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ASPECTS OF UNIVERSAL EXTRADIMENSIONAL MODELS AND THEIR LATTICIZED VERSIONS

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Ph. D. Thesis

Aspects of universal extra dimensional models and their latticized versions

Josep F. Oliver

ARCADI SANTAMARIA LUNA, Profesor titular del Departamento de Física Teórica de la Universidad de Valencia y **JOANNIS PAPAVASSILIOU**, Investigador Contratado del Departamento de Física Teórica de la Universidad de Valencia,

CERTIFICAN que la presente memoria **"Aspects of universal extra dimensional models and their latticized versions"** ha sido realizada bajo su dirección, en el Departamento de Física Teórica de la Universidad de Valencia, por D. **JOSÉ FRANCISCO OLIVER GUILLÉN**, y constituye su Tesis Doctoral para optar al grado de Doctor en Física.

Y para que así conste, en cumplimiento de la legislación vigente, presenta ante la Facultad de Física de la Universidad de Valencia la referida memoria, y firma el presente certificado

en Burjassot, a 22 de Septiembre de 2003

Arcadi Santamaria Luna | Joannis Papavassiliou

A mis abuelos
con cariño

Agraïments

Amb aquest treball s'acompleix un dels meus somnis: arribar algun dia a ser doctor en física teòrica. Des de ben xicotet sempre m'ha apassionat la física, en particular la física de partícules. El primer contacte amb ella va ser a través de certs llibres de divulgació on $W^{\pm}$ i Z^0 eren partícules exòtiques i misterioses fruit d'un principi, en aquells moments inintel.ligible per a mi, anomenat de Gauge. Eternament agraït li estaré a Ximo per haver-me'ls mostrat. Retrospectivament ell és el gran culpable que jo estiga ací escrivint aquests agraïments.

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Burjassot, a 23 d'Octubre de 2003
Josep F. Oliver

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Chapter 1

Introduction

The possible existence of more dimensions than those directly accessible to our senses has always fascinated mankind. They have spurred the imagination of many writers who have speculated about how to reach them, their nature and possible applications. Interesting as it would be, we will not take this path in this work; instead we will divert our attention to a more scientific point of view, namely what can we say about extra dimensions by using the scientific knowledge we have accumulated trough time. In the next lines we show, very briefly, the origin and evolution of the ideas related with extra dimensions. Finally, we sketch their present state to situate our work in context[1].

The extra dimensions are a relatively old idea among the scientific community. Its origins can be traced back to the year 1912 when the Finnish physicist Gunnar Nordström proposed a relativistic extra dimensional theory that described simultaneously gravity and electromagnetism. At that moment the general theory of relativity was not developed and therefore the geometric origins of gravity were not unveiled. Before Einstein, Nordström had developed a relativistic theory of gravitation based on the existence of a scalar potential, ϕ ; he unified this theory with the electromagnetism.

The ideas of Nordström were strongly influenced by Maxwell's theory of electromagnetism, which unified elegantly electric and magnetic phenomena by describing the electric and magnetic fields as components of a single six-component antisymmetric tensor, $F_{\mu\nu}$, while the corresponding potentials were unified into a four dimensional vector, A_μ. It became clear through Minkowski's work that the unification of electricity and magnetism entailed a unification of space and time in a four dimensional *space-time*. Nordström followed this reasoning and added an extra dimension to this space-time; the vectors of this new manifold allowed for one more scalar field that Nordström proposed to be precisely his gravity potential, ϕ. The action was taken to be the five dimensional version of electromagnetism, now built from the antisymmetric tensor $F_{\alpha\beta}$, where $\alpha, \beta = 0, 1, \ldots, 4$. Of course, the fifth dimension needed to be different from the rest, hence it was assumed to have a topology drastically different compared with the other four, and it was compactified on a circumference, i.e. the values of the coordinates in the fifth dimension were restricted. This topology allowed a Fourier expansion of the fields and when only the fundamental mode of each field was considered a four dimensional theory of electromagnetism and gravity emerged. In other words, Nordström showed that gravitation and electromagnetism could be understood as two different faces of a five dimensional electromagnetism.

[1]Part of the contents of this section has been extracted from [2]

It was in 1915 when Einstein proposed his awe-inspiring relativistic theory of gravitation, the "*general theory of relativity*", in which gravity is understood as geometrical deformations of the underlying space-time. In 1919, the mathematician Theodor Kaluza showed that a five dimensional theory of gravity, with the action taken as the Einstein-Hilbert action, would manifest itself as the electromagnetism and the gravity in a four dimensional world. In 1926, the same year Schrödinger proposed his famous equation, Oskar Klein and the Russian physicist H. Mandel independently rediscovered Kaluza's theory. They hoped this theory would underlie the quantum theory that at that moment was still under construction.

These first steps into the fifth dimension were rather hesitant. It was viewed as a mathematical trick that allowed a more concise formulation of the laws of Nature but was completely void of any physical interpretation. In addition, it posed a number of problems: the correct four dimensional limit was recovered when only the fundamental Fourier mode was retained, the so called *cylindricity condition*, and it was unclear the role of the rest of the modes, the reason why some fields must be constant was also obscure, just to cite some.

