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Evaporation of near-extremal Reissner-Nordström black holes

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The formation of near-extremal Reissner-Nordström black holes in the S-wave approximation can be described, near the event horizon, by an effective solvable model. The corresponding one-loop quantum theory remains solvable and allows to follow analytically the evaporation process which is shown to require an infinite amount of time.

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Black holes are the most fascinating objects in General Relativity. Since Hawking discovered that they emit thermal radiation [1], it has been a long standing puzzle to explain their thermodynamical properties, in terms of some microscopic structure, and to understand the dynamical evolution beyond Hawking initial scheme where the gravitational field was treated as a fixed background. Extremal and near-extremal charged black holes have recently played a fundamental role in String Theory, where in some special cases it has been possible to give a statistical explanation of the Bekenstein-Hawking area law for their entropy [2]. Moreover, the scattering of low-energy particles off extremal black holes also provides a convenient setting to study the evaporation process including back-reaction effects. By throwing a long-wavelength particle into an extremal hole, a non-extremal configuration is created and quantum-mechanically one expects it to decay via Hawking emission back to extremality. To render this problem tractable one can boil it down considering large q and incoming neutral matter with zero angular momentum. In this context, dilatonic black holes [3] were extensively considered since the scattering particle-hole and the ensuing information loss problem can be analyzed in an analytical framework. This is so because the problem can be reduced to study a two-dimensional effective theory [4] which turns out to be solvable both at the classical and at the one-loop quantum level [5–8] (see also the reviews [9] for a more detailed description and as useful references for the methods used in this work). Among the 'nice' properties of these dilatonic black holes, making the problem under study rather special, the extremal black holes are classically completely regular and, moreover, the temperature near extremality is constant. In contrast, the Reissner-Nordström (RN) black holes always have singularities (extremal case $lm = q$ included) and the temperature goes as $T_H \sim \sqrt{lm-q}/q^2$ near extremality. This case was reconsidered after the improvements in the physical understanding obtained with dilatonic black holes. However, only partial analytic (obtained by means of the adiabatic approximation) and numerical answers have been obtained [10,11]. The purpose of this letter is to present the first exact results on the evaporation process of a near-extremal RN black hole. If we consider the physics in the S-wave approximation, near the horizon we can describe it by an effective model (the Jackiw-Teitelboim model [12]) which is solvable even when back-reaction effects are included.

Let us consider the solutions of Einstein-Maxwell gravity with null infalling matter

$$d\bar{s}^2 = -\frac{(r-r_+)(r-r_-)}{r^2}\, dv^2 + 2drdv + r^2 d\Omega^2 \,, \tag{1}$$

$$F = q\epsilon_2 \,, \tag{2}$$

where ϵ_2 is the volume element of the unit S^2. The only nonzero component of the stress tensor is given by

$$T_{vv} = \frac{\partial_v (r_+(v) + r_-(v))}{8\pi l^2 r^2} \,, \tag{3}$$

where $l^2 = G$ is Newton's gravitational constant. One can describe the formation of a non-extremal black hole by sending a low-energy shock wave

$$T_{vv} = \frac{\Delta m}{4\pi r^2}\delta(v - v_0) \,, \tag{4}$$

in the extremal geometry ($v < v_0$). This model can be described by an effective two-dimensional theory given by

$$I = \int d^2x\sqrt{-g}\left[R\phi + l^{-2}V(\phi) - \frac{1}{2}|\nabla f|^2\right] \,, \tag{5}$$

where the field f represents the null matter with $(\partial_v f)^2 \equiv T^f_{vv} = 4\pi r^2 T_{vv}$ and

$$\phi = \frac{r^2}{4l^2} \,, \qquad V(\phi) = (4\phi)^{-\frac{1}{2}} - q^2(4\phi)^{-\frac{3}{2}} \,. \tag{6}$$

The two-dimensional metric is related to the $r - v$ projection of (1) by the conformal rescaling

$$ds^2 = \sqrt{\phi}\, d\bar{s}^2 \,. \tag{7}$$

The extremal black hole is recovered for the zero of the potential $V(\phi_0) = 0$, which corresponds to $\phi_0 = q^2/4$. This fact suggests us to consider the effective near-horizon and near-extremal theory defined by the expansion of (5) around ϕ_0

$$\phi = \phi_0 + \tilde{\phi}\,, \qquad m = lq + \Delta m\,. \tag{8}$$

We obtain

$$I = \int d^2x\sqrt{-g}\left[(R + \frac{4}{l^2q^3})\tilde{\phi} - \frac{1}{2}|\nabla f|^2\right] + \mathcal{O}(\tilde{\phi}^2)\,, \tag{9}$$

