Nucleon and delta masses in QCD [*]

M. Rafecas [**] and V.Vento [‡]

Departament de Física Teòrica and I.F.I.C.
Centre Mixt Universitat de València – C.S.I.C.
E-46100 Burjassot (València), Spain.

Abstract

Using the positivity of the path integral measure of QCD and defining a structure for the quark propagator in a background field according to the fluxon scenario for confinement, we calculate and compare the correlators for nucleon and delta. From their shape we elucidate about the origin of their mass difference, which in our simplified scenario is due to the tensor structure in the propagator. This term arises due to a dynamical mechanism which is responsible simultaneously for confinement and spontaneous chiral symmetry breaking. Finally we discuss, by comparing the calculated correlators with the Lehmann representation, the possibility that a strong CP and/or P violation occurs as a consequence of a specific mechanism for confinement.

1 Introduction

It was realized long after QCD was formulated that one could derive some exact inequalities between hadron masses [1] and other observables [2]. The key element in deriving them is that the Euclidean fermion determinant in vector like gauge theories (such as QCD) is positive definite and so the measure

$$d\mu = Z^{-1} D A_\mu^a(x) det(i\slashed{D} + M) \exp\left(-\frac{1}{2g^2} \int d^4 x Tr F_{\mu\nu}^2\right) \qquad (1)$$

for the A_μ^a integration obtained after integrating out the fermions is positive definite for $\Theta = 0$. Note that $\slashed{D} = \gamma^\mu D_\mu$, D_μ being the covariant derivative. Inequalities that hold pointwise continue to hold after integrating with respect to a positive measure. Thus any inequality among matrix elements that holds after performing the Fermi integral in a fixed background gauge field holds in the exact theory.

Mass inequalities have been obtained among the mesons and comparing baryons with mesons [1]. The important property in these calculations has been

$$S^+(x, y) = \gamma_5 S(y, x) \gamma_5 \qquad (2)$$

where $S(x, y)$ is the quark propagator in a background. Our aim is to discuss mass relations among baryons, but due to their current quark constituency the previous property is of no use. Some fine details of the structure of the quark propagator and of the baryon currents will be necessary to be able to address the issue.

We center our investigation in the nucleon–delta mass difference. This observable has become the first test of any hadron model, and they all agree by now that it is different from zero, even if the desintegration channel for the delta is decoupled. Therefore both particles shall be treated on an equal footing in our work.

The procedure to follow is straight forward. We calculate the baryon correlators for these two systems and study their asymptotic behavior which is dominated exponentially by their masses [1]. To do so we describe the baryon fields in terms of elementary quark fields à la Ioffe [3]. Some plausible assumptions on the structure of the quark propagator are required in order to be predictive. We will use in this paper the continuum language of Witten [1]. To preserve the positivity of the fermion determinant for a fixed A_μ^a and do the integral a suitable cut-off is required. Recent developments give us confidence in the result of our discussions [4].

2 The baryon correlators

The first step is to find composite gauge invariant operators which have all the quantum numbers of the nucleon and the delta. We choose the proton to perform the calculation. There are two such operators for it [3, 5],

$$O_1^{udu}(x) = \varepsilon^{abc}[u_a^T(x) C d_b(x)] \gamma_5 u_c(x) \qquad (3)$$

and

$$O_2^{udu}(x) = \varepsilon^{abc}[u_a^T(x)\,C\gamma_5 d_b(x)]u_c(x) \tag{4}$$

Here T means transposed and C is the charge conjugation operation. We take initially the general combination

$$O(x) = O_1(x) + tO_2(x) \tag{5}$$

where t can be considered the tangent of a mixing angle. However in our final discussion, we shall simplify our expressions to the chiral limit $(t = -1)$, because it is favored by the data [5].

For the delta states there is only one such operator. We work for simplicity, with that corresponding to the Δ^{++}, i.e.,

$$O_{uuu}^{\mu} = -i\varepsilon^{abc}[u_a^T C\gamma^{\mu} u_b]u_c \tag{6}$$

Once we substitute in the expression for the baryon fields the correlators become

$$< N_\alpha(x)\bar{N}_\beta(y) > \sim \int d\mu[(\gamma_5 S(x,y)\gamma_5)_{\alpha\beta}\, Tr(S(x,y)S(x,y))$$

