Parity Assisted Decision Making for QAM Modulation

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Abstract

A simple technique which involves the transmission of a Quadrature Amplitude Modulation (QAM) symbol and two parity bits in separate channels to improve the performance of communication systems is devised. When a symbol is received, a decision is made not solely by its Euclidean distances to the constellation points. Rather, the two parity bits are used to assist in making the decision. Unlike standard error correcting codes (ECC), the proposed method operates on the received symbols at the detector level and before the ECC. The parity bits and the information symbols can be sent in different channels (frequency division) or at different times on the same channel (time division). The available energy for transmission can be distributed unevenly among the information bits and the parity bits to improve the performance. Simulation results show large gains in required signal to noise ratios over uncoded system to achieve the same performance. The scheme is simple and is well suited for systems with low computational power.

1 Introduction

In traditional QAM, each k information bits are associated with a point in a $M-$point constellation diagram where $M = 2^k$. A waveform corresponding to the coordinates of the constellation point is transmitted to convey the information. In Additive White Gaussian Noise (AWGN) channels and when the symbols occur with equal probabilities, the receiver makes a decision as which symbol was transmitted according to the Euclidean distances of the received symbol to all constellation points.

In hard-decision decoding, the output of the demodulator is quantized into a small number, usually M, of discrete levels. This kind of decision-making highly simplifies the receiver structure. This simplification comes at a cost in terms of performance. Soft-decision decoding, on the other hand, does not quantize the output of the demodulator. The demodulator output is sent to the decoder as is for further processing. It is also possible to quantize the output of the demodulator to a large number of levels, much larger than M, and still achieve performance close to that of pure un-quantized output. In BPSK where $M = 2$, for example, the performance when 3-bit quantization, i.e., 8 levels, is used is similar to that of no quantization.

When hard decision is used instead of soft decision, there is a loss of about 2-3 dB per bit to achieve the same level of performance in terms of bit error rate. To compensate for the loss associated with hard decision, the authors investigated the use of *parity-assisted decoding* (PAD). This technique involves sending two parity bits in addition to the QAM symbol. The parity bits are sent using QPSK in a different channel. The parity bits can also be sent at alternating times with the information symbols on the same channel. The two parity bits are simply logical functions of the bits of the word corresponding to the symbol to be transmitted. This simple coding technique is particularly attractive for systems where high computational power is not available.

Standard error correcting codes (ECC) correct errors *after* the received symbols have been detected. The received word is compared, through certain algorithms, to all codewords. The codeword closest to the received one, in Hamming distances for hard decisions and Euclidean distances

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for soft decisions, is chosen as the most probable transmitted codeword. PAD, on the other hand, operates on the received symbols during detection and before they are passed to ECC decoders. It uses the two parity bits for decision making at the matched filter or correlator bank level before they are further processed by the ECC.

2 PAD Encoder

The operation of the PAD can be summarized as follows. At the transmitter, two parity bits are generated for each symbol to be transmitted. Only two parity bits are generated regardless of the number of points in the constellation. This enables the transmission of the parity bits using QPSK which inherently has a low bit error rate performance for a fixed SNR compared to other modulation techniques. It is also possible to generate three parity bits instead of two and sending them using a different modulation technique, like PSK-8 for example. However, PSK-8 has a high bit error rate compared to that of QPSK of the same SNR. Moreover, PSK-8 is slightly more complex than QPSK. The two parity bits are logic functions of the bits representing the symbol to be transmitted. If the bits of information symbol are $b_0,\ b_1,\ \ldots,\ b_{k-1}$, then the two parity bits are represented as

$$p_0 = f_0(b_0,\ b_1,\ \ldots, b_{k-1}), \quad (1)$$

$$p_0 = f_1(b_0,\ b_1,\ \ldots, b_{k-1}), \quad (2)$$

where f_0 and f_1 are logic functions. The two functions, f_0 and f_1 can be implemented either as combinational logic functions using for example sum of products or product of sums structures. They can also be implemented using look up tables (LUT).

Since a symbol error is most likely to occur when a symbol is received, due to additive noise, at a neighboring constellation point decision region, the functions are chosen to be as different as possible for neighboring points. A count of a simulation run showing where points are received when 10000 transmissions of the point in the center are transmitted at a fixed SNR are shown in Fig. 1. Mathematical expression for the expected number of point to fall in any region is simply the area (volume) under a two dimensional Gaussian distribution function centered at the coordinates of the central point and with a covariance matrix diag$\{\frac{N_0}{2}, \frac{N_0}{2}\}$ multiplied by the number of simulation runs.

