

Extraction of cluster parameters with future Sunyaev-Zel'dovich observations

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Abstract. The Sunyaev-Zel'dovich (SZ) effect of galaxy clusters is characterized by three parameters: Compton parameter, electron temperature and cluster peculiar velocity. In the present study we consider the problem of extracting these parameters using multi-frequency SZ observations only. We show that there exists a parameter degeneracy which can be broken with an appropriate choice of frequencies. As a result we discuss the optimal choice of observing frequencies from a theoretical point of view. Finally, we analyze the systematic errors (of the order μK) on the SZ measurement introduced by finite bandwidths, and suggest a possible method of reducing these errors.

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1. Introduction

The interaction of the Cosmic Microwave Background Radiation (CMBR) photons with the free electrons in the ionized gas of clusters of galaxies produces a small change in the intensity known as the Sunyaev-Zel'dovich (SZ) effect. This distortion arises from the transfer of photons from the low-energy to the high-energy or Wien side of the planckian spectrum. The underlying physics of the SZ effect is well understood and a quantitative description was given by Sunyaev & Zel'dovich [1], who already realized its cosmological significance. Nowadays there exists reliable observations of the SZ effect (see e.g. [2, 3, 4]) and dedicated experiments are being prepared, which will provide information about the kinematics and evolution of clusters, which in turn can be used to extract cosmological parameters (see e.g. [5, 6, 7] for recent reviews).

The intensity change of the CMBR caused by the SZ effect is proportional to the Comptonization parameter,

$$y_c = \int dl \frac{T_e}{m_e} n_e \sigma_{\text{Th}} , \quad (1)$$

where T_e is the temperature of the electron gas in the cluster, m_e the electron mass, n_e the electron number density, σ_{Th} the Thomson scattering cross section, and the integral is calculated along the line of sight through the cluster. We use units for which $k_B = \hbar = c = 1$. For an isothermal intra-cluster gas one has $y_c = \tau T_e/m_e$, where $\tau = \int dl n_e \sigma_{\text{Th}}$ is the optical depth. The distortion of the CMBR is then given by an intensity change

$$\Delta I_{\text{T}} = I_0 y_c (f_0(x) + \delta f(x, T_e)) , \quad (2)$$

where

$$f_0(x) = \frac{x^4 e^x}{(e^x - 1)^2} \left[\frac{x(e^x + 1)}{e^x - 1} - 4 \right] . \quad (3)$$

Here $x = \nu/T_0$ is the dimensionless frequency with $T_0 = 2.725$ K, and $I_0 = T_0^3/(2\pi^2)$. The intensity change is independent of the temperature for non-relativistic electrons, a limit which is valid for small frequencies ($\nu \lesssim 100$ GHz), but for high frequencies it must be corrected [8] with $\delta f(x, T_e)$ either by using an expansion in $\theta_e = T_e/m_e$ [9, 10, 11] or calculated exactly [12]. In some rich clusters the optical depth is large enough ($\tau \sim 0.02-0.03$) to require the inclusion of multiple scattering effects [12, 13, 14].

When the cluster has a relative motion with respect to the CMBR rest frame, there exists an additional distortion known as the kinetic SZ effect [15], which is identical to a change in the temperature of the CMBR. The corresponding intensity change, for a peculiar velocity v_p , is $\Delta I_{\text{K}} = I_0 y_c f_{\text{kin}}$, where

$$f_{\text{kin}} = -v_p \frac{m_e}{T_e} \frac{x^4 e^x}{(e^x - 1)^2} . \quad (4)$$

This expression is slightly modified by the corrections caused by the electron temperature [16, 17].

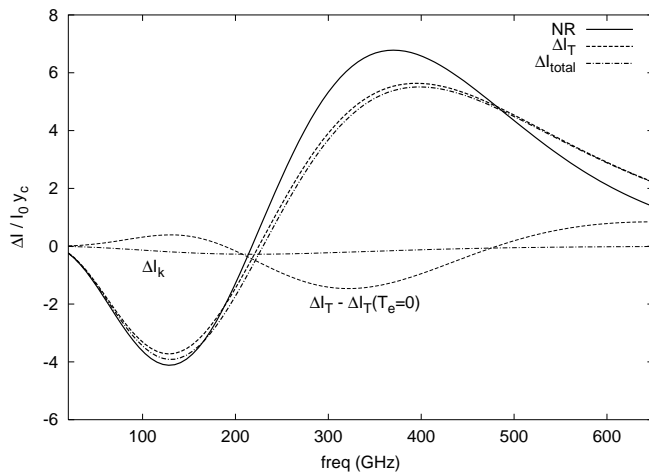


Figure 1. The normalized intensity change caused by the thermal and kinetic SZ effects for the choice $T_e = 15$ keV and $v_p = 500$ km/sec. The solid line is the non-relativistic SZ effect. The dashed lines are the relativistic corrections, both alone and together with the thermal effect. The dot-dashed lines are the kinetic SZ effect, both alone and together with the thermal effect.

The total intensity change is just the sum of the thermal and kinetic SZ effects,

$$\Delta I_{\text{total}} = \Delta I_T + \Delta I_K , \quad (5)$$

and it is characterized by three parameters: y_c , T_e and v_p . The different contributions to the SZ distortion are presented in figure 1, where the total intensity change in (5) is compared to the thermal effect with vanishing temperature (the first term in (2)). The kinetic SZ effect and the contribution of the relativistic corrections (the second term in (2)) are presented for the parameter choice $T_e = 15$ keV and $v_p = 500$ km/s.

Let us emphasise that all 3 SZ parameters can in principle be extracted from multi-frequency observations of the SZ effect. This is because the frequency dependence of the SZ effect is different for the different parameters. This is clear from figure 1, where one can see that the main effect (the non-relativistic SZ effect characterized by the comptonization parameter, y_c) has one crossing with the zero-line. The kinetic effect (governed by v_p) has no crossings of the zero-line, and the relativistic effect (governed by T_e) crosses the zero-line twice. Thus with several observing frequencies placed optimally and with good sensitivity, one can extract all 3 SZ parameters as we describe in detail in section 6.

