

Modelling tri-bimaximal neutrino mixing

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We model tri-bimaximal lepton mixing from first principles in a way that avoids the problem of the vacuum alignment characteristic of such models. This is achieved by using a softly broken A_4 symmetry realized with an isotriplet fermion, also triplet under A_4 . No scalar A_4 -triplet is introduced. This represents one possible realization of general schemes characterized by the minimal set of either three or five physical parameters. In the three parameter versions m_{ee} vanishes, while in the five parameter schemes the absolute scale of neutrino mass, although not predicted, is related to the two Majorana phases. The model realization we discuss is potentially testable at the LHC through the peculiar leptonic decay patterns of the fermionic and scalar triplets.

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So far the historical discovery of neutrino oscillations constitutes the only indication of physics beyond the standard model. It is especially intriguing that the pattern of neutrino mixing angles [1] implied by current neutrino data is in sharp contrast with the structure of the Cabibbo-Kobayashi-Maskawa quark mixing matrix [2]. Current data do not yet fully determine the absolute scale of neutrino masses, nor do they shed any light on the important issue of leptonic CP violation [3, 4].

Lacking a basic theory of flavor and the origin of mass it is useful to have theoretical models restricting the pattern of fermion masses and mixings and providing guidance for future experimental searches. A successful phenomenological ansatz for leptons has been proposed by Harrison, Perkins and Scott (HPS) and is given by [5]

$$U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad (1)$$

which corresponds to

$$\tan^2 \theta_{\text{atm}} = 1, \quad \sin^2 \theta_{\text{chooz}} = 0, \quad \tan^2 \theta_{\text{sol}} = 0.5,$$

providing a good first approximation to the values indicated by current neutrino oscillation data.

In this note we discuss the general form of the effective neutrino mass operator which yields tri-bimaximal lepton mixing U_{HPS} . In general it is characterized by 5 physical parameters. These determine the two neutrino mass square differences measured in current neutrino os-

cillation experiments. The third neutrino mass parameter, namely the absolute scale of neutrino masses, though not strictly predicted, correlates with the remaining two free parameters, namely two Majorana phases. Hence these would be indirectly tested by probing the absolute scale of neutrino mass in beta decay endpoint studies, neutrinoless double beta decay ($0\nu\beta\beta$) searches [6, 7], or cosmology [8].

We provide a possible gauge-theoretic realization based on the non-abelian discrete A_4 flavour symmetry. All A_4 models predicting U_{HPS} lepton mixing, need at least two A_4 -triplets whose vacuum expectation values (vevs) have different A_4 -alignments. Different papers have emphasized this problem [9, 10, 11, 12]. In order to account for such alignments, extra-dimensions [13, 14] and/or supersymmetry [15, 16] have been invoked. Here we show how U_{HPS} mixing can be obtained with just A_4 Higgs singlets in the framework of a softly broken A_4 theory. The model is similar in spirit, but inequivalent to the one suggested in Ref. [9].

We start by noting that if M_ν is

$$M_\nu = \begin{pmatrix} x(1+\alpha) & y(1+\alpha) & y(1+\alpha) \\ y(1+\alpha) & x+\alpha y & y+\alpha x \\ y(1+\alpha) & y+\alpha x & x+\alpha y \end{pmatrix} \quad (2)$$

in the basis where charged leptons are diagonal, the lepton mixing matrix is U_{HPS} independently of the values of x , y and α , since it is $\mu \leftrightarrow \tau$ exchange invariant and $M_{\nu_{11}} = M_{\nu_{22}} + M_{\nu_{33}} - M_{\nu_{13}}$. Here x , y and α are three

complex free parameters. In such “weak-basis” the physical parameters can be chosen as their *moduli* $|x|$, $|y|$, $|\alpha|$, the phase of α , which we denote with δ_1 and the relative phase between x and y , denoted as δ_2 . The eigenvalues of M_ν are

$$\begin{aligned} m_1 &= (1 + \alpha)(x - y), \\ m_2 &= (1 + \alpha)(x + 2y), \\ m_3 &= (1 - \alpha)(x - y). \end{aligned} \quad (3)$$