The first function, f_0, can be chosen to be checkered among the symbols on the constellation diagram. Fig. 2 shows a typical function truth table for QAM-16. This can be extended for constellations with a larger number of points. This arrangement ensures that most errors, which as shown in Fig. 1 occur at the four neighboring regions, top, bottom, left and right, can be detected and corrected. Errors occurring at diagonal neighbors or those falling in the four closest neighbors but further than half the decision regions width will not be corrected by f_0. The function f_1 augments the first function to overcome the above shortcomings.

2	110	2
131	9512	138
0	103	2

Figure 1: Simulation run of the number of symbols that are received in neighboring regions when 10000 symbols of the center point are transmitted.

The second function, f_1, can be chosen to be as shown in Fig. 3. This function ensures that diagonal neighboring regions have different parities. This function does not solve the problem when a received symbol arrives at a close neighbor but further than half the regions width. However, such errors occur very infrequently except for very low SNRs.

We note that f_0 has more effect in correcting errors than f_1. However, combining both functions increases the effectiveness of the coding scheme. One might also map the two parity bits so as to protect the bit representing f_0 more than the bit representing f_1. However, that increases

1	0	1	0
0	1	0	1
1	0	1	0
0	1	0	1

Figure 2: The first parity bit function f_0 truth table for a QAM-16. The function is simply checkered bits like in a chess-board.

0	1	0	1
0	1	0	1
0	1	0	1
0	1	0	1

Figure 3: The second parity bit function f_1 truth table for a QAM-16. This and in conjunction with f_0 constitute the parity bits of PAD.

the complexity of the coding method even further.

While mapping the bits to the constellation diagram can be done in a multitude of ways and yielding the same symbol error rate, it is customary to use Gray mapping. In Gray mapping neighboring symbols differ in a single bit. Therefore, a symbol error will most likely result is a single bit error. For a constellation of M points, the bit error rate P_e is approximated by

$$P_e \approx \frac{P_M}{\log_2 M}, \tag{3}$$

where P_M is the symbol error rate.

3 Decoding Using PAD

Unlike traditional QAM, the decoder of the proposed scheme uses both the Euclidean distance of the received symbol as well as the parity bits to make the decision. The distances of the received symbol to N closest constellation points are calculated and sorted in ascending order. Then a decision is made for the first point whose parity matches the parity bits received from the other channel. If the parity of none of the N points matches the received parity bits, then a decision is made to the one which matches the parity bit corresponding to f_0, which as shown above is more effective in correctly making decisions as compared to f_1. If that also is not met, then the first point–the one with shortest Euclidean distance–is selected. For a practical system and when the number of constellation points is small or medium, N can be less than 9. When the received symbol is close to the perimeter constellation points or outside the constellation diagram, then N can be made even smaller.

We note here that the proposed scheme is not restricted to QAM as it can be used in other modulation techniques like PSK and PAM. It can also be used in multi-dimensional, like lattices, modulation techniques.

4 Varying Relative Energies

In a typical M-point QAM modulation, each k bit of information where $k = \log_2 M$ is mapped to a point in the QAM constellation diagram. Associated with each point is a waveform whose amplitude and phase are determined by the coordinates of the point in the diagram. Let the average energy per symbol (i.e., waveform) be E_s. Then, the average bit energy is

$$E_b = \frac{E_s}{k}. \tag{4}$$

In the proposed system, the available symbol energy is divided among the k information bits and the two parity bits. The division, however, need not be equal. Since the parity bits are used to make decisions on the information symbols, they need to have a lower probability of error than those of the information symbols. That necessitates assigning the parity bits a higher portion of the available energy. However, assigning them a very large portion, will lead to low error rates for the parity but will leave a small amount of energy available to the information symbol. A possible solution to this problem is to divide the energy in such a way that the probability of error of the parity bits, P_p, is a small fraction of the probability of error of the information symbols. Thus, the average waveform energy is divided among the k information bits and the two parity bits like

$$E_s = k\widetilde{E}_b + 2E_p, \tag{5}$$

where $\widetilde{E}_b$ is the energy per bit of the information symbol and E_p is the alloted energy to each parity bit. Further, let the ratio of the energy allocated to each parity bit relative to each bit in the QAM symbol be α. Thus,

$$E_p = \alpha\widetilde{E}_b, \tag{6}$$

for some $\alpha > 0$. From (4)-(6)

$$kE_b = \widetilde{E}_b(k + 2\alpha). \tag{7}$$

The probability of error for the parity bits, P_p, assuming Gray coding is given by

$$P_p = Q\left(\sqrt{\frac{2E_p}{N_0}}\right), \tag{8}$$

where N_0 is the noise power spectral density. Similarly, the probability of symbol error for the QAM symbol P_{QAM} is [1]

$$P_{QAM} = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k\widetilde{E}_b}{(M-1)N_0}}\right). \tag{9}$$

