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Systematic decomposition of the neutrinoless double beta decay operator

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Abstract

We discuss the systematic decomposition of the dimension nine neutrinoless double beta decay operator, focusing on mechanisms with potentially small contributions to neutrino mass, while being accessible at the LHC. We first provide a ($d = 9$ tree-level) complete list of diagrams for neutrinoless double beta decay. From this list one can easily recover all previously discussed contributions to the neutrinoless double beta decay process, such as the celebrated mass mechanism or "exotics", such as contributions from left-right symmetric models, R-parity violating supersymmetry and leptoquarks. More interestingly, however, we identify a number of new possibilities which have not been discussed in the literature previously. Contact to earlier works based on a general Lorentz-invariant parametrisation of the neutrinoless double beta decay rate is made, which allows, in principle, to derive limits on all possible contributions. We furthermore discuss possible signals at the LHC for mediators leading to the short-range part of the amplitude with one specific example. The study of such contributions would gain particular importance if there were a tension between different measurements of neutrino mass such as coming from neutrinoless double beta decay and cosmology or single beta decay.

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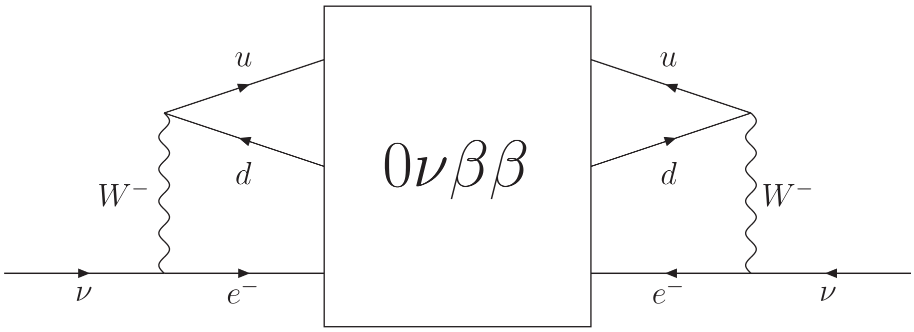

Figure 1: *Black Box diagram relating the Majorana nature of neutrinos with $0\nu\beta\beta$ decay.*

1 Introduction

Neutrinoless double beta ($0\nu\beta\beta$) decay is mostly known as a sensitive probe for Majorana neutrino masses [1–4]. However, the mass mechanism is only one out of many possible contributions to the $0\nu\beta\beta$ decay amplitude [5, 6]. The aim of the current paper is to provide a (tree-level) complete list of all possible contributions to the neutrinoless double beta decay dimension nine ($d = 9$) operator:

$$\mathcal{O} \propto \bar{u}\bar{u}\, dd\, \bar{e}\bar{e} \tag{1}$$

From this list one can easily recover all known contributions to $0\nu\beta\beta$ decay. More interestingly, however, we will identify a number of new possibilities to generate $0\nu\beta\beta$ decay not discussed in the literature previously.

The Black Box theorem [7–9] states that since observation of $0\nu\beta\beta$ indicates that lepton number is not conserved, it proves that neutrinos must be Majorana particles.[1] Graphically the theorem can be depicted as shown in Fig. 1: If $0\nu\beta\beta$ decay is observed, Majorana neutrino masses are generated at least at the 4-loop order, which is a model-independent statement. This does not mean that the $0\nu\beta\beta$ decay contribution from Fig. 1 is the leading contribution to neutrino mass; in fact, in specific models, neutrino mass is often generated at a lower loop order. A recent calculation [11] indeed confirms that the neutrino mass generated through this 4-loop diagram with the $0\nu\beta\beta$ operator of the size of the current limits is roughly $m_\nu \simeq \mathcal{O}(10^{-24})$ eV. Obviously, this number is too small to explain the neutrino masses observed in oscillation experiments, and also many orders of magnitude smaller than the current sensitivity of $0\nu\beta\beta$ decay via the mass mechanism. The correct interpretation of the black box theorem thus is: If $0\nu\beta\beta$ decay is observed, neutrinos are Majorana particles, whether the contribution from neutrino mass dominates $0\nu\beta\beta$ decay or not.

Up to now, $0\nu\beta\beta$ decay has not been observed. The best half-life limits on $0\nu\beta\beta$ decay come from experiments on two isotopes: ^{76}Ge and ^{136}Xe. The Heidelberg-Moscow collaboration gives $T_{1/2}^{0\nu\beta\beta}(^{76}\mathrm{Ge}) \geq 1.9 \cdot 10^{25}$ yr [12],[2] while the recent results from EXO-200 and KamLAND-ZEN quote $T_{1/2}^{0\nu\beta\beta}(^{136}\mathrm{Xe}) \geq 1.6 \cdot 10^{25}$ yr [14] and $T_{1/2}^{0\nu\beta\beta}(^{136}\mathrm{Xe}) \geq 1.9 \cdot 10^{25}$ yr [15], both at the 90 % CL. There is, however, reasonable hope that the half-lifes in excess of 10^{26} yr will be probed within the next few

[1]For a recent version of the black box theorem including lepton flavour violation, see [10].

[2]There is also a claim [13] for the observation of $0\nu\beta\beta$ decay in ^{76}Ge, which is, however, not supported by any other experiment.

years, since a number of next generation $0\nu\beta\beta$ experiments are under construction or already taking data. For recent reviews and a list of experimental references, see for example [2, 4]. Moreover, proposals for ton-scale next-to-next generation $0\nu\beta\beta$ experiments claim that even sensitivities in excess $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr can be reached for ^{136}Xe [16,17] and ^{76}Ge [18,19]. For a brief summary of the long-term prospects for $0\nu\beta\beta$ experiments, see for example [20].

The interpretation of these half-life limits in terms of particle physics parameters requires assumptions, such as which contribution dominates the $0\nu\beta\beta$ decay amplitude. If neutrinos have Majorana masses, $0\nu\beta\beta$ decay can be mediated by Majorana neutrino propagation, depending on the magnitude of the effective neutrino mass given by $\langle m_\nu \rangle = \sum_j U_{ej}^2 m_j$. This mechanism is hence forth referred to as the *the mass mechanism*. The mass mechanism has attracted most of the attention within the community. The reason for this "bias" is rather straightforward: Neutrinos exist and oscillation experiments [21–28] have shown that (at least two) neutrinos have non-zero masses. Thus, if neutrinos are indeed Majorana particles, the mass mechanism is guaranteed to give a contribution to the $0\nu\beta\beta$ decay amplitude — in this sense the mass mechanism is the minimal possibility to generate $0\nu\beta\beta$ decay. With the assumption of the mass mechanism being the dominant contribution, the current limits for the half-life of $0\nu\beta\beta$ decay [12, 14] correspond to the limits $\langle m_\nu \rangle \lesssim (0.2 - 0.35)$ eV [$\langle m_\nu \rangle \lesssim (0.17 - 0.30)$ eV] for ^{76}Ge [^{136}Xe] using the latest QRPA matrix elements of [29] to $\langle m_\nu \rangle \lesssim 0.53$ eV [$\langle m_\nu \rangle \lesssim 0.34$ eV] with the matrix elements calculated within the shell model [30,31]. Future limits of order $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr will then probe $\langle m_\nu \rangle \lesssim (0.02 - 0.06)$ eV that is of the order of the mass scale suggested by atmospheric neutrino oscillations, $\sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV [32]. If the next generation of $0\nu\beta\beta$ decay experiments detects a signal, one might expect that future cosmological data [33–35] also provide indications for non-zero neutrino masses. Contradicting results from $0\nu\beta\beta$ (observation) and cosmology (limit) then might point to a non-standard explanation for $0\nu\beta\beta$ decay, see [36] for a detailed discussion of the interplay of different data.

Contributions to the $0\nu\beta\beta$ decay rate can be divided into a short-range [37] and a long-range [38] part. The long-range contributions, which the neutrino mass mechanism belongs to, can lead in some cases to very stringent limits on the new physics scale $\Lambda \gtrsim \lambda_{\rm LNV}^{\rm eff} \times (10^2 - 10^3)$ TeV, where $\lambda_{\rm LNV}^{\rm eff}$ is some effective Lepton Number Violating (LNV) coupling that depends on the model under consideration. Therefore, for some of the "exotic" mechanisms discussed in the literature, falling into this category, the half-life limits from $0\nu\beta\beta$ decay themselves yield the most stringent bounds. On the other hand, in the case of the short-range contribution, i.e., the $0\nu\beta\beta$ amplitude is mediated only by heavy mediators with masses at a high energy scale Λ, the effective Lagrangian describing $0\nu\beta\beta$ decay is simply proportional to $1/\Lambda^5$. In that case, the next generation $0\nu\beta\beta$ decay experiments are sensitive to new physics at scales $\Lambda \gtrsim$ (few) TeV.

Data from the LHC will probe physics at the TeV scale, i.e., of a similar scale as the sensitivity of $0\nu\beta\beta$ decay experiments in case of short-range contributions. In fact, first limits on particular models have already been published. Just to mention one particular example, in Left-Right (LR) symmetric models, $0\nu\beta\beta$ decay can be generated by $W_R - N - W_R$ exchange, where N is a heavy Majorana neutrino and W_R is the charged gauge boson of the right-handed $SU(2)$ [39].[3] Using the nuclear matrix elements of [42], the limit on the half-life [14] corresponds to $\langle m_N \rangle = m_{W_R} \gtrsim 1.3$ TeV (assuming that the gauge coupling of the right-handed and left-handed $SU(2)$s are equal), while the

[3] The idea to use accelerator data to test $0\nu\beta\beta$ decay contributions in left-right symmetric models has a long history. "Inverse" neutrinoless double beta decay, i.e., $e^-e^- \to W_R W_R$ has been first discussed in [40]. W_R production at hadron colliders, followed by the decay $W_R \to \mu^+\mu^+ jj$ was first studied in [41].

recent analysis by the ATLAS [43] and the CMS collaborations [44] give $m_{W_R} \geq (2.3 - 2.5)$ TeV for $m_N \lesssim 1.3$ TeV. [4] Because of this complementarity of $0\nu\beta\beta$ decay and LHC, we will pay particular attention to the short-range part of the $0\nu\beta\beta$ decay amplitude. However, our decomposition of the $d = 9$ operator, which will be shown in Sec. 3, is general, and the corresponding limits for the long-range part can be easily derived as well, using the recipes described below and in the appendix.

Our paper is not the first work attempting a systematic analysis of the $d = 9$ $0\nu\beta\beta$ decay operator, relevant earlier papers include [37, 38, 45–47]. The authors of [37, 38] worked out a general Lorentz-invariant parametrisation for the $0\nu\beta\beta$ decay rate. This approach is motivated from the nuclear physics point of view of $0\nu\beta\beta$ decay. At low energies, adequate for $0\nu\beta\beta$ decay studies, any of the $0\nu\beta\beta$ diagrams in which heavy mediation fields are inserted among the six fermions $\bar{u}\bar{u}dd\bar{e}\bar{e}$ will be reduced to a finite set of combinations of the hadronic and the leptonic currents corresponding to a basic set of nuclear matrix elements. This approach leaves the LNV parameters in the $0\nu\beta\beta$ decay rate unspecified, and our current work can be understood as providing a (tree-level) complete list of all possible ultraviolet completions ("models") for $0\nu\beta\beta$ decay. As expected, at low energies any information on the particle models is reduced then to one (or a combination of more than one) of the coefficients ϵ_i of the effective $0\nu\beta\beta$ currents presented in [37, 38].

Our approach also has some overlap with [45, 46]. These authors write down all effective LNV operators from $d = 5$ (the famous Weinberg operator [48]) to $d = 11$. The main motivation of those papers is to identify all possible Majorana neutrino mass models via the effective LNV operators [45]. This effective operator treatment allows to estimate the scale Λ at which new physics appears, if these operators give neutrino masses or a $0\nu\beta\beta$ decay amplitude of the order of the current experimental sensitivity [46]. However, our current work is complementary to these papers, in that we list all possible *decompositions* of a particular LNV operator, that is the $d = 9$ $0\nu\beta\beta$ operator. We also go one step further than these works in the estimation of the bounds on the LNV operator, by making contact with the nuclear matrix element calculation of [37, 38], instead of simply relying on dimensional arguments.[5] Finally, there is also the recent paper [47], where the authors study $0\nu\beta\beta$ decay from an effective Lagrangian point of view. Operators of $d > 9$ are considered, in which the Standard Model (SM) Higgs doublets are additionally inserted to the $d = 9$ operator Eq. (1). Note, however, that [47] considers only the case when new physics is confined to the leptonic part of the $0\nu\beta\beta$ decay amplitude.

The rest of this paper is organised as follows. In the next section, as a preparation for our approach, we will first recapitulate the characteristics of the long-range and short-range contributions to $0\nu\beta\beta$ decay. Then, in Sec. 3 we will focus on the decomposition of effective operators to find all possible models generating the $d = 9$ $0\nu\beta\beta$ operator. Through an example, we will study the crucial role that the LHC can play in discriminating such models in Sec. 4. The relations between the list of particle models and the general decay rate of [37, 38] are given in tabular form in the appendix for the short-range part for the case of scalar exchange. The corresponding relations for the other parts of the decay rate can be easily derived from the recipes spelled out below and in the appendix.

[4] Here, $(\langle m_N \rangle)^{-1} \equiv \sum_j V_{ej}^2 / m_{N_j}$ is a sum over all heavy neutrinos coupling to the electron, while in the limit from the LHC experiments it is assumed that only one heavy neutrino (the lightest) has a mass below the mass of W_R.

[5] We also found a $d = 9$ LNV operator $\overline{u_R} \overline{u_R} d_R d_R \overline{e_R} \overline{e_R}$ that was not listed in the earlier papers [45, 46].