The discovery of new interactions, other than electromagnetism and gravitation, complicated more the overall picture: using a single extra dimension as a means of reaching a unified description was unnatural, because it was not able to accommodate the strong and weak forces. The latter were described by a class of theories called non-Abelian gauge theories, proposed by Yang and Mills in 1954, which became widely accepted in the seventies. Yang-Mills theories could be incorporated within an extra dimensional framework, at the price of extending the number of additional dimensions; but this could not be done straightforwardly, and posed a number of difficulties that had to be overcome. At this point the extra dimensions were still useful for grouping together equations in a unified mathematical framework, but had acquired a great degree of complexity, while not being predictive and presenting serious theoretical problems.

In the next years the interpretation of the extra dimensions changed, in the sense that they were given a physical meaning. It was due to the development of new theories; *supergravity* and *string theory*, where the extra dimensions played a key role. Both kind of theories provided a promising framework for achieving a quantum description of gravity. The natural energy scale for these theories is the Planck mass, $1.2\ 10^{19}$ GeV $\cdot\, c^2$, that is completely out of reach for the current particle accelerators. Nevertheless, in some string scenarios this energy scale can be as low as a few TeV what suggests that the associated phenomenology can be more accessible to observation.

In recent years, extra-dimensional quantum field theories have received a great deal of attention. On one hand, the recent interest is because the scale at which the extra dimensional effects can be relevant could be around a few TeV, even hundreds of GeV in some cases, clearly a challenging possibility for the next generation of accelerators. On the other hand, this new point of view has permitted to study many long-standing problems in physics from this new perspective. These problems cover many different fields of particle physics: the hierarchy problem, new neutrino physics, the masses of the fermions, the number of generations in SM, possible modifications in the running of the coupling constants, new candidates for dark matter, etc.

Extra dimensional theories offer a wide variety of scenarios and therefore have a rich phenomenology. For instance, in some scenarios (*large extra dimensions*) only the gravity field can probe the extra dimensions. In others gravity is not considered and only boson fields are allowed to propagate through the extra dimensions. Another possibility is to allow all the fields present in the theory to feel the extra dimensions (*universal extra dimensions*).

One of the reasons why this latter scenario is particularly interesting is because the corrections to the SM predictions appear for the first time at the one-loop level. As a consequence, the modifications to precision observables are small, what implies that the scale of this theory can be as low as hundreds of GeV. The fact that these models do not give any tree-level contribution, is due to the conservation of the so called Kaluza-Klein (KK) number (strictly, it is not a conserved number in the usual sense). Most of this thesis is devoted to study the phenomenology of theories with one universal extra dimension.

In particular, in chapter 2 we show how to treat the different fields (scalars, fermions and vector bosons) in an extra-dimensional quantum field theory formalism. The concept of dimensional reduction is introduced and we show how to obtain a four dimensional Lagrangian from the expression of the Lagrangian in $4+\delta$ dimensions. We also show how the extra dimensional fields are transformed into an infinite number of four-dimensional fields (the so called KK towers) with the same quantum numbers associated to all. By studying the low energy limit of these theories we stress the relevance of selecting a suitable topology for the compactified dimensions since different topologies have associated different degrees of freedom in this limit. We show that the orbifold topology selects the correct low energy degrees of freedom; specifically, it is possible to obtain chiral fermion fields and to remove the extra dimensional components of the vector fields in the low energy spectrum. Interacting theories are studied to demonstrate explicitly the KK number conservation, which is related to the local extra-dimensional Lorentz invariance of the theory and has a deep impact on the phenomenology of theories with universal extra dimensions.

In chapter 3 we use the ideas developed previously to construct an extra dimensional model that reduces to SM in the low-energy limit. We study the phenomenology of this model and focus on the observables that display a strong dependence on the mass of the top-quark because in this case the deviations from the SM predictions are more important. We compute the radiative corrections for the $Z \to b\overline{b}$ decay, $b \to s\gamma$, the $B^0 - \overline{B}^0$ mixing and the ρ parameter, and study their consequences.

In chapter 4 we construct in detail the latticized version of the previous model, i.e. the version in which the extra dimension is discretized. Latticized as well as deconstructed models were devised as ultraviolet completions of the extra dimensional models. The latter are not renormalizable because the coupling constants have dimensions of mass to some negative power. This is suggesting that they must be understood as low energy effective manifestations of a more complete theory. Models with deconstructed extra dimensions are usual four-dimensional theories, which, due to the special nature of the interactions present in the Lagrangian, display an extra dimensional behaviour in certain range of energies. These kind of models have received a great deal of interest because in some of their extensions the Higgs boson is a pseudo-Goldstone boson, what would explain why it is so light and stable against radiative corrections. This models are very similar to latticized models. The latter are still non-renormalizable but they can be understood as usual four-dimensional σ-models, and all the known possible ultraviolet completions for the σ-models can be applied now. In this thesis we study part of the phenomenology of the models with one latticized extra dimension.

The next chapter is devoted to investigate the modification of the running of the coupling constants in models with (continuous) extra dimensions. It has been pointed out that in extra dimensional theories the running can be accelerated, i.e. the dependence with the energy scale is not logarithmic, as usual, but can be power-like. This change could have as a major consequence that the unification of the three interactions could be achieved at a very low scale, of order a few TeV. We have studied in some detail, by resorting to simplified models, how reliably this power corrections can be computed without knowing the details of the more complete theory, i.e. the theory valid above the scale at which the extra dimensional theory ceases to be correct. We have found that the coefficients that govern this power corrections are sensitive to the details of the theory in which the extra dimensional theory is embedded. This is a completely different result compared with the situation in usual grand unification scenarios where the unification of the gauge coupling constants can be tested without knowing the details of the Grand Unification Theory.