In this paper we discuss the possibilities for future SZ observations to extract these 3 cluster parameters, taking into account the parameter degeneracies described in section 2. To this end we try to extract the 3 SZ parameters from a set of simulated galaxy clusters with realistic temperatures and peculiar velocities, using the sensitivities of upcoming experiments like ACT and Planck. In section 6, we discuss the optimal observing frequencies from a theoretical point of view. Finally, we analyze in section 7

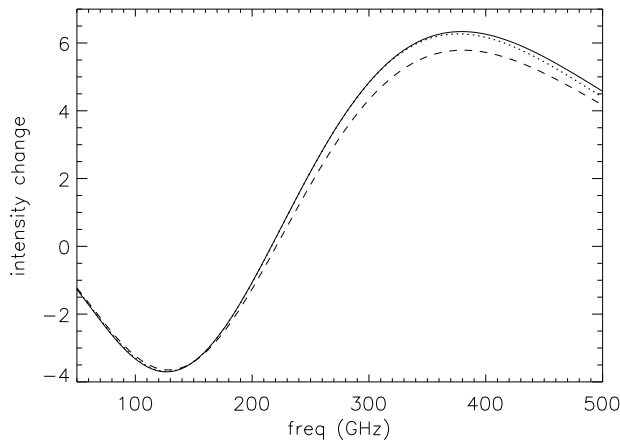


Figure 2. The intensity change for 3 different clusters, normalized at 150 GHz. The solid line is $T = 5$ keV, $v_p = -145$ km/sec and the dotted is $T = 7$ keV, $v_p = -260$ km/sec. These two are virtually indistinguishable. The dashed line is $T = 6$ keV, $v_p = 0$ km/sec, which differs significantly for large frequencies.

the systematic error introduced by a finite frequency bandwidth, which may be as large as few μK and hence important for the next generation of SZ observations.

2. Parameter degeneracies

When one estimates the expected error-bars of a given experiment, it is always important to consider the effect of parameter degeneracies. This is naturally also true when considering the SZ effect. Let us clarify that by degeneracies we mean that two sets of parameters give an indistinguishable intensity change due to the limited sensitivity of the observations. In other words, the extracted values of y_c, v_p and T_e from the same experiment will have larger error bars for some of the clusters than for others.

The degeneracy between the 3 SZ parameters depends on the chosen value of the parameters, e.g. if the peculiar velocity is large and positive, then it is virtually impossible to adjust the temperature and comptonization parameter to mimic the effect on the intensity change of the peculiar velocity. On the other hand, for large and negative peculiar velocities, such imitation is possible. In figure 2 the solid and dotted lines are virtually indistinguishable. They are with $T = 5$ keV, $v_p = -145$ km/sec and $T = 7$ keV, $v_p = -260$ km/sec respectively, the comptonization parameter is normalized to make the intensity change agree at 150 GHz. For comparison we see, that the dashed line, $T = 6$ keV, $v_p = 0$ km/sec, also normalized to agree at 150 GHz, can more easily be distinguished. As a specific example, let us consider a hypothetical experiment, with 4 frequencies at 90, 181, 220 and 330 GHz, with $1 \mu\text{K}$ sensitivity. With 3 test clusters all with the same temperature, $T = 6$ keV and comptonization parameter, $y_c = 2 \times 10^{-4}$,

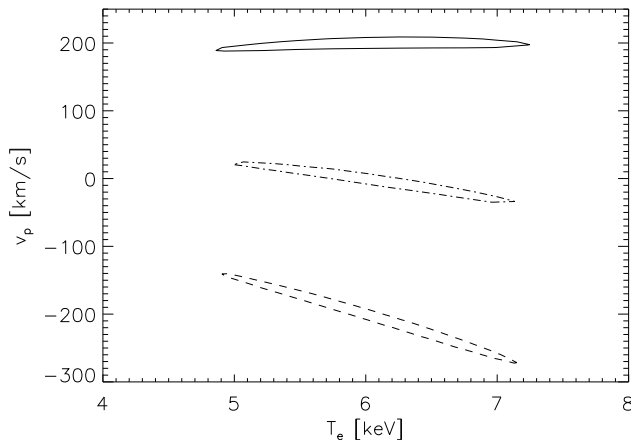


Figure 3. 2σ contour plots for an experiment with 4 frequencies at 90, 181, 220 and 330 GHz, with $1 \mu\text{K}$ sensitivity. The 3 clusters have the same temperature $T = 6 \text{ keV}$ and comptonization parameter, $y_c = 2 \times 10^{-4}$, but have different peculiar velocities, $v_p = -200, 0, +200 \text{ km/sec}$.

but with different peculiar velocities, $v_p = -200, 0, +200 \text{ km/sec}$. The resulting 2σ error-contours are given figure 3, and it is clear from the figure, that whereas the temperature error-bars are virtually independent of the real value of v_p (always about $\pm 1 \text{ keV}$), then the error-bar of v_p is strongly dependent, and may differ from 10 km/sec for $v_p = +200 \text{ km/sec}$ to 70 km/sec for $v_p = -200 \text{ km/sec}$, i.e. by a factor of 7. Hence it is important to consider a range of cluster parameters when one wants to make predictions about future experiments.

In this paper we therefore use realistic simulations of clusters of galaxies (described below), and the error-bars will not be for a specific cluster, but instead presented for a large samples of clusters.

3. Simulated clusters

In order to produce a large sample of clusters with realistic temperatures, comptonization parameters and peculiar velocities, one must basically use an extended Press–Schechter formalism. Let us here discuss the details of the used simulation which closely follows [18].

The physical parameters (temperature, core and virial radii, central electron density, ...) used to describe the clusters are computed according to their mass and redshift. In the following, we use the most favoured cosmological model with critical density described by the following cosmological parameters: the reduced Hubble constant $h = 0.70$, cold dark matter density parameter $\Omega_{cdm} = 0.12h^{-2}$, baryonic density parameter $\Omega_b = 0.02h^{-2}$, total matter density parameter $\Omega_M = \Omega_{cdm} + \Omega_b$, and a

cosmological constant $\Omega_\Lambda = 1 - \Omega_M = 0.714$.

The cluster core radius R_c is related to the virial radius R_v through the parameter $p = R_{vir}/R_c$, where we use $p = 10$ [19]. The virial radius is given by the following equation

$$R_{vir} = \frac{(GM)^{1/3}}{(3\pi H(z))^{2/3}}, \quad (6)$$

where G is the gravitational constant and $H(z)$ the Hubble constant. For a critical universe, R_v is fixed solely by the mass and the collapse redshift of the cluster,

$$R_{vir} = \frac{(GM)^{1/3}}{(3\pi H_0)^{2/3}} \frac{1}{1+z}, \quad (7)$$

with $H_0 = 100 \times h$ the value of the Hubble constant today. The electron density profile for the clusters is well described by the so-called phenomenological β profile as follows

$$n_e(R) = n_{e0} \left[1 + \left(\frac{R}{R_c} \right)^2 \right]^{-3\beta/2}, \quad (8)$$

where n_0 is the central electron density and β is a parameter describing the steepness of the profile. The X-ray brightness profiles of galaxy clusters mostly agree with $\beta = 2/3$, which is the value we use. Finally knowing the virial radius, we can derive the central electron density n_0 from the gas mass of the cluster M_G using the following relation

$$M_G \left(\frac{\Omega_b}{\Omega_M} \right) = m_p \mu \int_0^{R_v} n_e(R) 4\pi R^2 dR. \quad (9)$$

In this relation, m_p is the proton mass and $\mu = 0.6$ is the mean molecular weight of a plasma with primordial abundances. In our model, we use a gas fraction of $0.166(h/0.7)^{-1}$ (where the gas fraction equals the baryon fraction, and is slightly higher than some observed values [20]). We compute the temperature of the intra-cluster medium as a function of the mass, the redshift and the cosmological model

$$T = \frac{GM^{2/3} \mu m_p}{2\beta} \left[\frac{H^2(z) \Delta_c}{2G} \right]^{1/3}, \quad (10)$$

and thus

$$T = 1.39 f_T M_{15}^{2/3} (h^2 \Delta_c E^2(z))^{1/3}, \quad (11)$$

where T is given in keV and M_{15} is the mass of the structure in units of 10^{15} solar masses. $f_T = 0.78$ is a normalization factor, we take it as a unique value from Bryan & Norman [21] although it varies very slightly with the cosmological model. Δ_c is the average over-density ($\Delta_c = 18\pi^2$ for a critical universe) and $E^2(z)$ is a function such as $H^2(z) = H_0^2 E^2(z)$ which depends on the cosmology.