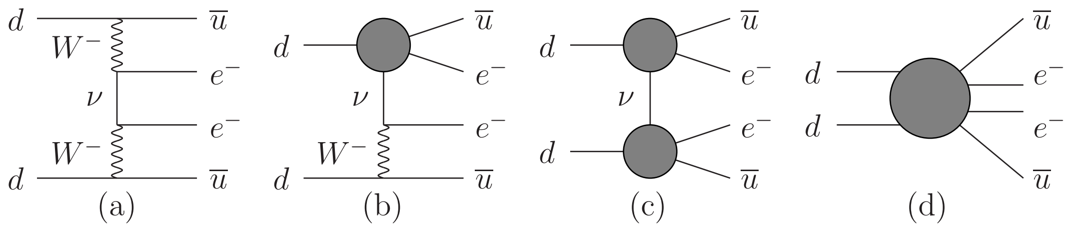

Figure 2: *Different contributions to* $0\nu\beta\beta$ *: (a)-(c) A light neutrino is exchanged between two point-like vertices, which are classified as "long-range". (d) Contributions mediated by heavy particles are classified as "short-range". Diagram (a) corresponds to the mass mechanism — the standard interpretation of* $0\nu\beta\beta$ *with Majorana neutrino propagation. See main text for details.*

2 Model-independent parametrisation of the $0\nu\beta\beta$ decay rate

A general Lorentz-invariant parametrisation of new physics contributions to $0\nu\beta\beta$ has been developed in [37,38]. This formalism allows to derive limits on any LNV new physics contributing to $0\nu\beta\beta$ decay without recalculation of nuclear matrix elements. In order to make contact with this formalism, we recapitulate the main results and definitions of [37, 38] in this section. The total amplitude of $0\nu\beta\beta$ is most conveniently divided into two parts: Long-range and short-range contributions, see Fig. 2.

2.1 Long-range contributions

Consider first the long-range part. Here, we can sub-divide the amplitudes into parts (a)-(c) as shown in the figure. In case (a), a massive Majorana neutrino is exchanged between two SM charged current vertices, while cases (b) and (c) contain one and two (unspecified) non-standard interactions respectively, indicated by the black blobs.

At low energy, we can write the relevant part of the effective Lagrangian with the leptonic (j) and hadronic (J) charged currents as

$$\begin{aligned}\mathcal{L}^{\text{4-Fermi}} &= \mathcal{L}^{\text{SM}} + \mathcal{L}^{\text{LNV}} \\ &= \frac{G_F}{\sqrt{2}}\left[j^\mu_{V-A} J_{V-A,\mu} + \sum_{\alpha,\,\beta\,\neq\,V-A} \epsilon_\alpha^\beta\, j_\beta J_\alpha \right] .\end{aligned} \tag{2}$$

Here, we follow the notations of j and J adopted in [38], which are[6]

$$\begin{aligned} J^\mu_{V\pm A} = (J_{R/L})^\mu \equiv \overline{u}\gamma^\mu(1\pm\gamma_5)d\,, &\qquad j^\mu_{V\pm A} \equiv \overline{e}\gamma^\mu(1\pm\gamma_5)\nu\,, \\ J_{S\pm P} = J_{R/L} \equiv \overline{u}(1\pm\gamma_5)d\,, &\qquad j_{S\pm P} \equiv \overline{e}(1\pm\gamma_5)\nu\,, \\ J^{\mu\nu}_{T_{R/L}} = (J_{R/L})^{\mu\nu} \equiv \overline{u}\gamma^{\mu\nu}(1\pm\gamma_5)d\,, &\qquad j^{\mu\nu}_{T_{R/L}} \equiv \overline{e}\gamma^{\mu\nu}(1\pm\gamma_5)\nu\ , \end{aligned} \tag{3}$$

[6]Note that the difference in normalisation of Eq. (3) and the normal convention for L/R in particle physics leads to various powers of two, see appendix, when relating models with the ϵ_α^β of Eq. (2).

Isotope	$\lvert\epsilon^{V+A}_{V-A}\rvert$	$\lvert\epsilon^{V+A}_{V+A}\rvert$	$\lvert\epsilon^{S+P}_{S-P}\rvert$	$\lvert\epsilon^{S+P}_{S+P}\rvert$	$\lvert\epsilon^{TR}_{TL}\rvert$	$\lvert\epsilon^{TR}_{TR}\rvert$
^{136}Xe	$2.8 \cdot 10^{-9}$	$5.6 \cdot 10^{-7}$	$6.8 \cdot 10^{-9}$	$6.8 \cdot 10^{-9}$	$4.8 \cdot 10^{-10}$	$8.1 \cdot 10^{-10}$

Table 1: *Limits on effective long-range interactions from $T^{0\nu\beta\beta}_{1/2}$ (^{136}Xe) $\gtrsim 1.6 \cdot 10^{25}$ ys [14] which corresponds to approximately $\langle m_\nu \rangle \lesssim 0.35$ eV in the mass mechanism. These limits are taken from [6] and are derived assuming only one ϵ is different from zero at a time.*

where the Lorenz tensor matrix $\gamma^{\mu\nu}$ is defined as $\gamma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]$. Recall that $P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$ and we will use the short-hand notation L and R for left-handed and right-handed fermions, respectively. The first term of Eq. (2) is the SM charged current interaction, and the second term contains the new physics contributions, which do not take the Lorenz structure of the standard four-Fermi interaction $(V - A)(V - A)$. The coefficients ϵ^β_α for the exotic four-Fermi interactions are normalised to the SM charged current strength $G_F/\sqrt{2}$. If these dimensionless coefficients take numbers smaller than one, diagram (c) in Fig. 2 is of order ϵ^2 and becomes immediately sub-dominant in the $0\nu\beta\beta$ amplitudes.

The neutrino propagator in diagrams (a)-(c) contains two terms: $m_\nu + \not{q}$, the mass and the momentum terms. Since the charged current for leptons in the SM is purely left-handed, it picks out the m_ν part, i.e., the amplitude of the (standard) mass mechanism of $0\nu\beta\beta$ is proportional to m_ν. Clearly, then not all ϵ^β_α can be constrained from $0\nu\beta\beta$ decay, due to the absence of a lower bound on m_ν. On the other hand, if the new physics in diagram (b) generates a right-chiral lepton interaction $(j_{V+A}, j_{S+P}, j_{T_R})$, the $\not{q}$-term in the neutrino propagator will enter the amplitude. The size of the 3-momentum $|\vec{q}|$ can be estimated from the typical inter-nucleon distance of two neutrons in the nucleus to be of the order of 100 MeV. Therefore, the amplitudes with $\not{q}$-terms are highly enhanced in comparison with those with the m_ν-term.[7] For this reason, the coefficients ϵ^β_α with right-chiral leptonic interactions, are heavily constrained by $0\nu\beta\beta$ decay. The hadronic and leptonic currents are best defined as currents of definite chirality (as defined at Eq. (3)) to take care of this fact.

In Table 1 we give the updated bounds for all ϵ^β_α, which are taken from [6]. Note that the index β for the leptonic current in the table takes neutrino interactions with the chirality R in the exotic four-Fermi interaction, while the hadronic currents can be of either L- or R-type. Note also that, while the ϵ^β_α are defined as dimensionless coefficients, they scale like $\epsilon^\beta_\alpha \propto (\lambda^{\mathrm{eff}}_{\mathrm{LNV}}/\Lambda)^2$.

2.2 Short-range contributions

The short-range contributions encompass all processes where no light neutrinos are exchanged, and can be understood as one $d = 9$ effective vertex diagram as shown in diagram (d) in Fig. 2. In this case, one can use the basis of low energy hadronic currents J as defined in Eq. (3), while for the currents j of two electrons, one defines

$$\begin{aligned} j_{L/R} &\equiv \overline{e}(1 \mp \gamma_5)e^c\,, \\ (j_{L/R})^\mu &\equiv \overline{e}\gamma^\mu(1 \mp \gamma_5)e^c, \end{aligned} \tag{4}$$

[7] This momentum-enhancement mechanism has also been discussed in the context of the other LNV processes, such as $\mu^- N \to e^+ N$ [49,50] and $\mu^- N \to \mu^+ N$ [51,52]. A possibility of a direct experimental test of the four-Fermi LNV interaction with a right-chiral lepton current is examined in [53].

Isotope	$\|\epsilon_1\|$	$\|\epsilon_2\|$	$\|\epsilon_3^{LLz(RRz)}\|$	$\|\epsilon_3^{LRz(RLz)}\|$	$\|\epsilon_4\|$	$\|\epsilon_5\|$
^{136}Xe	$2.6 \cdot 10^{-7}$	$1.4 \cdot 10^{-9}$	$1.1 \cdot 10^{-8}$	$1.7 \cdot 10^{-8}$	$1.2 \cdot 10^{-8}$	$1.2 \cdot 10^{-7}$

Table 2: *Limits on effective short-range interactions. These limits are taken from [6] and are derived assuming only one ϵ is different from zero at a time.*

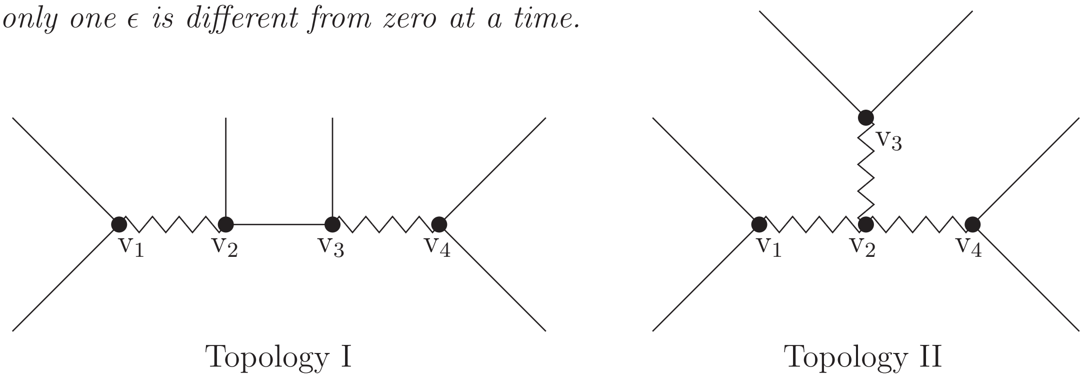

Figure 3: *The two basic tree-level topologies realizing a $d = 9$ $0\nu\beta\beta$ operator. External lines are fermions; internal lines can be fermions (solid), or scalars or vectors (zig zag).*

to express the effective Lagrangian for short-range $0\nu\beta\beta$ as [37]

$$\mathcal{L}^{\text{eff}} = \frac{G_F^2}{2} m_P^{-1} \left[\epsilon_1 J J j + \epsilon_2 J^{\mu\nu} J_{\mu\nu} j + \epsilon_3 J^\mu J_\mu j + \epsilon_4 J^\mu J_{\mu\nu} j^\nu + \epsilon_5 J^\mu J j_\mu\right] , \tag{5}$$

where m_P is the mass of proton. Here we omitted the indices for clarity. However, if chirality changes play a role in the value of the $0\nu\beta\beta$ decay rate, one needs to maintain the chirality indices and define $\epsilon_i = \epsilon_i^{xyz}$, with $x, y, z \in \{L, R\}$; cf., App. A.1 for details. Since $(j_R)^\mu = -(j_L)^\mu$, we define $(j)^\mu = (j_R)^\mu = -(j_L)^\mu$.

Current limits on short-range type ϵ_i are summarised in Tab. 2. Note that, different from the ϵ_α^β of the long-range part, here ϵ_i scale as $\epsilon_i \propto (\lambda_{\text{LNV}}^{\text{eff}})^4/\Lambda^5$. For $\lambda_{\text{LNV}}^{\text{eff}} \simeq g_L$ the limits given in Tab. 2 then correspond to $\Lambda \gtrsim (1-3)$ TeV.

3 General decomposition of the $d = 9$ $0\nu\beta\beta$ decay operator

The $d = 9$ effective operator generating $0\nu\beta\beta$ decay at the quark level can be written as in Eq. (1). In this section, we decompose this operator, following the techniques developed in [54–57], in terms of the SM quantum numbers of the mediators. Note that the results obtained in this section are valid for both short- and long-range contributions, while the quantitative impact of the contribution to $0\nu\beta\beta$ depends on the Lorentz nature. Details are left for the appendix, where we tabulate all possibilities for the short-range part mediated by fermions and scalars.

At tree-level, there are only two possible topologies for this operator, which are shown in Fig. 3. For topology I (T-I) the internal particles between vertices v$_1$ − v$_2$ and v$_3$ − v$_4$ can be either scalars (S) or vectors (V), while the particle between v$_2$ − v$_3$ must be a fermion (F). The topology thus contains three classes of diagrams: VFV, SFS and SFV. The inner particles for topology II (T-II), on the other hand, must all be either scalars or vectors, all possible combination in principle can occur (SSS, VVV, VVS and SSV).

One has multiple choices for assigning the fermions to the outer legs. Once a particular assignment is chosen, the electric charge and the colour of the internal particles are fixed (the latter up to a two-fold ambiguity), but not the $U(1)_Y$ hypercharge. The assignments of hypercharge are fixed as well, once the chiralities of the six outer fermions are determined. We will come back to this point later in the discussion, and more details are given in the appendix. We will now discuss the general decompositions of T-I and T-II in turn.

3.1 Decomposition of topology I

In Tab. 3 we list the general decompositions for the $0\nu\beta\beta$ decay operator for T-I. The chiralities of the outer fermions are left unspecified here for a more compact presentation. In total there are 18 (times 2 for the choice of colour) possibilities for realizing T-I. Because of the $SU(3)$ multiplication rules, $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3}_a \oplus \bar{\mathbf{6}}_s$ and $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$, there are always two possible colour assignments for the internal particles. The table is valid for all three possible classes of diagrams (VFV, SFS and SFV). In some of the cases listed in Tab. 3, only VFV or SFV exchange is possible, because in the decomposition $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3}_a \oplus \bar{\mathbf{6}}_s$ the coupling of a scalar to two identical quarks (two $\bar{\mathbf{3}}$) vanishes for the $\mathbf{3}_a$. These affects all the cases in T-I-3, T-I-4-ii and T-I-5-ii, see table for nomenclature.

Each of the models listed in this table leads to an effective operator with a different Lorentz structure, which needs to be projected onto the basis shown in Eqs. (2) and (5), once the chiralities are fixed. The projection can be done by Fierz transformations and the transformations of the SM gauge group indices. This allows to identify immediately if a model gives important contributions to $0\nu\beta\beta$ decay. More details and results of this procedure are shown in the appendix.