Finally, in chapter 6 we study a model with a non-universal extra dimension. In this case only the boson fields are allowed to propagate through the extra dimension, due to this the extra-dimensional Lorentz symmetry is broken and the KK number conservation rule does not apply. The results are compared with those obtained when the extra dimension is universal to show explicitly the importance of the KK number conservation. The bound on the compactification scale is clearly higher for the kind of models that lack this extra-dimensional Lorentz symmetry because the deviations from the SM predictions appear already at the tree-level.

Chapter 2

Quantum field theory with one universal extra dimension

In this chapter we study the main features of theories with one additional space dimension accessible to all fields, called universal extra dimension. To this end we study a number of toy models which we will use to show how to treat scalar, spinor and vector fields in five dimensions. We address the issue of compactification in this theories and study two different topologies: a sphere, S^1, as well as an *orbifold*, S^1/Z_2. It is shown that in the process, called *dimensional reduction*, one can trade the extra dimension for an infinite *tower* of fields, called Kaluza-Klein (KK) tower or KK modes. The different topologies provide different low energy theories even when one starts from the same five dimensional Lagrangian. We will show that the advantage of compactifing in an orbifold is double: on one hand, only four of the five components of the vector field are present in the low-energy spectrum, on the other, it can contain chiral fermions. This opens the door to identifying the SM with the low energy realization of an extra dimensional theory. Instead of studying possible extra dimensional extensions of the SM, we first propose some simple interactions to gain some insight into the properties of these theories while keeping the model as simple as possible. It is found that a new kind of conserved number appears, the *KK number*. It cames from the fact that the theories are locally invariant under the Lorentz group in five dimensions. Strictly, it is not conserved in the usual sense and therefore we study it in some detail. Its main contribution is to screen, to some extent, the impact of the KK towers in the low-energy effective theory.

2.1 Fields and interactions in five dimensions

In particle physics each particle is associated to the quanta of a field defined in the Minkowski space-time $\mathcal{M}^4$. The coordinates in this manifold are written as x^μ where $\mu = 0, \ldots, 3$. To extend this formulation to more dimensions one must define fields that depend on $4+d$ coordinates, say $\psi(x^\alpha)$, where $\alpha = 0, \ldots, 3+d$. All the extra coordinates are supposed to be associated with spatial dimensions, therefore the metric takes the form $g_{\alpha\beta} = (+, -, \ldots, -)$. Once this is done, a topology for the additional dimensions must be selected. This choice has important consequences in the low-energy spectrum. In the next sections all this process is performed in detail for different kind of fields.

2.1.1 Scalar field with self interaction

Let us define a complex scalar field $\Phi(x^\alpha)$ that depends on the $4+d$ coordinates $\alpha = 0, 1, \ldots, d$. The action is defined in the usual way through the standard Klein-Gordon Lagrangian density

$$S = \int d^{4+d}x \, \mathcal{L}^{4+d}(x^\alpha). \tag{2.1}$$

For a complex scalar field

$$\mathcal{L}^{4+d} = (\partial_\alpha \Phi)^\dagger (\partial^\alpha \Phi) - m^2 \Phi^\dagger \Phi, \tag{2.2}$$

The first consequence is that the canonical dimension of the field gets modified, now $[\Phi] = E^{1+d/2}$, what will be important when studying the renormalization of theories with extra dimensions. The next step to do is to specify the topology of the extra dimensions. The simplest choice is to associate a circumference, S^1, to each one, i.e. the full manifold is a direct product of the Minkowski space and d circumferences, $\mathcal{M} = \mathcal{M}^4 \times (S^1)^d$. This means that the extra coordinates are periodic with a periodicity of $2\pi R$, assuming the same *radius*, R. From this it follows that Φ can be expanded on its Fourier modes

$$\Phi(x^\mu, \vec{x}) = \sum_{n_1,\ldots,n_d=-\infty}^{\infty} \phi_{\vec{n}}(x^\mu) e^{i\vec{n}\vec{x}/R} \tag{2.3}$$

where $\vec{x} = (x^4, \ldots, x^{d-1})$ is a vector whose components are the coordinates in the extra dimensions and $\vec{n} = (n_1, \ldots, n_d)$ identifies unambiguously each Fourier mode. By using Eq. (2.3) the integration over the extra coordinates in Eq. (2.1) can be performed

$$S = \int_{-\infty}^{\infty} d^4x \int d^dx \, \mathcal{L}^{4+d} \equiv \int_{-\infty}^{\infty} d^4x \, \mathcal{L}. \tag{2.4}$$

This shows that this theory can be described by a four dimensional Lagrangian related with the original one by the equation

$$\mathcal{L} = \int d^dx \, \mathcal{L}^{4+d}. \tag{2.5}$$

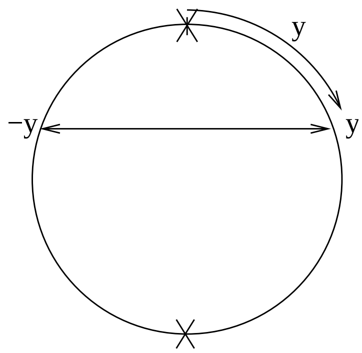
This process receives the name of *dimensional reduction*. It is independent of the kind of field(s) (scalar, fermion, vector, etc...) that $\mathcal{L}^{4+d}$ contains, it only depends on our ability to perform the integration over the coordinates of the extra dimensions.

