FTUV-04-0421

CP violation and electric-dipole-moment at low energy τ-pair production

J. Bernabéu [a], G. A. González-Sprinberg [b] and J. Vidal [a]

[a] *Departament de Física Teòrica Universitat de València, E-46100 Burjassot, València, Spain*
and
IFIC, Centre Mixt Universitat de València-CSIC, València, Spain

[b] *Instituto de Física, Facultad de Ciencias, Universidad de la República, Iguá 4225, 11400 Montevideo, Uruguay*

Abstract

CP violation at low energy is investigated at the τ electromagnetic vertex. High statistics at B factories, and on top of the Υ resonances, allows a detailed investigation of CP-odd observables related to the τ-pair production. The contribution of the tau electric dipole moment is considered in detail. We perform an analysis independent from the high energy data by means of correlation and linear spin observables at low energy. We show that different CP-odd asymmetries, associated to the normal-transverse and normal-longitudinal correlation terms can be measured at low energy accelerators, both at resonant and non resonant energies. These observables allow to put stringent and independent bounds to the tau electric dipole moment that are competitive with other high or low energy results.

1 Introduction

The time reversal odd electric dipole moment (EDM) of the τ is the source of CP violation in the τ-pair production vertex. In the framework of local quantum field theories the CPT theorem states that CP violation is equivalent to T violation. While the electric dipole moments (EDM) of the electron and muon have been extensively investigated both in experiment and theory, the case of the tau is somewhat different. The tau lepton has a relatively high mass: this means that tau lepton physics is expected to be more sensitive to contributions to chirality-flip terms coming from high energy scales and new physics. Furthermore, the tau decays into hadrons, so different techniques to those for the (stable) electron or muon case are needed in order to measure the dipole moments. There are very precise bounds on the EDM

Preprint submitted to Nuclear Physics B *14 July 2011*

magnitude of nucleons and leptons, and the most precise one is the electron EDM, $d^e_\gamma = (0.07 \pm 0.07) \times 10^{-26}$ $e\ cm$, while the looser one is the τ EDM [1], $-0.22\ e\ cm < Re(d^\tau_\gamma) \times 10^{16} < 0.45\ e\ cm$. The dipole moments flip chirality and are therefore related to the mass mechanism of the theory. ¿From the theoretical point of view the CP violation mechanisms in many models provide a kind of accidental protection in order to generate an EDM for quarks and leptons. This is the case in the CKM mechanism, where EDM and weak-EDM are generated only at very high order in the coupling constant. This opens a way to efficiently test many models: CP-odd observables related to EDM would give no appreciable effect from the standard model and any experimental signal should be identified with physics beyond the standard model. Following the ideas of [2] and [3], the tau weak-EDM has been studied in CP-odd observables [4,5] at high energies through terms involving spin linearly and spin-spin correlations. Electric dipole moment bounds for the tau, from CP-even observables such as total cross sections or decay widths, have been considered in [6,7,8]. While most of the statistics for the tau pair production was dominated by high energy physics, mainly at LEP, nowadays the situation has evolved. High luminosity B factories and their upgrades at resonant energies (Υ thresholds) have the largest τ pair samples. This calls for a dedicated study of the observables related to CP violation and the EDM of the τ lepton at low energies. In this paper we study different observables to the ones used, at high energies, in the past. For the tau lepton we present some of them that may lead to competitive results with the present bounds in the near future.

The paper is organized as follows: in the next section we present the effective Lagrangian description for the EDM, in section 3 we discuss the low energy observables, in section 4 we consider the resonant production energies, in section 5 we show how to measure the imaginary parts and finally we conclude with some comments.

2 $\mathcal{T}$ EDM at Low Energies

The standard model describes with high accuracy most of the physics found in present experiments. Nowadays, however, neutrino physics offers a first clue to physics beyond this low energy model [9]. Deviations from the standard model, at low energies, can be parametrized by an effective Lagrangian built with the standard model particle spectrum, having as zero order term just the standard model Lagrangian, and containing higher dimension gauge invariant operators suppressed by the scale of new physics, Λ [10]. The leading non-standard effects come from the operators with the lower dimension. For CP violation those are dimension six operators. There are only two operators of this type that contribute [11] to the tau EDM and weak-EDM:

$$\mathcal{O}_B = \frac{g'}{2\Lambda^2}\overline{L_L}\varphi\sigma_{\mu\nu}\tau_R B^{\mu\nu} \ , \qquad \mathcal{O}_W = \frac{g}{2\Lambda^2}\overline{L_L}\vec{\tau}\varphi\sigma_{\mu\nu}\tau_R \vec{W}^{\mu\nu} \ . \qquad (1)$$

Here $L_L = (\nu_L, \tau_L)$ is the tau leptonic doublet, φ is the Higgs doublet, $B^{\nu\nu}$ and $\vec{W}^{\mu\nu}$ are the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ field strength tensors, and g' and g are the gauge couplings.

Other possible operators that one could imagine reduce to the above ones of Eq.(1) after using the standard model equations of motion. In so doing, the couplings will be proportional to the tau-lepton Yukawa couplings.

Thus, we write our effective Lagrangian

$$\mathcal{L}_{eff} = i\alpha_B\mathcal{O}_B + i\alpha_W\mathcal{O}_W + \text{h.c.} \qquad (2)$$

where the couplings α_B and α_W are real. Note that complex couplings do not break CP conservation and lead to magnetic dipole moments which are not considered in this paper where we are mainly interested on CP-odd observables.

If these operators come from a low energy expansion of a renormalizable theory, in the perturbative regime one expects that they arise only as quantum corrections and therefore their contribution must be suppressed. However, this does not need necessarily to be the case, therefore we leave the couplings α_B and α_W as free parameters without any further assumption.