The number density of clusters are derived from a modified Press–Schechter mass function according to Sheth & Tormen [22] and Wu [23]. The numbers are normalized to Viana & Liddle [24] and we use $\sigma_8 = \sigma_8^0 \Omega_m^{-0.47}$ with $\sigma_8^0 = 0.6$.

On the scale of clusters of galaxies we assume that the density fluctuations are in the linear regime. In fact, under the assumption of an isotropic Gaussian distribution

of the initial density perturbations, the initial power spectrum $P(k)$ gives a complete description of the velocity field through the three-dimensional *rms* velocity (v_{rms}) predicted by the linear gravitational instability for an irrotational field at a given scale R [25]. This velocity is given by

$$v_{rms} = a(t) H f(\Omega, \Lambda) \left[\frac{1}{2\pi^2} \int_0^\infty P(k) W^2(kR) dk \right]^{1/2} \quad (12)$$

where $a(t)$ is the expansion parameter, the Hubble constant H and the density parameter Ω vary with time [26]. The function $f(\Omega, \Lambda)$ is accurately approximated by $f(\Omega, \Lambda) = \Omega^{0.6}$ [27]. Furthermore, under the assumptions of linear regime and Gaussian distribution of the density fluctuations, the structures move with respect to the global Hubble flow with peculiar velocities following a Gaussian distribution $f(v) = \frac{1}{v_{rms}\sqrt{2\pi}} \exp(\frac{-v^2}{2v_{rms}^2})$ which is fully described by v_{rms} . This prediction is in agreement with numerical simulations. The velocity of each cluster (at mass M associated with the scale R) is then randomly drawn from the Gaussian distribution.

Thus we have produced a large sample of realistic galaxy clusters each with their *true* comptonization parameter, temperature and peculiar velocity, and we will in the next sections consider this sample as being what can be observed in future all-sky observations.

4. General method

In this section, we present the method used to answer the following question: How well can a multi-frequency experiment measure the physical cluster parameters (namely T_e , v_p and y)? For a given simulated cluster, we have the *true* value of the parameters, T_e^t, v_p^t and y_c^t . With these values we can calculate what the *true* SZ signal should be at each frequency, and then the sensitivity of the experiment gives us an estimate of the expected observational error-bars at each frequency. Now given this *observed* SZ signal (with its error-bars), we can try to *deduce* the 3 parameters T_e^d, v_p^d and y_c^d , and corresponding error-bars, and we can then compare with the true values, T_e^t, v_p^t and y_c^t .

To deduce the parameters we have developed a method which in 3 steps finds the central value and 1σ error-bars of each parameter. The code is an extension of the method developed in Hansen et al. [28]. First, we find the approximate 5σ region in y_c (in the range $10^{-6.5} < y_c < 10^{-1.5}$). Next, we define a grid for all 3 parameters (using the above mentioned y_c -range, and $0 < T < 20$ keV and $-2000 < v_p < +2000$ km/sec), and we find an approximate 3σ range for all parameters. Finally, the code makes a new refined grid in the ranges derived from the previous step (3σ range), and calculates the central values and 1σ error-bars for T_e, v_p and y_c . The deduced central parameters will be unrealistically close to the true values, since we assume that the true intensity variation due to SZ effect is the central observational intensity variation. However, the estimated error-bars will nevertheless be realistic.

In the approach described above, we have assumed that no external information is known about the observed value. In that case, all three parameters are derived from

the SZ observations. We can relax this assumption by allowing the temperature T_e to be known at $\pm 10\%$ accuracy, e.g. from X-rays observations. The temperature is in that case in the limited range of $\pm 10\%$ around the *real* temperature. This would be true for a limited sample of clusters, however, the X-ray observations are time consuming, so one might not want to rely on them for large cluster surveys. By doing this, we estimate the importance of priors on the temperature on the determination of the deduced parameters.

5. Application to CMBR and SZ experiments

5.1. The ACT experiment

Let us consider the Atacama Cosmology Telescope (ACT), a proposed future SZ experiment [29]. This experiment will observe at 3 frequencies (150, 220 and 270 GHz), with $0.9 - 1.7'$ resolution, and an expected sensitivity of $2 \mu\text{K}$ per pixel [30]. The ACT team expects conservatively to observe $\sim 10^3$ galaxy clusters through the SZ effect.

In order to test the capabilities of ACT to the determination of the cluster parameters, we analyze a subsample with the 500 hottest clusters of our simulation (which with the chosen cosmological parameters lie in the range $3.6 < T_e < 7.4 \text{ keV}$). For each simulated cluster we have both the *true* values, and the ones *deduced* with our method. It turns out, that ACT will basically not be able to extract the temperature (within the reasonable range $0 < T_e < 20 \text{ keV}$), partly because of the range of temperatures of our cluster subsample. One should keep in mind, that e.g. during cluster mergers the temperatures will rise significantly, in which case the relativistic corrections become more prominent, and a temperature detection would be easier. Thus we have always left T_e as a free parameter, which must be marginalized over. We therefore focus on the two other parameters y_c and v_p , for which we have performed two runs, one where the temperature is known within 10%, and one with no prior knowledge of the temperature.

We find that the comptonization parameter can be deduced with 1σ error-bars of 10% when the temperature is unknown, whereas the results are about 2% with prior temperature information. These accuracies are obtained for a cluster with $y_c \approx 10^{-4}$. For a Compton parameter about ten times smaller the accuracy is still very good $\simeq 15\%$ without prior and $\simeq 6\%$ if the temperature is known to 10%. We also notice that the error-bars are not symmetric for the unknown temperature case, being larger for positive $\delta y_c / y_c^t$, which is because a larger y_c can be compensated by a simultaneous larger temperature and larger negative peculiar velocity.

The results for the peculiar velocity show a strong effect of the degeneracies discussed in section 2, namely positive peculiar velocities will be extracted with much smaller error-bars than negative peculiar velocities. In the case with known temperature (always within 10%) this degeneracy is almost completely broken, the error-bars are almost symmetric, and of the order 10 – 20% for most peculiar velocities.