In the case we are studying this process leads to

$$\mathcal{L} = \sum_{\vec{n}=-\infty}^{\infty} (\partial_\mu \phi^{(\vec{n})})^\dagger (\partial^\mu \phi^{(\vec{n})}) - (m^2 + m_{\vec{n}}^2) \phi^{(\vec{n})\dagger} \phi^{(\vec{n})} \qquad m_{\vec{n}}^2 = \vec{n}^2/R^2. \tag{2.6}$$

To obtain canonical kinetic terms, the original fields, $\phi^{(\vec{n})}$, must be redefined: $\phi^{(\vec{n})} \to (2\pi R)^{-1} \phi^{(\vec{n})}$. Eq. (2.6) shows one of the most important features of this kind of theories, specifically, the extra dimensions have been traded for an infinite tower of fields, called *Kaluza-Klein tower* or *KK tower*, with increasing masses. The lowest mass, the mass of the fundamental mode, $\phi^{(\vec{0})}$, is the one appearing initially in $\mathcal{L}^{4+d}$ and in principle is completely independent of R and insensitive to the compactification procedure. In particular, it could be much lower than R^{-1} or even zero. Notice the degeneracy in the spectrum, except for the fundamental mode.

Another important topology for the extra dimensions is the so called *orbifold*. It is a bit more complicate than S^1 and for the sake of simplicity we will concentrate on the case of a single extra dimension, which is the important case for this thesis. Its relevance will be manifest in the next sections when we study the spinor and gauge fields. From now on, when only a single extra dimension is present its coordinate, x^4, will be denoted by $y \equiv x^4$. The topology of the space-time is now $\mathcal{M}^4 \times (S^1/Z^2)$. S^1 means that the extra dimension is again periodic and Z_2 reflects the fact that the points $-y$ and y are identified. The orbifold is schematically represented in the figure.



The crosses in the figure represent two special points, called *fixed points*, that are mapped onto themselves under the orbifold symmetry: $y \to -y$. When we say that these points are identified we mean that the values of the fields in them are related, i.e. $\Phi(-y) = U\Phi(y)$, where U is a unitary transformation that is a symmetry in the original Lagrangian $\mathcal{L}^{4+d}$. As a result, the physics on one side and on the other is exactly the same, or more formally, the action can be computed restricting the integration to the interval $y \in [0, \pi R]$

$$S = \int d^4x \int_0^{2\pi R} dy\, \mathcal{L}^5 = 2 \int d^4x \int_0^{\pi R} dy\, \mathcal{L}^5. \quad (2.7)$$

If it is further imposed that $U^2 = \mathbb{1}$, then for a scalar field it is perfectly valid the choice $U = \pm 1$. This extended symmetry imposes further structure to the fields, for one extra dimension the S^1 topology implies that a field can be expanded as[1]

$$\Phi(x^\mu, y) = \phi^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \phi^{(n)+}(x^\mu) \cos\left(\frac{ny}{R}\right) + \sum_{n=1}^{\infty} \phi^{(n)-}(x^\mu) \sin\left(\frac{ny}{R}\right). \quad (2.8)$$

The orbifold topology requires that the fields are even or odd under the orbifold parity transformation ($y \to -y$), calling Φ^+ and Φ^- respectively

$$\begin{aligned} \Phi^+(-y) &= +\Phi^+(y), \\ \Phi^-(-y) &= -\Phi^-(y). \end{aligned} \quad (2.9)$$

Their expansions are now

$$\Phi^+(x^\mu, y) = \phi^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \phi^{(n)+}(x^\mu) \cos\left(\frac{ny}{R}\right), \quad (2.10)$$

$$\Phi^-(x^\mu, y) = \sum_{n=1}^{\infty} \phi^{(n)-}(x^\mu) \sin\left(\frac{ny}{R}\right). \quad (2.11)$$

This is of great importance, since only the even fields have a fundamental mode. Recall that the mass of the fundamental mode was only determined by the mass in the original Lagrangian and can be in principle as low as desired. On the contrary, the lower available mass for the odd modes is R^{-1}. Finally, taking $\mathcal{L}^5 = (\partial_\alpha \phi)^\dagger (\partial^\alpha \phi)$

$$\mathcal{L} = \int_0^{\pi R} \mathcal{L}^5(\Phi^+) = \sum_{n=0}^{\infty} \frac{1}{2} [(\partial_\mu \phi^{(n)})^\dagger (\partial^\mu \phi^{(n)}) - m_n^2 \phi^{(n)\dagger} \phi^{(n)}], \quad (2.12)$$

[1] Of course this is the (2.20), where the exponentials have been expressed in terms of sines and cosines.

while for Φ^- the expression is the same without the fundamental mode, hence the low-energy spectrum is radically different. It is worth to stress that these theories had a clear separation between low-energy and high-energy regimes due to the existence of a natural energy scale, $1/R$, which, excluding the zero mode, is the mass of the lightest mode. The particles we know could be identified with the fundamental mode of an even field, the smallness of R would explain why no KK mode has yet been detected. If this idea is true then we can make a rough estimate of the size of the extra dimension, $R^{-1} > 200$ GeV because this is the highest energy directly probed by accelerators. Of course, to obtain a serious bound it is required a more evolved model and the careful study of radiative contributions to precision observables since these KK modes would also modify SM predictions via virtual exchanges in loops. This detailed study is the main aim of this work.