In the spontaneous symmetry breaking regime the neutral scalar gets a vacuum expectation value $<\varphi^0>= u/\sqrt{2}$ with $u = 1/\sqrt{\sqrt{2}G_F} = 246$ GeV, and the interactions in Eq.(2) can be written in terms of the gauge boson mass eigenstates A^μ and Z^μ. Similar results, but for the magnetic moments, are found in [11] where the notation is the same. The Lagrangian, written in terms of the mass eigenstates, is then

$$\mathcal{L}_{eff}^{\gamma,Z} = -i\frac{e}{2m_\tau}\,F_\gamma^\tau\,\overline{\tau}\sigma_{\mu\nu}\gamma^5\tau F^{\mu\nu} - i\frac{e}{2m_\tau}\,F_Z^\tau\,\overline{\tau}\sigma_{\mu\nu}\gamma^5\tau Z^{\mu\nu} \qquad (3)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ are the abelian field strength tensors of the photon and Z gauge boson, respectively. We have not written in Eq.(3) some of the terms coming from Eq.(2) because they do not contribute at leading order to the observables we are interested in. These terms are a) the non-abelian couplings involving more than one gauge boson b) the Lagrangian related to the CP-odd $\nu_\tau - \tau - W^\pm$ couplings. As usual, we define the following dimensionless couplings

$$F_\gamma^\tau = (\alpha_W - \alpha_B)\frac{u m_\tau}{\sqrt{2}\Lambda^2} \ , \tag{4}$$

$$F_Z^\tau = (\alpha_B s_W^2 + \alpha_W c_W^2)\frac{u m_\tau}{\sqrt{2}\Lambda^2}\frac{1}{s_W c_W} \tag{5}$$

where $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ are the sine and cosine of the weak angle. In this effective Lagrangian approach the same couplings that contribute to the electric dipole moment form factor, $F^{\rm new}(q^2)$ also contribute to the electric dipole moment defined at $q^2 = 0$. Only higher dimension operators contribute to the difference $F^{\rm new}(q^2) - F^{\rm new}(0)$ and, if $|q^2| \ll \Lambda^2$, as required for the consistence of the effective Lagrangian approach, their effects will be suppressed by powers of q^2/Λ^2. This allows us to make no distinction between the electric *dipole moment* and the electric *form factor* in this paper so we may take the usual definitions of the electric and weak-electric dipole moments in terms of the form factors defined in Eqs.(4,5) as:

$$d_\gamma^\tau = \frac{e}{2m_\tau}F_\gamma^\tau, \qquad d_Z^\tau = \frac{e}{2m_\tau}F_Z^\tau. \tag{6}$$

The $e^+ e^- \longrightarrow \tau^+\tau^-$ cross section has contributions coming from the standard model and the effective Lagrangian Eq.(3). At low energies the tree level contributions come from γ exchange (off the Υ peak) or Υ (at the Υ peak) exchange in the s-channel. The interference with the Z-exchange ($\gamma - Z$, $\Upsilon - Z$ at the Υ peak) and the $Z - Z$ diagrams are suppressed by powers of (q^2/M_Z^2). The tree level contributing diagrams are shown in Fig.1 where diagrams (a) and (b) are standard model contributions, and $(c)$and (d) come from beyond the standard model terms in the Lagrangian. Notice that standard model radiative corrections that may contribute to CP-odd observables (for example, the ones that generate the standard model electric dipole moment for the τ) come in higher order in the coupling constant, and at present level of experimental sensitivity they are not measurable. On these grounds the bounds on the EDM that one may get are just coming from the physics beyond the standard model.

Electric dipole moment effects can be studied at leading order in the angular distribution of the $e^+e^- \longrightarrow \tau^+(s_+)\tau^-(s_-)$ differential cross section. The polarization of the final fermions is determined through the study of the angular distribution of their decay products. In our analysis we only keep linear terms in the EDM, neglecting terms proportional to the mass of the electron.

When considering the measurement of the polarization of just one of the taus, the normal -to the scattering plane- polarization (P_N) of each tau is the only component which is T-odd. For CP-conserving interactions, the CP-even term $(s_+ + s_-)_N$ of the normal polarization only gets contribution through the combined effect of both an helicity-flip transition and the presence of absorptive

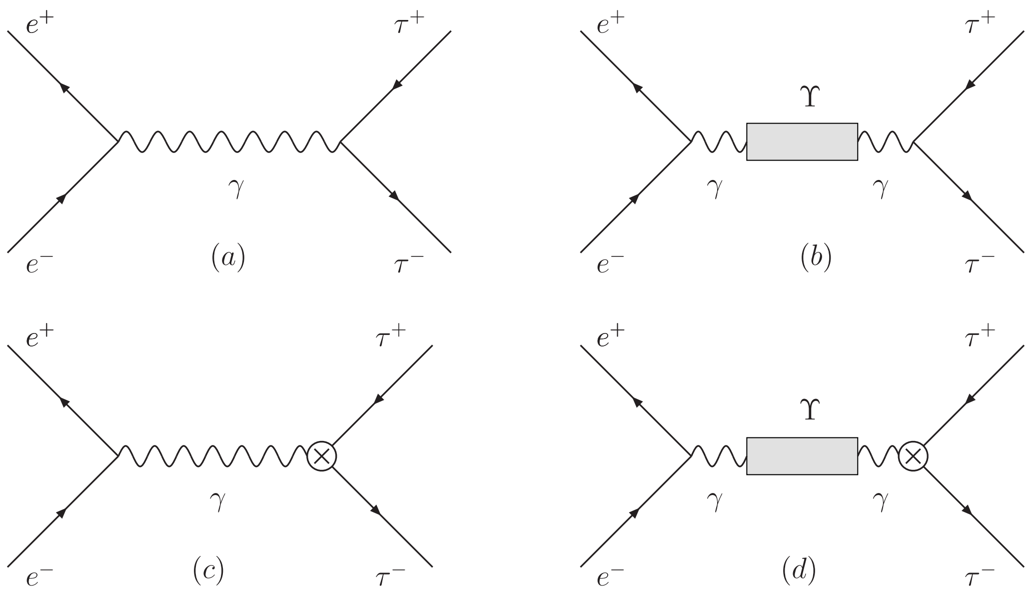

Fig. 1. Diagrams (a) direct γ exchange (b) Υ production (c) EDM in γ exchange (d) EDM in Υ production

parts, which are both suppressed in the standard model. For a CP-violating interaction, such as an EDM, the $(s_+ - s_-)_N$ CP-odd term gets a non-vanishing value without the need of absorptive parts.

As P_N is even under parity (P) symmetry, the observable sensitive to the EDM should also be proportional to a standard axial coupling in addition to d^τ_γ. This would need a Z-exchange in the s-channel, which is suppressed by powers of (q^2/M_Z^2) at low energies. In our case, with only γ and/or Υ exchange (the complete spin density matrix can be seen, for example, in the cross section formulas of ref. [12]) there is no such amplitudes and the EDM does not give any contribution to the single normal polarization (P_N) of the tau. As a consequence, we have to move to other observables associated with spin correlations of both taus.