where the leading order term is just the Jackiw-Teitelboim (JT) model, which now arises as the effective theory governing the dynamics near extremality and close to the horizon. A $d = 1$ realization of the $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ correspondence [13] in the JT model exactly accounts for the deviation from extremality of the Bekenstein-Hawking entropy of RN black holes [14]. Therefore one can also expect to obtain an exact picture of the evaporation process near the horizon. The formation of a near-extremal black hole due to a shock wave (4) can be pushed down to the JT metrics of constant negative curvature. For $v < v_0$ we have the extremal RN configuration and its near-horizon geometry is given by the Robinson-Bertotti anti-de Sitter geometry [15]

$$d\bar{s}^2 = -\frac{r^2}{r_0^2}dt^2 + \frac{r_0^2}{r^2}dr^2 + r_0^2 d\Omega^2\,, \tag{10}$$

where r_0 is the extremal radius. After the rescaling (7) the two-dimensional metric in null conformal coordinates becomes

$$ds^2 = -\frac{2}{l^2q^3}\tilde{x}^2 du dv\,, \tag{11}$$

with $u = v + \frac{l^2q^3}{\tilde{x}}$, $\tilde{x} = l\tilde{\phi}$. Proceeding in a similar way for the near-extremal configuration $v > v_0$, we obtain

$$ds^2 = -(\frac{2}{l^2q^3}\tilde{x}^2 - l\Delta m)d\bar{u}dv\,, \tag{12}$$

with

$$\bar{u} = v + \sqrt{\frac{2lq^3}{\Delta m}}\mathrm{arctanh}\sqrt{\frac{2}{l^3q^3\Delta m}}\,\tilde{x}\,. \tag{13}$$

The coordinates (v, u) and $(v, \bar{u})$ are the radial null coordinates corresponding to those of RN $(t + r^*, t - r^*)$ before and after the shock wave. Imposing the continuity of (11) and (12) along $v = v_0$ one obtains

$$u = v_0 + a\,\mathrm{cotanh}\frac{\bar{u} - v_0}{a}\,, \tag{14}$$

where $a = \sqrt{\frac{2lq^3}{\Delta m}}$. From this relation we can work out immediately the outgoing energy flux of Hawking radiation in terms of the Schwarzian derivative between the coordinates u and $\bar{u}$

$$\langle T^f_{\bar{u}\bar{u}}\rangle = -\frac{1}{24\pi}\{u, \bar{u}\} = \frac{1}{12\pi a^2} = \frac{\pi}{12}T_H^2\,. \tag{15}$$

We observe that this flux is constant and coincides with the thermal value of Hawking flux for near-extremal RN black holes, where T_H is Hawking's temperature. This fact can be understood easily since the $\mathrm{AdS}_2\times\mathrm{S}^2$ geometries associated to (11) and (12) represent indeed the near-horizon limit of the RN geometries (1) due to the shock wave (4). So the constant thermal flux for every value of $\bar{u}$ corresponds to the flux measured by an inertial observer at future null infinity approaching the event horizon of the RN black hole. In the light of this remark it is interesting to point out that the JT theory also describes the dynamics of extremal and near-extremal RN black holes close to the horizon in the presence of a spherically symmetric Klein-Gordon field. This is so because, as it is well-known, the scalar field propagates freely near the horizon.

Our purpose now is to analyze the back-reaction effects in the evaporation process. The one-loop effective theory is obtained by adding the non-local Liouville-Polyakov term to the classical action. We then get ($\lambda^2 = l^{-2}q^{-3}$)

$$I = \int d^2x\sqrt{-g}\left[R\tilde{\phi} + 4\lambda^2\tilde{\phi} - \frac{1}{2}\sum_{i=1}^{N}|\nabla f_i|^2\right] - \frac{N\hbar}{96\pi}\int d^2x\sqrt{-g}R\,\Box^{-1}R + \xi\frac{N\hbar}{12\pi}\int d^2x\sqrt{-g}\lambda^2\,, \tag{16}$$

where we have considered the presence of N scalar fields. The parameter N allows us to consider the theory in the large N limit, keeping $N\hbar$ fixed. Moreover we have also added a local conterterm (in the form of a 2d cosmological constant which corresponds to the freedom of adding a constant to the 2d conformal anomaly), mimicking the analysis of dilaton gravity theory [5], to ensure that the extremal geometry remains an exact solution of the one-loop theory at $\xi = 1$. Our results are the same irrespective of the value of ξ. We should mention now that for the region we are interested in, the conformal factor $\sqrt{\phi}$ of (7) is almost constant and therefore the semiclassical quantization in terms of the Einstein-Maxwell action and the JT action are equivalent. Far from the horizon this is no more true. The equations of motion derived from (16) in conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$ are (from now on we take $\xi = 1$)

$$2\partial_+\partial_-\rho + \lambda^2 e^{2\rho} = 0\,, \tag{17}$$

$$\partial_+\partial_-\tilde{\phi} + \lambda^2\tilde{\phi}e^{2\rho} = 0\,, \tag{18}$$

$$\partial_+\partial_- f_i = 0\,, \tag{19}$$

$$-2\partial_\pm^2\tilde{\phi} + 4\partial_\pm\rho\partial_\pm\tilde{\phi} = T^f_{\pm\pm} - \frac{N\hbar}{12\pi}t_\pm - \frac{N\hbar}{12\pi}\left((\partial_\pm\rho)^2 - \partial_\pm^2\rho\right)\,. \tag{20}$$