$$+\, t\,\{S(x,y),\gamma_5\}_{\alpha\beta}\, Tr(S(x,y)S(x,y)\gamma_5)$$

$$+\; t^2\, S(x,y)_{\alpha\beta}\, Tr(S(x,y)\gamma_5 S(x,y)\gamma_5)] \tag{7}$$

and

$$< \Delta_\alpha^\mu(x)\bar{\Delta}_\beta^\nu(y) > \sim \int d\mu[S_{\alpha\beta}Tr(\gamma^\mu S(x,y)\gamma^\nu S(x,y))$$

$$+\; (S(x,y)\gamma^\nu S(x,y)\gamma^\mu S(x,y))_{\alpha\beta}] \tag{8}$$

where $\{\,,\,\}$ represents an anticommutator. At large separation, where the spectrum becomes explicit, and taking into account the observed symmetries of the strong interactions, the structure of the correlator should be dominated by the Lehmann decomposition [6, 7]

$$< N_\alpha(x)\bar{N}_\beta(y) > \quad \longrightarrow \quad F_{\mu 1}(x-y)\gamma_{\alpha\beta}^\mu + F_2(x-y)\delta_{\alpha\beta} \tag{9}$$

and

$$< \Delta_\alpha^\mu(x)\bar{\Delta}_\beta^\nu(y) > \quad \longrightarrow \quad g^{\mu\nu}(F_{1\sigma}^\Delta(x-y)\gamma_{\alpha\beta}^\sigma + F_2^\Delta(x-y)\delta_{\alpha\beta}) + \dots \tag{10}$$

The F functions, which can be extracted from Eqs. (7) and (8), by comparison with Eqs.(9) and (10), contain information about the exponential fall off, i.e., *pole* in momentum space, and therefore on the masses. Among the tensorial structures which appear in Eq.(10) we are interested in the one proportional to $g_{\mu\nu}$, because it is the only one in which the delta gives a contribution to the *pole* but not the nucleon [5].

3 The mass relations

In the present approach the properties of the quark propagator are crucial, not only to determine relations among various observables, but also to establish the possible realization of the different symmetries of the theory [8, 9]. The most general quark propagator in a background field has the following structure

$$S(x,y) = s(x,y) + p(x,y)\gamma_5 + v^\mu(x,y)\gamma_\mu + a^\mu(x,y)\gamma_\mu\gamma_5 + t^{\mu\nu}(x,y)\sigma_{\mu\nu} \qquad (11)$$

Configurations to the p, a^μ and $t^{\mu\nu}$ terms arise only through complex field configurations: instantons, fluxons, etc... [9, 10] and, as we shall see, their behavior is restricted if QCD is to reproduce the behavior described by the Lehmann representation [6].

The fluxon mechanism has been proposed as a possible scenario where chiral symmetry breaking could occur in a confining theory [11]. Recent Lattice Montecarlo calculations point towards a mechanism similar to this scenario [12]. We take this ansatz, however, because it contains the minimal structure necessary to present our ideas and cannot be naively discarded. Under this circumstances the quark propagator could have the following form [9, 10] [1]

$$S(x,y) = s(x,y) + v^\mu(x,y)\gamma_\mu + t^{\mu\nu}(x,y)\sigma_{\mu\nu} \qquad (12)$$

Calculating the correlation functions we obtain for the proton

$$
\begin{aligned}
< N\bar{N} > \sim \frac{1}{2} \int d\mu \{ & [(s^2 + t^{\alpha\beta}t_{\alpha\beta})(1+t^2) + v^2(1-t^2)]s \\
& + [(s^2 + t^{\alpha\beta}t_{\alpha\beta})(1-t^2) + v^2(1+t^2)]v^\mu\gamma_\mu \\
& + [(s^2 + t^{\alpha\beta}t_{\alpha\beta})(1+t^2) + v^2(1-t^2)]t^{\delta\epsilon}\sigma_{\delta\epsilon} \}
\end{aligned}
\qquad (13)
$$

and for the delta

$$
\begin{aligned}
< \Delta^\mu\Delta_\mu > \sim \int d\mu \{ & s(\frac{4}{3}s^2 - v^2 - t^{\alpha\beta}t_{\alpha\beta}) - \frac{i}{2}s\varepsilon_{\alpha\beta\gamma\delta}t^{\alpha\beta}t^{\gamma\delta}\gamma_5 \\
& + [(\frac{5}{4}s^2 - \frac{3}{4}v^2 + \frac{1}{2}t^{\alpha\beta}t_{\alpha\beta})v^\gamma + \frac{1}{2}t^{\alpha\beta}v_\beta t_\alpha^\gamma]\gamma_\gamma \\
& + [(s^2 - \frac{1}{2}v^2)t^{\alpha\gamma} - 2ist^{\alpha\beta}t_\beta^\gamma]\sigma_{\alpha\gamma} \\
& - \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}[g^{\mu\delta}st^{\alpha\beta}v^\gamma + it^{\alpha\beta}t^{\delta\mu}v^\gamma + it^{\alpha\rho}v_\rho t^{\beta\gamma}g^{\mu\delta}]\gamma_\mu\gamma_5 \}
\end{aligned}
\qquad (14)
$$

Recalling at this point the Lehmann representation it is apparent that the above equations contain exotic terms not present in the latter and which imply violations, hopefully small, of some of the symmetries of the strong interactions [7].

