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The Weak-Magnetic Moment of Heavy Quarks

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ABSTRACT

With initial and final particles on-shell, the anomalous weak-magnetic dipole moments of b and c quarks are electroweak gauge invariant quantities of the effective couplings $Zb\bar{b}$ and $Zc\bar{c}$, respectively, and good candidates to test the Standard Model and/or new physics. Here we present a complete computation of these quantities within the Standard Model. We show that decoupling properties with respect to heavy particles do take place in the weak magnetic moment. The obtained values, $a_b(M_Z^2) = (2.98 - 1.56i) \times 10^{-4}$ and $a_c(M_Z^2) = (-2.80 + 1.09i) \times 10^{-5}$ are dominated by one-gluon exchange diagrams. The electroweak corrections are less than 1% of the total magnitude.

1   Introduction

The neutral current sector of the Standard Model (SM) has been subjected to a detailed precision scrutiny in the past few years [1]. This has led to establish definite quantum electroweak corrections to an impressive list of physical observables which see their tree-level values modified at the percent level. The agreement between the experiment and the theory proves the correctness of the SM and the machinery of renormalization in the quantum field theory. Although the issue is not still close, it seems [2] that even the Z-vertex to heavy quarks, which contains non-decoupling effects [3], is in agreement with the SM. An alternative to this methodology consists in isolating *new* observables in the quantum theory which were absent in the tree-level Lagrangian. In this paper we study the *anomalous* weak-magnetic moment (AWMM) of heavy quarks.

The anomalous weak-magnetic moment of fermions carries important information about their interactions with other particles. It may be seen as the coefficient of a chirality-flipping term in the effective Lagrangian of the Z coupled to fermions. Therefore, at $q^2 \neq 0$, it is expected to be proportional to the mass of the fermion, and only heavy fermions (leptons or quarks) are good candidates to have a measurable anomalous weak-magnetic moment. The already mentioned chirality properties indicate that some insight into the mechanism of mass generation may be obtained from it. These properties have also been considered in the context of extended models [4]. In previous work [5] we have studied the case of the *tau* and shown that it is possible to construct polarization observables sensitives to the AWMM. In this paper we focus on quarks, in particular on the AWMM of the b- and c-quarks. In Ref. [6] different strategies to detect polarization effects for the b-quark are suggested and discussed, so that the observables may become feasible in the future.

2   Anomalous Weak-Magnetic Moment

As the AWMM is proportional to the mass of the particle, in principle, only heavy fermions might have a sizeable value for it. The heaviest quark, the *top*-quark [7], would

seem to be the perfect candidate. The problem arises there in the electroweak gauge invariant properties of the defined form factor. As it is already well known [5, 8] only the on-shell definition of the AWMM is electroweak gauge invariant and free of uncertainties. Nevertheless, recently some procedures to move off-shell the gauge invariant form factors have been proposed [9], but their invariant properties and physical significance are still under discussion [10]. In this paper we concentrate ourselves in the study of the AWMM for the heavy quarks produced from on-shell Z's, i.e. bottom b and charm c quarks. This is of order α_s-strong or α-electroweak radiative correction to the $Zq\bar{q}$ vertex.

Using Lorentz covariance, the matrix element of the i-quark vector neutral current can be written in the form:

$$\bar{u}_i(p)\, V^\mu(p,\bar{p})\, v_i(\bar{p}) = e\, \bar{u}_i(p) \left[\frac{v_i(q^2)}{2s_w c_w}\gamma^\mu + i\frac{a_i^w(q^2)}{2m_i}\sigma^{\mu\eta}q_\eta \right] v_i(\bar{p}) \tag{1}$$

