Predicting Neutrinoless Double Beta Decay

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We give predictions for the neutrinoless double beta decay rate in a simple variant of the A_4 family symmetry model. We show that there is a *lower* bound for the $\beta\beta_{0\nu}$ amplitude even in the case of normal hierarchical neutrino masses, corresponding to an effective mass parameter $|m_{ee}| \geq 0.17\sqrt{\Delta m^2_{\rm ATM}}$. This result holds both for the CP conserving and CP violating cases. In the latter case we show explicitly that the lower bound on $|m_{ee}|$ is sensitive to the value of the Majorana phase. We conclude therefore that in our scheme, $\beta\beta_{0\nu}$ may be accessible to the next generation of high sensitivity experiments.

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Are neutrinos their own antiparticles? This is one of the most basic current unknowns in neutrino physics. Of all possible manifestations of neutrino masses, neutrinoless double beta decay ($\beta\beta_{0\nu}$, for short) offers – to date – the only potentially viable way to answer this question. If $\beta\beta_{0\nu}$ exists, then neutrino masses are Majorana in nature, irrespective of their ultimate origin [1]. Together with cosmology [2] and direct kinematical searches in tritium decay [3], $\beta\beta_{0\nu}$ offers one of the three main complementary ways to probe the absolute scale of neutrino masses.

Current experimental limits from $\beta\beta_{0\nu}$ on the effective Majorana mass of the neutrino m_{ee} are of order $m_{ee} \leq 0.3 - 1$ eV [4, 5]. We note that a claim for a finite $\beta\beta_{0\nu}$ rate has been published in [6], but this has so far not been confirmed by any other experiment. A recent proposal [7] aims explicitly at testing the half-life range suggested in [6]. However, future $\beta\beta_{0\nu}$ experiments may be able to reach down to much lower mass scales. Many experiments sensitive to $m_{ee} \simeq 0.05$ eV have already been discussed; see for example [8, 9]. In the longer-term future, even $m_{ee} \simeq 0.01$ eV [10, 11] does not seem impossible.

The historic confirmation of neutrino oscillations over the last few years [12], together with some basic theory, suggests that $\beta\beta_{0\nu}$ is expected, although in general no lower bound on the magnitude of the expected effect can be given. Theoretical input is therefore needed. Currently the origin of neutrino masses is completely unknown. The basic dimension–five operator which leads to neutrino masses [13] can arise from a variety of mech-

anisms characterized by vastly different scales. Alternatives include the seesaw mechanism [14, 15, 16] and low–energy R–parity violating supersymmetry [17]. In neither case is it possible, in general, to establish a lower bound on the magnitude of the $\beta\beta_{0\nu}$ rate.

Here we consider a simple phenomenological model based on a new realization of the A_4 family symmetry [18, 19, 20] in which no $SU(3) \otimes SU(2) \otimes U(1)$ singlet neutrinos are introduced. Instead, the small neutrino masses arise from the small induced vacuum expectation values (VEVs) generated for the neutral components of triplet Higgs bosons [15, 21], transforming nontrivially under the A_4 family symmetry. The lepton and Higgs particle content and their transformation properties under A_4 and $SU(2) \otimes U(1)$ are specified in Table I. With

Fields	L	l^c	ϕ_1	ϕ_2	ϕ_3	η_1	η_2	η_3	ξ
A_4	**3**	**3**	**1**	**1'**	**1''**	**1**	**1'**	**1''**	**3**
$SU(2)_L$	**2**	**1**		**2**			**3**		**3**
Y	–1	2		–1			2		2

TABLE I: Lepton and scalar boson quantum numbers

these transformation properties, the charged lepton mass matrix is already diagonal in the flavor basis, with

$$m_e = h_1 v_1 + h_2 v_2 + h_3 v_3$$
$$m_\mu = h_1 v_1 + \omega h_2 v_2 + \omega^2 h_3 v_3$$
$$m_\tau = h_1 v_1 + \omega^2 h_2 v_2 + \omega h_3 v_3$$

where h_i are charged lepton Yukawa couplings, $v_i = \langle \phi_i^0 \rangle$ and ω is a complex cubic root of unity satisfying $1 + \omega + \omega^2 = 0$. The neutrino mass matrix is then of the form

$$M_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix} \quad (1)$$

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where the only non-diagonal entries are those of the A_4 triplet ξ. Here we have defined