---

[1] We shall analyze the effects of a pseudoscalar term, its relation with the anomaly and some experimental consequences, in a future publication [13].

It has been shown [3, 5] that $t = -1$, the value in the chiral limit is close to that preferred by the data. Let us study therefore this limit and impose consistently the zero quark mass limit. The above expressions simplify notably and we thus obtain

$$F_2 = F_2^\Delta = 0 \tag{15}$$

$$F_1^\mu \sim -\int d\mu v^2 v^\mu \tag{16}$$

$$F_1^{\Delta\mu} \sim -\int d\mu (v^2 v^\mu + \frac{2}{3} v^\mu t^{\alpha\beta} t_{\alpha\beta} + \frac{2}{3} t^{\alpha\beta} v_\beta t_\alpha^\mu) \tag{17}$$

As has been shown in ref.([9]) a non vanishing tensor component leads to the spontaneous breaking of chiral symmetry. Thus the same mechanism which produces in our scenario chiral symmetry breaking leads to the mass difference between the nucleon and the delta [2].

In the fluxon scenario, the pion and σ correlators are given by [1]

$$< \pi\pi > \sim \int d\mu (|v|^2 + t^{\alpha\beta} t^*_{\alpha\beta}) \tag{18}$$

$$< \sigma\sigma > \sim \int d\mu (|v|^2 - t^{\alpha\beta} t^*_{\alpha\beta}) \tag{19}$$

In the Wigner realization of chiral symmetry, only the vector term does not vanish and the sigma and pion have the same mass. In the Goldstone realization of chiral symmetry the tensor component must cancel the exponential asymptotic behavior leading to a constant one, i.e., a Goldstone pion. The tensor interaction, which turns out to be attractive in the pion channel must be repulsive in the delta channel. Comparing the Lehmann representation with Eq.(14), we see that tensor terms must produce an asymptotically subleading behavior and therefore

$$\int d\mu |(v^2 v^\mu + \frac{2}{3} v^\mu t^{\alpha\beta} t_{\alpha\beta} + \frac{2}{3} t^{\alpha\beta} v_\beta t_\alpha^\mu)| < k | \int d\mu v^2 v^\mu | \tag{20}$$

where k is a constant. Thus the mass difference between the nucleon and the delta is expressing the fact, that the tensor components, which produce the leading behavior in the pionic sector, contribute destructively in the baryonic sector generating a subleading behavior.

4   Exotic terms

Let us look at Eqs.(13) and (14) as obtained from our calculation. As we have previously mentioned we obtain heterodox components in our correlator, assuming orthodoxy given by the Lehmann representation. In the chiral limit for example

$$Nucleon \sim \int d\mu [t^{\alpha\beta} t_{\alpha\beta} t^{\delta\epsilon} \sigma_{\delta\epsilon}] \tag{21}$$

---

[2]The pseudoscalar and scalar contributions vanish in the massless limit if one assumes, as we do here, that no nonperturbative phenomena associated with confinement occur in these terms (see also discussion in [9]). Moreover we work under the assumption that the tensor component corresponds to the scenario of strong confining fields and does not vanish [9, 12]. Therefore spontaneous chiral symmetry breaking is entirely described by the tensor component in this limit.

$$Delta \sim \frac{1}{2} \int d\mu [v^2 t^{\alpha\gamma}\sigma_{\alpha\gamma} + i\epsilon_{\alpha\beta\gamma\delta}(t^{\alpha\beta}t^{\delta\mu}v^\gamma + t^{\alpha\rho}v_\rho t^{\beta\gamma}g^{\mu\delta})\gamma_\mu\gamma_5] \qquad (22)$$

The deviations from the Lehmann representation due to a tensor structure imply a strong violation of CP, while those of an axial vector structure that of P [7]. We see that the exotic terms in the delta and in the nucleon do not correspond to the same Lorentz structure, having the delta an additional axial vector term. Although naive power counting of tensor and vector components might indicate that the axial vector term is of the same order as the non exotic vector contribution,the antisymmetric product is making this non-observed contribution vanishingly small, thus giving a hint about the structure of the fluxon mechanism. It is important to notice that such a term might have experimental consequences and one should analyze its implications. Besides it, both nucleon and delta contain an exotic tensor structure. By looking at the non exotic vector components we might infer that the magnitude in both cases must be similar, since the tensor and vector parts have similar leading behaviors. Thus one can use either nucleon or delta observables to detect this type of deviations from the experimentally dominant symmetry structures[3].