5.2. An experiment with 4 frequencies

As we saw above, a SZ experiment measuring at 3 frequencies like ACT produce large error-bars when trying to extract all 3 SZ parameters simultaneously. One way around this problem is to have prior knowledge of the temperature, which thus effectively reduces the number of parameters to two. This improves the accuracy with which the parameters are determined.

Another way around this problem is to increase the number of frequencies of observation. As an example, we show in figures 4 and 5 the 1σ error-bars resulting from 3 different runs. Two of these have no prior information about the SZ parameters. The first (open squares) is the same as discussed above, namely 3 frequencies centred on 150, 220 and 270 GHz and sensitivity $2 \mu\text{K}$. The second run (stars) is an extension of ACT, with one extra observing frequency added at 90 GHz. As can be seen on the figures, the error-bars for both y_c and v_p become smaller and more symmetric, an indication of breaking of the degeneracy. Unfortunately, the level of accuracy reached in this way is far from what was achieved using the prior on the temperature. This is because contrary to the previous case, one then has to solve for the values of all three parameters. One should keep in mind that it may not be realistic to expect X-ray measured temperatures for that many clusters, and that additional observing frequencies thus might be an interesting alternative. For comparison, we have also calculated the 1σ error-bars when the electron temperature is restricted to the range $T_e < 5 \text{ keV}$ (which is true for most clusters in our subsample), presented in figures 4 and 5 with triangles. Making such model-dependent assumption (temperatures restricted to the range $T_e < 5 \text{ keV}$) is essentially the same as having prior temperature knowledge.

An alternative to add a new frequency band to the ACT experiment could be to measure the SZ effect of the same clusters with a different experiment. An example is the Atacama Large Millimetre Array (ALMA) project, which has a frequency centred on 35 GHz with precision up to a few μK [31]. Such low frequencies are very good in restricting the dominating comptonization parameter. The main feature of ALMA will be a very good angular resolution up to scales of a few to tens of arc-seconds, which will lead to high resolution SZ images.

5.3. Planck mission

The Planck satellite [32] will be observing the millimetre and sub-millimetre wavelengths in nine frequencies. It is thus natural to investigate how well the three SZ parameters can be extracted with the Planck sensitivity. Such analysis was first made by Pointecouteau et al. [33], with the simplifying assumption of vanishing peculiar velocity. As discussed above, due to the parameter degeneracy such simplification should not be made. In the present study, we rather take explicitly into account the kinetic SZ effect in order to *deduce* all three cluster parameters. It is worth making the remark that the component separation algorithms for Planck are so efficient that even though the exact shape of the SZ effect is unknown (due to relativistic corrections) the SZ contamination will not

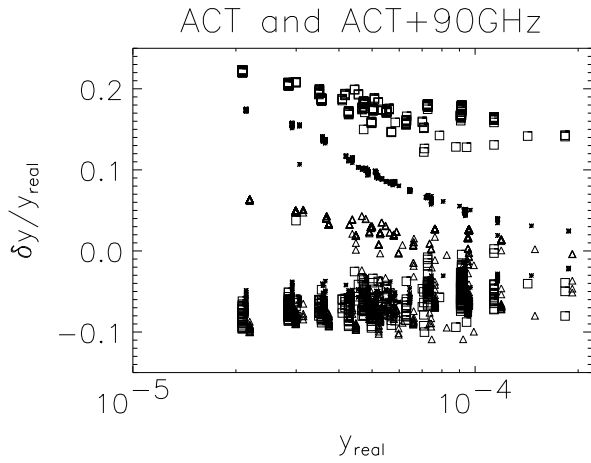


Figure 4. The *deduced* versus the *real* comptonization parameter for the upcoming SZ survey ACT with 3 observing frequencies 150, 220 and 270 GHz, and an extension of it with one extra channel at 90 GHz. The open squares (stars) are the 1σ error-bars for ACT (ACT+90) when we have no prior information on the SZ parameters. For comparison, the triangles are the 1σ error-bars when the electron temperature is restricted to the range $T_e < 5$ keV.

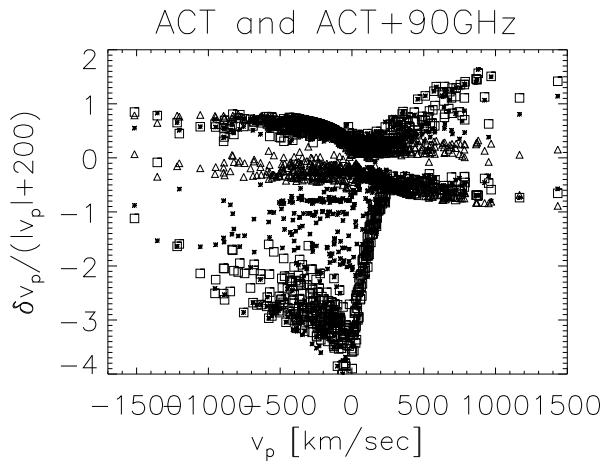


Figure 5. Same as figure 4 for the peculiar velocity v_p .

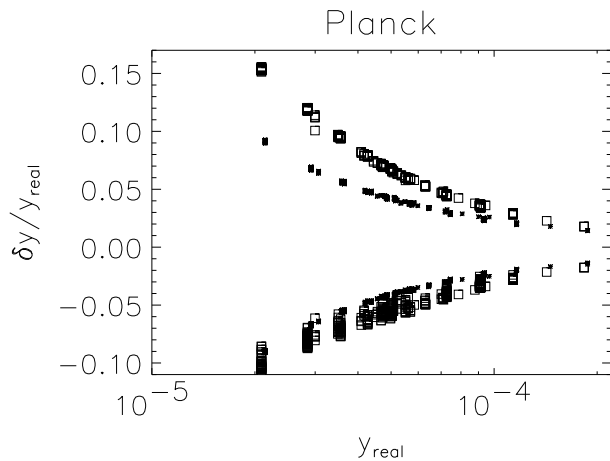


Figure 6. The 1σ error-bar on the *deduced* Compton parameter y_c for the Planck satellite. The open squares are with no prior temperature knowledge, and the stars are when the temperature is known within 10%.

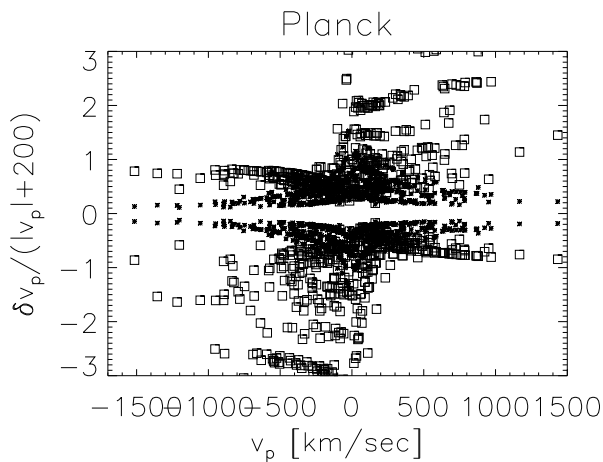


Figure 7. Same as figure 6 for the peculiar velocity v_p .

be a problem for the CMBR analysis [34].