$$
\begin{aligned}
a &= \lambda_1 \langle \eta_1^0 \rangle & d &= \kappa \langle \xi_1^0 \rangle \\
b &= \lambda_2 \langle \eta_2^0 \rangle & e &= \kappa \langle \xi_2^0 \rangle \\
c &= \lambda_3 \langle \eta_3^0 \rangle & f &= \kappa \langle \xi_3^0 \rangle
\end{aligned} \tag{2}
$$

where λ_i, κ and $\langle \eta_i^0 \rangle$, $\langle \xi_i^0 \rangle$ are triplet Yukawa couplings, and VEVs, respectively. Let us further assume that the conditions $b = c$, and $d = e = f$ hold. Whereas the former is an *ad hoc* assumption, the latter can be maintained naturally because of a residual Z_3 symmetry. Note that M_ν has a very remarkable (and possibly unique) property here in that each entry is renormalized by the charged-lepton Yukawa couplings in the same way, i.e. $|h_1|^2 + |h_2|^2 + |h_3|^2$, instead of being proportional to $m_i^2 + m_j^2$ as in the Standard Model.

It is easy to see that in the above limit we have the prediction

$$
\theta_{23} = \pi/4, \quad \theta_{13} = 0
$$

which matches well with the neutrino oscillation data [12]. Furthermore it can be shown that, in the limit where the solar mass splitting is neglected, b and d can be made real, so that the atmospheric neutrino mass splitting takes on a very simple form

$$
\Delta m_{32}^2 = 6bd \equiv \Delta m_{\text{ATM}}^2 \tag{3}
$$

The solar neutrino mass splitting $\Delta m_{\text{SOL}}^2 \ll \Delta m_{\text{ATM}}^2$ can be expressed as

$$
\Delta m_{21}^2 = \sqrt{T_1^2 + T_2^2 + T_3^2} \equiv \Delta m_{\text{SOL}}^2 \tag{4}
$$

where

$$
T_1 \equiv 6\sqrt{2}|b||d|\sin(\phi_2) \tag{5}
$$

$$
T_2 \equiv 2\sqrt{2}|d|\Big(2|a|\cos(\phi_1) + |b|\cos(\phi_2) + |d|\Big) \tag{6}
$$

$$
\begin{aligned}
T_3 \equiv &-3|b|^2 + |d|^2 - 6|a||b|\cos(\phi_1 + \phi_2) \\
&+ 2|a||d|\cos(\phi_1) - 2|b||d|\cos(\phi_2)
\end{aligned} \tag{7}
$$

with $\phi_1 \equiv \phi_a - \phi_d$ and $\phi_2 \equiv \phi_d - \phi_b$, where $a = |a|\exp(i\phi_a)$, $b = |b|\exp(i\phi_b)$ and $d = |d|\exp(i\phi_d)$. The condition in Eq. (4) leads to three inequalities

$$
|T_i| \leq \Delta m_{\text{SOL}}^2
$$

which, normalized by Δm_{ATM}^2, can be expressed in terms of the small parameter $\alpha \equiv \Delta m_{\text{SOL}}^2 / \Delta m_{\text{ATM}}^2$ as

$$
\sqrt{2}|\sin(\phi_2)| \leq \alpha \tag{8}
$$

$$
\frac{\sqrt{2}}{3|b|}\Big|2|a|\cos(\phi_1) + |b|\cos(\phi_2) + |d|\Big| \leq \alpha \tag{9}
$$

$$
\frac{1}{6|b||d|}\Big| -3|b|^2 + |d|^2 - 6|a||b|\cos(\phi_1 + \phi_2) \tag{10}
$$

$$
+2|a||d|\cos(\phi_1) - 2|b||d|\cos(\phi_2)\Big| \leq \alpha
$$

where the current allowed values of α are shown in Fig.13 of Ref. [12].

As for the solar mixing angle, it is given here by

$$
t_{2S} \equiv \tan(2\theta_{12}) = \frac{2\sqrt{2}d}{3b - d} \tag{11}
$$

which reduces to

$$
\tan^2 \theta_{12} = 1/2 \tag{12}
$$

in two ways, namely,

$$
b = 0, \qquad b = 2d/3 \tag{13}
$$

Current fits of solar, reactor, atmospheric and accelerator neutrino oscillation data lead to a best fit point to the solar mixing angle for which $\tan^2 \theta_{12}$ is slightly less than $1/2$. Performing a series expansion of $\tan^2 \theta_{12}$ around the two solutions in Eq. (13), we get

$$
\tan^2 \theta_{12} \simeq \frac{1}{2} + \frac{b}{d} \tag{14}
$$

$$
\tan^2 \theta_{12} \simeq \frac{1}{2} - \frac{1}{d}\left(b - \frac{2}{3}d\right) \tag{15}
$$