The functions $t_\pm(x^\pm)$ are related with the boundary conditions of the theory and depend on the quantum state of the system. The Liouville equation (17) has the general solution

$$ds^2 = -\frac{\partial_+A_+\partial_-A_-}{(1 + \frac{\lambda^2}{2}A_+A_-)^2}dx^+dx^-\,, \tag{21}$$

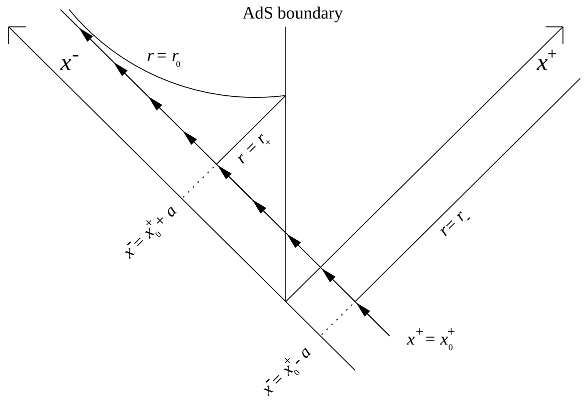

FIG.1. Kruskal diagram of near-extremal black hole. The two time-like boundaries of near-horizon geometry AdS_2 are represented by the vertical line $x^- = x^+$. The infalling shock wave emerges from one boundary (left side of $x^- = x^+$ line) and, crossing the outer and inner horizons, reaches the other boundary (right side of $x^- = x^+$ line).

where $A_\pm(x^\pm)$ are arbitrary chiral functions. We can choose a particular form of the functions $A_\pm$ as a way to fix completely the conformal coordinates. We find convenient to choose

$$A_+ = x^+ \,, \qquad A_- = \frac{-2}{\lambda^2 x^-} \,. \tag{22}$$

Before the shock wave these coordinates $x^\pm$ correspond to the RN coordinates (v, u) and after ($v > v_0$) the relation is

$$v = x_0^+ + a \,\mathrm{arctanh} \frac{x^+ - x_0^+}{a} \,, \tag{23}$$

together with $u = x^-$. Then both metrics (11) and (12) are brought into (21) and the physical information is encoded in the field $\tilde{\phi}$. At the classical level the solution for it is given by

$$\tilde{\phi} = lq^3 \frac{1 - \Theta(x^+ - x_0^+)\frac{\Delta m}{2lq^3}(x^+ - x_0^+)(x^- - x_0^+)}{x^- - x^+} \,. \tag{24}$$

After the shock wave ($x^+ > x_0^+$) the extremal radius is given by curve $\tilde{\phi} = 0$: $(x^+ - x_0^+)(x^- - x_0^+) = a^2$, and the outer and inner apparent horizons $r = r_\pm$ are given by the condition $\partial_+ \tilde{\phi} = 0$: $x^- = x_0^+ \pm a$. The corresponding Kruskal diagram (in this region the coordinates $x^\pm$ are regular at the horizon and therefore they represent a sort of Kruskal frame) is given by Fig.1.

At the quantum level we have to solve equations (17-20) and the crucial point is to choose the adequate functions $t_\pm(x^\pm)$ for the physical situation. The natural choice is $t_v = t_{x^+} = 0$ and $t_u = t_{x^-} = 0$ before the shock-wave and $t_{x^+} = \frac{1}{2}\{v, x^+\}$ and $t_{x^-} = 0$ after, where now v is the light-cone coordinate of the evaporating Vaidya-type metric

$$ds^2 = -(\frac{2\tilde{x}^2}{l^2 q^3} - lm(v))dv^2 + 2d\tilde{x}dv \,. \tag{25}$$