We assume that the central Planck frequencies (44, 70, 100, 143, 217 and 353 GHz) are observing with sensitivities (2.4, 3.6, 1.7, 2.0, 4.3 and 14.4 μK). We do not include the lowest and highest frequencies (30, 545, and 857 GHz) which are dominated by other astrophysical sources (point sources, dust), and will thus be used to remove the contaminants. Assuming that the 6 central frequencies have a clean SZ signal is naturally too ambitious, and our results will therefore provide an upper bound on the abilities of Planck. Also Planck will for most clusters not provide any temperature determination, although we note again that this is partly because of the low

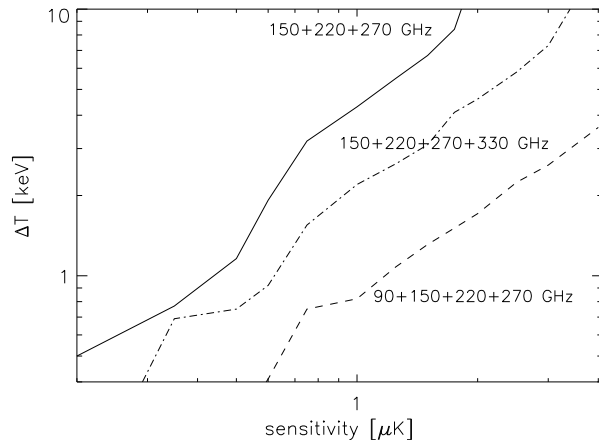


Figure 8. 1σ error-bars on T_e as a function of the observing sensitivity for a cluster with $T_e = 5$ keV, $v_p = 200$ km/sec and $y = 2 \times 10^{-4}$. The solid line is for an experiment with 3 observing frequencies centred at 150, 220 and 270 GHz (ACT-like). The dashed and dot-dashed lines include one additional observing frequency, at 90 GHz or 330 GHz respectively.

temperatures of our subsample. The 1σ error-bars on the deduced Compton parameter and peculiar velocities are shown in figures 6 and 7.

The comptonization parameter will be determined to about 2% and 10-15% for $y_c = 2 \times 10^{-4}$ and $y_c = 2 \times 10^{-5}$ respectively. A 10% temperature knowledge will only improve these numbers slightly. The peculiar velocity will only be determined within a factor of a few, however, a 10% temperature knowledge will improve this to about 20-50%.

5.4. Extracting the cluster temperatures

Our previous results tell us that extracting the electron temperatures from the data of the studied SZ experiments is problematic due to the limited sensitivities. We have calculated the error-bars on T_e for a specific cluster ($T_e = 5$ keV, $v_p = 200$ km/sec and $y = 2 \times 10^{-4}$) as a function of the sensitivity. The result is shown in figure 8. The solid line is for an ACT-like experiment with 3 observing frequencies placed at 150, 220 and 270 GHz. As one sees, to reach ΔT_e of the order 1 keV one must have a sensitivity of about $0.5 \mu\text{K}$. However, by introducing one additional channel the errors can be significantly reduced. In particular with an extra measurement at 90 GHz, one can reach ΔT_e of 1 keV (4 keV) already with $1 \mu\text{K}$ ($4 \mu\text{K}$) sensitivity. We thus emphasize that future SZ observations with several observing frequencies should be able to extract the electron temperature directly without the need for independent X-ray information.

Good temperature sensitivities are expected for the next generation of SZ experiments. For instance the South Pole Telescope (SPT) is a new and ambitious

project [35], that will use the SZ effect to search for and count 10,000 clusters of galaxies and cover 4000 squared degrees per year. This telescope will consist of a single 8 meter primary dish, accompanied by an array of 1,000 bolometers, and will measure temperature differences with an accuracy of $0.1 \mu\text{K}$. With several observing frequencies and such impressive sensitivity one will be able to deduce the electron temperature purely from SZ observations.

We remark that SZ can probe the outermost regions of clusters better than X-ray observations, simply because the SZ effect decreases like n_e , whereas X-ray brightness decreases as n_e^2 . Thus with direct SZ temperature observations and good angular resolution one can find the density profile of the outer cluster region, which directly allows one to infer the outer dark matter density profile [36]. This dark matter profile has not been measured yet at such very large radii, but is predicted from N-body simulations to be $\rho \sim r^{-3}$.

6. Optimal frequency choice for a SZ experiment

The question of an optimal choice of observing frequencies for the SZ effect has been posed a long time ago and is motivated by the fact that one wants to measure both the thermal and kinetic SZ effects. The measurement of both effects has great importance for cosmology since it allows to probe the clusters physics (through thermal SZ) and the matter distribution at large scales (through kinetic SZ, see for instance Doré et al. [37]). In order to attain this goal, one needs to have observations at several frequencies in the positive and negative part of the intensity change. In particular, one should have an observation near 218 GHz, where the thermal effect vanishes (figure 1, solid line). At this frequency one can theoretically measure the kinetic effect directly (figure 9, dashed line) which happens to be maximal at this frequency. In a recent paper [38] it was pointed out, that 217 GHz may not be an optimal frequency when one observes with only 3 frequencies. We will discuss observations with more than 3 frequencies for which such arguments may not apply.

A future, optimal and dedicated SZ experiment could possibly have many observing frequencies. Let us here discuss, from a theoretical point of view, which frequencies would be preferable. Basically we want to extract all 3 SZ parameters, and the actual frequency choice depends upon which parameters we want to extract most precisely, and whether we have prior information such as a temperature determination from X-rays. In order to address the question of optimal frequency choice, we start by re-computing the frequency dependence of both thermal and kinetic SZ effects in view of the most recent developments in the subject. Namely, we compute the thermal effect in its exact form, i.e. including the corrections for the relativistic effects according to Dolgov et al. [12]. The 3 dash-dotted lines in figure 9 are the relativistic contributions to the thermal effect, for cluster temperatures $T_e = 5, 10$ and 15 keV. That is, for a given temperature one can calculate $\Delta I(T_e)$, and the 3 dot-dashed curves are thus $\Delta I(T_e) - \Delta I(T_e = 0)$, normalized to $I_0 y_e$. As is clear from the figure, this quantity is approximately zero

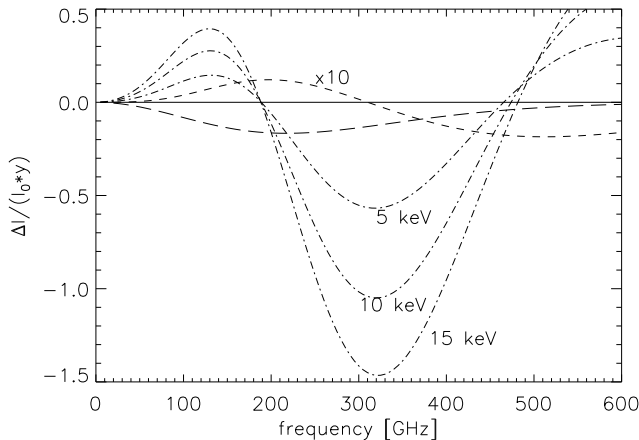


Figure 9. The 3 dash-dotted lines are the relativistic contributions to the thermal effect for temperatures 5, 10 and 15 keV. The short-dashed line is ten times the relativistic correction to the kinetic SZ effect (long-dashed line) for $v_p = 200$ km/sec and $T_e = 10$ keV.

around 181 GHz and again near 475 GHz. This feature is almost independent of the temperature in the first case, but not for the frequency region around 475 GHz.