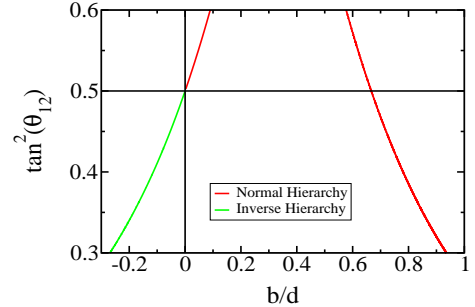These two branches are depicted in Fig. 1. The positive

FIG. 1: Solar mixing angle, $\tan^2 \theta_{12}$, vs. b/d.

horizontal axis corresponds to a normal hierarchy (NH) neutrino spectrum, while the negative part corresponds to inverse hierarchy (IH), as depicted in Fig. 1. This behavior is also recognized in the model of [22].

We now turn to neutrinoless double beta decay. The neutrino–exchange amplitude for this process is simply given by

$$
\langle m_\nu \rangle = m_{ee} = a + 2b \tag{16}
$$

In the case of real parameters, we can solve the following

system of equations

$$\left.\begin{array}{r} \Delta m_{\text{SOL}}^2 \equiv \Delta m_{21}^2 = |2a + b + d|\sqrt{(d - 3b)^2 + 8d^2} \\ \Delta m_{\text{ATM}}^2 \equiv \Delta m_{32}^2 = 6bd \\ t_{2S} \equiv \tan(2\theta_{12}) = \dfrac{2\sqrt{2}d}{3b - d} \end{array}\right\}$$

and express the parameters a, b and d in terms of experimentally measurable ones Δm_{SOL}^2, Δm_{ATM}^2 and t_{2S}. Substituting in Eq. (16) we can therefore express $\langle m_\nu \rangle$ in terms of these measured observables. We then obtain, up to an overall sign,

$$\frac{m_{ee}}{\sqrt{\Delta m_{\text{ATM}}^2}} = \text{Sign}[\Delta m_{\text{ATM}}^2]\frac{1}{\sqrt{2\sqrt{2}t_{2S} + t_{2S}^2}} \qquad (17)$$

$$\pm\text{Sign}[2\sqrt{2}t_{2S} + t_{2S}^2]\frac{\alpha\sqrt{2\sqrt{2}t_{2S} + t_{2S}^2}}{4\sqrt{1 + t_{2S}^2}}$$

The calculated values of $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$ as functions of t_{2S} according to Eq. (17) are shown in Fig. 2. It can
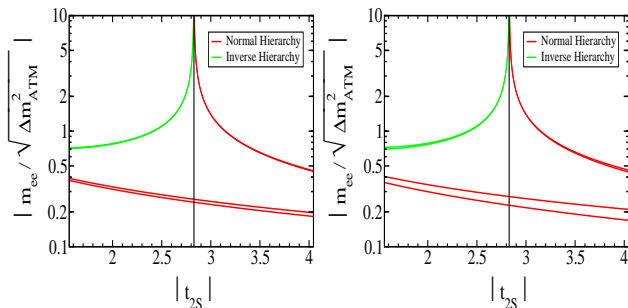

FIG. 2: $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$ vs. $|t_{2S}|$ according to Eq. (17). The bound depends slightly on the value of α: the left panel corresponds to $\alpha = 0.022$ and the right one to $\alpha = 0.065$. The dark (red) lines correspond to normal hierarchy, while the grey (green) line is for the inverse hierarchy case. The vertical line corresponds to the horizontal line in Fig. 1 at $\tan^2\theta_{12} = 1/2$.

be seen that given the currently allowed experimental 3σ range

$$\tan^2\theta_{12} \in [0.30, 0.61] \qquad (18)$$

we can set lower bounds for $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$:

$$0.17 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| \qquad \text{for NH} \qquad (19)$$

$$0.70 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| \qquad \text{for IH} \qquad (20)$$

Note that the lower bound on $\beta\beta_{0\nu}$ for the NH case is especially relevant, as it is absent in the generic case. It results from this specific realization of the A_4 symmetry and is also in contrast with previous A_4–based models that led to quasi–degenerate neutrinos [19, 23].