Note that, the vanishing of θ_{13} removes the possibility of Dirac-type leptonic CP violation. Hence the two phases δ_1 and δ_2 are directly (but not trivially) related to the two physical Majorana phases of the lepton mixing matrix [17]. On the other hand we can rewrite the *moduli* $|x|$, $|y|$, $|\alpha|$ as the three neutrino mass parameters, which may be chosen as the two mass square splittings that enter the oscillations,

$$\begin{aligned} |m_3|^2 - |m_1|^2 &= \Delta m_{atm}^2 = \\ &= -4|\alpha| \cos \delta_1 (|x|^2 + |y|^2 - 2|x||y| \cos \delta_2), \end{aligned} \quad (4)$$

$$\begin{aligned} |m_2|^2 - |m_1|^2 &= \Delta m_{sol}^2 = \\ &= 3(1 + 2|\alpha| \cos \delta_1 + |\alpha|^2)(2|x||y| \cos \delta_2 + |y|^2), \end{aligned}$$

plus the absolute mass scale that governs neutrinoless double beta decay

$$|m_{ee}|^2 = |x|^2(1 + 2|\alpha| \cos \delta_1 + |\alpha|^2). \quad (5)$$

First we note that in the limit $|y| = 0$ the neutrino mass matrix (2) is diagonalized by a maximal rotation in the 23 plane, but the “solar sector” is clearly inconsistent with experiment. On the other hand, when $|\alpha| = 0$ the unitary diagonalizing matrix can not be fixed since there are two degenerate eigenvalues. Thus $|\alpha| \neq 0$ and $y \neq 0$ are required by experimental data. In contrast $|x|$ can be taken equal to zero, in fact in this case the three eigenvalues are distinct and the neutrino mass matrix M_ν is still diagonalized by the U_{HPS} matrix. The most relevant phenomenological consequence of taking $|x| = 0$ is that the neutrinoless double beta decay amplitude $\propto m_{ee}$ is zero, see Eq. (5).

We can solve the system in Eq. (4) with respect to $|x|$ and $|y|$ and write $m_{1,2,3}$ and m_{ee} as function of δ_1 , δ_2 and $|\alpha|$. For any value of $\cos \delta_2$, we have three possibilities: *i*) for $\cos \delta_1 \sim 0$ we have $|m_1| \sim |m_3|$ that correspond

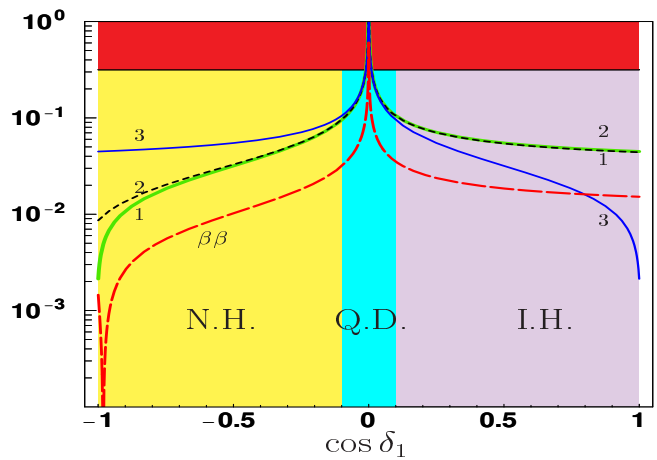


FIG. 1: $|m_1|$ (solid green), $|m_2|$ (small dashed dark), $|m_3|$ (solid blue) and $|m_{ee}|$ (large dashed red) vs $\cos \delta_1$ with $|\alpha| = 1.1$ and $\cos \delta_2 = 1$.

to the degenerate spectrum, *ii*) if $\cos \delta_1 < 0$ we have $|m_1| < |m_3|$ or normal hierarchy and *iii*) if $\cos \delta_1 > 0$ we have $|m_1| > |m_3|$ or inverse hierarchy. These cases are illustrated in Fig. (1) where we plot the three masses versus $\cos \delta_1$ for a fixed value of $|\alpha|$ and $\cos \delta_2$. We note that for $\alpha \sim 0$ we have only the degenerate case since $|m_1| \sim |m_3|$.