Up to this point only the free part of the theory has been investigated. As an example of interaction we will use a five dimensional Φ^4 self-interaction in an orbifold; therefore we add to the Lagrangian the next interaction term

$$\mathcal{L}_I^5 = -\frac{\tilde{\lambda}}{4!}\Phi^4, \tag{2.13}$$

where Φ is assumed to be a real and even field. Notice that $\tilde{\lambda}$ is not dimensionless, it is easy to derive that in general $[\tilde{\lambda}] = E^{-d}$. By using the decomposition given in Eq. (2.10) one finds after dimensional reduction that the Lagrangian of this theory can be written as

$$\mathcal{L} = \sum_{n=0}^{\infty}\frac{1}{2}[\partial_\mu\phi^{(n)}\partial^\mu\phi^{(n)} - m_n^2\phi^{(n)}\phi^{(n)}] - \sum_{n,m,p,q=0}^{\infty}\frac{\lambda}{4!}\,\Theta_{nmpq}\,\phi^{(n)}\phi^{(m)}\phi^{(p)}\phi^{(q)}, \tag{2.14}$$

where a new dimensionless coupling constant has been defined, $\lambda = \tilde{\lambda}/\sqrt{\pi R}$. The function Θ is decomposed as the product $\Theta_{nmpq} = \theta\Delta_{nmpq}$. The first is just a numerical factor due to the fact that the fundamental mode has different normalization than the rest, it depends on the number of fundamental modes present in the vertex, f, as $\theta = 2^{-\frac{3f+1}{2}}$. The second is more interesting because it forbids certain combinations of indices

$$\Delta_{nmpq} = \begin{cases} 1 & \pm n \pm m \pm p \pm q = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.15}$$

i.e. if any of the possible combinations is zero then the vertex exists otherwise it is forbidden. The Feynman rule for the vertex of the theory is otherwise straightforward, see Fig. 2.1. One of the most interesting properties of the interaction is that there is no vertex that couples three fundamental modes with one excited. This means that to create particles with $n \geq 1$ from the particles of the fundamental mode they must be created in pairs, for instance through the process $\phi_0\phi_0 \to \phi_n\phi_n$. Therefore the threshold for creating the new particles is a least twice the mass of the first mode, $\sqrt{s} \geq 2m_1$. The situation is reminiscent of other occasions in physics where also new particles had to be created in pairs, as for instance the creation of the charm quark. In that case there was a symmetry behind: the charm quark had to be created via strong interactions which are flavour-symmetric; thus, in order to create a charm quark it was necessary to create also an anticharm quark.

In extra dimensions the pair production can also be related to a symmetry: the local five-dimensional Lorentz symmetry of the tree level Lagrangian. However, this symmetry is broken by the compactification. This breaking is a non-local effect. Since the vertices

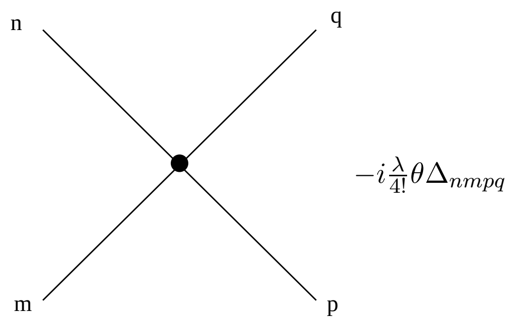


Figure 2.1: Feynman Rule for Φ^4 theory.

describe always local interactions, all of them must conserve momentum. This is not true for correlators of the theory, as for instance the propagators or in general any n-point Green function, because they are extended objects. From Eq. (2.10) one can see that each mode $\phi^{(n)}$ is associated with a stationary wave in the fifth dimension (the cosine in its exponential form contains $e^{i\frac{n}{R}y}$ and $e^{-i\frac{n}{R}y}$ with the same amplitude). This is consistent with the fact that the fifth dimension is compactified because it shows that the momentum in the fifth direction is quantized, $p^4 = \pm n/R$. The term Φ^4 in the original Lagrangian couples locally this waves. The function Δ_{nmpq} in the Feynman rule just checks if there is a choice among all the possible momenta in the vertex that preserves momentum.

Quantization of momentum in the direction of the extra dimension provides another way of understanding why the masses of the modes are m_n; recall that due to Eq. (2.2) $p^\alpha p_\alpha = m^2$, so

$$\begin{aligned} p_\alpha p^\alpha &= m^2 \\ E^2 - |\vec{p}|^2 - p_4^2 &= m^2 \\ p_\mu p^\mu &= m^2 + p_4^2 \\ p^2 &= m^2 + m_n^2 \end{aligned} \tag{2.16}$$

As stated, momentum is violated by quantum corrections because these are inherently non-local, thus the masses, i.e. the spectrum of the theory, will also get radiative corrections $m_n = n/R + \delta_n^{1-loop}$, and in general it ceases to be true that the difference between to consecutive modes is R^{-1}, see Ref. [3].

2.1.2 Spinor field and Yukawa interactions

The Dirac equation in d dimensions reads

$$(i\gamma_\alpha^{(d)}\partial^\alpha - m)\Psi(x^\alpha) = 0 \tag{2.17}$$

where $\alpha = 0, \ldots, 3 + d$. The quanta of the field Ψ are the particles described by it and will obey the dispersion relation $p^\alpha p_\alpha = m^2$ provided the $\gamma^{(d)}$ matrices obey the Clifford algebra

$$\{\gamma_\alpha^{(d)}, \gamma_\beta^{(d)}\} = 2g_{\alpha\beta}\mathbb{1} \tag{2.18}$$

where $g^{\alpha\beta} = diag(+1, -1, \ldots, -1)$. So the problem of constructing representations of the Lorentz group in d dimensions is equivalent to looking for a set of matrices that satisfy

Eq. (2.18). One important outcome is that the number of γ matrices is equal to the number of dimensions since they always come paired with a derivative, Eq. (2.17).