Moreover, should future precision experiments narrow down the experimental range for $\tan^2\theta_{12}$ then we might

be able to distinguish between both neutrino mass hierarchies. For example, if $\tan^2\theta_{12} \leq 1/2$ could ever be established, then we would have

$$0.23 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| < 0.41 \qquad \text{for NH} \qquad (21)$$

$$0.70 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| \qquad \text{for IH} \qquad (22)$$

It can also be seen that, up to order α corrections, m_{ee} can only be zero if $\tan^2\theta_{12} = 1$, now strongly rejected experimentally. Note that the two solutions in Fig. 2 correspond to the two branches depicted in Fig. 1. One can see that only in the branch corresponding to the solution $b = 0$ and for values of $\tan^2\theta_{12}$ that are less than $1/2$, there is a relative minus sign between b and d, which is the condition for inverse hierarchy, as can be seen from Eq. (3).

In the general case of complex parameters, lower bounds on $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$ can also be established for each hierarchy. This task is simplified greatly by taking into account the reliable approximations $\sin\phi_2 \simeq 0$, and $2|a|cos\phi_1 + |b|cos\phi_2 + |d| \simeq 0$. From Fig. 3, we see that in the complex case, lower bounds on $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$ are indeed also established for each hierarchy. By comparing Fig. 3 with the right panel in Fig. 2 we find that these lower bounds are in fact exactly the same as obtained in the real case. The robustness of these bounds is easily understood. It follows from the fact that the maximum degree of destructive interference between the three neutrino–exchange contributions occurs in the real case with appropriate CP parities [24].
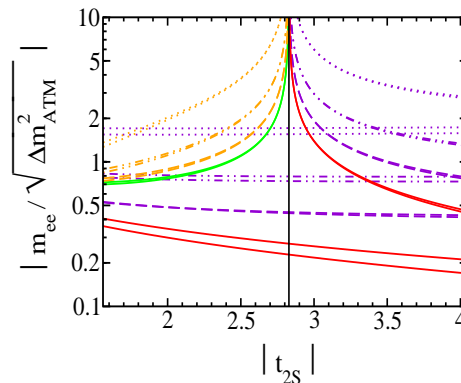

FIG. 3: $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$ vs. $|t_{2S}|$ for $\alpha = 0.065$ when complex parameters are allowed.

The left-rising lines correspond to inverse hierarchy, while all others refer to the case of normal hierarchy. The lower–lying pairs of solid lines correspond to the lower bounds already discussed in Fig. 2. Again, the vertical line corresponds to $\tan^2\theta_{12} = 1/2$. Finally, the remaining lines correspond to non-zero values of the relevant CP-violating phase varying $\cos(\phi_1)$ over the range [0,1] in equally–spaced steps. It is conceptually interesting to note that this phase is "Majorana type" [15, 25], as we are still considering the case $\theta_{13} = 0$.

Last, but not least, we find it very interesting also that the lower bound on $|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}|$ which we have obtained depends on the value of the Majorana violating phase $|\cos(\phi_1)|$. We can see from Fig. 4 that the lower bounds on $|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}|$ for each hierarchy become weaker for $\cos(\phi_1) = 1$. Finally, we mention that,
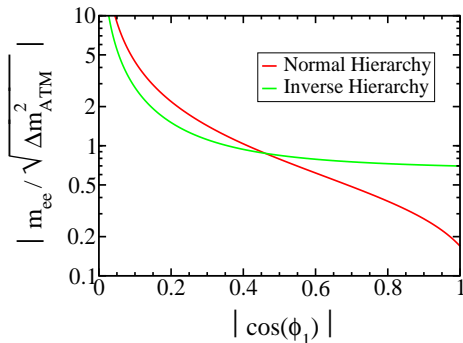

FIG. 4: Lower bound on $|m_{ee}/\sqrt{\Delta m^2_{\rm ATM}}|$ vs. $|\cos(\phi_1)|$, given current t_{2S} uncertainty. The lines in dark (red) and grey (green) correspond to normal and inverse hierarchy respectively.

even though for simplicity we have focused on the case $b = c$, all results can be generalized to the case $b \neq c$, in which case θ_{13} is allowed to be nonzero.

In summary, we have given predictions for the neutrinoless double beta decay rate in a simple hierarchical variant of the A_4 family symmetry model. We showed that there is a *lower* bound for the $\beta\beta_{0\nu}$ amplitude even in the case of normal hierarchical neutrino masses. We have seen that the bound is robust as it holds irrespective of whether CP is conserved or not. In the latter case we show explicitly how the lower bound on $|m_{ee}|$ is sensitive to the value of the Majorana phase. Our scheme suggests that neutrinoless double beta decay may be within reach of the next generation of high sensitivity experiments.

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