as a systematic error due to NNLO effects.

Our results are summarized in Fig. 2 for the Wilson (2a) and overlap (2b) simulation, respectively. For the Wilson case we show, as an example, the RCs for the scalar density and vector current. We also show the ratio Z_P/Z_S . This ratio (as well as Z_V/Z_A) allows to extract information on the renormalization window, since it should be a plateau up to $O(a)$ and NP effects. From Fig. 2a) we see that in the range $x^2 = 9 \div 21$ we observe a reasonable good plateau: For $x^2 \leq 9$ large $O(a)$ effects are present, and above $x^2 \sim 21$ the presence of NP effects begins to be visible (see Z_P/Z_S). Therefore, we choose to obtain our final results for the RCs in the above range. We also plot the scalar and vector RCs, the first in $\overline{\text{MS}}$ at 2 GeV and in the X-space scheme. For Z_V we see a good plateau as one would expect. $Z_S(1/x)$ in the X-space exhibits an anomalous dimension behaviour as it should be. On the contrary in the value of $Z_S^{\overline{\text{MS}}}(2 \text{ GeV})$ one expects that the anomalous dimension cancels (up to NNLO correction) and indeed a plateau is observed.

For the overlap simulation, we show the currents RCs, together with the ratio Z_V/Z_A . Since chirality holds exactly, one expect $Z_V = Z_A$ up to

Table 1

Wilson results and comparison with other methods: RI-MOM [2] and Schrödinger Functional [5].

Z_Γ	Z_V	Z_A	Z_P/Z_S	Z_S	Z_P	Z_T
This work	0.801(2)(18)(6)	0.833(2)(27)(6)	0.888(2)(8)	0.702(4)(27)(23)	0.624(3)(19)(21)	0.895(2)(21)
RI-MOM	0.803(3)	0.833(3)	0.897(4)	0.679(8)	0.609(8)	0.898(6)
SF	0.808(1)	0.825(8)	0.912(9)	--	0.61(1)	--

NP effects. From Fig. 2b) one see that indeed this is the case for $x^2 \lesssim 10$, and we choose the range $6 \div 10$ to obtain the values of Z_V , Z_A . Contrary to the Wilson results, no renormalization window has been found for the other bilinears in the overlap case. This is due to the larger lattice spacing (twice what we have for Wilson data) which consequently makes much narrower the renormalization window. In Table 1 we present our results for the Wilson case and those with other NP methods. All the RCs are in $\overline{\text{MS}}$ at 2 GeV. The first error in our results is statistic, the second one is an estimate of discretization errors and the last one comes from NNLO effects due to the choice of the scale (see above). For the overlap case we quote the results obtained for Z_V and Z_A :

$$Z_V = 1.471(2)(10)(20) \quad Z_A = 1.494(2)(9)(20), \quad (5)$$

the tiny difference between the results for Z_V and Z_A is due to small NP effects. In any case they are compatible within the errors. Our results for Z_A is comparing with $Z_A = 1.55(4)$ obtained with the WI method in [4]. From Table 1 and the above results in the overlap case we conclude that although the errors are, in most cases, larger than those obtained from other NP techniques, good agreement is found for all the cases which we can compare. We stress that one could reduce the errors carrying out a recursive volume matching technique.

We present a preliminary result for the chiral quark condensate, obtained in a novel way. The idea is very simple and consists on studying the chiral limit of the trace of the quark propagator in X-space together with its OPE:

$$\text{Tr}S(x) \sim C_1(x^2) \frac{m}{x^2} - C_2(x^2) \langle \bar{\psi}\psi \rangle + \dots \quad (6)$$

We stress that in (6), once the l.h.s. is renormalized, the r.h.s. gets renormalized too, and no

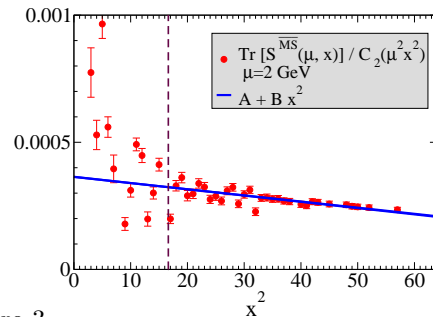


Figure 3.

other divergence (apart from the natural one at $x = 0$) is present, thus only the renormalization constant for the quark field is needed. We have computed the needed Wilson coefficients factors in eq. (6). From eq. (6) and in the chiral limit one can obtain an estimate of the chiral condensate. This is shown in Fig. 3. From that figure we see large discretization effects for $x^2 \leq 15$. One can see also that there is a contribution from the next order term which goes as x^2 . From a linear fit we obtain our (preliminary) result:

$$\langle \bar{\psi}\psi \rangle^{\overline{\text{MS}}}(2 \text{ GeV}) = -(276(5)(13) \text{ MeV})^3, \quad (7)$$

in good agreement with other lattice determinations. The first error is statistic and the second is due to the uncertainty in the value of a^{-1} .

REFERENCES

1. G. Martinelli *et al.*, Phys. Lett. B **411** (1997) 141.
2. D. Becirevic, *et al.*, [SPQcdR Collaboration], arXiv:hep-lat/0209168.
3. V. Giménez *et al.*, in preparation.
4. L. Giusti, C. Hoelbling and C. Rebbi, Nucl. Phys. Proc. Suppl. **106** (2002) 739.
5. M. Lüscher *et al.*, Nucl. Phys. **B491** (1997) 344. M. Guagnelli *et al.*, Nucl. Phys. **B535** (1998) 389. M. Guagnelli *et al.*, Nucl. Phys. **B595** (2001) 344.