The rate for neutrinoless double beta decay (5) vanishes when $|x| = 0$. Solving the system in Eq. (4) for $|x| = 0$ we find the relation

$$\cos \delta_1 = -\frac{3(1 + |\alpha|^2)}{2|\alpha|(3 + 2r)} \quad (6)$$

where $r = \Delta m_{sol}^2 / \Delta m_{atm}^2$. This special point correspond to the narrow dip in Fig. (1). By changing the value of $|\alpha|$ in Eq. (6) we have $-1 \leq \cos \delta_1 < -0.975$ for $0.8 < |\alpha| < 1.24$ (we have used the best fit value for $r = 0.032$, for different values the allowed range for $|\alpha|$ and $\cos \delta_1$ changes only slightly). For other values of $|\alpha|$ and $\cos \delta_1$ the neutrinoless double beta decay rate can not cancel. All the general considerations given here are quite independent of the value of $\cos \delta_2$ that is related to the difference between $|m_{ee}|$ and $|m_1|$. In fact if $\cos \delta_2 \sim 0$ then $|y| \ll |x|$ and $|m_{ee}| \sim |m_1|$. We have taken $\cos \delta_2 = 1$ in Fig. (1) since it gives the lowest value of m_{ee} .

We now turn to a theoretical derivation of the neutrino

	L_i	l_i^c	$h_{1,2,3}$	Δ_s
$SU(2)$	2	1	2	3
A_4	3	3	1, 1'', 1'	1

TABLE I: Lepton multiplet structure of our simple model, see text.

mass matrix in Eq. (2) based on the flavor symmetry A_4 , the finite group of the even permutations of four objects (for a short introduction to A_4 , see for instance [16] and references therein). It has four irreducible representations, one triplet 3 and three singlets 1, 1', 1''. Let χ and φ be two triplets of A_4 then

$$\begin{aligned}
1 &\sim (\chi\varphi) = (\chi_1\varphi_1 + \chi_2\varphi_3 + \chi_3\varphi_2), \\
1' &\sim (\chi\varphi)' = (\chi_3\varphi_3 + \chi_1\varphi_2 + \chi_2\varphi_1), \\
1'' &\sim (\chi\varphi)'' = (\chi_2\varphi_2 + \chi_1\varphi_3 + \chi_3\varphi_1), \\
3_s &\sim \begin{pmatrix} 2\chi_1\varphi_1 - \chi_2\varphi_3 - \chi_3\varphi_2 \\ 2\chi_3\varphi_3 - \chi_1\varphi_2 - \chi_2\varphi_1 \\ 2\chi_2\varphi_2 - \chi_1\varphi_3 - \chi_3\varphi_1 \end{pmatrix}, \quad 3_a \sim \begin{pmatrix} \chi_2\varphi_3 - \chi_3\varphi_2 \\ \chi_1\varphi_2 - \chi_2\varphi_1 \\ \chi_1\varphi_3 - \chi_3\varphi_1 \end{pmatrix}
\end{aligned} \tag{7}$$

and for instance [18]

$$1 \sim (\chi\bar{\varphi}) = (\chi_1\varphi_1 + \chi_2\varphi_2 + \chi_3\varphi_3).$$

Consider as starting point the model defined in Table (I). We have three Higgs doublets $h_{1,2,3}$ in the 1, 1'' and 1' representations of A_4 and one Higgs triplet Δ_s transforming as a singlet with respect to A_4 . The A_4 invariant renormalizable Lagrangian reads

$$\begin{aligned}
\mathcal{L} = & y_{1_{ij}}(L_i l_j^c)h_1 + y_{2_{ij}}(L_i l_j^c)'h_2 + y_{3_{ij}}(L_i l_j^c)''h_3 + \\
& + y_{ijk}^{\Delta_s}(L_i L_j)\Delta_{s_k}
\end{aligned} \tag{8}$$

where the structure of the Yukawa interaction can be easily obtained from the product rule in Eq. (7). Defining

$$\langle h_1 \rangle = v_1, \quad \langle h_2 \rangle = v_2, \quad \langle h_3 \rangle = v_3 \tag{9}$$