Then, the EDM term only shows up in the spin-spin correlation matrix. The T-odd, P-odd Normal-Transverse $(\overrightarrow{s}_+ \times \overrightarrow{s}_-)_{NT}$ and Normal-Longitudinal $(\overrightarrow{s}_+ \times \overrightarrow{s}_-)_{NL}$ spin correlation terms will be proportional to the EDM interfering with photon exchange. These two spin correlations receive standard model contributions to their symmetric CP-even terms through absorptive parts generated in radiative corrections. At leading order it is the imaginary part of the Z propagator that produces a contribution to these correlations with the interference of the amplitudes of direct γ and Z-exchange. This term is suppressed at low energies by the factor $(\frac{q^2}{M_Z^2}\frac{\Gamma_Z}{M_Z})$, and has been calculated in [13], so that it can be subtracted if necessary. The Transverse-Longitudinal term $(\boldsymbol{s}_+ \times \boldsymbol{s}_-)_{TL}$ is T-even, P-even and it can contain a term proportional to the EDM only through its interference with Z amplitudes carrying an axial coupling. As mentioned before, these are suppressed at low energies at least by (q^2/M_Z^2), and thereby not considered in what follows. Our aim is to identify genuine CP-odd observables linear in the EDM and not (additionally) suppressed by either (q^2/M_Z^2) or unitarity corrections.

3 Low energy observables

Following the notation of references [11] and [14], we now show how to measure the EDM using low energy CP-odd observables. At low energies and in the hypothesis stated above, the EDM gives contributions to leading order in the Normal-Transverse and Normal-Longitudinal correlation terms of the $e^+e^- \longrightarrow \tau^+(s_+)\tau^-(s_-)$ differential cross section.

Working in the center of mass (CM) reference frame we choose the orientation of our coordinate system so that the outgoing τ^- momenta is along the positive z axis and the vector $\boldsymbol{p}_{\tau^-} \times \boldsymbol{p}_{e^-}$ defines the positive y axis. The $\boldsymbol{s}^\pm$ are the $\tau^\pm$ spin vectors in the $\tau^\pm$ rest system, $s_\pm = (0, s_\pm^x, s_\pm^y, s_\pm^z)$. This frame has the axes parallel to the CM frame and the only difference between them and the CM frame of reference is the boost in the $z > 0$ direction in the case of the τ^- and the boost in the $z < 0$ direction for the τ^+ frame. With this setting, polarization along the directions x, y, z correspond to what is called transverse (T), normal (N) and longitudinal (L) polarizations, respectively.

We consider the τ-pair production in e^+e^- collisions though direct γ exchange (diagrams (a) and (c) in Fig. 1.). In the next section we will show that the basic results of this section still hold for resonant Υ production.

Let us assume from now on that the tau production plane and direction of flight can be fully reconstructed. This can be easily done [15] if both τ's decay semileptonically. Following the ideas of [2,14] this technique was applied by the L3-Collaboration [4] in the search of bounds on the tau weak electric and magnetic dipole moments.

The differential cross section for τ pair production is:

$$\frac{d\sigma}{d\Omega_{\tau^-}} = \frac{d\sigma^0}{d\Omega_{\tau^-}} + \frac{d\sigma^S}{d\Omega_{\tau^-}} + \frac{d\sigma^{SS}}{d\Omega_{\tau^-}} + \dots \tag{7}$$

The first three terms come from leading order standard model and effective operator (EDM) contributions. The dots take account for higher order terms in the effective Lagrangian that are beyond experimental sensitivity and which are not considered in this paper.

The first term of Eq. (7) represents the spin independent differential cross section

$$\frac{d\sigma^0}{d\Omega_{\tau^-}} = \frac{\alpha^2}{16\ s}\,\beta\,(2-\beta^2\sin^2\theta_{\tau^-}) \tag{8}$$

where α is the fine structure constant, the squared center of mass energy $s=q^2$ is also the square of the 4-momentum carried by the photon, θ_{τ^-} is the angle defined by the electron and the τ^- directions, and $\gamma=\frac{\sqrt{s}}{2m_\tau}$, $\beta=\sqrt{1-\frac{1}{\gamma^2}}$, are the dilation factor and τ velocity, respectively.

The second term $\dfrac{d\sigma^S}{d\Omega_{\tau^-}}$, involves linear terms in spin and has no contribution to CP-odd observables in our treatment.

The last term of Eq.(7) is proportional to the product of the spins of both τ's and it is written as:

$$\begin{aligned}\frac{d\sigma^{SS}}{d\Omega_{\tau^-}} = \frac{\alpha^2}{16s}\beta\Big(& s_+^x s_-^x C_{xx} + s_+^y s_-^y C_{yy} + s_+^z s_-^z C_{zz} + \\ & (s_+^x s_-^y + s_+^y s_-^x)C^+_{xy} + (s_+^x s_-^z + s_+^z s_-^x)C^+_{xz} + (s_+^y s_-^z + s_+^z s_-^y)C^+_{yz} + \\ & (\overrightarrow{s}_+ \times \overrightarrow{s}_-)_x C^-_{yz} + (\overrightarrow{s}_+ \times \overrightarrow{s}_-)_y C^-_{xz} + (\overrightarrow{s}_+ \times \overrightarrow{s}_-)_z C^-_{xy}\Big)\end{aligned} \tag{9}$$

where

$$\begin{aligned} C_{xx} &= (2-\beta^2)\sin^2\theta_{\tau^-} & C_{yy} &= -\beta^2\ \sin^2\theta_{\tau^-} \\ C_{zz} &= (\beta^2+(2-\beta^2)\cos^2\theta_{\tau^-}) & C^-_{xy} &= 2\beta\left(\sin^2\theta_{\tau^-}\right)\frac{2m_\tau}{e}\,d^\gamma_\tau \\ C^-_{yz} &= -\gamma\beta\left(\sin(2\theta_{\tau^-})\right)\frac{2m_\tau}{e}\,d^\gamma_\tau & C^+_{xz} &= \tfrac{1}{\gamma}\sin(2\theta_{\tau^-}) \end{aligned} \tag{10}$$

and $C^+_{xy}=C^+_{yz}=C^-_{xz}=0$. C^+_{xy} and C^+_{yz} correlation terms are zero in our hypothesis. They are CP-even and P-odd but we only consider photon exchange in the s-channel and there is no source of P violation to produce these CP-even terms. C^-_{xz} is zero because it is CP-odd and P-even, while the EDM in interference with photon exchange would be CP-odd but P-odd instead and cannot give contribution to this term.

The above equations show that the EDM modifies the spin properties of the produced taus and this translates into the angular distribution of both tau decay products. As can be seen from Eq.(10), the EDM is the leading contribution to the Normal-Transverse $(y-x)$ and Normal-Longitudinal $(y-z)$ correlations.