The remarkable property of the equations of the near-horizon effective theory is that one can solve them also in conformal gauge. We find that

$$\tilde{\phi} = \frac{F(x^+)}{x^- - x^+} + \frac{1}{2}F'(x^+) \,, \tag{26}$$

$\tilde{x} = l\tilde{\phi}$, where the function $F(x^+)$ satisfies the following differential equation

$$F''' = \frac{N\hbar}{24\pi}\left(-\frac{F''}{F} + \frac{1}{2}\left(\frac{F'}{F}\right)^2\right) \,, \tag{27}$$

and relates the x^+ and v coordinates

$$\frac{dv}{dx^+} = \frac{lq^3}{F} \,. \tag{28}$$

The evaporating mass is then given by

$$m(x^+) = \frac{24\pi}{N\hbar l q^3} F^2 F''' \,, \tag{29}$$

and can be related to the boundary function t_{x^+}

$$t_{x^+} = \frac{lq^3 m(x^+)\Theta(x^+ - x_0^+)}{2F^2} \,. \tag{30}$$

The fact that the functions $t_\pm$ can be discontinuous for coordinates $x^\pm$ associated to free (or Liouville) fields was pointed out in [16]. The expression (30), and also the function $F(x^+)$, admits a series expansion in powers of $\hbar$, where the classical term, obtained using the classical relation (23), is given by

$$t_{x^+} = \frac{a^2 \Theta(x^+ - x_0^+)}{(a^2 - (x^+ - x_0^+)^2)^2} \,. \tag{31}$$

As before, the curve $\tilde{\phi} = 0$

$$x^- = x^+ - \frac{2F}{F'} \,, \tag{32}$$

represents the location of the extremal radius and $\partial_+ \tilde{\phi} = 0$ defines the inner and outer apparent horizons in the spacetime of the evaporating black hole

$$x^- = x^+ - \frac{F'}{F''} \pm \frac{\sqrt{F'^2 - 2FF''}}{F''} \,. \tag{33}$$

The intersection of these three curves takes place when

$$F'^2 - 2FF'' = 2lq^3 m(x^+) = 0 \,, \tag{34}$$

i.e. at the end of the evaporation. On the other hand , it is easy to show that $\partial_+ m(x^+) = -\frac{N\hbar}{24\pi}\frac{m(x^+)}{F}$ which can be readily integrated in v coordinate giving $m(v) = \Delta m e^{-\frac{N\hbar}{24\pi l q^3}(v - v_0)}$ and so $m = 0$ is given by $v = +\infty$ and not before (had we started with the classical boundary term (31) we would have obtained a finite evaporation time). From the numerical graph of the function $F(x^+)$ it is clear that this happens at a finite

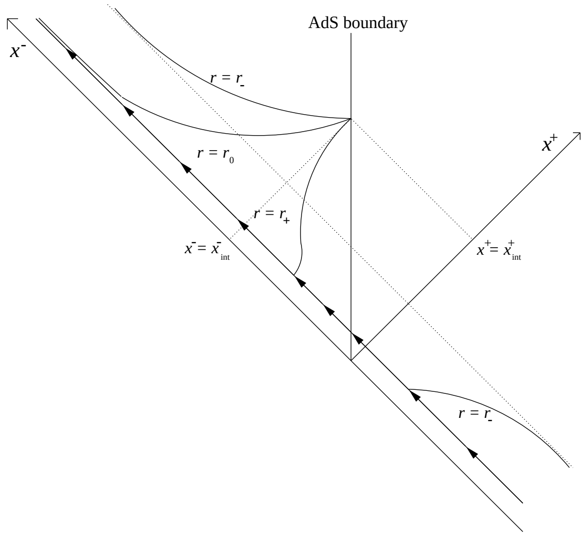

FIG.2. Kruskal diagram of semiclassical evolution of RN black hole. The outer apparent horizon shrinks until it meets both inner horizon and extremal radius curve at the AdS boundary.

value of $x^+ = x^+_{int}$ for which $F(x^+_{int}) = 0$ and moreover from eq. (34) it also follows that $F'(x^+_{int}) = 0$. One can also show that $F''(x^+_{int})$ is nonzero while all the other derivatives vanish, so that locally close to the intersection point $F(x^+)$ behaves as a parabola with exponentially suppressed corrections. This is enough to prove , see (32) and (33), that the intersection point belongs to the AdS boundary, i.e. $x^+_{int} = x^-_{int}$. The graph of all these curves is shown in Fig. 2, where the saddle point in the outer apparent horizon curve r_+ signals the transition from the strong to the weak back-reaction regimes as described in [11]. At the end-point the curves $\phi = 0$ and $\partial_+\phi = 0$ are null and the dilaton function is well represented asymptotically by the extremal solution

$$\phi = \frac{\partial_+^2 F(x^+_{int})}{2} \frac{(x^+ - x^+_{\rm int})(x^- - x^+_{\rm int})}{x^- - x^+} + ... \qquad (35)$$

where the dots are nothing but exponentially small corrections. So, as time passes the solution becomes 'more and more extremal' but without actually coming back to the extremal state due to the infinite evaporation time. Due to the fact that $T_H = 0$ for the extremal black hole our exact results are in agreement with the third law of thermodynamics applied to black holes (see for instance [17]), despite the fact that the weak energy condition is violated close to the horizon, as well as with those obtained using the adiabatic approximation [10].

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