the charged lepton mass matrix is given as

$$M_l = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} \tag{10}$$

where $a = y_1 v_1$, $b = y_2 v_2$, $c = y_3 v_3$. It is diagonalized as

$$\begin{aligned}
U_\omega^\dagger M_l U_\omega^\dagger &= \begin{pmatrix} a+b+c & 0 & 0 \\ 0 & a+\omega b+\omega^2 c & 0 \\ 0 & 0 & a+\omega^2 b+\omega c \end{pmatrix}, \\
U_\omega &= \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}
\end{aligned} \tag{11}$$

with $\omega^3 = 1$. As required, the charged lepton mass matrix has three distinct eigenvalues.

From the Lagrangian in Eq. (8) we have the tree level contribution to M_ν shown in Fig.(2):

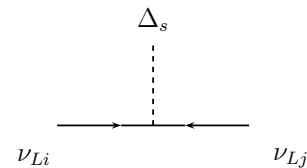


FIG. 2: Tree level contribution to M_ν .

This diagram is the type-II seesaw [17] contribution to M_ν . The tree-level diagram yields

$$M_\nu^0 = \begin{pmatrix} z & 0 & 0 \\ 0 & 0 & z \\ 0 & z & 0 \end{pmatrix} \tag{12}$$

where $z = y^{\Delta_s} v_s$.

Consider now adding an A_4 triplet of fermion isotriplets $\Sigma \sim 3$ with hypercharge zero, together with one Higgs doublet being a singlet under A_4 , $\eta \sim 1$. Both Σ and η carry an extra Z_2 quantum number. Since Σ is in the adjoint representation of the electroweak group, it has a Majorana mass term, see for instance [19]. We have the following extra terms in the Lagrangian

$$y_{\Sigma ik}(L_i \Sigma_k)\eta + \mu_{kl}\Sigma_k \Sigma_l \tag{13}$$

where we assume μ to break softly A_4 , that is

$$\mu = m_\Sigma \begin{pmatrix} 1 & 0 & 0 \\ 0 & \xi & 1 \\ 0 & 1 & \xi \end{pmatrix}. \tag{14}$$

When $\xi \rightarrow 0$, μ respects A_4 . In general the fermion triplets Σ_i induce one-loop neutrino wave-function corrections, see Fig.(3), giving

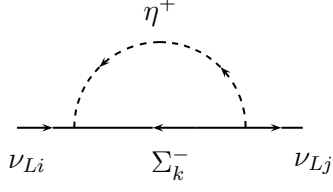


FIG. 3: One loop corrections to M_ν induced by fermion triplets Σ_i .

$$R_{ij} = \frac{y_\Sigma^2}{16\pi^2} \sum_k R_{ik}^\mu R_{jk}^{\mu*} B_1(m_{\Sigma_k}^2, m_\eta^2) \quad (15)$$

where $m_{\Sigma_1} = m_\Sigma$, $m_{\Sigma_2} = m_\Sigma(-1 + \xi)$, $m_{\Sigma_3} = m_\Sigma(1 + \xi)$ are the eigenvalues of the matrix μ in Eq. (14) and

$$R^\mu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

is the orthogonal matrix that diagonalizes μ . Since the fermion triplet masses are not degenerate, the loop correction is off-diagonal,

$$R = \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & \delta_3 \\ 0 & \delta_3 & \delta_2 \end{pmatrix} \quad (16)$$

with

$$\begin{aligned} \delta_1 &= \frac{y_\Sigma^2}{16\pi^2} B_1(m_{\Sigma_1}^2, m_\eta^2), \\ \delta_2 &= \frac{y_\Sigma^2}{32\pi^2} (B_1(m_{\Sigma_2}^2, m_\eta^2) + B_1(m_{\Sigma_3}^2, m_\eta^2)), \\ \delta_3 &= \frac{y_\Sigma^2}{32\pi^2} (B_1(m_{\Sigma_3}^2, m_\eta^2) - B_1(m_{\Sigma_2}^2, m_\eta^2)), \end{aligned} \quad (17)$$

The neutrino mass matrix M_ν^0 in Eq. (12), is corrected by the one loop diagram Fig. (3) as follows

$$M_\nu = M_\nu^0 + R M_\nu^0 + M_\nu^0 R^T. \quad (18)$$

After some algebra Eqs.(11) and (18) are diagonalized by U_{HPS} and M_ν is of the form of Eq. (2) as required.