Angular asymmetries of the tau decay product distributions allow to select observable EDM effects. In order to enlarge the sensitivity to these terms we will sum in all kinematic variables when possible.

The complete cross section for the process $e^+e^- \to \gamma \to \tau^+\tau^- \to h^+\bar{\nu}_\tau h^-\nu_\tau$ can be written as a function of the kinematical variables of the hadrons into which each tau decays [16] as:

$$d\sigma\left(e^+e^- \to \gamma \to \tau^+\tau^- \to h^+\bar{\nu}_\tau h^-\nu_\tau\right) = 4\, d\sigma\left(e^+e^- \to \tau^+(\vec{n}^*_+)\, \tau^-(\vec{n}^*_-)\right)$$
$$\times Br(\tau^+ \to h^+\bar{\nu}_\tau) Br(\tau^- \to h^-\nu_\tau) \frac{d\Omega_{h^+}}{4\pi}\frac{d\Omega_{h^-}}{4\pi} \tag{11}$$

with

$$\vec{n}^*_\pm = \mp\alpha_\pm \frac{\vec{q}^{\,*}_\pm}{|\vec{q}^{\,*}_\pm|} = \mp\alpha_\pm(\sin\theta^*_\pm\, \cos\phi_\pm, \sin\theta^*_\pm\, \sin\phi_\pm, \cos\theta^*_\pm) \tag{12}$$

$\alpha_\pm$ are the polarization parameters of the τ decay and $\vec{q}^{\,*}_\pm$ are the momenta of the hadrons with moduli fixed to $P_\pm = \dfrac{m_\tau^2 - m_{h^\pm}^2}{2m_\tau}$. The $*$ means that all affected quantities are given in the respective τ-at-rest reference frame. Notice that the boost on the *taus* is along the z axis, so the $\phi^*_\pm$ angles do no change when referred to the LAB or the CM reference frame and we can just use $\phi_\pm$ to refer to them. Both hadron energies are fixed by energy conservation and the neutrino, in each channel, was integrated out in the cross section (11).

3.1 Normal-Transverse correlation azimuthal asymmetry

We now show how to get an observable proportional to the EDM term from the NT correlation. The correlation terms in the cross section depend on several kinematic variables that we have to take into account: the CM polar angle θ_{τ^-} of production of the τ^- with respect to the electron, the azimuthal $\phi_\pm$ and polar $\theta^*_\pm$ angles of the produced hadrons $h^\pm$ in the $\tau^\pm$ rest frame (see Fig.2). These angles appear in a different way on each term. The θ_{τ^-} angle enters in the cross section (spin independent, linear and correlation terms) as coefficients (such as C^-_{xy}, for example) while the hadron's angles appear in the cross section through the $\vec{n}^*$ vectors. The whole angular dependence of each correlation term is unique and it is this dependence that allows to select one of the correlation terms in the cross section. Indeed, it is by a combination of an integration on the hadronic angles plus, eventually, an integration on the θ_{τ^-} angle, that one can select a polarization or correlation term, and there, the contribution of the EDM.

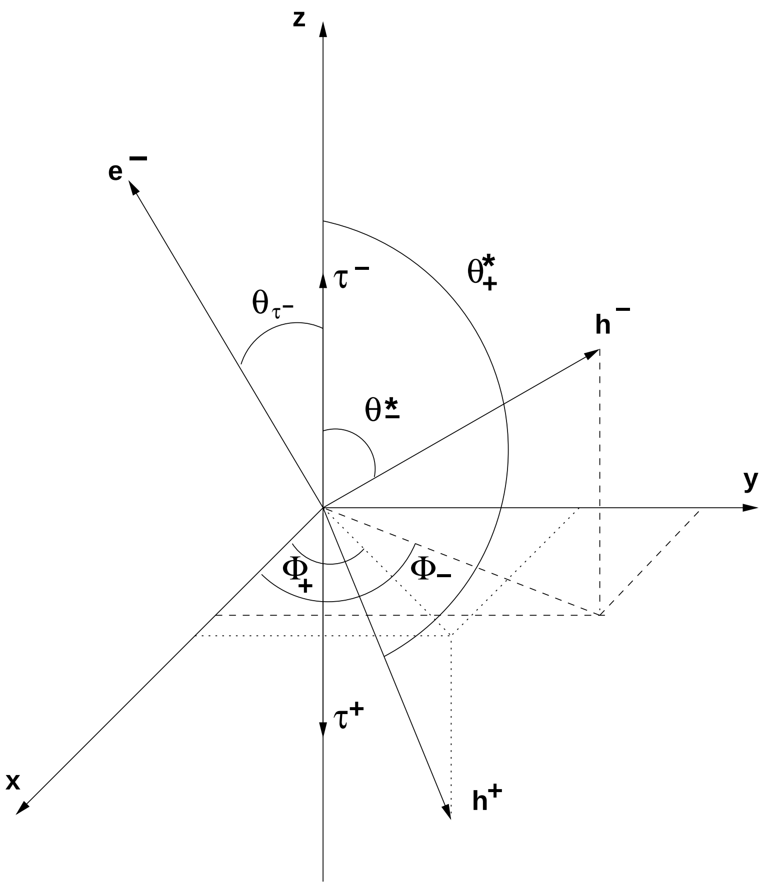

Fig. 2. Coordinate system for $h^{\pm}$ production from the $\tau^{\pm}$

For the NT term, for example, this works as follows. The integration over the τ^- variables $d\Omega_{\tau^-}$ erases all the information on the EDM in the Normal-Longitudinal (C^-_{zy}) correlation, together with the C^+_{xz} term of the cross section. Then, the cross section can be written only in terms of the surviving correlation terms as:

$$
\begin{aligned}
d^4\sigma^{SS} = & \frac{\pi\alpha^2\beta}{2\,s}\, Br(\tau^+ \to h^+\bar{\nu}_\tau) Br(\tau^- \to h^-\nu_\tau)\, \frac{d\Omega_{h^+}}{4\pi}\, \frac{d\Omega_{h^-}}{4\pi} \;\times \\
& \left\{ \frac{4}{3}(2-\beta^2)\, \left[(n^*_-)_x (n^*_+)_x\right] - \frac{4}{3}\beta^2\, \left[(n^*_-)_y (n^*_+)_y\right] + \right. && (13) \\
& 2\left(\beta^2 + (2-\beta^2)\frac{1}{3}\right)\, \left[(n^*_-)_z (n^*_+)_z\right] + && (14) \\
& \left. \frac{4}{3}\, 2\beta\, \left[(n^*_+)_x (n^*_-)_y - (n^*_+)_y (n^*_-)_x\right]\, \frac{2m_\tau}{e}\, d^\gamma_\tau \right\} && (15)
\end{aligned}
$$

Up to this point, linear polarization terms may also survive (to this order, in fact, it is only a longitudinal term that is studied in section 5) but a dedicated integration of the hadronic angles ends up with the NT correlation as the only surviving term.