From this table we can identify all (T-I) contributions discussed in the literature, once the nature of the internal bosons and the chirality of the outer fermions are chosen. For example, the mass mechanism corresponds to T-I-1-i, with the bosons being vectors ($W^{\pm}$) and all the outer fermions being left-handed. In this case, the quantum numbers of the internal fermion ψ are equal to those of a (light) neutrino, and the mass term is picked out from the propagator.

Other examples can be identified as easily. To list a few more, the afore-mentioned $W_R - N - W_R$ exchange diagram is also contained in T-I-1-i with vectors ($W_R^{\pm}$), all outer fermions now being right-handed. Since N must be heavy, however, this is now a short-range contribution. Concrete models can lead to the occurrence of more than one of the operators listed, an example is provided by (trilinear) R-parity violating (RPV) supersymmetry (SUSY). The six short-range diagrams [59, 60] for the RPV SUSY mechanism of $0\nu\beta\beta$ decay [58], correspond to class SFS and are identified as T-I-1-i ($\tilde{e} - \tilde{\chi}^0 - \tilde{e}$ diagram), T-I-2-i-b ($\tilde{e} - \tilde{\chi}^0 - \tilde{d}$), T-I-2-ii-b ($\tilde{e} - \tilde{\chi}^0 - \tilde{u}$), T-I-2-iii-a ($\tilde{u} - \tilde{\chi}^0/\tilde{g} - \tilde{d}$), T-I-4-i ($\tilde{u} - \tilde{\chi}^0/\tilde{g} - \tilde{u}$) and T-I-5-i ($\tilde{d} - \tilde{\chi}^0/\tilde{g} - \tilde{d}$). In those diagrams, the neutralino $\tilde{\chi}^0$ corresponds to $\psi(0, \mathbf{1})$, while the gluino $\tilde{g}$ corresponds to $\psi(0, \mathbf{8})$. Finally, the leptoquark (LQ) mechanism of [65, 66] is a long-range contribution of the class SFV with T-I-2-i-b and T-I-2-ii-b. The internal fermion $\psi(0, \mathbf{1})$ is again identified with a light neutrino.

A few more comments on Tab. 3 might be in order. There are a total of six possibilities in which the intermediate fermion transforms $\psi(0, \mathbf{1})$ under the gauge symmetries $(U(1)_{\mathrm{em}}, SU(3)_c)$. Only they can lead to long-range contributions, all the other models in the list are necessarily of the short-range type. Among those six, only the cases marked (a) or (b) in the column "Long Range?" can lead to interesting constraints, since the remaining three cases marked as (c) are suppressed with ϵ^2. Note, however, that all of these cases can also be of short-range type. There are 12 (times two)

#	Decomposition	Long Range?	Mediator ($U(1)_{\rm em}, SU(3)_c$) S or V_ρ	ψ	S' or V'_ρ	Models/Refs./Comments
1-i	$(\bar{u}d)(\bar{e})(\bar{e})(\bar{u}d)$	(a)	$(+1,\mathbf{1})$	$(0,\mathbf{1})$	$(-1,\mathbf{1})$	Mass mechan., RPV [58–60], LR-symmetric models [39], Mass mechanism with ν_S [61], TeV scale seesaw, e.g., [62, 63]
			$(+1,\mathbf{8})$	$(0,\mathbf{8})$	$(-1,\mathbf{8})$	[64]
1-ii-a	$(\bar{u}d)(\bar{u})(d)(\bar{e}\bar{e})$		$(+1,\mathbf{1})$	$(+5/3,\mathbf{3})$	$(+2,\mathbf{1})$	
			$(+1,\mathbf{8})$	$(+5/3,\mathbf{3})$	$(+2,\mathbf{1})$	
1-ii-b	$(\bar{u}d)(d)(\bar{u})(\bar{e}\bar{e})$		$(+1,\mathbf{1})$	$(+4/3,\overline{\mathbf{3}})$	$(+2,\mathbf{1})$	
			$(+1,\mathbf{8})$	$(+4/3,\overline{\mathbf{3}})$	$(+2,\mathbf{1})$	
2-i-a	$(\bar{u}d)(d)(\bar{e})(\bar{u}\bar{e})$		$(+1,\mathbf{1})$	$(+4/3,\overline{\mathbf{3}})$	$(+1/3,\overline{\mathbf{3}})$	
			$(+1,\mathbf{8})$	$(+4/3,\overline{\mathbf{3}})$	$(+1/3,\overline{\mathbf{3}})$	
2-i-b	$(\bar{u}d)(\bar{e})(d)(\bar{u}\bar{e})$	(b)	$(+1,\mathbf{1})$	$(0,\mathbf{1})$	$(+1/3,\overline{\mathbf{3}})$	RPV [58–60], LQ [65, 66]
			$(+1,\mathbf{8})$	$(0,\mathbf{8})$	$(+1/3,\overline{\mathbf{3}})$	
2-ii-a	$(\bar{u}d)(\bar{u})(\bar{e})(d\bar{e})$		$(+1,\mathbf{1})$	$(+5/3,\mathbf{3})$	$(+2/3,\mathbf{3})$	
			$(+1,\mathbf{8})$	$(+5/3,\mathbf{3})$	$(+2/3,\mathbf{3})$	
2-ii-b	$(\bar{u}d)(\bar{e})(\bar{u})(d\bar{e})$	(b)	$(+1,\mathbf{1})$	$(0,\mathbf{1})$	$(+2/3,\mathbf{3})$	RPV [58–60], LQ [65, 66]
			$(+1,\mathbf{8})$	$(0,\mathbf{8})$	$(+2/3,\mathbf{3})$	
2-iii-a	$(d\bar{e})(\bar{u})(d)(\bar{u}\bar{e})$	(c)	$(-2/3,\overline{\mathbf{3}})$	$(0,\mathbf{1})$	$(+1/3,\overline{\mathbf{3}})$	RPV [58–60]
			$(-2/3,\overline{\mathbf{3}})$	$(0,\mathbf{8})$	$(+1/3,\overline{\mathbf{3}})$	RPV [58–60]
2-iii-b	$(d\bar{e})(d)(\bar{u})(\bar{u}\bar{e})$		$(-2/3,\overline{\mathbf{3}})$	$(-1/3,\mathbf{3})$	$(+1/3,\overline{\mathbf{3}})$	
			$(-2/3,\overline{\mathbf{3}})$	$(-1/3,\overline{\mathbf{6}})$	$(+1/3,\overline{\mathbf{3}})$	
3-i	$(\bar{u}\bar{u})(\bar{e})(\bar{e})(dd)$		$(+4/3,\overline{\mathbf{3}})$	$(+1/3,\overline{\mathbf{3}})$	$(-2/3,\overline{\mathbf{3}})$	only with V_ρ and V'_ρ
			$(+4/3,\mathbf{6})$	$(+1/3,\mathbf{6})$	$(-2/3,\mathbf{6})$	
3-ii	$(\bar{u}\bar{u})(d)(d)(\bar{e}\bar{e})$		$(+4/3,\overline{\mathbf{3}})$	$(+5/3,\mathbf{3})$	$(+2,\mathbf{1})$	only with V_ρ
			$(+4/3,\mathbf{6})$	$(+5/3,\mathbf{3})$	$(+2,\mathbf{1})$	
3-iii	$(dd)(\bar{u})(\bar{u})(\bar{e}\bar{e})$		$(+2/3,\mathbf{3})$	$(+4/3,\overline{\mathbf{3}})$	$(+2,\mathbf{1})$	only with V_ρ
			$(+2/3,\overline{\mathbf{6}})$	$(+4/3,\overline{\mathbf{3}})$	$(+2,\mathbf{1})$	
4-i	$(d\bar{e})(\bar{u})(\bar{u})(d\bar{e})$	(c)	$(-2/3,\overline{\mathbf{3}})$	$(0,\mathbf{1})$	$(+2/3,\mathbf{3})$	RPV [58–60]
			$(-2/3,\overline{\mathbf{3}})$	$(0,\mathbf{8})$	$(+2/3,\mathbf{3})$	RPV [58–60]
4-ii-a	$(\bar{u}\bar{u})(d)(\bar{e})(d\bar{e})$		$(+4/3,\overline{\mathbf{3}})$	$(+5/3,\mathbf{3})$	$(+2/3,\mathbf{3})$	only with V_ρ
			$(+4/3,\mathbf{6})$	$(+5/3,\mathbf{3})$	$(+2/3,\mathbf{3})$	see Sec. 4 (this work)
4-ii-b	$(\bar{u}\bar{u})(\bar{e})(d)(d\bar{e})$		$(+4/3,\overline{\mathbf{3}})$	$(+1/3,\overline{\mathbf{3}})$	$(+2/3,\mathbf{3})$	only with V_ρ
			$(+4/3,\mathbf{6})$	$(+1/3,\mathbf{6})$	$(+2/3,\mathbf{3})$	
5-i	$(\bar{u}\bar{e})(d)(d)(\bar{u}\bar{e})$	(c)	$(-1/3,\mathbf{3})$	$(0,\mathbf{1})$	$(+1/3,\overline{\mathbf{3}})$	RPV [58–60]
			$(-1/3,\mathbf{3})$	$(0,\mathbf{8})$	$(+1/3,\overline{\mathbf{3}})$	RPV [58–60]
5-ii-a	$(\bar{u}\bar{e})(\bar{u})(\bar{e})(dd)$		$(-1/3,\mathbf{3})$	$(+1/3,\overline{\mathbf{3}})$	$(-2/3,\overline{\mathbf{3}})$	only with V'_ρ
			$(-1/3,\mathbf{3})$	$(+1/3,\mathbf{6})$	$(-2/3,\mathbf{6})$	
5-ii-b	$(\bar{u}\bar{e})(\bar{e})(\bar{u})(dd)$		$(-1/3,\mathbf{3})$	$(-4/3,\mathbf{3})$	$(-2/3,\overline{\mathbf{3}})$	only with V'_ρ
			$(-1/3,\mathbf{3})$	$(-4/3,\mathbf{3})$	$(-2/3,\mathbf{6})$	

Table 3: *General decomposition of the $d = 9$ operator $\bar{u}\bar{u}dd\bar{e}\bar{e}$ for topology I. Here we do not specify the chirality of outer fermions, and the mediators are given with the charge of electromagnetic $U(1)_{\rm em}$ and that of colour $SU(3)_c$. The symbols S and S' denote scalars, V_ρ and V'_ρ vectors, and ψ a fermion. The column "Long Range?" indicates if and which type of long-range diagram in Fig. 2 can be constructed, apart from the short-range diagram (d). The column "Models/Refs./Comments" lists possible models, discussed previously in the literature, and references, and comments on possible limitations for the mediators. Here "RPV" stands for R-parity violating SUSY models, and "LQ" for "leptoquarks".*

		Mediator $(Q_{\rm em}, Q_{\rm colour})$			
#	Decomposition	S or V_ρ	S' or V'_ρ	S'' or V''_ρ	Models/Refs./Comments
1	$(\bar{u}d)(\bar{u}d)(\bar{e}\bar{e})$	$(+1,\mathbf{1})$	$(+1,\mathbf{1})$	$(-2,\mathbf{1})$	Addl. triplet scalar [69] LR-symmetric models [40,42]
		$(+1,\mathbf{8})$	$(+1,\mathbf{8})$	$(-2,\mathbf{1})$	
2	$(\bar{u}d)(\bar{u}\bar{e})(\bar{e}d)$	$(+1,\mathbf{1})$	$(-1/3,\mathbf{3})$	$(-2/3,\overline{\mathbf{3}})$	
		$(+1,\mathbf{8})$	$(-1/3,\mathbf{3})$	$(-2/3,\overline{\mathbf{3}})$	
3	$(\bar{u}\bar{u})(dd)(\bar{e}\bar{e})$	$(+4/3,\overline{\mathbf{3}})$	$(+2/3,\mathbf{3})$	$(-2,\mathbf{1})$	only with V_ρ and V'_ρ
		$(+4/3,\mathbf{6})$	$(+2/3,\overline{\mathbf{6}})$	$(-2,\mathbf{1})$	
4	$(\bar{u}\bar{u})(\bar{e}d)(\bar{e}d)$	$(+4/3,\overline{\mathbf{3}})$	$(-2/3,\overline{\mathbf{3}})$	$(-2/3,\overline{\mathbf{3}})$	only with V_ρ
		$(+4/3,\mathbf{6})$	$(-2/3,\overline{\mathbf{3}})$	$(-2/3,\overline{\mathbf{3}})$	
5	$(\bar{u}\bar{e})(\bar{u}\bar{e})(dd)$	$(-1/3,\mathbf{3})$	$(-1/3,\mathbf{3})$	$(+2/3,\mathbf{3})$	only with V''_ρ
		$(-1/3,\mathbf{3})$	$(-1/3,\mathbf{3})$	$(+2/3,\overline{\mathbf{6}})$	[70,71]

Table 4: *Decomposition of topology II. As in Tab 3, we do only give electric and colour charges of the internal bosons here. The mediators can be either scalars (S, S', S'') or vectors (V_ρ, V'_ρ, V''_ρ). All listed possibilities give short-range contributions.*

cases listed, which require that the internal fermion has a fractional electric charge. As far as we know, none of them have been discussed in the literature before. All of these new “models” not only require fractionally charged fermions, but also exotic bosons. The latter can be doubly-charged bileptons [67], diquarks, or leptoquarks [68].

The $0\nu\beta\beta$ decay process violates lepton number L. We can easily identify the different possibilities for LNV from Tab. 3 (cf., Fig. 3, left panel). If the internal fermion is neutral, it can have a Majorana mass and a mass insertion leads then to $\Delta(L) = 2$. This is the case, for example, in the mass mechanism and in the $W_R - N - W_R$ diagram of LR-symmetric models. The other possibility is to have LNV vertices. For the cases with a doubly charged bilepton $S'(+2,\mathbf{1})$, for example, one can have one $\Delta(L) = 2$ vertex. And, finally, it is possible to have models with two $\Delta(L) = 1$ vertices. An example is trilinear RPV SUSY, e.g., from the superpotential $\mathcal{W} = \lambda' \widehat{L}\widehat{Q}\widehat{D}^c$.