If the Z_2 symmetry is broken, η takes a vacuum expectation value $\langle \eta \rangle = u$ and the diagram of Fig. (4) gives a type-III seesaw contribution [20] to the neutrino mass matrix where the heavy Majorana neutrino of type-I seesaw is simply replaced with Σ^0 and is given by

$$-y_\Sigma \cdot \mu^{-1} \cdot y_\Sigma^T. \quad (19)$$

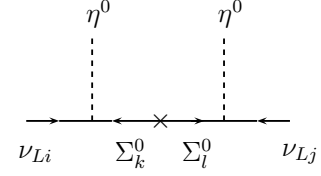


FIG. 4: Type III seesaw contribution to M_ν .

Then from Eqs. (12) and (19) we have

$$M_\nu = \begin{pmatrix} z + \beta & 0 & 0 \\ 0 & \frac{\beta\xi}{\xi^2-1} & z - \frac{\beta}{\xi^2-1} \\ 0 & z - \frac{\beta}{\xi^2-1} & \frac{\beta\xi}{\xi^2-1} \end{pmatrix} \quad (20)$$

where $\beta = y_\Sigma^2 u^2 / m_\Sigma$. Again M_ν is diagonalized by the U_{HPS} in the basis where charged leptons are diagonal since in such a basis it has the form of matrix in Eq. (2) with the eigenvalues given by

$$\begin{aligned} m_1 &= \frac{\beta+z(1+\xi)}{1+\xi} \\ m_2 &= \beta + z \\ m_3 &= \frac{\beta+z(1-\xi)}{1-\xi}. \end{aligned} \quad (21)$$

Eqs.(21) are compatible with normal, degenerate and inverse neutrino mass spectra. For instance when $\beta = -z(1 + \xi)$ then $m_1 = 0$ and we have the simple relation

$$4r = |1 - \xi|^2.$$

If $\beta = -z(1 - \xi)$ then $m_3 = 0$. This correspond to the inverse hierarchy and we have

$$r = 1 - \frac{4}{|1 - \xi|^2}.$$

In both cases we need $\xi \sim O(1)$ to reproduce data. For $|\xi| \ll 1$ we have degenerate neutrino masses.

We mention in passing that in general one could also have the situation where both tree-level, Fig. (4), and wave function correction, Fig. (3), contribute to M_ν , leading however to the same overall structure in Eq. (2). We therefore do not discuss this ‘‘mixed’’ case in detail.

The above is just the simplest model with U_{HPS} mixing that meets all phenomenological requirements. Other variants are easy to obtain. We could for example replace the Higgs triplet with two Higgs doublets. They contribute proportional to z to the neutrino mass matrix via a dimension five operator.

Finally we mention possible accelerator tests of our model realization. If the members of the scalar triplet are light enough to be produced at LHC [22], the decay pattern of the Δ_s^{++} is completely fixed by the A_4 symmetry, and can be predicted to have only flavour diagonal decays $Br(\Delta_s^{++} \rightarrow e^+e^+) = Br(\Delta_s^{++} \rightarrow \mu^+\mu^+) = Br(\Delta_s^{++} \rightarrow \tau^+\tau^+)$ with all $Br(\Delta_s^{++} \rightarrow l_i^+l_j^+) = 0$. This is different from, for example, the case discussed in [23], where the type-II seesaw is assumed to be the only source of neutrino mass. Also the cross section for the fermionic triplet at LHC has been recently studied [24, 25]. If $m_\Sigma \lesssim 1 TeV$, $\Sigma^+\Sigma^-$ pairs will be produced decaying to $l^+l^- + 2\eta$. Again the branching ratios to the different lepton flavors are completely fixed by the A_4 symmetry. Pair produced Σ^\pm will decay to two charged leptons plus two η with branching ratios to the different lepton flavours approximately fixed by our ansatz for A_4 breaking. Since the charged lepton mass matrix is diagonalized by U_ω , the final states can be predicted to be $Br(e\mu) \simeq Br(e\tau)$, $Br(\mu\mu) \simeq Br(\tau\tau)$ and $Br(ee) \simeq Br(\mu\mu) + Br(\mu\tau) - Br(e\tau)$.