where $q^2 = (p+\bar{p})^2$ is the 4-momentum squared in the center of mass frame, e is the proton charge and s_w, c_w are the weak mixing angle sine and cosine, respectively. The first term $v_i(q^2)$ is the Dirac vertex (or charge radius of the fermion i) form factor and it is present at tree level with a value $v_i(q^2) = T_{i3} - 2Q_i\, s_w^2$, whereas the second form factor, $a_i^w(q^2)$, is the AWMM and it appears due to quantum corrections. As already mentioned, at $q^2 = M_Z^2$, it is a linearly independent and gauge invariant form factor of the Lorentz covariant matrix element, contributing to the physical $Z \longrightarrow q\,\bar{q}$ decay amplitude.
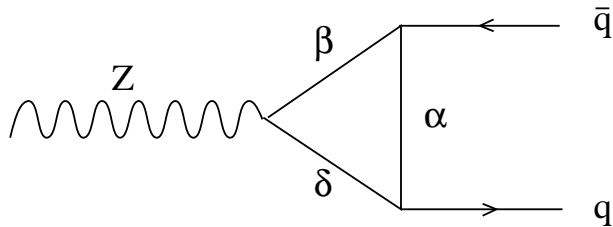

Figure 1: Contributing Feynman diagrams to the anomalous weak magnetic moment.

In the t'Hooft-Feynman gauge, there are 14 diagrams that contribute to a_i^w. In Fig. 1 we show the generic one-loop diagram contribution. From now on we denote by q_i (q_I) the internal quark in the loop with the same (different) charge as the external quark; α,

β and δ are the particles circulating in the loop as shown in Fig. 1; χ and Φ are the neutral would-be Goldstone boson and physical Higgs, and $\sigma^{\pm}$ are the charged would-be Goldstone bosons. Then, all the contributions may be written with the compact notation:

$$[a_i]^{\alpha\beta\delta} = \frac{\alpha}{4\pi} \ \frac{m_i^2}{M_Z^2} \ \sum_{jk} c_{jk} I_{jk}{}^{\alpha\beta\delta}(i) \tag{2}$$

with $(\alpha\,\beta\,\delta)$ standing for: $(N\,\bar{q}_i\,q_i)$, $(C^{\pm}\,\bar{q}_I\,q_I)$, $(q_I\,C^+\,C^-)$ and $(q_i\,N\,N')$. N and N' are the neutral particles γ, Z, χ, Φ (with $N \neq N'$), and $C^{\pm}$ are the charged bosons $W^{\pm}$ and $\sigma^{\pm}$. c_{jk} are coefficients, and

$$I_{jk}{}^{\alpha\beta\delta}(i) \equiv I_{jk}(m_i^2, q^2, m_i^2, m_\alpha^2, m_\delta^2, m_\beta^2) \tag{3}$$

are the on-shell ($p^2 = m_i^2$, $\bar{p}^2 = m_i^2$ and $q^2 = (p + \bar{p})^2 = M_Z^2$) scalar, vector and tensor functions defined, from the one-loop 3-point functions

$$I_{00;\ \mu;\ \mu\nu}\,(p^2, (p+\bar{p})^2, \bar{p}^2, m_A{}^2, m_B{}^2, m_C{}^2) =$$

$$\frac{1}{i\pi^2} \ \times \ \int d^n k \frac{\{1\,;\,k_\mu\,;\,k_\mu k_\nu\}}{(k^2 - m_A{}^2)((k-p)^2 - m_B{}^2)((k+\bar{p})^2 - m_C{}^2)} \tag{4}$$

as [5]:

$$I^\mu = (p - \bar{p})^\mu I_{10} + (p + \bar{p})^\mu I_{11}$$

$$I^{\mu\nu} = (\bar{p}^\mu \bar{p}^\nu + p^\mu p^\nu) I_{21} + (\bar{p}^\mu p^\nu + p^\mu \bar{p}^\nu) I_{22} + (\bar{p}^\mu \bar{p}^\nu - p^\mu p^\nu) I_{2-1} + g^{\mu\nu} I_{20} \tag{5}$$

We are only interested in the AWMM so that, for each diagram, we have to pick up only the $\sigma^{\mu\nu} q_\nu$ coefficient shown in Eq.(1). Though the AWMM receives its leading contribution from one loop diagrams (renormalizability excludes $\bar{\psi}\sigma_{\mu\nu}\psi\,Z^{\mu\nu}$ terms in the Lagrangian), it is finite, and can be extracted from them with no need of renormalization. Notice also that only vertex corrections may contribute to the AWMM, because the renormalization of the external legs does not change the (V-A) Lorentz structure of the vertex.