5   Conclusion

The fact that the QCD measure is positive has been used to establish exact relations among hadronic observables. These results, although exact, are not quantitative, like those obtained by lattice calculations. Moreover, in the case of baryons, due to their quark constituency, no relations among observables has been established. In this work we initiate a new line of thought, based on these ideas, but which requires from some additional input in the form of the quark propagator. One looses exactitude, but in principle one might get quantitative predictions. We have applied the technique to the baryon case, and by formulating a precise structure of the quark propagator we have analyzed some of the implications of the baryon spectrum into the behavior of the theory. We have not aimed at quantitative results in this first attempt. One could do so by postulating a precise model for the fermionic propagator in terms by weak field expansions [10], semiclassical fields as hinted by lattice calculations [12] or postulating some stromg field limit, and in so doing define models of hadron structure, at the level of the full quantum theory. Here we have only described immediate implications of the tensor structure in the fermion propagator, which we next recall.

We have assumed a quark propagator as would appear from a fluxon picture of confinement. We have taken a simplified scenario which contains in a relatively manageable calculation all possible exotic expectations of the theory. It is immediate to see, by looking into the mesonic sector, that such structure produces spontaneous chiral symmetry breaking. We have analyzed the nucleon and delta correlators and discovered that this same mechanism should be reponsible for their mass difference. We have seen the appearence of exotic components, both in the nucleon and delta channels, reflecting the violation (hopefully small) of some crucial symmetries of the

---

[3] We have analyzed the contribution from the pseudoscalar also [13] and an analogous conclusion can be drawn. Concrete experimental proposals for both cases are being investigated.

strong interactions, in particular P and CP, without the necessity of Θ angle. Since we have not been able to solve QCD exactly, but have simply postulated an ansatz, it is clear that the non-appearence of these symmetry violations could not be, in principle, attributable to QCD, but to the inconvenience of our ansatz. In particular a naive quark propagator structure with only scalar and vector components does not lead to any symmetry violations.

Finally if confinement would behave in a non-naive manner, as in the model presented here, our calculation shows that violation of symmetries may not occur in all channels. For example, in our case, violation of P only occurs if deltas are present. Thus one should study effects in different channels if one wants to gain a full understanding of the fundamental properties of QCD. The advent of high duty cycle machines might allow for searches, which could be carried out in parallel with the more traditional neutron experiments.

Acknowledgement

References

[1] D. Weingarten, Phys. Rev. Lett. **51** (1983) 1830; S. Nussinov, Phys. Rev. Lett. **51** (1983) 2081; E. Witten, Phys. Rev. Lett. **51** (1983) 2351.

[2] D. Espriu, M. Gross and J.F. Wheater, Phys. Lett. **146B** (1984) 67; J. Alfaro, R.C. Brower and M.B. Gavela, Phys. Lett. **147B** (1984) 357; S. Nussinov and B. Sathiapalan, Nucl. Phys. **B256** (1985) 285; S. Nussinov, Phys. Lett. **164B** (1985) 367.

[3] B.L. Ioffe, Nucl. Phys. **B188** (1981) 317.

[4] M. Asorey and F. Falceto, Nucl. Phys. **B327** (1989) 427.

[5] D. Espriu, P. Pascual and R. Tarrach, Nucl. Phys. **B214** (1983) 285.

[6] H. Lehmann, Nuov. Cim. **11** (1954) 342.

[7] S.S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row and Peterson, New York 1961), Chapter 17b.

[8] C. Vafa and E. Witten, Phys. Rev. Lett. **53** (1984) 5

[9] S. Nussinov and M. Spiegelglas, Phys. Lett. **181B** (1986) 134.

[10] J. Schwinger, Phys. Rev. **82** (1951) 664.

[11] T. Banks and A. Casher, Nucl. Phys. **B169** (1980) 103.

[12] L. Delbio, A. Di Giacomo, M. Maggiore and S. Olejnik, IFUP-TH33/90, CERN-TH 6103/91.

[13] M. Rafecas and V. Vento, work in preparation.