In order to enhance and select the corresponding NT observable we must integrate as much as kinematic variables as possible without erasing the signal of the EDM. Keeping only azimuthal angles and integrating all other variables we get:

$$\frac{d^2\sigma^{SS}}{d\phi_- d\phi_+} = -\frac{\pi\alpha^2\beta}{96s}\;(\alpha_-\,\alpha_+)\;Br(\tau^+\to h^+\bar{\nu}_\tau)Br(\tau^-\to h^-\nu_\tau)\times$$
$$\Big[(2-\beta^2)\cos(\phi_-)\;\cos(\phi_+) - \beta^2\,\sin(\phi_-)\;\sin(\phi_+) + \tag{16}$$
$$2\beta\,\sin(\phi_-^* - \phi_+^*)\,\frac{2m_\tau}{e}\,d_\tau^\gamma\Big] \tag{17}$$

Now, to get only sensitivity to the EDM in the NT correlation we can define the azimuthal asymmetry as:

$$A_{NT} = \frac{\sigma_{NT}^+ - \sigma_{NT}^-}{\sigma_{NT}^+ + \sigma_{NT}^-} \tag{18}$$

where

$$\sigma_{NT}^{\pm} = \int\limits_{w\gtrless 0} \frac{d^2\sigma}{d\phi_- d\phi_+}\,d\phi_-\,d\phi_+\;, \quad \text{with} \qquad w = \sin(\phi_- - \phi_+) \tag{19}$$

so that one gets that the Normal-Transverse correlation azimuthal asymmetry is:

$$A_{NT} = -\alpha_-\alpha_+\,\frac{\pi\beta}{4(3-\beta^2)}\,\frac{2m_\tau}{e}\,d_\tau^\gamma \tag{20}$$

It is easy to verify that all other terms in the cross section, i.e. the spin independent ones (σ^0), the ones coming with the linear polarization (σ^S) and those not relevant of the spin-spin correlation term (Eq.(16) in σ^{SS}) are eliminated when we integrate in the way we have shown above. Notice that this integration procedure also erases any possible contribution coming from the CP-even term C_{xy}^+ (zero in our approach) of the NT polarization. This means that the only source for this azimuthal asymmetry is exactly the term C_{xy}^- we are interested in, so that we have ended up with a genuine CP-odd Normal-Transverse correlation observable which is directly proportional to the EDM.

3.2 *Normal-Longitudinal correlation asymmetry*

In a similar way, we can now define an observable related to the Normal-Longitudinal correlation term. In this case the angular dependence on the decay product of both τ is different:

$$\left.\frac{d\sigma}{d\left(\cos\theta_{\tau^-}\right)}\right|_{C^-_{yz}} = \frac{\pi\alpha^2\beta^2\gamma}{2\,s}\frac{2m_\tau}{e}\; d^\gamma_\tau\; \sin(2\theta_{\tau^-})\; \Big[(n^*_-)_y(n^*_+)_z - (n^*_-)_z(n^*_+)_y\Big] \times$$
$$Br(\tau^+ \to h^+\bar{\nu}_\tau)Br(\tau^- \to h^-\nu_\tau)\,\frac{d\Omega_{h^+}}{4\pi}\,\frac{d\Omega_{h^-}}{4\pi} \tag{21}$$

We can not just integrate over the θ_{τ^-} variable because this erases all the information on the EDM of Eq.(21), so we must do a forward-backward (with respect to the e^- direction) integration of the τ :

$$d\sigma(\pm) \equiv \left[\int_0^1 d(\cos\theta_{\tau^-}) \pm \int_{-1}^0 d(\cos\theta_{\tau^-})\right] d\sigma \tag{22}$$

Then, from Eq.(9), only terms on $\sin(2\theta_{\tau^-})$ survive for $d\sigma(-)$,

$$d\sigma(-)^{SS} \equiv \left[\int_0^1 d(\cos\theta_{\tau^-}) - \int_{-1}^0 d(\cos\theta_{\tau^-})\right] d\sigma^{SS} = \frac{2\pi\alpha^2\beta}{3\,s} \times$$
$$\Big\{\Big[(n^*_-)_y(n^*_+)_z - (n^*_-)_z(n^*_+)_y\Big]\;\gamma\beta\,\frac{2m_\tau}{e}\; d^\gamma_\tau \tag{23}$$
$$+\Big[(n^*_-)_x(n^*_+)_z + (n^*_-)_z(n^*_+)_x\Big]\,\frac{1}{\gamma}\Big\} \times \tag{24}$$
$$Br(\tau^+ \to h^+\bar{\nu}_\tau)Br(\tau^- \to h^-\nu_\tau)\,\frac{d\Omega_{h^+}}{4\pi}\,\frac{d\Omega_{h^-}}{4\pi} \tag{25}$$

Again, this is not enough to select the NL correlation. The dependence on the n^* produces an angular dependence on the angles of the form

$$(n^*_-)_y(n^*_+)_z - (n^*_-)_z(n^*_+)_y =$$
$$\alpha_+\alpha_-\,\Big(\sin\theta^*_+\;\cos\theta^*_-\;\sin\phi_+ - \cos\theta^*_+\;\sin\theta^*_-\;\sin\phi_-\Big) \tag{26}$$
$$(n^*_-)_x(n^*_+)_z + (n^*_-)_z(n^*_+)_x =$$
$$-\alpha_+\alpha_-\,\Big(\sin\theta^*_-\;\cos\theta^*_+\;\cos\phi_- + \cos\theta^*_-\;\sin\theta^*_+\;\cos\phi_+\Big) \tag{27}$$