The parity bits are used to make a decision for the QAM symbol. Hence, it is imperative to have a probability of error for the parity bits P_p to be much lower than the expected error for the QAM symbol. We let P_p be a small fraction of P_{QAM}. Let the two probabilities be related by

$$P_p = \beta P_{QAM}, \tag{10}$$

where $0 < \beta \ll 1$. It follows from (4)-(10) that

$$Q\left(\sqrt{\frac{2E_p}{N_0}}\right) = 4\beta\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k\widetilde{E}_b}{(M-1)N_0}}\right). \tag{11}$$

Since $\widetilde{E}_b$ is related to E_b, viz,

$$\widetilde{E}_b = \frac{k}{k+2\alpha}E_b, \tag{12}$$

and from (6), we can write (11) as

$$Q\left(\sqrt{\frac{2\alpha k E_b}{(k+2\alpha)N_0}}\right) = 4\beta\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k^2 E_b}{(k+2\alpha)(M-1)N_0}}\right). \tag{13}$$

For a given signal to noise ratio $\frac{E_b}{N_0}$, this equation can be solved numerically for α and hence for $\widetilde{E}_b$. The parameter β is a design adjustable value. A value of 0.1 implies that the parity bits errors will occur approximately a tenth of the times less frequently than those of the QAM symbol. Currently, there is no algorithm to obtain the optimal value of β. The choice is made mainly from simulations.

5 Coding the Parity Bits

Since the parity bits control the decision making for the QAM symbols, they need to have as low an error rate as possible. From (8) we see that the error rate of the parity bits can be decreased by increasing the portion of energy assigned to the parity bits, E_p, because the Q function is a monotonically decreasing function. However, increasing E_p may not be desirable as that would reduce the energy available to the QAM symbol. Another way to reduce the parity error probability P_p and hence improve the overall performance is by coding the parity bits using either convolutional or block codes. Such coding, however, will necessitate expanding the required bandwidth even further. One can, however, use Trellis-Coded Modulation (TCM) for the parity bits [2]. By adding a little bit complexity at the receiver, it is possible to gain as much as 3-6 dB for the parity bits to achieve the same error performance without bandwidth expansion. We note, however, that when M is large, then the overall performance improves only slightly as the saved parity energy is distributed among a large $k = \log_2 M$ bits of the information symbols.

6 Simulation Results

The proposed method was simulated under AWGN. The system was tested using QAM-16 and QAM-64 for modulation and two parity bits for the PAD. Fig. 4 show the bit error rate for the PAD system when QAM-16 was used. Similarly, fig. 5 demonstrates the performance under QAM-64.

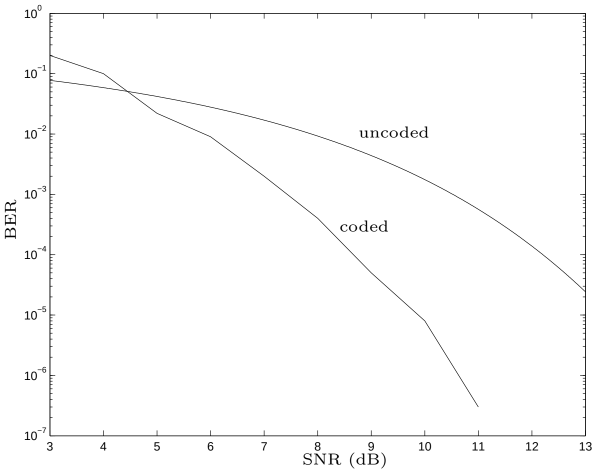

Figure 4: The performance of PAD on QAM-16 in terms of bit error probability Pb.

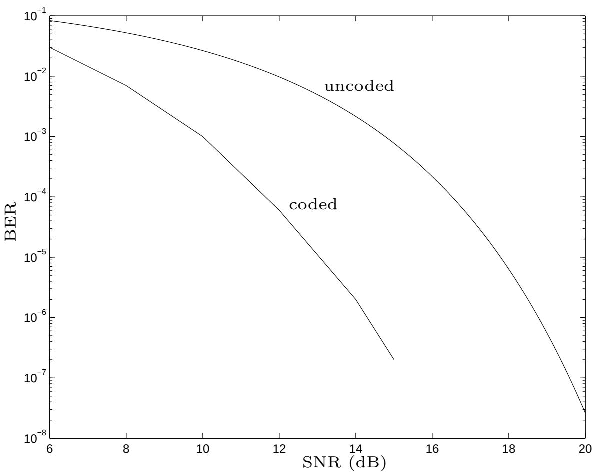

Figure 5: The performance of PAD on QAM-64 in terms of bit error probability Pb.

References

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