In this work we concentrate almost exclusively on the case of five dimensions, so we will not look for representations in an arbitrary dimension d, this can be found for instance in Ref. [4]. Instead we will look for a set of five gamma matrices, $\gamma_\alpha^{(5)}$, denoted for simplicity in the following by Γ_α, that fulfil Eq. (2.18). From now on, the indices denoted by the first letters in the Greek alphabet will take the values $\alpha, \beta = 0, \ldots, 4$, while as usual $\mu, \nu = 0, \ldots, 3$. The Γ's can be found in terms of the usual γ matrices. It is easy to check that the assignments $\Gamma_\mu = \gamma_\mu$ and $\Gamma_4 = i\gamma^5$ indeed work, where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

The Dirac's equation (2.17) can be obtained from the Lagrangian density

$$\mathcal{L}^5 = \overline{\Psi}(i\Gamma^\alpha \partial_\alpha - m)\Psi. \tag{2.19}$$

Of course, it is equivalent to solving the equation of motion of Eq. (2.17) and to working with the action obtained from Eq. (2.19), but working with $\mathcal{L}^5$ will simplify the calculations, hence we will use it in the following.

First of all, notice that Ψ is a four component spinor, even if it is associated with a five dimensional representation. If we assume that the extra dimension is compactified on a sphere, S^1, then the field must be periodic in y and can be expanded in its Fourier modes

$$\Psi(x^\mu, y) = \psi^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \eta^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right) + \varepsilon^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right) \tag{2.20}$$

Integrating on the extra dimension and rescaling the fields

$$\psi^{(0)} \to \frac{1}{\sqrt{\pi R}}\psi^{(0)} \qquad \eta^{(n)} \to \sqrt{\frac{2}{\pi R}}\,\eta^{(n)} \qquad \varepsilon^{(n)} \to \sqrt{\frac{2}{\pi R}}\,\varepsilon^{(n)}, \tag{2.21}$$

the four-dimensional Lagrangian density reads

$$\mathcal{L} = \overline{\psi}^{(0)}(i\partial\!\!\!/ - m)\psi^{(0)} + \sum_{n=1}^{\infty} \overline{\eta}^{(n)}(i\partial\!\!\!/ - m)\eta^{(n)} + \overline{\varepsilon}^{(n)}(i\partial\!\!\!/ - m)\varepsilon^{(n)} + m_n(\overline{\eta}^{(n)}\gamma^5\varepsilon^{(n)} - \overline{\varepsilon}^{(n)}\gamma^5\eta^{(n)}) \tag{2.22}$$

the fields η and ε are four-component spinors because so it was Ψ. Using this, one can write them in terms of their chirality components, $\eta = \eta_R + \eta_L$ and similarly for ε.

$$\mathcal{L} = \overline{\psi}^{(0)}(i\partial\!\!\!/ - m)\psi^{(0)} + \sum_{n=1}^{\infty} \overline{\eta}^{(n)} i\partial\!\!\!/ \eta^{(n)} + \overline{\varepsilon}^{(n)} i\partial\!\!\!/ \varepsilon^{(n)} - \left[\begin{array}{cc} \overline{\eta}_L^{(n)} & \overline{\varepsilon}_L^{(n)} \end{array}\right] \left[\begin{array}{cc} m & -m_n \\ m_n & m \end{array}\right] \left[\begin{array}{c} \eta_R^{(n)} \\ \varepsilon_R^{(n)} \end{array}\right] + \text{h.c.} \tag{2.23}$$

The mass matrix has to be diagonalized with a bi-unitary transformation. If we call M the mass matrix in Eq. (2.23) then $U^\dagger M V = m_D$. It turns out that $U = \mathbb{1}$ because $M^\dagger M$ is diagonal, therefore only the right-handed fields are changed by V:

$$\left[\begin{array}{c} \eta_R^{(n)} \\ \varepsilon_R^{(n)} \end{array}\right] = \frac{1}{\sqrt{m^2 + m_n^2}} \left[\begin{array}{cc} m & m_n \\ -m_n & m \end{array}\right] \left[\begin{array}{c} \eta_R'^{(n)} \\ \varepsilon_R'^{(n)} \end{array}\right]. \tag{2.24}$$

Finally, if we define the Dirac (or vector-like) fields $\psi^{(n)} \equiv \eta_R'^{(n)} + \eta_L^{(n)}$ and $\xi^{(n)} \equiv \varepsilon_R'^{(n)} + \varepsilon_L^{(n)}$ the Lagrangian is written

$$\mathcal{L} = \overline{\psi}^{(0)}(i\partial\!\!\!/ - m)\psi^{(0)} + \sum_{n=1}^{\infty} \overline{\psi}^{(n)}(i\partial\!\!\!/ - m'_n)\psi^{(n)} + \overline{\xi}^{(n)}(i\partial\!\!\!/ - m'_n)\xi^{(n)}, \tag{2.25}$$

where as in the case of the boson field $m'_n = +\sqrt{m^2 + m_n^2}$. From the above result one can see that there are two infinite KK towers formed by vector-like spinors, $\psi^{(n)}$ and $\xi^{(n)}$, with masses m_n. Eq. (2.25) also shows that the fundamental low-energy spectrum is formed by a vector-like field $\psi^{(0)}$ which mass is the one appearing in the original Lagrangian.