Most of the new models we find are of the short-range type and thus should be testable at the LHC. We will discuss one particular example in greater detail in Sec. 4.

3.2 Decomposition of topology II

There are a total of five (again times two due to colour) possibilities to assign the outer fermions to T-II. These are listed together with the electric and colour charges of the possible mediators in Tab. 4. As before, this table is valid for both V and S intermediate states and we do not specify the chiralities of the outer fermions here. In the appendix, we give a table for the operator decompositions with fixed chiralities.

Much fewer models with T-II have been studied in the literature than for T-I. In fact, we have found only the cases T-II-1 and T-II-5 (with a scalar $\overline{\mathbf{6}}$) have been considered previously. The best known is the case T-II-1 with bosons transforming as $V(+1,\mathbf{1})$. This diagram can be produced by adding an $SU(2)_L$ triplet scalar to the SM particle content [69] (case SSS).[8] The same diagram can occur in LR-symmetric models [40] (as VVS). In this case, the outer fermions are again all right-handed, and this possibility has been considered in [42]. Note that in all these models, there is always an additional contribution from T-I, which is not necessarily present for all T-II models. In the case of the LR-symmetric model, T-II can be comparable with T-1, but can never completely dominate the contributions to $0\nu\beta\beta$ decay.

Finally there is the recent paper [70], in which the author constructs a model with the particle content corresponding to the scalars in T-II-5 (with $\bar{\mathbf{6}}$ representation under $SU(3)_c$). In this model neutrinos are pseudo-Dirac, such that T-II-5 gives the dominant contribution to $0\nu\beta\beta$ decay. This is the only example in the literature, which we are aware of that a T-II contribution dominates the $0\nu\beta\beta$ decay amplitude.[9] Note, however, that our Tab. 4 allows to construct a number of additional models with this property.

4 An example for short-range $0\nu\beta\beta$ decay, and its test at the LHC

As mentioned above, one expects that exotic models with LNV, which lead to short-range contributions for $0\nu\beta\beta$ decay, yield testable phenomenology at the LHC. The classical example is the LR-symmetric model, and there is also a recent paper that has made a study for trilinear RPV SUSY and $0\nu\beta\beta$ decay [74]. In this section we will discuss basic LHC phenomenology of one particular example in our Tab. 3, based on the decomposition T-I-4-ii-a. We have chosen this particular case basically for two reasons: (i) it belongs to the class of models, which have not been studied in the literature before, and (ii) it leads to richer phenomenology at the LHC than either the LR-symmetric model or RPV SUSY. By using the different signals which we will discuss in the following, one could distinguish this model from the other possibilities, such as the LR-symmetric model and RPV SUSY.

For the decomposition T-I-4-ii-a one needs to introduce three new particles to the SM particle content, which are (a) a diquark, which is either a vector $V_{\mathrm{DQ}}^{4/3}$ or a scalar $S_{\mathrm{DQ}}^{4/3}$, (b) an exotic coloured ($\mathbf{3}$) vector-like fermion $\Psi^{5/3}$ with electric charge 5/3, and (c) a leptoquark, which is again either a vector $V_{\mathrm{LQ}}^{-2/3}$ or a scalar $S_{\mathrm{LQ}}^{2/3}$. We will concentrate on the case where both the diquark and the leptoquark are scalars. The vector case is qualitatively similar from the point of view of LHC phenomenology.[10] The $0\nu\beta\beta$ decay is generated through the diagram shown in Fig. 4.

So far, the chiralities of the outer fermions are not determined. When taking into account the fact that the effective operator should be a component of a SM gauge invariant operator, the number of the choices is limited. There are three possibilities of the SM gauge invariant operators

[8]However, it was shown in [7, 72, 73] that this contribution is always sub-dominant.

[9]A similar diagram can be found in Fig. 2 of [71]. The model therein was constructed to describe neutrino mass at two-loop order. Note that neutrino mass may be obtained at a lower than 4-loop order (as postulated by the black box diagram), in a specific model. That, however, necessarily requires that the effective operator can be promoted to an $SU(2)$ invariant operator with one or two lepton doublets, which cannot be done for every model. The $SU(2)$ invariant effective operator with one lepton doublet and a right-handed electron may induce neutrino mass at the 3-loop level (as our example in Sec. 4), and the operator with two lepton doublets may induce neutrino mass at the 2-loop level. If one assumes that the SM gauge invariant effective operator is constrained by the current limit of $0\nu\beta\beta$ decay, the 2- and 3-loop-induced neutrino masses via the associated operators $SU(2)_L$ should be sub-dominant.

[10]Of course, some numerical factors are different in the vector cases from the scalar case.

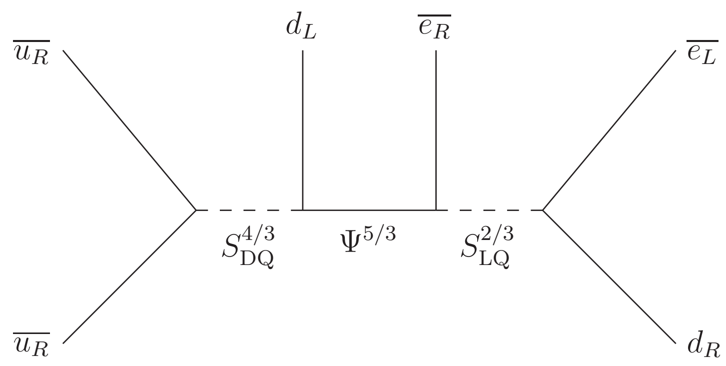

Figure 4: *The diagram of $0\nu\beta\beta$ for the example T-I-4-ii-a with two scalar mediators (SFS type diagram).*

that can contain the T-I-4-ii-a operator,[11] which are $(\overline{QQ})(d_R)(\overline{L})(\overline{L}d_R)$, $(\overline{u_R u_R})(Q)(\overline{e_R})(\overline{L}d_R)$, and $(\overline{u_R u_R})(d_R)(\overline{L})(\overline{e_R}Q)$. We note in passing that they correspond to the effective operators (#11, #20, and also #20 respectively) shown in [45]. Here, we take the second one for our example, and consequently the chiralities of the outer fermions are fixed as $(\overline{u_R u_R})(d_L)(\overline{e_R})(\overline{e_L}d_R)$. Note that in this example, neutrino mass can be generated at the 3-loop order, with one effective vertex given by the $0\nu\beta\beta$ operator. This contribution should be negligible by similar arguments as in Ref. [11].

Diquarks at the LHC have been studied recently in [75]. Among them, the relevant one for our $0\nu\beta\beta$ example is a scalar colour-sextet ($\mathbf{6}$) diquark $S_{\rm DQ}^{+4/3}(\mathbf{6},\mathbf{1})_{+4/3}$ which is an $SU(2)_L$ singlet ($\mathbf{1}$) and has $+4/3$ $U(1)_Y$-hypercharge.[12] It interacts with two right-handed up-quarks as

$$\mathcal{L}_{\rm DQ} = \left[\lambda_{\rm DQ}^{\alpha\beta}(\overline{u_{\alpha R}})^{Ia}(T_{\bar{\mathbf{6}}})^X_{IJ}({u_{\beta R}}^c)^J_a(S_{\rm DQ}^{+4/3})_X + h.c.\right] - m_{\rm DQ}^2(S_{\rm DQ}^{-4/3})^X(S_{\rm DQ}^{+4/3})_X. \tag{6}$$

where the indices α and β indicate the generations of the up-type quarks, I and J label the fundamental representations ($\mathbf{3}$ and $\bar{\mathbf{3}}$) of $SU(3)_c$ ($I,J=1,2,3$), and a is the index for the 2-component left-handed (=conjugate of right-handed) spinor. Here, the matrices $(T_{\bar{\mathbf{6}}})^X_{IJ}$ (X =1-6) provides $SU(3)_c$ Clebsch-Gordan coefficients, which are symmetric under the exchange of I and J. The concrete form of $T_{\bar{\mathbf{6}}}$ is given in App. A.2. If the diquark is chosen as a colour $\bar{\mathbf{3}}$ (i.e., $\epsilon_{IJK}(\overline{u_{\alpha R}})^I({u_{\beta R}}^c)^J(S_{\rm DQ}^{+4/3})^K)$ instead of a $\mathbf{6}$, the coefficient $\lambda_{\rm DQ}^{\alpha\beta}$ must be antisymmetric in the indices, due to the transformation properties of the $\mathbf{3}$ that is made from an antisymmetric combination in colour indices of two $\bar{\mathbf{3}}$ (two $\bar{u}$). For $0\nu\beta\beta$ decay, only the choice $\alpha=\beta=u$ is relevant. Thus, the scalar diquark $S_{\rm DQ}$ must be a colour $\mathbf{6}$ representation.[13]

A diquark with couplings as in Eq. (6) will be copiously produced at the LHC. In [75], the authors evaluated the production cross sections σ, which are as large as $\sigma/(\lambda_{\rm DQ}^2 {\rm BR}_{\rm jj}) = 400$ (1) pb for $m_{\rm DQ} = 1$ (3) TeV. Here, ${\rm BR}_{\rm jj}$ is the branching ratio for the diquark decaying to two jets. Due to the s-channel resonance in the cross section, σ scales approximately as $\sigma \propto \lambda_{\rm DQ}^2$ (in the narrow width approximation). Recently, the CMS [76] and ATLAS [77, 78] collaborations have searched for resonances in the dijet mass spectrum and upper limits on $\sigma \times {\rm BR}_{\rm jj} \times \mathcal{A}$ have been derived as a

[11] The chiralities of the outer fermions on the vertices v_2 and v_3 are chosen so that the mass term in the propagator of the fermion mediator $\Psi^{5/3}$ is picked out. See appendix for more details.

[12] Here we use the notation $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ from the appendix.

[13] This is strictly true only for unmixed (valence) quarks. Note also that for vector diquarks in principle both, the $\mathbf{6}_s$ and the $\bar{\mathbf{3}}_a$ can contribute.

function of the invariant dijet mass. Here, $\mathcal{A}$ is the acceptance, which is estimated to be $\mathcal{A} \simeq 0.6$ for isotropic decays [76]. The experimental upper limits range from $\sigma \times \mathrm{BR_{jj}} \times \mathcal{A} \simeq 1$ (0.01) pb for $m_{\mathrm{DQ}} = 1$ (3) TeV. These limits get stronger for larger values of m_{DQ}, because of the larger QCD background for smaller invariant masses. These limits, together with the theoretical calculation of the cross sections [75], imply upper limits on $\lambda_{\mathrm{DQ}}^{uu}$ of the order of roughly $\lambda_{\mathrm{DQ}}^{uu} \lesssim 0.2$ over the whole mass range explored ($m_{\mathrm{DQ}} \sim (1-4)$ TeV).

Note that a scalar diquark has been proposed [79–81] as a possible explanation for the unexpectedly larger $t\bar{t}$ asymmetry observed at the Tevatron. However, these papers consider only a scalar $\mathbf{3}_a$, which will not contribute to $0\nu\beta\beta$ decay, as explained above. A $\bar{\mathbf{6}}_s$ would probably be able to give a similar enhancement, but a recent paper by the ATLAS collaboration claims that most of the parameter space of [80] is now ruled out by LHC data [82]. We will therefore not enter into a detailed discussion of this possibility.

The mediator $\Psi^{5/3}$ is a heavy vector-like coloured ($\mathbf{3}$) fermion, aka Vector-like Quark (VLQ). The LHC phenomenology of such states has been recently studied by a number of authors [83–86]. From the SM gauge invariance, this exotic fermion should be a component field of an $SU(2)_L$ doublet $\Psi = (\Psi^{5/3}, \Psi^{2/3})^T$ with hypercharge 7/6. Current limits from pair production have been summarised recently in [85]. For the $\Psi^{5/3}$ the ATLAS search for pair-produced heavy quarks decaying to $WqWq$ gives $m_{\Psi^{5/3}} \gtrsim 350$ GeV [85]. Note that vector-like quarks have received a lot of attention recently [87–99] as a possibility to explain the larger than expected event rate in $h \to \gamma\gamma$ observed by the ATLAS [100] and CMS [101] collaborations.

The other scalar mediator $S_{\mathrm{LQ}}^{2/3}$, which interacts with d_R and L, can be identified as so-called a first generation Leptoquark (LQ) which interacts only with the first generation fermions and it comes from the $SU(2)_L$ doublet with hypercharge, 1/6, $S_{\mathrm{LQ}} = (S_{\mathrm{LQ}}^{2/3}, S_{\mathrm{LQ}}^{-1/3})^T$ [68]. The Lagrangian relevant for generating the $0\nu\beta\beta$ decay diagram of Fig. 4 contains

$$\mathcal{L}_{\mathrm{LQ}} = \left[\lambda_{\mathrm{LQ}}(\overline{L})^i_{\dot{a}}(d_R)^{\dot{a}}_I(\mathrm{i}\tau^2)_{ij}(S^*_{\mathrm{LQ}})^{Ij} + \mathrm{h.c.}\right] - m^2_{\mathrm{LQ}}(S^\dagger_{\mathrm{LQ}})^{Ii}(S_{\mathrm{LQ}})_{Ii}\,, \tag{7}$$

where $(\mathrm{i}\tau^2)$ is an antisymmetric tensor for the $SU(2)_L$ indices. At the LHC, the first generation LQs are studied through pair production via the strong interaction. The produced LQs can then decay into eq or νq pairs through the interaction shown in Eq. (7). This allows to derive absolute bounds on the LQ mass (nearly independent of λ_{LQ}). The current bounds [102] from ATLAS are $m_{\mathrm{LQ}} > 660\,\mathrm{GeV}$. The CMS searches for the LQ give the limits which range from 830 GeV to 640 GeV for first generation LQs with $\mathrm{Br}_{eq} = 1$ to $\mathrm{Br}_{eq} = 0.5$. The HERA experiment [103] has also searched for LQs, but via single LQ production. This leads to limits in the parameter plane λ_{LQ}-m_{LQ}. However, practically all the HERA-excluded combinations are now superseded by the recent LHC limits.