As a test of our calculation we have verified the conservation of the vector current

$$q^\mu \ \bar{u}(p)\, V^\mu(p, \bar{p})\, v(\bar{p}) \ = \ 0 \tag{6}$$

by explicitly checking that the coefficient of the q^μ term of the matrix element (1) vanishes. This conservation does not occur on each diagram, but it can be confirmed by considering cancellations among some of them and, of course, in the overall sum.

In the following, we list all contributions to the AWMM that are written, in a self-explanatory notation, as:

$$[a_b^w]^{\gamma bb} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{4v_b Q_b^2}{s_w c_w} M_Z^2 [I_{10} + I_{22} - I_{21}]^{\gamma bb} \tag{7}$$

$$[a_b^w]^{Zbb} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{v_b}{s_w^3 c_w^3} M_Z^2 \left[v_b^2 (I_{22} - I_{21} + I_{10}) + \right.$$
$$\left. a_b^2 (3I_{22} - 3I_{21} + 11I_{10} - 4I_{00})\right]^{Zbb} \tag{8}$$

$$[a_b^w]^{\chi bb} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{m_b^2}{M_Z^2} \frac{v_b}{2s_w^3 c_w^3} M_Z^2 [I_{22} - I_{21}]^{\chi bb} \tag{9}$$

$$[a_b^w]^{\Phi bb} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{m_b^2}{M_Z^2} \frac{v_b}{2s_w^3 c_w^3} M_Z^2 [I_{22} - I_{21} + 2I_{10}]^{\Phi bb} \tag{10}$$

$$[a_b^w]^{Wtt} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{(v_t + a_t)}{s_w^3 c_w} |V_{tb}|^2 M_Z^2 [I_{22} - I_{21} + 3I_{10} - I_{00}]^{Wtt} \tag{11}$$

$$[a_b^w]^{\sigma tt} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \left(\frac{m_t}{M_Z}\right)^2 \frac{1}{2s_w^3 c_w^3} |V_{tb}|^2 M_Z^2 \left[v_t \left(I_{22} - I_{21} - I_{10} + \right.\right.$$
$$\left.\left. \left(\frac{m_b}{m_t}\right)^2 (I_{22} - I_{21} + I_{10})\right) - a_t \left(1 - \left(\frac{m_b}{m_t}\right)^2\right)(I_{22} - I_{21} + I_{10})\right]^{\sigma tt} \tag{12}$$

$$[a_b^w]^{tWW} = \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{c_w}{s_w^3} |V_{tb}|^2 M_Z^2 [I_{10} + 2I_{21} - 2I_{22}]^{tWW} \tag{13}$$

$$[a_b^w]^{t\sigma\sigma} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \left(\frac{m_t}{M_Z}\right)^2 \frac{1 - 2c_w^2}{2s_w^3 c_w^3} |V_{tb}|^2 M_Z^2 \left[I_{00} - I_{22} + I_{21} - 3I_{10} + \right.$$
$$\left. \left(\frac{m_b}{m_t}\right)^2 (I_{21} - I_{22} - I_{10})\right]^{t\sigma\sigma} \tag{14}$$

$$[a_b^w]^{bZ\Phi} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{v_b}{2s_w^3 c_w^3} M_Z^2 [I_{11} - I_{10}]^{bZ\Phi} \tag{15}$$

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$$[a_b^w]^{b\Phi Z} = \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{v_b}{2s_w^3 c_w^3} M_Z^2 \left[I_{11} + I_{10}\right]^{b\Phi Z} \tag{16}$$

$$[a_b^w]^{tW\sigma} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{1}{2s_w c_w} |V_{tb}|^2 M_Z^2 \left[I_{10} - I_{11}\right]^{tW\sigma} \tag{17}$$

$$[a_b^w]^{t\sigma W} = -\left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{1}{2s_w c_w} |V_{tb}|^2 M_Z^2 \left[I_{10} + I_{11}\right]^{t\sigma W} \tag{18}$$

$$[a_b^w]^{b\Phi\chi} = [a_b^w]^{b\chi\Phi} = 0 \tag{19}$$

with $a_{i,I}$, V_{Ii} being the axial vector $Zq\bar{q}$ coupling, and the Kobayashi-Maskawa $q_I q_i$ mixing matrix element, respectively.