This poses a serious problem if one wants to identify the fundamental mode with any of the known particles. Specifically, we would want to identify the fundamental modes of a set of fields as the fields appearing in the SM Lagrangian. But to achieve this, it is essential that the fundamental modes are chiral. In four dimensions, a chiral field can be defined as the field that fulfils simultaneously $\gamma^5\Psi = \pm\Psi$ as well as the Dirac equation $i\partial\!\!\!/\Psi = 0$. One could try to impose the same definition in five dimensions, but now the situation is completely different because in four dimensions γ^5 anticommutes with all the γ matrices, a fact that ceases to be true in five dimensions. This is because, in the former case, the representation of the Lorentz group that comes from the Dirac's equation is not irreducible, but it is a direct sum of two irreducible representations that can be distinguished by their different eigenvalues under the action of γ^5. In five dimensions γ^5 is one of the γ matrices, hence it no longer anticommutes with *all* the γ matrices, and as a consequence the two equations can not be fulfilled simultaneously. To see this in detail, let Ψ be a four-component spinor field that fulfils the five dimensional Dirac's equation. Then the transformed field $\gamma^5\Psi$ does not obey Dirac's equation

$$\begin{aligned} i\Gamma^\alpha\partial_\alpha(\gamma^5\Psi) &= 0 && (2.26)\\ \gamma^5(-i\partial\!\!\!/ + i\Gamma^4\partial_y)\Psi &= 0 && (2.27)\\ (i\partial\!\!\!/ - i\Gamma^4\partial_y)\Psi &= 0. && (2.28)\end{aligned}$$

Notice that it is the relative sign between Γ^4 and the rest of γ matrices what prevents $\gamma^5\Psi$ to be a valid solution. But this sign can be reabsorbed by the derivative ∂_y if the action of γ^5 is accompanied by a parity transformation in the fifth direction, i.e. $\Psi'(y) = \gamma^5\Psi(-y)$ will be a solution of the Dirac's equation if previously $\Psi(y)$ is a solution. The presence of a mass term in Eq. (2.26) would invalidate this last conclusion.

This result can be exploited to obtain chiral fundamental modes. Recall that we had assigned the topology of a circumference to the fifth dimension S^1; now suppose that impose in addition that the action computed with the values of the fields in one side $y \in [0, \pi R]$ is the same as in the other $y \in [-\pi R, 0]$, then to extract the physics one only needs to look for extremals of the action S in only one side. This means that the value of a field in one side must be related with the values it takes in the other side by a transformation that is a symmetry of the original Lagrangian, $\Psi(-y) = U\Psi(y)$. For a spinor field we choose $U = \pm\gamma^5$, or what is the same we impose that the combined action of γ^5 and $y \to -y$ should leave Ψ invariant $\Psi(y) = \pm\gamma^5\Psi(-y)$, with this, the Fourier modes are all chiral. Take for concreteness the minus sign

$$\Psi(x^\mu, y) = \psi_L^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \eta_L^{(n)}(x^\mu)\cos\left(\frac{ny}{R}\right) + \varepsilon_R^{(n)}(x^\mu)\sin\left(\frac{ny}{R}\right). \qquad (2.29)$$

So the modes that are even under $y \to -y$ have the same chirality as the fundamental one while the ones that are odd have opposite chirality. Since one only need to know the values that the field takes on one side, say $y \in [0, \pi R]$, this means that the new topology is no other than an orbifold, S^1/Z_2, see previous section. The Lagrangian in five dimensions is taken to be

$$\mathcal{L}_F^5 = \overline{\Psi} i \Gamma^\alpha \partial_\alpha \Psi, \tag{2.30}$$

and it has no mass term because it breaks the orbifold symmetry. After the dimensional reduction the four-dimensional Lagrangian reads

$$\mathcal{L}_F = \overline{\psi}_L^{(0)} i\partial\!\!\!/ \psi_L^{(0)} + \sum_{n=1}^{\infty} \overline{\psi}^{(n)} [i\partial\!\!\!/ - m_n] \psi^{(n)}, \tag{2.31}$$

where $\psi^{(n)} = \eta_L^{(n)} + \varepsilon_R^{(n)}$. So we have finally achieved a chiral fundamental mode, $\psi_L^{(0)}$. On top of it, it has appeared a KK-tower of vector-like fields, $\psi^{(n)}$, with masses m_n.

If instead of the minus sign we had taken the plus, then we would have ended with a similar equation but with sign of the mass reversed, i.e. for each mode we would have obtained $\overline{\psi}^{(n)}[i\partial\!\!\!/ + m_n]\psi^{(n)}$. Of course, the sign of the mass is unobservable. So in addition of the required field redefinitions to obtain a canonical kinetic term, the modes of the fields with a right-handed fundamental mode must be further redefined to get the right sign of the mass.

Now that we have succeeded in constructing a theory with chiral fundamental modes, the next step is to study possible interactions that may involve this kind of fields. We will study in the following the Yukawa couplings for two reasons: it is the easiest interaction between fermions and bosons and because it is present in the SM. The interacting fields will be a spinor field Ψ with a right-handed fundamental mode, another spinor field X with a left-handed fundamental mode and a boson field ϕ even under Z_2. The orbifold topology is assumed. The Lagrangian is taken to be

$$\mathcal{L}_Y^5 = \widetilde{Y}\, \overline{X} \phi \Psi + \text{h.c.} \tag{2.32}$$

which, as we will see, will provide a four-dimensional Yukawa interaction for the zero modes.