For the diagram generating $0\nu\beta\beta$ shown in Fig. 4, two more interaction terms among the different mediators are needed:

$$\mathcal{L}_\Psi = \lambda^\alpha_{\mathrm{DQ}\Psi}(\overline{Q_\alpha{}^c})^a_{Ii}(T_{\mathbf{6}})^{IJ}_X(\mathrm{i}\tau^2)^{ij}(\Psi_L)_{Jja}(S_{\mathrm{DQ}}^{-4/3})^X + \lambda_{\mathrm{LQ}\Psi}(\overline{\Psi_R})^{Iia}(e_R{}^c)_a(S_{\mathrm{LQ}})_{Ii} + h.c. \tag{8}$$

Note that Eq. (8), together with the previously specified pieces of Lagrangians, necessarily violates lepton (but not baryon) number, as is necessary for the generation of a finite $0\nu\beta\beta$ decay amplitude. No constraints on $\lambda_{\mathrm{DQ}\Psi}$ and $\lambda_{\mathrm{LQ}\Psi}$ exist in the literature up to now.

After integrating out all the heavy fields, the LNV $d = 9$ effective Lagrangian for $0\nu\beta\beta$ decay is given as

$$\mathcal{L}^{\rm eff} = \frac{\lambda_{\rm DQ}\lambda_{\rm DQ\Psi}\lambda_{\rm LQ\Psi}\lambda_{\rm LQ}}{m_{\rm DQ}^2 m_\Psi m_{\rm LQ}^2}\left[(\overline{u_R})^{I'a}(T_{\bar{\mathbf{6}}})^X_{I'J'}(u_R{}^c)^{J'}_a\right]\left[(\overline{d_L{}^c})^b_I(T_{\mathbf{6}})^{IJ}_X(e_R{}^c)_b\right]\left[(\overline{e_L})_{\dot{c}}(d_R)^{\dot{c}}_J\right] + h.c. \tag{9}$$

As demonstrated in App. A.2, we arrive at a linear combination of the basis operators — the effective current description of [37] — after Fierz and the colour-index transformation, which is,

$$\mathcal{L}^{\rm eff} = \frac{\lambda_{\rm DQ}\lambda_{\rm DQ\Psi}\lambda_{\rm LQ\Psi}\lambda_{\rm LQ}}{m_{\rm DQ}^2 m_\Psi m_{\rm LQ}^2}\frac{1}{32}\left[{\rm i}(\mathcal{O}_4)_{LR} - (\mathcal{O}_5)_{LR}\right] \equiv \mathcal{C}_4\,(\mathcal{O}_4)_{LR} - \mathcal{C}_5\,(\mathcal{O}_5)_{LR} \tag{10}$$

with correspondingly defined coefficients $\mathcal{C}_i$ ($\propto \Lambda^{-5}$) and $|\mathcal{C}_4| = |\mathcal{C}_5|$ in this particular model. The basis operators are defined with the chirality indices L and R as described in App. A.1, cf., Eqs. (18) and (19). Note that the transition from Eq. (9) to the basis in Eq. (10) can be directly read off from our tables in App. A.3. This example corresponds to the second line of T1-4-ii-2 in Tab. 9, where Eq. (10) can be read off from the last column. Therefore, this example serves to illustrate how to use the tables in our appendix.

The general formula to calculate the half-life time is shown in [37], cf., Eq. (20) in App. A.1, and is given with the normalised (mass dimensionless) coefficients $\epsilon_i \equiv 2m_P\mathcal{C}_i/G_F^2$. The relevant part is

$$(T_{1/2}^{0\nu\beta\beta})^{-1} = G_2\left|\sum_{i=4}^{5}\epsilon_i\mathcal{M}_i\right|^2 \tag{11}$$

in this example. Here G_2 [yr^{-1}] is a phase space factor, and $\mathcal{M}_i$ are the nuclear matrix element parts of the total amplitude, which are normalised to be mass dimensionless.

The half-life Eq. (11) is dominated by the ϵ_4 contribution, because of $\mathcal{M}_4 \gg \mathcal{M}_5$.[14] Using the experimental bounds to ϵ_4 listed in Tab. 2, we obtain the bounds for the masses of the heavy particles as a function of the couplings involved:

$$|\mathcal{C}_4| = \frac{|\lambda_{\rm DQ}\lambda_{\rm DQ\Psi}\lambda_{\rm LQ\Psi}\lambda_{\rm LQ}|}{m_{\rm DQ}^2 m_\Psi m_{\rm LQ}^2}\frac{1}{32} = \frac{G_F^2}{2m_P}\epsilon_4 < \frac{G_F^2}{2m_P}1.2\cdot 10^{-8} \tag{12}$$

which, assuming that all masses are of order Λ, leads to

$$\Lambda \gtrsim 2.0\lambda_{\rm eff}^{\frac{4}{5}}\ {\rm TeV}, \tag{13}$$

where $\lambda_{\rm eff} \equiv (\lambda_{\rm DQ}\lambda_{\rm DQ\Psi}\lambda_{\rm LQ\Psi}\lambda_{\rm LQ})^{(1/4)}$.

While the individual masses involved in these expressions have been constrained from the searches at colliders, which were discussed above, the most direct test of this model — a signal directly related to the diagram shown in Fig. 4 — can be done at the LHC in the following way. In the case where $m_{\rm LQ} < m_\Psi < m_{\rm DQ}$, once a diquark $S_{\rm DQ}^{4/3}$ is produced, it will decay to the VLQ $\Psi^{5/3}$ with a branching ratio of roughly (neglecting kinematical factors) ${\rm Br} \sim \frac{\Gamma(S_{\rm DQ}\to\Psi Q)}{\Gamma(S_{\rm DQ}\to\Psi Q)+\Gamma(S_{\rm DQ}\to u_R u_R)} \sim \frac{\lambda_{\rm DQ\Psi}^2}{\lambda_{\rm DQ\Psi}^2+\lambda_{\rm DQ}^2}$, i.e.,

[14] In fact, given our tables in the appendix, one does not need to rely on the assumption of only one dominating NME since the coefficients are explicitely given, and one can directly translate the half-life time into the model constraints for a specific model. In this specific example, we use this dominance for the sake of simplicity.

Br $\sim 1/2$ for $\lambda_{\rm DQ\Psi} = \lambda_{\rm DQ}$, and the VLQ Ψ will then further decay to the LQ $S_{\rm LQ}^{2/3}$ plus a lepton. This decay channel will usually dominate over the 3-body decay $\Psi^{5/3} \to 3j$ via an off-shell diquark. The total signal for this decay chain is then e^+e^+jj.[15] This signal is the same as the process searched for by the ATLAS [43] and the CMS collaborations [44] in the context of the LR-symmetric model, and thus the result of the search can already be used to derive limits on the parameter space of the mediator fields, DQ, VLQ, and LQ. The upper limit on this channel reported in [44] is around 2.5 fb with an assumption of $m_{W_R} = 3$ TeV in the LR symmetric model. This bound corresponds then to roughly $\lambda_{\rm DQ\Psi} = \lambda_{\rm DQ} \lesssim 0.07$ for $m_{\rm DQ} = 3$ TeV (with an assumption on the mass difference $m_{\rm DQ} - m_\Psi \gtrsim 100$ GeV due to the experimental cuts). While this limit provides already some interesting constraints on this example "model" shown in Fig. 4, a more detailed analysis is required, before it is ruled out as the dominant mechanism of $0\nu\beta\beta$ decay process. We expect, of course, that much more stringent limits will be provided by the forth-coming LHC run with $\sqrt{s} = 14$ TeV.

5 Summary and conclusions

We have systematically decomposed the $0\nu\beta\beta$ operator with mass dimension nine ($d = 9$), resulting in a tree-level complete list of possible contributions to $0\nu\beta\beta$ decay. Our main results are summarised in Tables 3 and 4.

Our list encompasses all previously discussed contributions to $0\nu\beta\beta$ decay and, more interestingly, demonstrates that actually most cases have not been discussed yet. The new options typically require not only fractionally charged fermions, but also exotic bosons. The latter can be doubly-charged bileptons, diquarks, or leptoquarks. For topology II (cf., right panel of Fig. 3), we have also found a possibility with integer charges and scalars (or vectors) only. In fact, almost all of the topology II possibilities have not been discussed in the literature before as leading contribution to $0\nu\beta\beta$ decay.

The $d = 9$ $0\nu\beta\beta$ decay operator is genuinely suppressed by $1/\Lambda^5$ if the lightest mediator is heavier than a few GeV. On the other hand, the $0\nu\beta\beta$ decay operator mediated by a light neutrino with right-chiral interactions (long-range contribution) is weaker suppressed, and therefore, the new physics scale Λ could be quite high, unreachable in current collider experiments. $0\nu\beta\beta$ decay itself does very likely give the strongest constraint on such mediators. In the short-range case, Λ points towards the TeV scale, if a signal is detected at the next generation $0\nu\beta\beta$ decay experiments, which means that the mediators leading to $0\nu\beta\beta$ decay may be constrained at the LHC. We have therefore focused on the short-range case for a more detailed analysis.

In order to translate the bound on the half-life $T_{1/2}^{0\nu\beta\beta}$ into a bound for the masses and couplings of a particular model, the Lorentz structure of the effective $0\nu\beta\beta$ decay operator is important, since each model will be sensitive to a distinct combination of Nuclear Matrix Elements (NME). For the short-range case mediated by scalars and a fermion, we have therefore expanded all models in terms of the chiralities of their effective low-energy operators. The results are given in tabular form, and can be used to directly translate new bounds or re-computed NMEs into a mass and coupling limit for a specific model. From our lists and the recipes discussed in the appendix, also the corresponding vector cases can be derived in a straightforward manner.

[15] If $m_\Psi < m_{\rm LQ}$ both the $3j$ and the eej final states of the Ψ decay will occur.

We have also worked out one example which can be tested at the LHC in greater detail. This example requires a diquark, an exotic colour-triplet vector-like fermion with electric charge 5/3, and a leptoquark. While the individual mediators can be produced and tested at the LHC, not all couplings needed for $0\nu\beta\beta$ decay are directly accessible in all kinematically possible configurations. However, in case the exotic fermion is lighter than the diquark, the $0\nu\beta\beta$ decay diagram can be directly tested by processes with two like-sign leptons and two jets in the final state. While our example only serves as a prototype, we expect that a more systematic study of all short-range $0\nu\beta\beta$ decay contributions for the LHC is feasible.

In conclusion, a discovery of $0\nu\beta\beta$ guarantees physics beyond the Standard Model. Whether this new physics is due to the $d = 5$ Weinberg operator which implies heavy mediators, such as heavy right-handed neutrinos in the famous (type I) seesaw mechanism, or some other mechanism, is an open question. There are many different possibilities to mediate $0\nu\beta\beta$ decay without Majorana neutrinos, even at tree level. We have discussed that many of these options have in common that the mediators should be found at the LHC, or that LHC will provide very stringent constraints. Finally, the most interesting case may be that $0\nu\beta\beta$ decay is discovered in conflict with neutrino mass bounds from tritium endpoint experiments [104] or cosmology [33–35] which would point towards one of our exotic mechanisms.

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A From effective Lagrangians to the decay rate

Our general decomposition of the $0\nu\beta\beta$ decay operator is given in terms of the quark (and the lepton) currents. However, $0\nu\beta\beta$ is a low-energy process in which we must treat hadronic currents — neutrons are converted into protons in a nucleus. Therefore, the derivation of the decay rate for any model leading to an effective Lagrangian of the form Eqs. (2) and (5), involves a number of steps. However, since this derivation has been studied several times in the literature, we summarise only the relevant definitions necessary for making contact with the general Lorentz-invariant description of [37, 38] in this appendix.

A.1 Decay rate

Although we have already given the effective Lagrangian for short-range contributions in Eq. (5), here we re-define them with the chiralities:

$$\mathcal{L}^{\text{eff}} = \frac{G_F^2}{2} m_P^{-1} \left[\sum_{i=1}^{3} \epsilon_i^{\{XY\}Z} (\mathcal{O}_i)_{\{XY\}Z} + \sum_{i=4}^{5} \epsilon_i^{XY} (\mathcal{O}_i)_{XY} \right], \tag{14}$$

where the effective operators are described as

$$(\mathcal{O}_1)_{\{XY\}Z} \equiv J_X J_Y j_Z, \tag{15}$$
$$(\mathcal{O}_2)_{\{XY\}Z} \equiv (J_X)^{\mu\nu} (J_Y)_{\mu\nu} j_Z, \tag{16}$$
$$(\mathcal{O}_3)_{\{XY\}Z} \equiv (J_X)^{\mu} (J_Y)_{\mu} j_Z, \tag{17}$$
$$(\mathcal{O}_4)_{XY} \equiv (J_X)^{\mu\nu} (J_Y)_{\mu} (j)_{\nu}, \tag{18}$$
$$(\mathcal{O}_5)_{XY} \equiv J_X (J_Y)^{\mu} (j)_{\mu}. \tag{19}$$

The following formula directly relates the inverse half-life with the effective Lagrangian Eq. (14): [16]

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G_1 \left| \sum_{i=1}^{3} \epsilon_i \mathcal{M}_i \right|^2 + G_2 \left| \sum_{i=4}^{5} \epsilon_i \mathcal{M}_i \right|^2 + G_3 \text{Re} \left[\left(\sum_{i=1}^{3} \epsilon_i \mathcal{M}_i \right) \left(\sum_{i=4}^{5} \epsilon_i \mathcal{M}_i \right)^* \right], \tag{20}$$

Eq. (20) contains the product of three distinct factors, G_i, $\mathcal{M}_i$ and ϵ_i. Here, $G_{i\in\{1,2,3\}}$ are the leptonic phase space integrals, which can be calculated accurately, e.g., [50]. The nuclear Matrix Elements (NME) $\mathcal{M}_{i\in\{1\text{-}5\}}$ are different for different short-range contributions $\epsilon_{i\in\{1\text{-}5\}}$, detailed definitions can be found in [37], numerical values are given in [6, 37]. The contribution from the mass mechanism can be expressed as

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G_1 \left| \frac{\langle m_\nu \rangle}{m_e} \left[\mathcal{M}_{\text{GT}} - \frac{g_V^2}{g_A^2} \mathcal{M}_{\text{F}} \right] \right|^2, \tag{21}$$

where $\mathcal{M}_{\text{F}}$ and $\mathcal{M}_{\text{GT}}$ are the standard Fermi and Gamow-Teller transition matrix elements, g_V and g_A are the vector and axial-vector couplings of the hadron current, and m_e is the mass of electron.[17]

Given the numerical values of G_i and $\mathcal{M}_i$, once the combination of ϵ_i's for a given model have been identified (see the tables below), the derivation of the limits from $T_{1/2}^{0\nu\beta\beta}$ becomes straightforward.