By an appropriate integration of the remaining variables one may get rid of the irrelevant (for our purposes) term Eq.(24) and get sensitivity to the EDM. For example one may integrate in such a way as to select the first term in Eq.(26). This can be done computing

$$\sigma_{NL}(-)^{\pm}_{-} = \int\limits_{w \gtrless 0} \frac{d^2\sigma}{d(\cos\theta^*_-) d\phi_+} \, d(\cos\theta^*_-) \, d\phi_+ \ , \quad \text{with} \quad w = \cos\theta^*_- \sin\phi_+ \tag{28}$$

which amounts to calculate

$$\sigma_{NL}(-)^{+}_{\mp} \equiv \left[\left(\int_0^{\pi} \mathrm{d}\phi_\pm \int_0^1 d(\cos\theta^*_\mp) + \int_{\pi}^{2\pi} d\phi_\pm \int_{-1}^{0} d(\cos\theta_\mp) \right) \right] \frac{d^2\sigma^{SS}}{d(\cos\theta^*_\mp) d\phi_\pm} \tag{29}$$

$$\sigma_{NL}(-)^{-}_{\mp} \equiv \left[\left(\int_0^{\pi} \mathrm{d}\phi_\pm \int_{-1}^0 d(\cos\theta^*_\mp) + \int_{\pi}^{2\pi} d\phi_\pm \int_{0}^{1} d(\cos\theta_\mp) \right) \right] \frac{d^2\sigma^{SS}}{d(\cos\theta^*_\mp) d\phi_\pm} \tag{30}$$

so that

$$\begin{aligned} &\sigma_{NL}(-)^{+}_{-} - \sigma_{NL}(-)^{-}_{-} = \\ &\frac{\pi\alpha^2\beta^2\gamma}{6s} (\alpha_+\alpha_-) \, \frac{2m_\tau}{e} \, d^\gamma_\tau \; Br(\tau^+ \to h^+\bar{\nu}_\tau) Br(\tau^- \to h^-\nu_\tau) \end{aligned} \tag{31}$$

Then one can construct the corresponding asymmetry as:

$$A^{+}_{NL} = \frac{\sigma_{NL}(-)^{+}_{+} - \sigma_{NL}(-)^{-}_{+}}{\sigma_{NL}(-)^{+}_{+} + \sigma_{NL}(-)^{-}_{+}} = \frac{\beta\gamma}{4(3-\beta^2)} \, \alpha_+\alpha_- \, \frac{2m_\tau}{e} \, d^\gamma_\tau \tag{32}$$

Notice that a similar asymmetry can be build by interchanging $\phi_+ \leftrightarrow \phi_-$, $\theta^*_- \leftrightarrow \theta^*_+$ and the $\pm$ signs of the σ sub-indexes in the above expressions:

$$A^{-}_{NL} = \frac{\sigma_{NL}(-)^{+}_{-} - \sigma_{NL}(-)^{-}_{-}}{\sigma_{NL}(+)^{+}_{-} + \sigma_{NL}(+)^{-}_{-}} = -A^{+}_{NL} \tag{33}$$

As $C^{+}_{yz} = 0$ in our approach, along this process of integration only the CP-odd C^{-}_{yz} term survives. We have verified that all other terms of Eq. (10) are annihilated in the definition of this asymmetry.

Differently to what happened in the NT asymmetry of Eq. (20), the NL asymmetry defined in Eq. (32) is not a genuine CP-violation observable because it can get possible contributions from the CP-even sector C^{+}_{yz} (zero in our case) of the cross section. To get a genuine CP-odd observable one has to test in this case both τ's decaying into the same kind of hadrons ($\alpha_- = \alpha_+ \equiv \alpha_h$) and then define the NL asymmetry,

$$A_{NL} = \frac{1}{2}\left(A^+_{NL} - A^-_{NL}\right) = \frac{\beta\gamma}{4(3-\beta^2)}\,\alpha_h^2\,\frac{2m_\tau}{e}\,d^\gamma_\tau \tag{34}$$

that exactly tests the CP-odd sector (*i.e.* C^-_{yz} only) of the Normal-Longitudinal correlation.

Notice that the above expressions for the asymmetries are linear in the $\alpha_\pm$ factors so that we can consider all the decaying channels of both τ's (to π, ρ...) in order to increase statistics and enlarge the signal.

4 Observables at the Υ resonances

All these ideas can be applied for e^+e^- collisions at the Υ peak where the τ pair production is mediated by the resonance: $e^+e^- \to \Upsilon \to \tau^-\tau^-$. At the Υ production energies we have an important tau pair production rate. We are interested in τ pairs produced by the decays of the Υ resonances, therefore we can use $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ where the decay rates into tau pairs have been measured. At the $\Upsilon(4S)$ peak, although it decays dominantly into $B\overline{B}$, high luminosity B-Factories have an important direct tau pair production. Except for this last case, that can be studied with the results of the preceding sections, we assume that the resonant diagrams (b) and (d) of Fig. 1. dominate the process on the Υ peaks. The Breit-Wigner propagator of the Υ is

$$P_\Upsilon(s) = \frac{1}{(s - M_\Upsilon^2) + iM_\Upsilon\Gamma_\Upsilon} \tag{35}$$

The $F_\Upsilon(q^2)$ vector form factor is defined as

$$\langle\Upsilon(w,\boldsymbol{q})|\bar{\psi}_b\gamma_\mu\psi_b(0)|0\rangle = F_\Upsilon(q^2)\epsilon^*_\mu(w,\boldsymbol{q}) \tag{36}$$

with $\epsilon^*_\mu(w,\boldsymbol{q})$ the polarization four-vector. This form factor can be related to the partial width of $\Upsilon \to e^+e^-$,

$$\Gamma_{ee} = \frac{1}{6\pi}Q_b^2\frac{(4\pi\alpha)^2}{M_\Upsilon^4}|F_\Upsilon(M_\Upsilon)|^2\frac{M_\Upsilon}{2}\ , \tag{37}$$

where $Q_b = -1/3$ is the electric charge of the b quark. Notice that all the hadronic uncertainties in our process are included in this unique form factor.

With this parameterization, the amplitudes A_b and A_d for the tau pair production diagrams (see Fig. 1.) through the Υ can be related to those of the direct production (A_a and A_c) as:

$$A_{\binom{b}{d}} = A_{\binom{a}{c}} \cdot H(s)\,, \quad \text{with} \quad H(s) \equiv \frac{4\pi\alpha Q_b^2}{s}\, |F_\Upsilon(s)|^2\, P_\Upsilon(s)\,, \tag{38}$$

so that the tau pair production at the Υ peak introduces the same tau polarization matrix terms as the direct production with γ exchange (diagrams (a) and (c)). The only difference is the overall factor $|H(s)|^2$ introduced in the cross section which is responsible for the enhancement at the resonant energies,

$$H(M_\Upsilon^2) = \frac{4\pi\alpha Q_b^2}{M_\Upsilon^2} \frac{|F_\Upsilon(M_\Upsilon^2)|^2}{i\,\Gamma_\Upsilon\, M_\Upsilon} = -i\,\frac{3}{\alpha} Br\left(\Upsilon \to e^+e^-\right) \tag{39}$$

¿From Eqs. (38) and (39) it is easy to show that, at the *Upsilon* peak, the interference of diagrams (a) and (d) plus the interference of diagrams (b) and (c) is exactly zero and so it is the interference of diagrams (a) and (b). Finally, the only contributions with EDM in polarization terms come with the interference of diagrams (b) and (d), while diagram (b) squared gives the leading contribution to the cross section.