In summary we have pointed out new ways to model U_{HPS} lepton mixing from first principles. The minimal scheme is characterized by only three parameters and requires $m_{ee} = 0$. Five parameter schemes can produce an arbitrary neutrino mass spectrum where m_{ee} is not strictly predicted but related to the two Majorana phases of the lepton mixing matrix. We have discussed one model realization of these schemes, which interestingly avoids the vev misalignment problem present in other models based on A_4 , as a consequence of not using A_4 triplet Higgses. This model is potentially testable at the LHC if either the scalar triplet or the fermionic triplet (or both) have mass below approximately $1 TeV$.

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- [1] T. Schwetz, M. Tortola and J. W. F. Valle, *New J. Phys.* **10**, 113011 (2008), [arXiv:0808.2016 [hep-ph]]; for previous analysis and relevant experimental references see also, e. g. M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, *New J. Phys.* **6**, 122 (2004), [hep-ph/0405172 v6]
- [2] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [3] H. Nunokawa, S. J. Parke and J. W. F. Valle, *Prog. Part. Nucl. Phys.* **60**, 338 (2008), [arXiv:0710.0554 [hep-ph]].
- [4] ISS Physics Working Group, A. Bandyopadhyay *et al.*, arXiv:0710.4947 [hep-ph].
- [5] P. F. Harrison, D. H. Perkins and W. G. Scott, *Phys. Lett.* **B530**, 167 (2002), [hep-ph/0202074].
- [6] F. T. Avignone, S. R. Elliott and J. Engel, arXiv:0708.1033 [nucl-ex].
- [7] M. Hirsch, arXiv:0609146 [hep-ph]
- [8] J. Lesgourgues and S. Pastor, *Phys. Rept.* **429** (2006) 307 [arXiv:astro-ph/0603494].
- [9] E. Ma, arXiv:0808.1729 [hep-ph]
- [10] W. Grimus and L. Lavoura, arXiv:0809.0226 [hep-ph]
- [11] A. Zee, *Phys. Lett.* **B630**, 58 (2005), [hep-ph/0508278].
- [12] E. Ma, *Phys. Rev.* **D70**, 031901 (2004), [hep-ph/0404199].
- [13] G. Altarelli and F. Feruglio, *Nucl. Phys.* **B720**, 64 (2005), [hep-ph/0504165].
- [14] T. Kobayashi, Y. Omura and K. Yoshioka, arXiv:0809.3064 [hep-ph]
- [15] K. S. Babu and X. G. He, arXiv:hep-ph/0507217.
- [16] G. Altarelli and F. Feruglio, *Nucl. Phys.* **B741**, 215 (2006), [hep-ph/0512103].
- [17] J. Schechter and J. W. F. Valle, *Phys. Rev.* **D22**, 2227 (1980).
- [18] F. Feruglio, C Hagedorn, Y. Lin and L. Merlo, to appear.
- [19] A. Abada, C. Biggio, F. Bonnet, M. B. Gavela and T. Hambye, *Phys. Rev. D* **78**, 033007 (2008)
- [20] R. Foot, H. Lew, X. G. He and G. C. Joshi, *Z. Phys.* **C44**, 441 (1989).
- [21] E. Ma and D. Suematsu, arXiv:0809.0942 [hep-ph]
- [22] J. F. Gunion, C. Loomis and K. T. Pitts, arXiv:9610237 [hep-ph]
- [23] E. J. Chun, K. L. Lee, Y. Y. Kang and S. C. Park, *Phys. Lett.* **B566**, 142 (2003), arXiv:0304069 [hep-ph]
- [24] R. Franceschini, T. Hambye and A. Strumia, *Phys. Rev.* **D78**, 033002 (2008), 0805.1613
- [25] F. del Aguila and J. A. Aguilar-Saavedra, arXiv:0808.2468.