A.2 Fierz and colour-index transformations

The general effective Lagrangians given in Eqs. (2) and (5) are described with *the standard form J of the quark current*, which (i) is a singlet under the colour $SU(3)_c$ and (ii) takes the bi-linear form $(\bar{u}\Gamma d)$ in terms of the Lorentz structure, where $\Gamma \in \{1 \pm \gamma^5, \gamma^\mu(1 \pm \gamma^5), \gamma^{\mu\nu}(1 \pm \gamma^5)\}$. It is suitable

[16] The formulae for the long-range contributions are given in [38]. Note that, when one decomposes the amplitudes into a long-range and a short range parts, one assumes implicitly that there is no new physics with a mass scale similar to the nuclear Fermi scale, i.e., $\mathcal{O}(100)$ MeV.

[17] The NME for the mass mechanism have been calculated in a number of papers in the literature. Unfortunately, however, some of the NME of the general decay rate have so far been calculated only in [6, 37, 38].

for the calculation of the hadron transition amplitudes of the type $\langle P(p)|(\bar{u}\Gamma d)|N(p')\rangle$, to describe the conversion of neutrons to protons. On the other hand, the decomposed effective operators which are listed in Tabs. 3 and 4 do not take this standard form (except for T-I-1 and T-II-1). In order to bring them to the standard form shown as the effective Lagrangian Eq. (5), we need to transform the Lorentz and the colour indices in the effective operators.

First, we discuss the treatment of colour indices. In the operator decomposition, we introduce the antisymmetric tensors ϵ_{IJK} and ϵ^{IJK} ($I, J, K = $ 1-3), the symmetric matrices $(T_{\mathbf{6}})^{IJ}_{X}$ and $(T_{\bar{\mathbf{6}}})^{X}_{IJ}$ ($X = $ 1-6) under the exchange of I and J, and the Gell-Mann matrices $(\lambda^A)_I{}^J$ ($A = $ 1-8), where the lower I-index is for $\mathbf{3}$ representation (d_I), while the upper one for $\bar{\mathbf{3}}$ ($\bar{u}^I$). Here, the matrices $T_{\mathbf{6}}$ and $T_{\bar{\mathbf{6}}}$ are explicitly defined as

$$
\begin{aligned}
(T_{\mathbf{6}})^{IJ}_{1} = (T_{\bar{\mathbf{6}}})^{1}_{IJ} &= \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}, &
(T_{\mathbf{6}})^{IJ}_{2} = (T_{\bar{\mathbf{6}}})^{2}_{IJ} &= \begin{pmatrix} 0 & 1/\sqrt{2} & \\ 1/\sqrt{2} & 0 & \\ & & 0 \end{pmatrix}, \\
(T_{\mathbf{6}})^{IJ}_{3} = (T_{\bar{\mathbf{6}}})^{3}_{IJ} &= \begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix}, &
(T_{\mathbf{6}})^{IJ}_{4} = (T_{\bar{\mathbf{6}}})^{4}_{IJ} &= \begin{pmatrix} 0 & & 1/\sqrt{2} \\ & 0 & \\ 1/\sqrt{2} & & 0 \end{pmatrix}, \\
(T_{\mathbf{6}})^{IJ}_{5} = (T_{\bar{\mathbf{6}}})^{5}_{IJ} &= \begin{pmatrix} 0 & & \\ & 0 & 1/\sqrt{2} \\ & 1/\sqrt{2} & 0 \end{pmatrix}, &
(T_{\mathbf{6}})^{IJ}_{6} = (T_{\bar{\mathbf{6}}})^{6}_{IJ} &= \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}.
\end{aligned} \tag{22}
$$

The transformation rules relevant to our work are summarised as

$$\epsilon^{IJK}\epsilon_{KI'J'} = \delta^I_{I'}\delta^J_{J'} - \delta^J_{I'}\delta^I_{J'}, \tag{23}$$

$$(T_{\mathbf{6}})^{IJ}_{X}(T_{\bar{\mathbf{6}}})^{X}_{I'J'} = \frac{1}{2}\left[\delta^I_{I'}\delta^J_{J'} + \delta^J_{I'}\delta^I_{J'}\right], \tag{24}$$

$$(\lambda^A)_{I'}{}^{I}(\lambda^A)_{J'}{}^{J} = -\frac{2}{3}\delta^I_{I'}\delta^J_{J'} + 2\delta^J_{I'}\delta^I_{J'}. \tag{25}$$

Next, we transform the spinor indices. General formulas for Fierz transformations can be found in the literature in many references, see for example [105] for 2-component spinor representations. In some decompositions, we must Fierz-transform all the six fermions, to which the well-known transformation rules for four fermions are not applicable. However, after appropriate successive transformations with the formulae shown in [105], one can reach the standard form.

Here, we demonstrate the procedure of the operator projection with an operator as an example, which is we saw in Sec. 4, $(\overline{u_R} \overline{u_R})(d_L)(\overline{e_R})(\overline{e_L} d_R)$. Writing down the operator again with all the indices explicitly, we have

$$\mathcal{O}_{\text{example}} \equiv \left[(\overline{u_R})^{I'a}(T_{\bar{\mathbf{6}}})^{X}_{I'J'}(u_R{}^c)^{J'}_{a}\right]\left[(\overline{d_L{}^c})^b_I(T_{\mathbf{6}})^{IJ}_{X}(e_R{}^c)_b\right]\left[(\overline{e_L})_{\dot{c}}(d_R)^{\dot{c}}_J\right]. \tag{26}$$

Applying Eq. (24), we obtain the following colour-singlet $\bar{u}d$ combinations, that however do not form standard quark currents yet:

$$
\begin{aligned}
\mathcal{O}_{\text{example}} =& \frac{1}{2}\left[\delta^I_{I'}\delta^J_{J'} + \delta^J_{I'}\delta^I_{J'}\right](\overline{u_R})^{I'a}(u_R{}^c)^{J'}_a(\overline{d_L{}^c})^b_I(e_R{}^c)_b(\overline{e_L})_{\dot{c}}(d_R)^{\dot{c}}_J \\
=& (\overline{u_R})^{Ia}(\overline{d_L{}^c})^b_I(u_R{}^c)^J_a(d_R)^{\dot{c}}_J(\overline{e_L})_{\dot{c}}(e_R{}^c)_b.
\end{aligned} \tag{27}
$$

In order to obtain the standard form, we transform also the spinor indices. The formulae that are necessary for this transformation will be shown later.

$$\begin{aligned}
&\text{(RHS) of Eq. (27)} \\
=&\delta_b^d \delta_{\dot{f}}^{\dot{c}} (\overline{u_R})^{Ia} (\overline{d_L{}^c})_I^b (u_R{}^c)_a^J (d_R)_J^{\dot{f}} (\overline{e_L})_{\dot{c}} (e_R{}^c)_d \\
=&-\frac{1}{32}\left[J_L (J_R)^\mu (j)_\mu + \frac{1}{\mathrm{i}} (J_L)^{\mu\nu} (J_R)_\mu (j)_\nu\right].
\end{aligned} \tag{28}$$

Here, Fierz transformations are carried out in the 2-component representation (as in [105]), which is related to the 4-component representation in the following manner. We take so-called chiral representation for a 4-component spinor, i.e., the Lorentz vector matrices $\sigma^\mu \equiv (1, \sigma^a)$ and $\overline{\sigma}^\mu \equiv (1, -\sigma^a)$ for 2-component spinors are introduced, which are given as the components of the γ^μ matrices as

$$\gamma^\mu = \begin{pmatrix} & (\sigma^\mu)_{a\dot{b}} \\ (\overline{\sigma}^\mu)^{\dot{a}b} & \end{pmatrix}. \tag{29}$$

The Lorentz tensor matrices $\sigma^{\mu\nu}$ for 2-component spinors, which appear in Eq. (28), are defined with σ^μ and $\overline{\sigma}^\mu$ as[18]

$$(\sigma^{\mu\nu})_a{}^b \equiv \frac{1}{4}\left[(\sigma^\mu)_{a\dot{a}} (\overline{\sigma}^\nu)^{\dot{a}b} - (\sigma^\nu)_{a\dot{a}} (\overline{\sigma}^\mu)^{\dot{a}b}\right], \tag{30}$$

$$(\overline{\sigma}^{\mu\nu})^{\dot{a}}{}_{\dot{b}} \equiv \frac{1}{4}\left[(\overline{\sigma}^\mu)^{\dot{a}a} (\sigma^\nu)_{a\dot{b}} - (\overline{\sigma}^\nu)^{\dot{a}a} (\sigma^\mu)_{a\dot{b}}\right], \tag{31}$$

which are related to the matrices $\gamma^{\mu\nu}$ for 4-component spinors as

$$\gamma^{\mu\nu} = 2\mathrm{i} \begin{pmatrix} (\sigma^{\mu\nu})_a{}^b & \\ & (\overline{\sigma}^{\mu\nu})^{\dot{a}}{}_{\dot{b}} \end{pmatrix}. \tag{32}$$

The relations between the currents in the 2-component representation and those (defined at Eq. (3)) in the 4-component representation are explicitly described as

$$(\overline{u_R})^{Ia} (d_L)_{Ia} = \frac{1}{2} J_L, \tag{33}$$

$$(\overline{u_R})^{Ia} (\sigma^\mu)_{a\dot{b}} (d_R)_I^{\dot{b}} = \frac{1}{2} (J_R)^\mu, \tag{34}$$

$$(\overline{u_R})^{Ia} (\sigma^{\mu\nu})_a{}^b (d_L)_{Ib} = \frac{1}{4\mathrm{i}} (J_L)^{\mu\nu}, \tag{35}$$

$$(\overline{e_L})_{\dot{a}} (\overline{\sigma}^\mu)^{\dot{a}b} (e_R{}^c)_b = \frac{1}{2} (j_L)^\mu. \tag{36}$$

In the steps of the transformation shown in Eq. (28), we applied the following formulae of Fierz-transformation,

$$\delta_a^d \delta_{\dot{b}}^{\dot{c}} = \frac{1}{2} (\sigma^\mu)_{a\dot{b}} (\overline{\sigma}^\mu)^{\dot{c}d}, \tag{37}$$

$$\epsilon_{ac}\epsilon^{bd} = -\frac{1}{2}\left[\delta_a^b \delta_c^d + (\sigma^{\rho\sigma})_a{}^b (\sigma_{\rho\sigma})_c{}^d\right], \tag{38}$$

[18] Note that the definition of 2-component tensor matrices here is different from [105] by an imaginary unit i.

and the nature of the sigma matrices,

$$(\sigma^{\mu\nu})_a{}^b(\sigma^\rho)_{b\dot{b}} = \frac{1}{2}\left[(\sigma^\mu)_{a\dot{b}}g^{\nu\rho} - (\sigma^\nu)_{a\dot{b}}g^{\mu\rho} + \mathrm{i}\epsilon^{\mu\nu\rho\sigma}(\sigma_\sigma)_{a\dot{b}}\right], \tag{39}$$

$$\mathrm{i}\epsilon^{\mu\nu\rho\sigma}(\sigma_{\rho\sigma})_a{}^b = -2(\sigma^{\mu\nu})_a{}^b. \tag{40}$$

A.3 Full decompositions for the short-range scalar-mediated topology I

In case of topology I the intermediate states contain a fermion, leading to a different treatment in case of long-range and short-range contributions. For the long-range part of the amplitude, in order to derive interesting constraints, one has to pick out the neutrino momentum q_ν from the propagator, because of $q_\nu \gg m_\nu$. On the other hand, in the short-range case, the relevant amplitude must take the mass part m_ψ in the propagator of the fermion mediator ψ, because of $q_\psi \ll m_\psi$. Thus, not all possibilities to assign chiralities of the six outer fermions lead to interesting models. When the decomposition is symbolically written as $(ab)(c)(d)(ef)$ where each parenthesis corresponds to each vertex v_i in the left panel of Fig. 3, i.e., (ab) corresponds to the outer fermions on the vertex v_1, and (c) does to v_2, and so on, in order to pick out m_ψ from the propagator, only the combinations $(\overline{c_L{}^c})(d_L)$ and $(\overline{c_R{}^c})(d_R)$ need to be considered. Note, that $\overline{a_R} \equiv \overline{a}P_L$. Since we concentrate on the SFS case, in which both the mediators between the vertices v_1 and v_2 and that between v_3 and v_4 are scalars, the chirality structures of the outer fermions on v_1 and v_4 are also restricted to be $(ab) \in \{(\overline{a_L{}^c}b_L), (\overline{a_R{}^c}b_R)\}$ and $(ef) \in \{(\overline{e_L{}^c}f_L), (\overline{e_R{}^c}f_R)\}$. At this stage, there are eight choices for the combinations of chiralities of the six outer fermions. However, some of them are not generated from the $d = 9$ SM gauge invariant operators which are listed in [45,46]. We exclude the chirality choices which require an additional Higgs doublet(s) (i.e., those originated from the operators of $d > 9$), and list all the possibilities inspired from the $d = 9$ SM gauge invariant operators.

The results are summarised in Tabs. 5-9. The id-numbers indicated in the column "#" correspond to those in Tab. 3. One can find the chirality choices explicitly in the column "Operators". The id-numbers in the column "BL" tells the correspondence to the $d = 9$ SM gauge invariant operators listed by Babu and Leung [45]. The SM charges of all possible mediators are fully identified in the column "Mediators".[19] The basis operators, cf. Eq. (5), resulting after Fierz and the colour-index transformations are given in the column "Basis op.". Some of the decompositions appear necessarily with different ones at the same time. These associated decompositions are listed in the column "Appears with".