Diagrams with the Higgs (Φ) and the neutral would-be Goldstone boson (χ) coupled to the Z only contribute to the axial form factor and not to the magnetic moment, so that one gets the result of Eq.(19).

The *natural* scale of each diagram is $\left(\frac{m_b}{M_{Z,\Phi}}\right)^2$ but those with an exchange of a Higgs (physical or not) between the two b's (see $[a_b^w]^{\chi bb}$, $[a_b^w]^{\Phi bb}$) are suppressed by an extra $\left(\frac{m_b}{M_{Z,\Phi}}\right)^2$ factor coming from the Higgs-b-b coupling. For similar reasons, due to the high value of the top mass [7] one could then think that those diagrams with Higgs particles coupled to the t-quark ($[a_b^w]^{\sigma tt}$, $[a_b^w]^{t\sigma\sigma}$) would be the dominant ones. In fact, Eqs. (12) and (14) show the $\left(\frac{m_t}{M_Z}\right)^2$ expected factor, which should make sizeable the contribution coming from these diagrams. Nevertheless, contrary to what happens in the charge radius (γ^μ) form factor [3], where non-decoupling effects take place, the behaviour with m_t of the I_{jk} integrals given in Eqs. (12) and (14) prevents the product $m_t^2 I_{jk}$ to have a hard dependence with large m_t. An expansion of the scalar functions $I_{ij}^{t\sigma\sigma}$ and $I_{ij}^{\sigma tt}$ –up to leading order– in terms of $1/t \equiv (M_Z/m_t)^2$ gives:

$$M_Z^2 I_{00}^{t\sigma\sigma} = \frac{1}{t}\left(\log\frac{c_w^2}{t} + 2f_w - 1\right) + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{20}$$

$$M_Z^2 I_{10}^{t\sigma\sigma} = \frac{1}{t}\left(\frac{1}{2}\log\frac{c_w^2}{t} + f_w - \frac{1}{4}\right) + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{21}$$

$$M_Z^2 I_{21}^{t\sigma\sigma} = \frac{1}{t}\left(\frac{1}{3}\log\frac{c_w^2}{t} + \frac{2(1-c_w^2)}{3}f_w - \frac{1}{9} + \frac{2c_w^2}{3}\right) + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{22}$$

$$M_Z^2 I_{22}^{t\sigma\sigma} = -\frac{1}{t}\left(\frac{1}{6}\log\frac{c_w^2}{t} + \frac{1+2c_w^2}{3}f_w - \frac{2c_w^2}{3} + \frac{1}{36}\right) + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{23}$$

$$M_Z^2 I_{00}^{\sigma tt} = -\frac{1}{t} + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{24}$$

$$M_Z^2 I_{10}^{\sigma tt} = -\frac{1}{4t} + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{25}$$

$$M_Z^2 I_{21}^{\sigma tt} = -\frac{1}{9t} + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{26}$$

$$M_Z^2 I_{22}^{\sigma tt} = \frac{1}{18t} + \mathcal{O}\left(\frac{1}{t^2}\log\frac{c_w^2}{t}\right) \tag{27}$$

with $f_w = \sqrt{4c_w^2 - 1}\arctan\left(1/\sqrt{4c_w^2 - 1}\right)$. As can be seen from the previous expressions, only a mild $(M_Z/m_t)^2\log(m_t/M_Z)^2$ dependence is got from the four diagrams that may give non-decoupling effects with the *top*-quark mass. The chirality flipping property of the magnetic moment makes the difference with respect to the charge radius, where non-decoupling effects are seen. In addition, for the $[a_b^w]^{tW\sigma}$ and $[a_b^w]^{t\sigma W}$ amplitudes, the AWMM selects a product of left and right projectors that gives no linear contribution on m_t^2.

Adding all these terms, we end up with the following result:

$$[a_b^w]_{lead.ord.in\,m_t}^{(\sigma tt)+(t\sigma\sigma)+(t\sigma W)+(tW\sigma)} =$$

$$= \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 \frac{1}{2s_w^3 c_w^3}\,|V_{tb}|^2\left[\frac{23}{36} - \frac{11c_w^2}{9} + \mathcal{O}\left(\frac{M_Z^2}{m_t^2}\log\left(\frac{M_Z^2}{m_t^2}\right)\right)\right] \tag{28}$$

and we conclude that the non-decoupling of a heavy *top* reduces to a constant term for the AWMM.