We have also included in figure 9 the kinetic SZ effect (long-dashed line) for a peculiar velocity of 200 km/sec, while the short-dashed line is the small relativistic correction to it [17, 16], enlarged 10 times. That is, for a given temperature and peculiar velocity one can calculate $\Delta I_{\text{kin}}(v_p, T_e)$, and the short-dashed curve is thus, $\Delta I_{\text{kin}}(v_p, T_e) - \Delta I_{\text{kin}}(v_p, T_e = 0)$. The particular case shown was calculated for $T_e = 10$ keV. Thus, if one wants to differentiate between variations in the background CMBR signal and the kinetic SZ signal, then one should use these corrections, and place observing frequencies at the zero and extrema of the short-dashed curve. The relativistic correction to the kinetic effect is proportional to $v_p T_e$, and for a 10 keV cluster with very large peculiar velocity 1000 km/sec, this effect is about a factor 10 smaller than the relativistic correction to the thermal effect. We have seen that the temperature roughly can be detected through the relativistic correction to the thermal effect with an observing sensitivity about $1 \mu\text{K}$, so one should have at least $0.1 \mu\text{K}$ sensitivity (the expected sensitivity of SPT) in order to distinguish between variations in the background CMBR signal and the kinetic SZ effect.

First of all, to extract the dominating comptonization parameter y_c , one should pick a frequency where the relativistic corrections to the thermal and the kinetic SZ effects give small contributions. Clearly, which is well known, the smaller frequencies are good in this respect (see e.g. [38] for a recent discussion). Thus if one wants to remain in the atmospheric window, then a frequency of 90 GHz is good (a smaller frequency is even better if the point sources can be acceptably removed). It turns out, however, that an

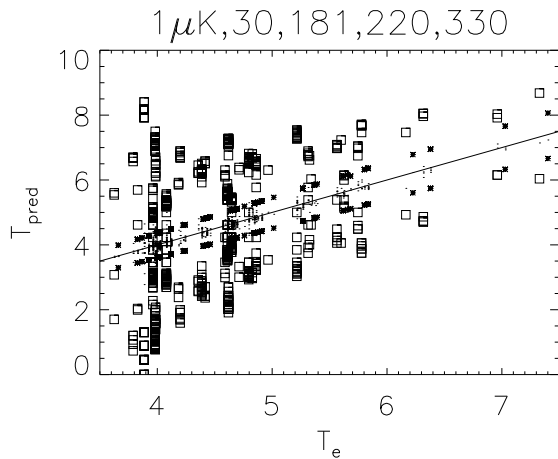


Figure 10. A hypothetical experiment with sensitivity of $1 \mu\text{K}$, with only 4 frequencies placed for optimal temperature determination. Open squares are with no prior temperature knowledge, stars are with 10% temperature information.

equally good frequency is near 475 GHz (figure 9). At this frequency, the contributions from both the kinetic SZ effect and the relativistic corrections are very small, while at the same time the thermal SZ effect is still quite significant. Observations at such large frequencies would require a careful removal of dust contamination, which can be achieved by fitting the observations at several even higher frequencies to a dust model. The resulting fit can be extended down to lower frequencies in order to extract the dust from the signal [33, 39].

Let us now concentrate on the cluster temperature. One can in theory extract the temperature using the relativistic corrections to the thermal effect. To do so one should choose frequencies where the relativistic contribution (dot-dashed lines in figure 9) is maximal and zero [40]. The maxima are near 120 and 330 GHz, and as discussed above the zero points are at 181 GHz and near 475 GHz. On the other hand, in order to extract the peculiar velocity one has to measure the kinetic SZ effect (dashed line, figure 9) which reaches its maximum near 217 GHz. The relativistic corrections to the kinetic SZ effect (long dashed line, figure 9) are maximal near 200 and 510 GHz, and disappear near 310 GHz.

As a specific example of the parameter extraction, let us consider an experiment with 4 frequencies and sensitivity of $1 \mu\text{K}$. Let us try to extract the temperature as well as possible. Due to the importance of the comptonization parameter we choose 30 GHz (assuming that point sources can be removed). For the temperature itself we choose the maximum and zero of the dot-dashed lines, namely 181 and 330 GHz. To remove the effect of peculiar velocity we include as the last frequency 220 GHz. The results are presented in figure 10. One finds that for a cluster of 5 keV and comptonization parameter of 3×10^{-5} , that with only 4 frequencies, one can extract

all 3 parameters simultaneously with 1σ error-bars (open squares) of about 2 keV, 10^{-6} for y_c , and v_p about 50%. For larger cluster temperatures the temperature will be extracted with better precision, since the relativistic corrections to the thermal SZ effect are more important. With prior knowledge about the temperature of 10% (stars), then the precision on y_c is improved by almost a factor of 2, and the precision on v_p with almost a factor of 5. It is therefore clear again, that prior temperature knowledge is very important and should be included if one considers small cluster samples. It is even crucial when one wants to deal with the proper motion of the galaxy clusters and the large scale motions.

7. Bandwidth and precision of measurements

We have so far been discussing how well a given experiment with few μK sensitivity can extract the cluster parameters, however, for this discussion to be relevant we must make sure that μK sensitivity is achievable. As we will see below, systematic errors as large as μK can appear simply through the finite bandwidths, and such systematic errors cannot easily be removed even in principle.

The distortion of the CMBR caused by the SZ effect can be formally defined through an effective change of temperature for each frequency

$$\Delta T(x) = T_0 \frac{\Delta I(x)}{I_0} \frac{(e^x - 1)^2}{x^4 e^x}, \quad (13)$$

which is only a formal representation of the SZ distortion, since strictly speaking temperature is not a well defined quantity when the photons are not in equilibrium. The above expression is derived from the relation $\Delta I(x) = I_0 x^3 (f - f_0)$ assuming that $x\Delta T(x)/T_0 \ll 1$, and can have corrections of the order $x\Delta T(x)/T_0$, which can be important for precise measurements. However, we will consider eq. (13) as an exact definition of the effective temperature difference in order to avoid confusions.

The SZ effect in terms of temperature difference from eq. (13) is shown in figure 11 for cluster parameters $y_c = 2 \times 10^{-4}$, $T = 5$ keV and $v_p = -200$ km/sec. The typical values of $\Delta T \sim 100 - 500 \mu\text{K}$ are more than two orders of magnitude larger than the desired precision of future experiments, which are of the order $0.1 - 5 \mu\text{K}$. In this section, we investigate the effects of a finite frequency bandwidth, which can be comparable to, or even larger than, the expected error-bars of future SZ experiments.