The use and application of our tables is illustrated with the example in Sec. 4, see discussion after Eq. (9). Once a specific model is chosen, the basis operators with the corresponding coefficients can be directly read off from the table. These basis operators are directly related with the NMEs, i.e., the table lists which NMEs specific models are testing. As a consequence, the lifetime bounds can be translated into constraints on masses and couplings for a specific model — as it is illustrated in our example.

#	Operators	BL	Mediators $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ S	ψ	S'	Basis op.
1-i	$(\overline{u_L}d_R)(\overline{e_L})(\overline{e_L})(\overline{u_L}d_R)$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$\frac{1}{8}(\mathcal{O}_1)_{\{RR\}R}$
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$	s.a.a
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\mathbf{8},\mathbf{2})_{-1/2}$	s.a.a
	$(\overline{u_L}d_R)(\overline{e_L})(\overline{e_L})(\overline{u_R}d_L)$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$	s.a.a
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$-\frac{1}{12}(\mathcal{O}_1)_{\{LR\}R} - \frac{1}{8}(\mathcal{O}_3)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\mathbf{8},\mathbf{2})_{-1/2}$	s.a.a
	$(\overline{u_R}d_L)(\overline{e_L})(\overline{e_L})(\overline{u_R}d_L)$	#12	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LL\}R}$
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{2})_{-1/2}$	s.a.a
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{LL\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{LL\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\mathbf{8},\mathbf{2})_{-1/2}$	s.a.a
1-ii-a	$(\overline{u_L}d_R)(\overline{u_L})(d_R)(\overline{e_L e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{RR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_L}d_R)(\overline{u_R})(d_L)(\overline{e_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{12}(\mathcal{O}_1)_{\{LR\}R} - \frac{1}{8}(\mathcal{O}_3)_{\{LR\}R}$
	$(\overline{u_R}d_L)(\overline{u_L})(d_R)(\overline{e_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{12}(\mathcal{O}_1)_{\{LR\}R} - \frac{1}{8}(\mathcal{O}_3)_{\{LR\}R}$
	$(\overline{u_R}d_L)(\overline{u_R})(d_L)(\overline{e_L e_L})$	#12	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LL\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{LL\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{LL\}R}$
1-ii-b	$(\overline{u_L}d_R)(d_L)(\overline{u_R})(\overline{e_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{12}(\mathcal{O}_1)_{\{LR\}R} - \frac{1}{8}(\mathcal{O}_3)_{\{LR\}R}$
	$(\overline{u_L}d_R)(d_R)(\overline{u_L})(\overline{e_L e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{RR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R}d_L)(d_L)(\overline{u_R})(\overline{e_L e_L})$	#12	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LL\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{LL\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{LL\}R}$
	$(\overline{u_R}d_L)(d_R)(\overline{u_L})(\overline{e_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{12}(\mathcal{O}_1)_{\{LR\}R} - \frac{1}{8}(\mathcal{O}_3)_{\{LR\}R}$

Table 5: *The results of decomposition and projection of the operators categorised to #1. We also show the ID-number of the lepton number violating operators listed by Babu and Leung (BL) [45] (see also Ref. [46]). For T-I-i-1, there are only three independent choices of chiralities. Here, we assume that the effective operators are originated from the SM gauge ($SU(3)_c \times SU(2)_L \times U(1)_Y$) invariant $d = 9$ operators. Note that the scalar mediators S and S' of #1-i-(2) take interactions with different combinations of quarks, although their SM charges are the same. The abbreviation "s.a.a" in "Basis op." column means "same as above". The hypercharge Y is defined as $Y \equiv Q_{\rm em} - I_3$.*

#	Operators	BL	Mediators $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ S	ψ	S'	Basis op.	Appears with
2-i-a	$(\overline{u_L}d_R)(d_R)(\overline{e_L})(\overline{u_L e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R}$	
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{5}{48}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
	$(\overline{u_L}d_R)(d_R)(\overline{e_L})(\overline{u_R e_R})$	#19	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16}(\mathcal{O}_5)_{RR}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{RR}$ $-\frac{5}{48}(\mathcal{O}_5)_{RR}$	
	$(\overline{u_R}d_L)(d_R)(\overline{e_L})(\overline{u_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LR\}R}$	
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{24}(\mathcal{O}_1)_{\{LR\}R}$ $+\frac{1}{16}(\mathcal{O}_3)_{\{LR\}R}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
	$(\overline{u_R}d_L)(d_R)(\overline{e_L})(\overline{u_R e_R})$	#20	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16}(\mathcal{O}_5)_{LR}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{LR}$ $-\frac{5}{48}(\mathcal{O}_5)_{LR}$	
2-i-b	$(\overline{u_L}d_R)(\overline{e_L})(d_R)(\overline{u_L e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R}$	1-i & 5-i
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	1-i & 5-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{5}{48}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$	1-i & 5-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	1-i & 5-i
	$(\overline{u_L}d_R)(\overline{e_L})(d_R)(\overline{u_R e_R})$	#19	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16}(\mathcal{O}_5)_{\{RR\}}$	1-i & 5-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{RR}$ $-\frac{5}{48}(\mathcal{O}_5)_{RR}$	1-i & 5-i
	$(\overline{u_R}d_L)(\overline{e_L})(d_R)(\overline{u_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LR\}R}$	1-i & 5-i
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	1-i & 5-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{24}(\mathcal{O}_1)_{\{LR\}R}$ $+\frac{1}{16}(\mathcal{O}_3)_{\{LR\}R}$	1-i & 5-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	1-i & 5-i
	$(\overline{u_R}d_L)(\overline{e_L})(d_R)(\overline{u_R e_R})$	#20	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16}(\mathcal{O}_5)_{LR}$	1-i & 5-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{LR}$ $-\frac{5}{48}(\mathcal{O}_5)_{LR}$	1-i & 5-i

Table 6: *Decomposition #2-i. In case of decomposition #2-i-b, we will have not only #2-i-b, but also #1-i and #5-i.*

#	Operators	BL	Mediators $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ S	ψ	S'	Basis op.	Appears with
2-ii-a	$(\overline{u_L}d_R)(\overline{u_L})(\overline{e_L})(d_R\overline{e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{5}{48}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$	
	$(\overline{u_L}d_R)(\overline{u_R})(\overline{e_R})(d_R\overline{e_L})$	#19	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{16}(\mathcal{O}_5)_{RR}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{RR}$ $-\frac{5}{48}(\mathcal{O}_5)_{RR}$	
	$(\overline{u_R}d_L)(\overline{u_L})(\overline{e_L})(d_R\overline{e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LR\}R}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{24}(\mathcal{O}_1)_{\{LR\}R}$ $+\frac{1}{16}(\mathcal{O}_3)_{\{LR\}R}$	
	$(\overline{u_R}d_L)(\overline{u_R})(\overline{e_R})(d_R\overline{e_L})$	#20	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{16}(\mathcal{O}_5)_{LR}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{LR}$ $-\frac{5}{48}(\mathcal{O}_5)_{LR}$	
2-ii-b	$(\overline{u_L}d_R)(\overline{e_L})(\overline{u_L})(d_R\overline{e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R}$	1-i & 4-i
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	s.a.a	1-i & 4-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{5}{48}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$	1-i & 4-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	s.a.a	1-i & 4-i
	$(\overline{u_L}d_R)(\overline{e_R})(\overline{u_R})(d_R\overline{e_L})$	#19	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{16}(\mathcal{O}_5)_{RR}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{RR}$ $-\frac{5}{48}(\mathcal{O}_5)_{RR}$	
	$(\overline{u_R}d_L)(\overline{e_L})(\overline{u_L})(d_R\overline{e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LR\}R}$	1-i & 4-i
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	s.a.a	1-i & 4-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{24}(\mathcal{O}_1)_{\{LR\}R}$ $+\frac{1}{16}(\mathcal{O}_3)_{\{LR\}R}$	1-i & 4-i
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{3})_0$	$(\mathbf{3},\mathbf{2})_{+1/6}$	s.a.a	1-i & 4-i
	$(\overline{u_R}d_L)(\overline{e_R})(\overline{u_R})(d_R\overline{e_L})$	#20	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{16}(\mathcal{O}_5)_{LR}$	
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{LR}$ $-\frac{5}{48}(\mathcal{O}_5)_{LR}$	

Table 7: *Decomposition #2-ii. In case of decomposition #2-ii-b, we will have not only #2-ii-b, but also #1-i and #4-i.*

#	Operators	BL	Mediators $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ S	ψ	S'	Basis op.	Appears with
2-iii-a	$(d_R\overline{e_L})(\overline{u_L})(d_R)(\overline{u_L}e_L)$	#11	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$	4-i & 5-i
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{3})_0$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	4-i & 5-i
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{7}{48}(\mathcal{O}_1)_{\{RR\}R}$ $-\frac{1}{192}(\mathcal{O}_2)_{\{RR\}R}$	4-i & 5-i
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{3})_0$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	4-i & 5-i
	$(d_R\overline{e_L})(\overline{u_L})(d_R)(\overline{u_R}e_R)$	#19	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{32\mathrm{i}}(\mathcal{O}_4)_{RR}$ $-\frac{1}{32}(\mathcal{O}_5)_{RR}$	4-i & 5-i
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{1})_0$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$+\frac{1}{48\mathrm{i}}(\mathcal{O}_4)_{RR}$ $+\frac{7}{48}(\mathcal{O}_5)_{RR}$	4-i & 5-i
	$(d_R\overline{e_L})(\overline{u_R})(d_L)(\overline{u_L}e_L)$	#14	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{32}(\mathcal{O}_3)_{\{LR\}R}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$ $-\frac{1}{48}(\mathcal{O}_3)_{\{LR\}R}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
	$(d_R\overline{e_L})(\overline{u_R})(d_L)(\overline{u_R}e_R)$	#20	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{32\mathrm{i}}(\mathcal{O}_4)_{LR}$ $-\frac{1}{32}(\mathcal{O}_5)_{LR}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{48\mathrm{i}}(\mathcal{O}_4)_{LR}$ $+\frac{7}{48}(\mathcal{O}_5)_{LR}$	
2-iii-b	$(d_R\overline{e_L})(d_L)(\overline{u_R})(\overline{u_L}e_L)$	#14	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{16}(\mathcal{O}_1)_{\{LR\}R}$ $+\frac{1}{32}(\mathcal{O}_3)_{\{LR\}R}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{32}(\mathcal{O}_1)_{\{LR\}R}$ $+\frac{1}{64}(\mathcal{O}_3)_{\{LR\}R}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{6}},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
	$(d_R\overline{e_L})(d_L)(\overline{u_R})(\overline{u_R}e_R)$	#20	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{1})_{-1/3}$	$\frac{1}{32\mathrm{i}}(\mathcal{O}_4)_{LR}$ $-\frac{3}{32}(\mathcal{O}_5)_{LR}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{1})_{-1/3}$	$\frac{1}{64\mathrm{i}}(\mathcal{O}_4)_{LR}$ $+\frac{1}{64}(\mathcal{O}_5)_{LR}$	
	$(d_R\overline{e_L})(d_R)(\overline{u_L})(\overline{u_L}e_L)$	#11	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{3}{32}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{6}},\mathbf{2})_{+1/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{64}(\mathcal{O}_1)_{\{RR\}R}$ $+\frac{1}{256}(\mathcal{O}_2)_{\{RR\}R}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{6}},\mathbf{2})_{+1/6}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	s.a.a	
	$(d_R\overline{e_L})(d_R)(\overline{u_L})(\overline{u_R}e_R)$	#19	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{32\mathrm{i}}(\mathcal{O}_4)_{RR}$ $-\frac{3}{32}(\mathcal{O}_5)_{RR}$	
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{6}},\mathbf{2})_{+1/6}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{64\mathrm{i}}(\mathcal{O}_4)_{RR}$ $+\frac{1}{64}(\mathcal{O}_5)_{RR}$	

Table 8: *Decomposition #2-iii. In case of decomposition #2-iii-a, we will have not only #2-iii-a, but also #4-i and #5-i.*