The $I_{jk}^{\alpha\beta\gamma}$ functions are analytically computed in terms of dilogarithm functions. As a check we confronted the result with a numerical integration in the $m_b \to 0$ limit. For $m_t = 174$ GeV, $M_Z = 91.19$ GeV, $s_w^2 = 0.232$, $\alpha = 1/127.9$ and $m_b = 4.5$ GeV, the following numerical contributions for each diagram are found:

$$[a_b^w]^{\gamma bb} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (1.10 - 0.57i) = (1.66 - 0.87i) \times 10^{-6} \tag{29}$$

$$[a_b^w]^{Zbb} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (1.6 + 0.71i) = (2.42 + 1.07i) \times 10^{-6} \tag{30}$$

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$$[a_b^w]^{\chi bb} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (0.31 + 4.79i) \times 10^{-4} = (4.69 + 72.5i) \times 10^{-10} \tag{31}$$

$$[a_b^w]^{\Phi bb} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (-1.86 - 5.98i;\ -1.43 - 1.95i;\ -0.91 - 0.92i) \times 10^{-3} =$$
$$(-2.81 - 9.07i;\ -2.16 - 2.96i;\ -1.37 - 1.40i) \times 10^{-9} \tag{32}$$

$$[a_b^w]^{Wtt} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (-0.54) = (-0.81) \times 10^{-6} \tag{33}$$

$$[a_b^w]^{\sigma tt} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (-0.71) = (-1.07) \times 10^{-6} \tag{34}$$

$$[a_b^w]^{tWW} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (-2.99) = (-4.53) \times 10^{-6} \tag{35}$$

$$[a_b^w]^{t\sigma\sigma} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (-0.81) = (-1.22) \times 10^{-6} \tag{36}$$

$$[a_b^w]^{bZ\Phi} = [a_b^w]^{b\Phi Z} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (0.57;\ 0.34;\ 0.22) =$$
$$(0.98;\ 0.52;\ 0.33) \times 10^{-6} \tag{37}$$

$$[a_b^w]^{tW\sigma} = [a_b^w]^{t\sigma W} \simeq \left(\frac{\alpha}{4\pi}\right)\left(\frac{m_b}{M_Z}\right)^2 (0.17) = (2.59) \times 10^{-7} \tag{38}$$

where the values between parenthesis in Eqs. (32) and (37) correspond to $\frac{M_\Phi}{M_Z} = 1, 2, 3$. The other values agreee with the result of Ref. [4] for the SM. We have taken the Kobayashi-Maskawa matrix being unity ($V_{Ii} = \text{diag}(1, 1, 1)$) for numerical results.

Finally, the electroweak contribution to the b-AWMM is

$$a_b^w(M_Z^2) = [\ -(1.1;\ 2.0;\ 2.4) + 0.2i] \times 10^{-6}, \qquad [M_\Phi = M_Z,\ 2M_Z,\ 3M_Z] \tag{39}$$

An immediate consequence of these results is that the AWMM contribution to the total electroweak width is very small. This is easily seen just by considering that the ratio $\Gamma(a_b^w)/\Gamma_{\text{Tree}}^w$ is given by the interference with the AWMM amplitude.

$$\frac{\Gamma(a_b^w)}{\Gamma_{\text{Tree}}^w} \approx 6\frac{s_w c_w v_b}{v_b^2 + a_b^2}\,\text{Re}(a_b^w) \tag{40}$$

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Then, Eq. (39) shows that only approximately 1 over 10^6 parts of the width is given by the electroweak contribution to the AWMM.

Contrary to what happened for the *tau* weak magnetic moment, where the lepton vector neutral coupling was responsible for the suppression of the Higgs mass dependence, we observe here that the mass of the physical Higgs has a sizeable effect on the final electroweak magnetic moment (39) for the *b*-quark. For the selected range of M_Φ, it changes the real part of the AWMM in more than 100%. This is so because, as can be seen from Eq. (37), the contribution of the $bZ\Phi$ diagrams –Eqs. (15) and (16)– are almost of the same order as the leading ones. Unfortunately, these effects will not be observable because, as we will show in the following, the magnetic moment is dominated by the QCD contributions.