The computations we did following Eqs.(7), (8) and (9) can be repeated here, and finally we obtain no changes in the asymmetries: the only difference is in the value of the resonant production cross section at the Υ peak that is multiplied by the overall factor $|H(M_\Upsilon^2)|^2$ given in Eq. (39). In this way all the asymmetries defined by Eqs. (20) and (32) do not change, and their expressions at the Υ peak are the same as before.

In fact, one can take the four diagrams (a,b,c,d) together and still get the same results of this section and section 3. Energies *off* or *on* the resonance will automatically select the significant diagrams.

5 Imaginary EDM observables

The imaginary part of the EDM does not appear in the effective Lagrangian approach and deserves a separate treatment. This is a T-even quantity and it can contribute to the cross section in the CP-odd components of the transverse (within the production plane) $(s_+ - s_-)_T$ and longitudinal $(s_+ - s_-)_L$ polarizations. These are P-odd observables so that the interference of the EDM with photon exchange will originate a non vanishing contribution. As a consequence, the leading contribution to the single polarization terms in the cross section are given by:

$$\frac{d\sigma^S}{d\Omega_{\tau^-}} = \frac{\alpha^2\beta^2}{16\,s}\left[(s_-^x - s_+^x)C_x + (s_-^z - s_+^z)C_z\right] \tag{40}$$

where

$$C_x = -\gamma\sin(2\theta_{\tau^-})\,\frac{2m_\tau}{e}\,\mathrm{Im}\left\{d_\tau^\gamma\right\}, \quad C_z = 2\left(\sin^2\theta_{\tau^-}\right)\frac{2m_\tau}{e}\,\mathrm{Im}\left\{d_\tau^\gamma\right\} \tag{41}$$

Contributions to the $d\sigma^S$ could also come from the CP-odd interference of the real part of the EDM with absorptive parts and from the CP-even $\gamma - Z$ interference.

¿From Eq. (40), one can see that the transverse polarization term has an angular dependence of the form

$$(\alpha_+\sin\theta_+^*\cos\phi_+ + \alpha_-\sin\theta_-^*\cos\phi_-)\,\sin(2\theta_{\tau^-}) \tag{42}$$

Integrating the cross section in all angles except θ_{τ^-} and $\phi_\pm$ we can define an asymmetry

$$A_T^\pm = \frac{\sigma(+)^\pm - \sigma(-)^\pm}{\sigma(+)^\pm + \sigma(-)^\pm} \tag{43}$$

where

$$\begin{aligned}
\sigma(+)^\pm &\equiv \int_{w>0} \frac{d^2\sigma}{d(\cos\theta_{\tau^-})d\phi_\pm}\,d(\cos\theta_{\tau^-})\,d\phi_\pm, \\
\sigma(-)^\pm &\equiv \int_{w<0} \frac{d^2\sigma}{d(\cos\theta_{\tau^-})d\phi_\pm}\,d(\cos\theta_{\tau^-})\,d\phi_\pm
\end{aligned} \tag{44}$$

and $w = \sin(2\theta_{\tau^-})\cos\phi_\pm$, so that

$$A_T^\pm = -\frac{\beta\gamma}{2(3-\beta^2)}(\alpha_\pm)\,\frac{2m_\tau}{e}\,\mathrm{Im}\left\{d_\tau^\gamma\right\} \tag{45}$$

This asymmetry receives also standard contributions form the $\gamma - Z$ interference term to $(s_-^x + s_+^x)$. We want to isolate the EDM signal only, so one has to define a genuine CP-odd transverse asymmetry. We assume that both τ's decay into the same kind of hadrons ($\alpha_- = \alpha_+ \equiv \alpha_h$),

$$A_T = \frac{1}{2}\left(A_T^+ + A_T^-\right) = -\frac{\beta\gamma}{2(3-\beta^2)}(\alpha_h)\,\frac{2m_\tau}{e}\,\mathrm{Im}\,\{d_\tau^\gamma\} \tag{46}$$

which eliminates the standard model contribution.

A similar procedure can be done for the longitudinal polarization. In that case the angular dependence is of the form:

$$\left(\alpha_- \cos\theta_-^* + \alpha_+ \cos\theta_+^*\right)\,\sin^2\theta_{\tau^-} \tag{47}$$

so that θ_{τ^-} and $\phi_\pm$ variables can be integrated out without erasing the signal. The asymmetry is then defined to be

$$A_L^\pm = \frac{\sigma_L(+)^\pm - \sigma_L(-)^\pm}{\sigma_L(+)^\pm + \sigma_L(-)^\pm} = \frac{\beta}{3-\beta^2}\,(\alpha_\pm)\,\frac{2m_\tau}{e}\,\mathrm{Im}\,\{d_\tau^\gamma\} \tag{48}$$

with

$$\sigma_L(+)^\pm \equiv \int_0^1 d(\cos\theta_\pm^*)\,\frac{d\sigma}{d(\cos\theta_\pm^*)}, \qquad \sigma_L(-)^\pm \equiv \int_{-1}^0 d(\cos\theta_\pm^*)\,\frac{d\sigma}{d(\cos\theta_\pm^*)} \tag{49}$$

¿From these observables one can again define a genuine CP-odd longitudinal asymmetry

$$A_L = \frac{1}{2}\left(A_L^+ + A_L^-\right) = \frac{\beta}{3-\beta^2}\,(\alpha_h)\,\frac{2m_\tau}{e}\,\mathrm{Im}\,\{d_\tau^\gamma\} \tag{50}$$

that erases standard model contributions $(s_-^z + s_+^z)$ coming from $\gamma - Z$ interference.

In each one of these cases we have verified that all the other terms in the cross section do not contribute to the asymmetries and are eliminated when we integrate in the angles. When measuring these single-tau asymmetries, for each decaying channel of the observed *tau*, one may increase the statistics by summing up over the π, ρ... semileptonic decay channels of the τ for which the angular distribution is not observed.