#	Operators	BL	Mediators $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ S	ψ	S'	Basis op.
3-i	$(\overline{u_L u_L})(\overline{e_L})(\overline{e_L})(d_R d_R)$	#11	$(\mathbf{6},\mathbf{3})_{+1/3}$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R u_R})(\overline{e_L})(\overline{e_L})(d_L d_L)$	#12	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{6},\mathbf{2})_{+5/6}$	$(\mathbf{6},\mathbf{3})_{+1/3}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LL\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{LL\}R}$
	$(\overline{u_R u_R})(\overline{e_R})(\overline{e_R})(d_R d_R)$	—	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{6},\mathbf{1})_{+1/3}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{16}(\mathcal{O}_3)_{\{RR\}L}$
3-ii	$(\overline{u_L u_L})(d_R)(d_R)(\overline{e_L e_L})$	#11	$(\mathbf{6},\mathbf{3})_{+1/3}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R u_R})(d_L)(d_L)(\overline{e_L e_L})$	#12	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LL\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{LL\}R}$
	$(\overline{u_R u_R})(d_R)(d_R)(\overline{e_R e_R})$	—	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{3},\mathbf{1})_{+5/3}$	$(\mathbf{1},\mathbf{1})_{+2}$	$\frac{1}{16}(\mathcal{O}_3)_{\{RR\}L}$
3-iii	$(d_L d_L)(\overline{u_R})(\overline{u_R})(\overline{e_L e_L})$	#12	$(\overline{\mathbf{6}},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{3})_{+1/3}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LL\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{LL\}R}$
	$(d_R d_R)(\overline{u_L})(\overline{u_L})(\overline{e_L e_L})$	#11	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{+5/6}$	$(\mathbf{1},\mathbf{3})_{+1}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$
	$(d_R d_R)(\overline{u_R})(\overline{u_R})(\overline{e_R e_R})$	—	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$(\overline{\mathbf{3}},\mathbf{1})_{+4/3}$	$(\mathbf{1},\mathbf{1})_{+2}$	$\frac{1}{16}(\mathcal{O}_3)_{\{RR\}L}$
4-i	$(d_L \overline{e_R})(\overline{u_R})(\overline{u_R})(d_R \overline{e_L})$	#20	$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{32i}(\mathcal{O}_4)_{LR} - \frac{1}{32}(\mathcal{O}_5)_{LR}$
			$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	$(\mathbf{8},\mathbf{2})_{-1/2}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{24i}(\mathcal{O}_4)_{LR} - \frac{1}{24}(\mathcal{O}_5)_{LR}$
	$(d_R \overline{e_L})(\overline{u_L})(\overline{u_L})(d_R \overline{e_L})$	#11	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{1},\mathbf{1})_{0}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$
			$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\mathbf{8},\mathbf{1})_{0}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{24}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{96}(\mathcal{O}_2)_{\{RR\}R}$
4-ii-a	$(\overline{u_L u_L})(d_R)(\overline{e_L})(\overline{e_L} d_R)$	#11	$(\mathbf{6},\mathbf{3})_{+1/3}$	$(\mathbf{3},\mathbf{3})_{+2/3}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R u_R})(d_L)(\overline{e_R})(\overline{e_L} d_R)$	#20	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{32i}(\mathcal{O}_4)_{LR} - \frac{1}{32}(\mathcal{O}_5)_{LR}$
	$(\overline{u_R u_R})(d_R)(\overline{e_L})(\overline{e_R} d_L)$	#20	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{3},\mathbf{1})_{+5/3}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$-\frac{1}{32i}(\mathcal{O}_4)_{LR} - \frac{1}{32}(\mathcal{O}_5)_{LR}$
4-ii-b	$(\overline{u_L u_L})(\overline{e_L})(d_R)(\overline{e_L} d_R)$	#11	$(\mathbf{6},\mathbf{3})_{+1/3}$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R u_R})(\overline{e_R})(d_L)(\overline{e_L} d_R)$	#20	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{6},\mathbf{1})_{+1/3}$	$(\mathbf{3},\mathbf{2})_{+1/6}$	$-\frac{1}{32i}(\mathcal{O}_4)_{LR} - \frac{1}{32}(\mathcal{O}_5)_{LR}$
	$(\overline{u_R u_R})(\overline{e_L})(d_R)(\overline{e_R} d_L)$	#20	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\mathbf{6},\mathbf{2})_{+5/6}$	$(\mathbf{3},\mathbf{2})_{+7/6}$	$-\frac{1}{32}(\mathcal{O}_5)_{LR} - \frac{1}{32i}(\mathcal{O}_4)_{LR}$
5-i	$(\overline{u_L e_L})(d_R)(d_R)(\overline{u_L e_L})$	#11	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{1},\mathbf{1})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{1},\mathbf{3})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	s.a.a
			$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{8},\mathbf{1})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{24}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{96}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{8},\mathbf{3})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	s.a.a
	$(\overline{u_R e_R})(d_R)(d_R)(\overline{u_L e_L})$	#19	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{1},\mathbf{3})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{32i}(\mathcal{O}_4)_{RR} - \frac{1}{32}(\mathcal{O}_5)_{RR}$
			$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{8},\mathbf{3})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$\frac{1}{24i}(\mathcal{O}_4)_{RR} - \frac{1}{24}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R e_R})(d_R)(d_R)(\overline{u_R e_R})$	—	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{1},\mathbf{3})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{32}(\mathcal{O}_3)_{\{RR\}L}$
			$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{8},\mathbf{3})_{0}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$	$-\frac{1}{24}(\mathcal{O}_3)_{\{RR\}L}$
5-ii-a	$(\overline{u_L e_L})(\overline{u_L})(\overline{e_L})(d_R d_R)$	#11	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	s.a.a
	$(\overline{u_L e_L})(\overline{u_R})(\overline{e_R})(d_R d_R)$	#19	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{6},\mathbf{1})_{+1/3}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{32i}(\mathcal{O}_4)_{RR} - \frac{1}{32}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R e_R})(\overline{u_L})(\overline{e_L})(d_R d_R)$	#19	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{6},\mathbf{2})_{-1/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{32i}(\mathcal{O}_4)_{RR} - \frac{1}{32}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R e_R})(\overline{u_R})(\overline{e_R})(d_R d_R)$	—	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{6},\mathbf{1})_{+1/3}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$-\frac{1}{32}(\mathcal{O}_3)_{\{RR\}L}$
5-ii-b	$(\overline{u_L e_L})(\overline{e_L})(\overline{u_L})(d_R d_R)$	#11	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{2})_{-5/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{32}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{128}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\mathbf{3},\mathbf{2})_{-5/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	s.a.a
	$(\overline{u_L e_L})(\overline{e_R})(\overline{u_R})(d_R d_R)$	#19	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{1})_{-4/3}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{32i}(\mathcal{O}_4)_{RR} - \frac{1}{32}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R e_R})(\overline{e_L})(\overline{u_L})(d_R d_R)$	#19	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{2})_{-5/6}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$\frac{1}{32i}(\mathcal{O}_4)_{RR} - \frac{1}{32}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R e_R})(\overline{e_R})(\overline{u_R})(d_R d_R)$	—	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{1})_{-4/3}$	$(\mathbf{6},\mathbf{1})_{-2/3}$	$-\frac{1}{32}(\mathcal{O}_3)_{\{RR\}L}$

Table 9: *Decompositions #3, #4, and #5. The operators with two $\overline{e_R}$'s are not listed in the paper by Babu and Leung (BL) [45].*

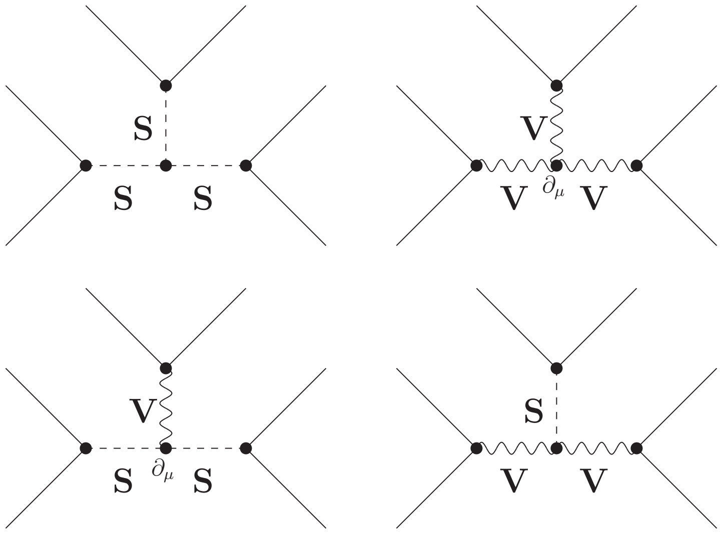

Figure 5: The four different insertions for topology-II: SSS, VVV, SSV and VVS. Note, that both VVV and SSV have necessarily derivative couplings. For a discussion see text.

A.4 Topology II

We can decompose the $0\nu\beta\beta$ operator with T-II in practically the same way as T-I. However, there are also some differences, which we will briefly discuss. As mentioned above, and shown in Fig. 5, there are four possible combinations of scalars and vectors to complete the T-II decomposition. As shown in the figure, both VVV and SSV necessarily involve derivative couplings. They lead to an effective $d = 10$ operator,

$$\mathcal{O}_{d=10} \propto \frac{1}{\Lambda^6}\partial_\rho(\bar{u}\bar{u}dd\bar{e}\bar{e})^\rho = \frac{q}{\Lambda}\frac{1}{\Lambda^5}(\bar{u}\bar{u}dd\bar{e}\bar{e}), \tag{41}$$

where q is a typical momentum of the $0\nu\beta\beta$ process. Therefore, they are suppressed by a factor of q/Λ in comparison with decompositions of the $d = 9$ operators without derivatives and can be safely neglected.

Diagrams of the type SVV come from vectors being gauge bosons, i.e., from the covariant derivative of the scalar field $\mathcal{S}$:

$$\mathcal{L}_{\rm gauge} = (D_\mu\mathcal{S})^\dagger(D^\mu\mathcal{S}) \supset g^2\mathcal{S}^\dagger\mathcal{S}V_\mu V^\mu \supset g^2\langle\mathcal{S}\rangle SV_\mu V^\mu, \tag{42}$$

if the scalar $\mathcal{S}$ can take a vacuum expectation value $\langle\mathcal{S}\rangle$. Here, S is a fluctuation around the vev $\mathcal{S} = \langle\mathcal{S}\rangle + S$, which would be a scalar mediator. If this vev breaks $SU(2)_L$, this leads to a suppression order v/Λ, but, as can be seen from the example of LR symmetry, an SM singlet vev can produce a coupling whose order is of Λ, such that the total amplitude for T-II is again proportional to Λ^{-5}. Similarly, for diagrams of the type SSS, the coupling has a dimension of mass, leading potentially to a Λ^{-5} total factor for the diagram.

Since new vectors require an extension of the gauge group, we consider the SSS case to be the more easily motivated choice. In our detailed decomposition of T-II, presented in Tab. 10, we therefore

[19]Here, the charges are fixed, following the charge flow $\mathrm{v}_1 \leftarrow \mathrm{v}_2 \leftarrow \mathrm{v}_3 \leftarrow \mathrm{v}_4$.

			Mediators $(SU(3)_c, SU(2)_L)_{U(1)_Y}$			
#	Operators	BL	S	S'	S''	Basis op.
1	$(\overline{u_L}d_R)(\overline{u_L}d_R)(\overline{e_L e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_{-1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{RR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_{-1}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R}d_L)(\overline{u_L}d_R)(\overline{e_L e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_{-1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_{-1}$	$-\frac{1}{12}(\mathcal{O}_1)_{\{LR\}R} - \frac{1}{8}(\mathcal{O}_3)_{\{LR\}R}$
	$(\overline{u_R}d_L)(\overline{u_R}d_L)(\overline{e_L e_L})$	#12	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_{-1}$	$\frac{1}{8}(\mathcal{O}_1)_{\{LL\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{1},\mathbf{3})_{-1}$	$-\frac{5}{24}(\mathcal{O}_1)_{\{LL\}R} - \frac{1}{32}(\mathcal{O}_2)_{\{LL\}R}$
2	$(\overline{u_L}d_R)(\overline{u_L e_L})(d_R\overline{e_L})$	#11	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{RR\}R}$
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	s.a.a
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\frac{5}{48}(\mathcal{O}_1)_{\{RR\}R} + \frac{1}{64}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	s.a.a
	$(\overline{u_L}d_R)(\overline{u_R e_R})(d_R\overline{e_L})$	#19	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\frac{1}{16}(\mathcal{O}_5)_{RR}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$-\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{RR} - \frac{5}{48}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R}d_L)(\overline{u_L e_L})(d_R\overline{e_L})$	#14	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$-\frac{1}{16}(\mathcal{O}_1)_{\{LR\}R}$
			$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	s.a.a
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\frac{1}{24}(\mathcal{O}_1)_{\{LR\}R} + \frac{1}{16}(\mathcal{O}_3)_{\{LR\}R}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	s.a.a
	$(\overline{u_R}d_L)(\overline{u_R e_R})(d_R\overline{e_L})$	#20	$(\mathbf{1},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\frac{1}{16}(\mathcal{O}_5)_{LR}$
			$(\mathbf{8},\mathbf{2})_{+1/2}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\frac{1}{16\mathrm{i}}(\mathcal{O}_4)_{LR} - \frac{5}{48}(\mathcal{O}_5)_{LR}$
3	$(\overline{u_L u_L})(d_R d_R)(\overline{e_L e_L})$	#11	$(\mathbf{6},\mathbf{3})_{+1/3}$	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$(\mathbf{1},\mathbf{3})_{-1}$	$-\frac{1}{24}(\mathcal{O}_1)_{\{RR\}R} + \frac{1}{96}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R u_R})(d_L d_L)(\overline{e_L e_L})$	#12	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\overline{\mathbf{6}},\mathbf{3})_{-1/3}$	$(\mathbf{1},\mathbf{3})_{-1}$	$-\frac{1}{24}(\mathcal{O}_1)_{\{LL\}R} + \frac{1}{96}(\mathcal{O}_2)_{\{LL\}R}$
	$(\overline{u_R u_R})(d_R d_R)(\overline{e_R e_R})$	—	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$(\mathbf{1},\mathbf{1})_{-2}$	$\frac{1}{24}(\mathcal{O}_3)_{\{RR\}L}$
4	$(\overline{u_L u_L})(d_R\overline{e_L})(d_R\overline{e_L})$	#11	$(\mathbf{6},\mathbf{3})_{+1/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$\frac{1}{48}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{192}(\mathcal{O}_2)_{\{RR\}R}$
	$(\overline{u_R u_R})(d_L\overline{e_R})(d_R\overline{e_L})$	#20	$(\mathbf{6},\mathbf{1})_{+4/3}$	$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	$(\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	$-\frac{1}{48\mathrm{i}}(\mathcal{O}_4)_{LR} - \frac{1}{48}(\mathcal{O}_5)_{LR}$
5	$(\overline{u_L e_L})(\overline{u_L e_L})(d_R d_R)$	#11	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$\frac{1}{48}(\mathcal{O}_1)_{\{RR\}R} - \frac{1}{192}(\mathcal{O}_2)_{\{RR\}R}$
			$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\mathbf{3},\mathbf{3})_{-1/3}$	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	s.a.a
	$(\overline{u_L e_L})(\overline{u_R e_R})(d_R d_R)$	#19	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$+\frac{1}{96\mathrm{i}}(\mathcal{O}_4)_{RR} - \frac{1}{48}(\mathcal{O}_5)_{RR}$
	$(\overline{u_R e_R})(\overline{u_R e_R})(d_R d_R)$	—	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\mathbf{3},\mathbf{1})_{-1/3}$	$(\overline{\mathbf{6}},\mathbf{1})_{+2/3}$	$-\frac{1}{48}(\mathcal{O}_3)_{\{RR\}L}$

Table 10: *Decomposition and operator projection for the three-scalar case of T-II.*

concentrate on the case of SSS. The results for SVV can be derived easily from the recipes discussed above.

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