In addition to the purely electroweak contributions to the AWMM of the *b*-quark given above, we now consider the QCD contributions to a_b. To lowest order, there is only one relevant diagram of the type shown in Fig. 1: the one with α being now a *gluon*. The evaluation of that diagram only differs from the $\gamma b\bar{b}$ diagram (Eq. (7)) in the couplings, so that it is straightforward to find the result

$$
\begin{aligned}
\left[a_b^{QCD}\right]^{gbb} &= \frac{\alpha_s}{\alpha} \frac{4}{3Q_b^2} [a_b^w]^{\gamma bb} = \left(\frac{\alpha_s}{4\pi}\right) \left(\frac{m_b}{M_Z}\right)^2 \frac{v_b}{s_w c_w} \frac{8}{3\beta} \left(\log\frac{1-\beta}{1+\beta} + i\pi\right) \\
&= (2.99 - 1.56i) \times 10^{-4}
\end{aligned}
\tag{41}
$$

with $\beta = \sqrt{1 - 4(m_b/M_Z)^2}$ and $\alpha_s = 0.117$, which is in good agreement with the analytical expression found in Ref. [11], when expressed in terms of an AWMM.

The final value we get for the weak magnetic moment of the *b*-quark is then

$$
a_b(M_Z^2) = a_b^w(M_Z^2) + a_b^{QCD}(M_Z^2) = (2.98 - 1.56i) \times 10^{-4}
\tag{42}
$$

for $M_\Phi = M_Z$.

Eqs. (39) and (41) show that even though different values of the Higgs mass modify considerably the purely electroweak AWMM, this effect does not translate into an appreciable change of the total AWMM (for which only a 0.4% of variation is found if M_Φ moves from M_Z to $3\,M_Z$) because the electroweak contribution is less than 1% (for $M_\Phi = 3M_Z$) of the total one.

Due to the fact that there is no enhancement of the electroweak contributions coming from the presence of a heavy *top*-quark, all the electroweak diagrams (except those already mentioned with Higgs exchanged between two b's) are of the same order, in particular the $\gamma b \bar{b}$ diagram. Then, Eq. (41) leads to the conclusion that the QCD contribution is two orders of magnitude bigger than the electroweak one. In fact, the next to leading order contribution in perturbative QCD would be probably comparable to the computed leading order in the electroweak sector.

For the c-quark, all the previous discussion holds, and one expects the electroweak contribution to be of the order $\frac{\alpha}{4\pi}\left(\frac{m_c}{M_Z}\right)^2$. That is

$$a_c^w(M_Z^2) \approx \mathcal{O}\left(\left(\frac{m_c}{m_b}\right)^2 a_b^w(M_Z^2)\right) \approx \mathcal{O}\left(10^{-7}\right) \tag{43}$$

The one-loop QCD contribution will also be dominant, and its magnitude can be easily computed from the analytic expression of Eq. (41), adapted to the c-quark. For $m_c = 1.6$ GeV, we get the value:

$$a_c(M_Z^2) \approx a_c^{QCD}(M_Z^2) = \frac{\alpha_s}{\alpha} \frac{4}{3Q_c^2} [a_c^w]^{\gamma cc} = (-2.80 + 1.09i) \times 10^{-5} \tag{44}$$

3   Conclusions

We have calculated the electroweak contributions to the anomalous weak magnetic moment of the b-quark, within the Standard Model, and found that it is of the order 10^{-6}. One loop QCD contributions to the AWMM are dominant and increase its value to 10^{-4}. The result tells us that in the magnetic moment form factor: 1) the contributions from new physics to the electroweak sector are hidden by the dominant strong interaction contribution, 2) the $Zb\bar{b}$ width is rather insensitive to electroweak contributions in the AWMM sector, and 3) contrary to what happens for the charge radius form factor, non-decoupling effects do not take place in the AWMM. The value of the AWMM for the c-quark is also computed (up to first order in QCD) and it is, as expected, smaller than that for the b-quark by a factor $(m_c/m_b)^2 \times v_c/v_b$.

Acknowledgments

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