Let us start with the SZ effect in the non-relativistic approximation, and emphasize the well known fact that one must use the exact shape of the SZ effect when reducing the observations in a given bandwidth to just one frequency. Usually the experiments detect all photons which fall inside of a bandpass with finite width $\pm\delta\nu$ around a central frequency ν_{mid} and extrema ν_{max} and ν_{min} . However, because the energy of photons in the atmospheric window is unknown, the resulting flux should be rescaled to the central frequency. In the non-relativistic case this is simple, because the functional shape of the

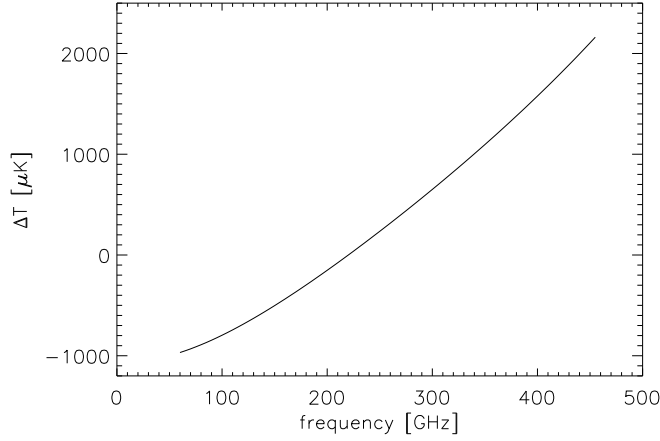


Figure 11. The SZ effect in terms of an effective ΔT from (13) for $y_c = 2 \times 10^{-4}$, $T_e = 5$ keV and $v_p = -200$ km/sec.

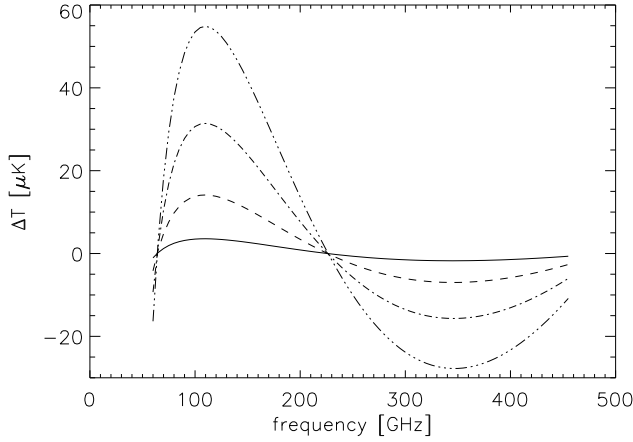


Figure 12. Shift in the non-relativistic SZ effect from finite bandwidth ($\delta\nu = 10, 20, 30$ and 40 GHz) with a broad top-hat filter, if the shape of the SZ effect is ignored. The 40 GHz filter gives the larger effect. Parameters are the same as in figure 11.

SZ effect is known,

$$\frac{\Delta I_{\text{exp}}(\nu_{\text{mid}})}{I_0} = y_c (f_0(\nu_{\text{mid}}) + \epsilon_0(\delta\nu)) , \quad (14)$$

where f_0 is given by (3) and $\epsilon_0(\delta\nu)$ is the shift due to a non-zero bandwidth

$$\epsilon_0(\delta\nu) = \frac{1}{\nu_{\text{max}} - \nu_{\text{min}}} \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} d\nu (f_0(\nu) - f_0(\nu_{\text{mid}})) . \quad (15)$$

There are two possible sources of error in (15). First, the central frequency might not be at the exact centre of the band, $\nu_{\text{mid}} \neq (\nu_{\text{min}} + \nu_{\text{max}})/2$. It was recently shown

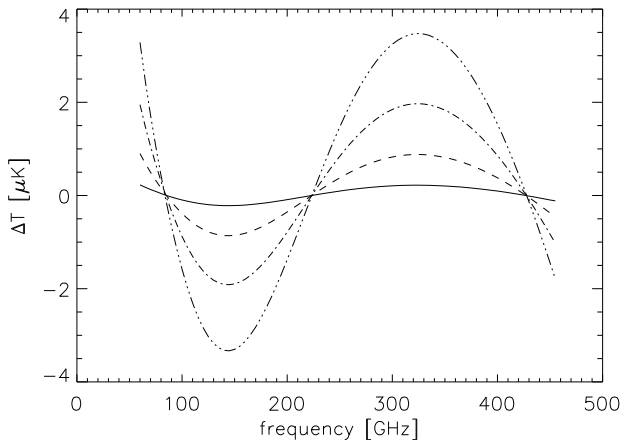


Figure 13. The systematic error in the SZ effect from finite bandwidth with a broad top-hat filter that arises from the shape of the SZ effect when taking into account the relativistic corrections for $y_c = 2 \times 10^{-4}$ and $T_e = 5$ keV ($\delta\nu = 10, 20, 30$ and 40 GHz top-hat filter, where the 40 GHz filter gives the larger effect).

that bandpass errors at the 10% level are acceptably small for Planck [41]. Second, the band can be wide enough that the shape of the SZ effect becomes important. We show in figure 12 what the error would be if one does not take the shape of $f(x)$ into account. The curves correspond to $\delta\nu = 10, 20, 30$ and 40 GHz. Fortunately, such error can be taken into account exactly from equations 3, 14 and 15.

Now, let us consider the case including relativistic corrections, where the exact shape is a priori unknown,

$$\frac{\Delta I_{\text{exp}}(\nu_{\text{mid}})}{I_0} = y_c (f_0(\nu_{\text{mid}}) + \epsilon_0(\delta\nu) + \delta f(\nu_{\text{mid}}, T_e) + \epsilon_1(\delta\nu)) , \quad (16)$$

where $\delta f(\nu, T_e)$ are temperature corrections and

$$\epsilon_1(\delta\nu) = \frac{1}{\nu_{\text{max}} - \nu_{\text{min}}} \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} d\nu (\delta f(\nu, T_e) - \delta f(\nu_{\text{mid}}, T_e)) \quad (17)$$

is the bandwidth error from the relativistic corrections to the thermal SZ effect. This error is shown for bandwidths $\delta\nu = 10, 20, 30$ and 40 GHz in figure 13. From this figure we conclude that, if the experimental precision is better than $\sim 5 \mu\text{K}$ and the frequency bands are large enough, one should take into account the correction from (17). This error will increase for larger temperatures and may also have important contribution from non-zero peculiar velocities, which will shift the shape and zero-point of figure 13 slightly. The main problem of taking into account the error $\epsilon_1(\delta\nu)$ in (17) is that the electron temperature is a priori unknown. Thus, by using the non relativistic form in eq. (3), to reduce the observations in a given bandwidth to one single frequency, one will always have a systematic error as large as shown in figure 13.

