Pion-Photon TDAs in the NJL Model *

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Abstract

The pion-photon Transition Distribution Amplitudes (TDAs) are studied, treating the pion as a bound state in the sense of Bethe-Salpeter, in the formalism of the NJL model. The results obtained explicitly verify support, sum rules and polynomiality conditions. The role of PCAC is highlighted.

Hard reactions provide important information for unveiling the structure of hadrons. The large virtuality, Q^2, involved in the processes allows the factorization of the hard (perturbative) and soft (non-perturbative) contributions in their amplitudes. In recent years a large variety of processes governed by the Generalized Parton Distributions (GPDs), like the Deeply Virtual Compton Scattering, has been considered. A generalization of GPDs to non-diagonal transitions has been proposed in [1]. In particular, the easiest case to consider is the pion-photon TDA, governing processes like $\pi^+\pi^- \to \gamma^*\gamma$ or $\gamma^*\pi^+ \to \gamma\pi^+$ in the kinematical regime where the virtual photon is highly virtual but with small momentum transfer. At leading-twist, the vector and axial TDAs, respectively $V(x,\xi,t)$ and $A(x,\xi,t)$, are defined as [2]

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \gamma(p') \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ \tau^- q\left(\frac{z}{2}\right) \left|\pi^+(p)\right\rangle\Big|_{z^+=z^\perp=0} = i\, e\, \varepsilon_\nu\, \epsilon^{+\nu\rho\sigma}\, P_\rho\, \Delta_\sigma\, \frac{V^{\pi^+}(x,\xi,t)}{\sqrt{2} f_\pi}\,, \tag{1}$$

$$\begin{aligned}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \gamma(p') \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+\gamma_5 \tau^- q\left(\frac{z}{2}\right) \left|\pi^+(p)\right\rangle\Big|_{z^+=z^\perp=0} &= e\left(\vec{\varepsilon}^{\perp} \cdot \vec{\Delta}^{\perp}\right) \frac{A^{\pi^+}(x,\xi,t)}{\sqrt{2} f_\pi} \\ &+ e\,(\varepsilon \cdot \Delta)\, \frac{2\sqrt{2} f_\pi}{m_\pi^2 - t}\, \epsilon(\xi)\ \phi\left(\frac{x+\xi}{2\xi}\right)\,, \end{aligned} \tag{2}$$

where $t = \Delta^2 = (p'-p)^2$, $P = (p+p')/2$, $\xi = (p-p')^+/2P^+$, $\epsilon(\xi) = 1$ for $\xi > 0$ and -1 for $\xi < 0$ and where $f_\pi = 93$ MeV. For any four-vector v^μ, we have the light-cone coordinates $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and the transverse components $\vec{v}^\perp = (v^1, v^2)$. Finally, $\phi(x)$ is the pion distribution amplitude (PDA).

Apart from the axial TDA $A(x,\xi,t)$, the axial current Eq.(2) contains a pion pole contribution, which can be understood as a consequence of PCAC because the axial current must be coupled to the pion. This second term has been isolated in a model independent way. Therefore, all the structure of the incoming pion remains in $A(x,\xi,t)$. The pion pole term is not a peculiarity of the pion-photon TDAs: a similar contribution would be present in the Lorentz decomposition, in terms of distribution amplitudes, of the axial current for any pair of external particles. This term is only non-vanishing in the ERBL region, i.e. the $x \in [-\xi, \xi]$ region, whose kinematics allow the emission or absorption of a pion from the initial state, which is described through the PDA.

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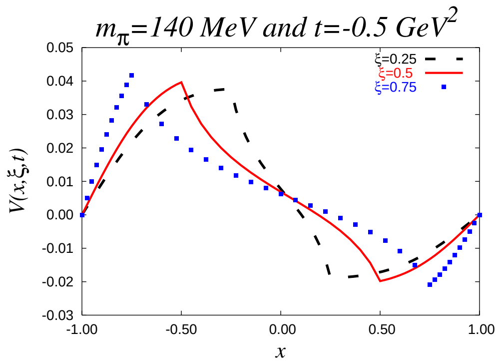

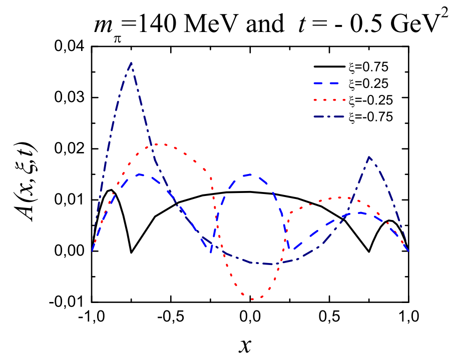

The π-γ TDAs are related to the vector and axial transition form factors through the sum rules

$$\int_{-1}^{1} dx\ V^{\pi^+}(x,\xi,t) = \frac{\sqrt{2}f_\pi}{m_\pi} F_V(t) \ , \qquad \int_{-1}^{1} dx\ A^{\pi^+}(x,\xi,t) = \frac{\sqrt{2}f_\pi}{m_\pi} F_A(t) \quad . \tag{3}$$

As usual, we consider that the currents present in Eqs. (1) and (2) are dominated by the handbag diagram. The method of calculation developed in [3] is here applied. The pion is treated as a bound-state in a fully covariant manner using the Bethe-Salpeter equation and solving it in the NJL model. Gauge invariance is ensured by using the Pauli-Villars regularization scheme. All the invariances of the problem are then preserved. As a consequence, the correct support is obtained, i.e. $x \in [-1,1]$, vector and axial TDAs obey the sum rules, Eq.(3), and the polynomiality expansion is recovered in both cases. Moreover, for the DGLAP region, we have obtained the isospin relations

$$V(-x,\xi,t) = -2V(x,\xi,t)\,, \qquad A(-x,\xi,t) = 2A(x,\xi,t)\,, \qquad\qquad |\xi| < x < 1 \quad . \tag{4}$$

In the figures are depicted both the vector and axial TDAs, which explicit expression are given in [2], for $m_\pi = 140$ MeV, $t = -0.5$ GeV^2 and different values of ξ ranging between $t/(2m_\pi^2 - t) < \xi < 1$: the process here does not constrain the skewness variable to be positive. The vector TDA is mainly a function of ξ^2 and we have depicted only positive values of ξ. For the axial TDA, two quite different behaviours are observed according to the sign of ξ. The value we numerically obtain for $F_V(0)$ is in agreement with [4], while the one we obtain for $F_A(0)$ is twice the expected value [4].

Previous studies of the pion-photon TDAs have been released [5]. Since both these studies parametrize TDAs by means of double distributions, Ref. [2] is the first study of the polynomiality property of TDAs. Moreover, in Ref. [2], the support, sum rules and polynomiality expansion are results (and not inputs) of the calculation. The study of TDAs should lead to interesting estimates of cross-sections for exclusive meson pair production in $\gamma\gamma^*$ scattering [1]. In particular, a deeper study of the pion pole contribution should allow us to give a cross-section estimate for the $\pi\pi$ pair case.

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