Let us finally point out that the CP-odd Transverse and Longitudinal polarization asymmetries of Eqs. (46) and (50) get a contribution from the EDM real part through its interference with the Z-exchange. They give, however, a vanishing small contribution. These terms are

$$
{}^{(\gamma-Z)}A_T^{\pm} = \frac{\beta\gamma}{2(3-\beta^2)}(\alpha_\pm)\,\frac{2m_\tau}{e}\,\overbrace{\left[\frac{s\Gamma_Z M_Z}{(s-M_Z^2)^2+(\Gamma_Z M_Z)^2}\,\frac{v_e\,v_\tau}{4s_w^2\,c_w^2}\right]}^{R}\mathrm{Re}\,\{d_\tau^\gamma\} \quad (51)
$$

$$
{}^{(\gamma-Z)}A_L^{\pm} = -\frac{2\beta}{3-\beta^2}\,(\alpha_\pm)\,\frac{2m_\tau}{e}\,\underbrace{\left[\frac{s\Gamma_Z M_Z}{(s-M_Z^2)^2+(\Gamma_Z M_Z)^2}\,\frac{v_e\,v_\tau}{4s_w^2\,c_w^2}\right]}_{R}\mathrm{Re}\,\{d_\tau^\gamma\} \quad (52)
$$

and they are suppressed, respect to the single photon exchange (45) and (48), by the factor R of the order $(\frac{q^2}{M_Z^2}\frac{\Gamma_Z}{M_Z})$. At Upsilon energies this factor is of the order $6\cdot 10^{-7}$, so that one can safely conclude that $(\gamma - Z)$ contributions can be neglected within the present experimental accuracy.

6 Bounds on the EDM and final remarks

We can now estimate the bounds on the EDM that can be achieved using these observables. We assume a conservative set of data of 10^6 (10^7) tau pairs produced from all Upsilon resonances. This would presume a collection of 4×10^7 (4×10^8) $\Upsilon(1S)$ for example. The bounds one gets for the EDM are:

$$
\text{NT asymmetry and } \pi^\pm \textit{ tau} \text{ decay channel: } |d_\tau^\gamma| < \begin{cases} 1.5\times 10^{-16}\ e\,cm \\ (4.9\times 10^{-17}\ e\,cm) \end{cases} \quad (53)
$$

$$
\text{NL asymmetry and } \pi^\pm \textit{ tau} \text{ decay channel: } |d_\tau^\gamma| < \begin{cases} 1.7\times 10^{-16}\ e\,cm \\ (5.4\times 10^{-17}\ e\,cm) \end{cases} \quad (54)
$$

While for the imaginary part of the EDM the bounds are:

$$
\left.\begin{array}{l}\text{T asymmetry and} \\ \pi^\pm \textit{ tau} \text{ decay channels}\end{array}\right\} : \quad |\mathrm{Im}\,\{d_\tau^\gamma\}| < \begin{cases} 8.3\times 10^{-17}\ e\,cm \\ (2.6\times 10^{-17}\ e\,cm) \end{cases} \quad (55)
$$

$$
\left.\begin{array}{l}\text{L asymmetry and} \\ \pi^\pm \textit{ tau} \text{ decay channels}\end{array}\right\} : \quad |\mathrm{Im}\,\{d_\tau^\gamma\}| < \begin{cases} 1.2\times 10^{-16}\ e\,cm \\ (3.7\times 10^{-17}\ e\,cm) \end{cases} \quad (56)
$$

To conclude, we would like to point out:

- with low energy data we may have an independent analysis of the EDM from that obtained with high energy data,

- low energy data makes possible a clear separation of the effects coming from the electromagnetic-EDM and the weak-EDM,
- high statistics can compensate the suppression factor q^2/Λ^2 in the low energy regime for the effective operators,
- low energy observables, as defined in this paper, are different and complementary to the high energy ones studied in the past, and
- competitive bounds can be obtained from this new set of low energy observables.

Acknowledgements

This work has been supported by CONICYT-6052-Uruguay, by MCyT Spain, under the grants BFM2002-00568 and FPA2002-00612, and by the Agencia Española de Cooperación Internacional.

References

[1] S. Eidelman *et al.*, *Phys. Lett.* **B592** (2004) 1; K. Inami *et al.* [BELLE Collaboration], *Phys. Lett.* **B551** (2003) 16.

[2] J. Bernabéu, G.A. González-Sprinberg and J. Vidal, *Phys. Lett.* **B326** (1994) 168.

[3] W. Bernreuther, U. Low, J. P. Ma and O. Nachtmann, Z. Phys. C **43**, 117 (1989).

[4] M. Acciarri *et al.* [L3 Collaboration], *Phys. Lett.* **B434** (1998) 169.

[5] D. Buskulic *et al.* [ALEPH Collaboration], Phys. Lett. B **346**, 371 (1995); K. Ackerstaff *et al.* [OPAL Collaboration], Z. Phys. C **74**, 403 (1997); H.Albrecht *et al.* [ARGUS Collaboration], *Phys. Lett.* **B485** (2000) 37.

[6] F. del Aguila and M. Sher, *Phys. Lett.* **B252** (1990) 116.

[7] J.A. Grifols and A. Mendez, *Phys. Lett.* **B255** (1991) 611 and Erratum *Phys. Lett.* **B259** (1991) 512.

[8] R. Escribano and E. Masso, *Phys. Lett.* **395** (1997) 369.

[9] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 1562.

[10] W. Buchmuller and D. Wyler, *Nucl. Phys.* **B268** (1986) 621; C.N. Leung, S.T. Love and S. Rao, *Zeit. für Physik* **C31** (1986) 433; M. Bilenky and A. Santamaria, *Nucl. Phys.* **B420** (1994) 47.

[11] G.A. González-Sprinberg, A. Santamaria, J. Vidal, *Nucl. Phys.* **B582** (2000) 3.

[12] W. Bernreuther, O. Nachtmann and P. Overmann, Phys. Rev. D **48** (1993) 78.

[13] J. Bernabéu and N. Rius, *Phys. Lett.* **B232** (1989) 127; J. Bernabéu, N. Rius and A. Pich, *Phys. Lett.* **B257** (1991) 219.

[14] J. Bernabéu, G.A. González-Sprinberg, M. Tung and J. Vidal, *Nucl. Phys.* **B436** (1995) 474.

[15] J. H. Kuhn, Phys. Lett. B **313** (1993) 458.

[16] Y.S. Tsai *Phys. Rev.* **D4** (1971) 2821.