The simplest way to avoid such error is to reduce the bandwidth, but this would lead to a reduction in the signal to noise and would increase the statistical error-bars. A possible procedure to significantly reduce this systematic errors could be the following. First, reduce to one frequency, taking into account only ϵ_0 from (15) (that is, assume the non-relativistic form). Afterwards, one finds the best fit point in the parameter space (y_c, v_p, T_e) . Finally, one can recalculate everything, taking into account $\epsilon_1(\delta\nu)$ for the reduction to one frequency (that is, using the real form of the SZ effect including temperature corrections and peculiar velocity), and subsequently find the new best fit point values for the SZ parameters.

8. Conclusions

We have pointed out that there are parameter degeneracies for the Sunyaev-Zel'dovich effect. This in particular implies that the error-bars on the peculiar velocities for clusters with positive peculiar velocities is significantly smaller than for cluster with negative peculiar velocities. This parameter degeneracy can be broken either by having prior temperature knowledge, or by observing at several frequencies. For an experiment with 2 μK sensitivity and 3 observing frequencies (ACT-like), we find that a Compton parameter of the order of 10^{-4} is often deduced with a good accuracy 2% (10%) with (without) prior information on the electron temperature. The prior information on the electron temperature is even more important for the extraction of the peculiar velocity for which the accuracy reaches 10–20%. Additional observing frequencies also break the parameter degeneracies but the accuracies are not better than with the prior temperature knowledge. In the particular case of Planck, the Compton parameter of 10^{-4} is extracted very accurately (2%) even without information on the temperature. Prior temperature knowledge is, however, necessary to go from a peculiar velocity determination of a factor of a few to 20–50%.

We have discussed the needed sensitivity in order to extract the cluster temperature using SZ observations only. We have seen that experiments with only 3 observing frequencies need very good sensitivity ($\sim 0.5 \mu\text{K}$) in order to determine the cluster temperature with 1 keV error-bars. Already with 4 frequencies one can reach 1 keV (4 keV) temperature error-bars with only 1 μK (4 μK) sensitivity.

We have identified the frequencies which are optimal for deducing the 3 cluster parameters from a theoretical point of view. This optimal choice of 4 observing frequencies and sensitivity of 1 μK allows one to deduce the parameters of a 5 keV, $y_c = 3 \times 10^{-5}$ cluster with 1σ error bars of about 2 keV and 10^{-6} . The velocity is extracted with a 50% accuracy. The optimal frequencies for an actual experiment will depend both on the sensitivity of the experiment and also on which cluster parameters are of primary interest.

We have studied the systematic error which appears due to the finite band-width. This error is related to the shape of the SZ effect. The exact shape of the SZ effect depends on the cluster temperature and peculiar velocity, which cannot be known a

priori. This systematic error can be of the order of a few μK , and must therefore be considered seriously by future experiments which aim at such impressive sensitivity.

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References

- [1] Sunyaev R A and Zel'dovich Ya B 1972 *Comments Astrophys. Space Phys.* **4** 173
- [2] LaRoque S J *et al* 2002. *Astrophys. J.* (submitted), *Preprint astro-ph/0204134*
- [3] De Petris M *et al* 2002 *Astrophys. J.* **574** L119
- [4] Joy M *et al* 2001 *Astrophys. J.* **551** L1
- [5] Rephaeli Y 1995 *Ann. Rev. Astron. & Astrophys.* **33** 541
- [6] Birkinshaw M 1999 *Phys. Rep.* **310** 97
- [7] Carlstrom J E, Holder G P and Reese E D 2002 *Ann. Rev. Astron. & Astrophys.* **40** 643
- [8] Rephaeli Y 1995 *Astrophys. J.* **445** 33
- [9] Challinor A D and Lasenby A N 1998 *Astrophys. J.* **499** 1
- [10] Itoh N, Kohyama Y and Nozawa S 1998 *Astrophys. J.* **502** 7
- [11] Nozawa S, Itoh N, Kawana Y and Kohyama Y 2000 *Astrophys. J.* **536** 31
- [12] Dolgov A D, Hansen S H, Pastor S and Semikoz D V 2001 *Astrophys. J.* **554** 74
- [13] Itoh N, Kawana Y, Nozawa S and Kohyama Y 2001 *Month. Not. Roy. Astron. Soc.* **327** 567
- [14] Colafrancesco S, Marchegiani P and Palladino E 2003 *Astron. & Astrophys.* **397** 27
- [15] Sunyaev R A and Zel'dovich Ya B 1980 *Ann. Rev. Astron. & Astrophys.* **18** 537
- [16] Sazonov S Y and Sunyaev R A 1998 *Astrophys. J.* **508** 1
- [17] Nozawa S, Itoh N and Kohyama Y 1998 *Astrophys. J.* **508** 17
- [18] Aghanim N, Górski K M and Puget J L 2001 *Astron. & Astrophys.* **374** 1
- [19] Arnaud M 2001 *Proc. Rencontres de Moriond 2001*
- [20] Grego L *et al* 2001 *Astrophys. J.* **552** 2
- [21] Bryan G L and Norman M L 1998 *Astrophys. J.* **495** 80
- [22] Sheth R K and Tormen G 1999 *Month. Not. Roy. Astron. Soc.* **308** 119
- [23] Wu J H 2001 *Month. Not. Roy. Astron. Soc.* **327** 629
- [24] Viana P T P and Liddle A R 1999 *Month. Not. Roy. Astron. Soc.* **303** 535
- [25] Peebles P J 1993 *Principles of Physical Cosmology* (Princeton University Press)
- [26] Carroll S M, Press W H and Turner W L 1992 *Ann. Rev. Astron. & Astrophys.* **30** 499
- [27] Peebles P J 1980 *The large-scale structure of the universe* (Princeton University Press)
- [28] Hansen S H, Pastor S and Semikoz D V 2002 *Astrophys. J.* **573** L69
- [29] ACT <http://www.hep.upenn.edu/~angelica/act/act.html>
- [30] Kosowsky A 2002 Private communication
- [31] ALMA <http://www.eso.org/projects/alma/>
- [32] Planck <http://astro.estec.esa.nl/SA-general/Projects/Planck/>
- [33] Pointecouteau E, Giard M and Barret D 1998 *Astron. & Astrophys.* **336** 44
- [34] Diego J M, Hansen S H and Silk J 2003 *Month. Not. Roy. Astron. Soc.* **338** 796
- [35] South Pole Telescope <http://astro.uchicago.edu/spt/>
- [36] Hansen S H and Stadel J 2003 *Preprint astro-ph/0303416*

- [37] Doré O, Knox L and Peel A 2003 *Astrophys. J.* **585** L81
- [38] Holder G P 2002 *Preprint* astro-ph/0207600
- [39] Lamarre J M *et al* 1998 *Astrophys. J.* **507** L5
- [40] Melchiorri F 2002 Private communication
- [41] Church S, Knox L and White M J 2002 *Astrophys. J.* **582** L63