The canonical dimensions of the Yukawa coupling constant are $[\widetilde{Y}] = E^{\frac{1}{2}}$; this is the reason why this theory is not renormalizable. Now, after dimensional reduction and field redefinitions to get canonical kinetic terms, the Lagrangian reads

$$\mathcal{L} = Y \phi^{(0)} \overline{X}_R^{(0)} \Psi_L^{(0)} + Y \sum_{n=1}^{\infty} [\phi^{(0)} \overline{X}^{(n)} \psi^{(n)} + \phi^{(n)} \overline{X}^{(0)} \psi^{(n)} + \phi^{(n)} \overline{X}^{(n)} \psi^{(0)}] + \frac{Y}{\sqrt{2}} \sum_{n,p,q=1}^{\infty} \Delta_{npq} \phi^{(n)} \overline{X}^{(p)} \psi^{(q)} + \text{h.c.} \tag{2.33}$$

where $y = \widetilde{y}/\sqrt{\pi R}$ and Δ_{npq} is defined in Eq. (2.15). Notice that there is no vertex that couples the fundamental modes $n = 0$ with only one single mode of the KK tower $n > 0$. The last ones appear at least paired, and as a consequence the energy threshold to produce them using only the fundamental fields is twice the mass of the lightest KK mode, $\sqrt{s} \geq 2m_1$. This feature also appeared when we studied the case of the boson field, and the same reasoning we gave there applies now here.

2.1.3 Vector bosons and gauge theories

In the previous section we have constructed a scenario in which the fundamental modes of the extra dimensional scalar and spinor fields can be identified with the fields in the SM. To achieve the full SM as a low-energy realization of a five-dimensional theory we still need to do a similar construction for the vector fields. In five dimensions these fields have five components, A^α, corresponding to the five possible polarizations. Therefore, to associate A^α to one of the vector fields in the SM we need to remove one of its components from the low-energy spectrum. As we will see, this can be done when we compactify the fifth dimension on the orbifold. The free Lagrangian in five dimensions is taken to be

$$\mathcal{L}_G^5 = -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} = -\frac{1}{4}(\partial^\alpha A^\beta - \partial^\beta A^\alpha)(\partial_\alpha A_\beta - \partial_\beta A_\alpha). \tag{2.34}$$

To perform the dimensional reduction we impose that the fifth component of the field A_4 is odd under the action of Z_2 while the rest A_μ are even. On the following the fifth components will be denoted by A_5 because it is done so in the literature.

$$\begin{aligned} A_\mu(x^\mu, -y) &= A_\mu(x^\mu, y), & (2.35)\\ A_5(x^\mu, -y) &= -A_5(x^\mu, y). & (2.36)\end{aligned}$$

The above prescription is not gauge invariant, it breaks gauge symmetry in five dimensions but preserves it in four. Different components of the strength tensor transform differently:

$$\begin{aligned} F^{\mu\nu} &\to F^{\mu\nu}, & (2.37)\\ F^{\alpha\nu} &\to -F^{\alpha\nu}. & (2.38)\end{aligned}$$

Despite of this, $\mathcal{L}_G^5$ is invariant under Eq.(2.35,2.36), as it should, since both sides of S^1 must be physically equivalent. After dimensional reduction the Lagrangian reads

$$\mathcal{L}_G = -\frac{1}{4}F^{(0)}\cdot F^{(0)} + \sum_{n=1}^{\infty} -\frac{1}{4}F^{(n)}\cdot F^{(n)} + \frac{1}{2}m_n^2 A^{(n)\mu}A_\mu^{(n)} + \frac{1}{2}\partial_\mu A_5^{(n)}\partial^\mu A_5^{(n)} + m_n^2\partial_\mu A_5^{(n)}A^{(n)\mu}, \tag{2.39}$$

where $F_{\mu\nu}^{(n)} \equiv \partial_\mu A_\nu^{(n)} - \partial_\mu A_\nu^{(n)}$ and m_n is defined in Eq. (2.6). $\mathcal{L}_G$ shows that the A_5 components are not present in the low-energy spectrum. Moreover, the modes $A_\mu^{(n)}$ have acquired a mass , m_n, $A_5^{(n)}$ being the Goldstone boson eaten by $A_\mu^{(n)}$. The appearance of a tree-level mixing between $A_\mu^{(n)}$ and $A_5^{(n)}$ suggest a gauge fixing term of the form:

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}(\partial_\mu A^{(n)\mu} - \xi m_n A_5^{(n)})^2. \tag{2.40}$$

This is very similar to the usual R_ξ type of gauge fixing, known from the SM. When $\mathcal{L}_{gf}$ is taken in this way the propagators are

$$\begin{aligned} A_\mu^{(0)} &: \quad = \tfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\tfrac{k^\mu k^\nu}{k^2}\right],\\ A_\mu^{(n)} &: \quad = \tfrac{-i}{k^2-m_n^2}\left[g^{\mu\nu} - (1-\xi)\tfrac{k^\mu k^\nu}{k^2-\xi m_n^2}\right],\\ A_5^{(n)} &: \quad = \tfrac{i}{k^2-\xi m_n^2}. \end{aligned} \tag{2.41}$$

Eq. (2.40) fixes the gauge *after* compactification. One may wonder if it is equivalent to fixing it *before* compactification. We will not study this issue in detail here, we refer the interested reader to Ref.[5], where it is treated. It is shown there that the choices of the Feynman gauge and unitary gauge in Eq. (2.40) can be obtained through suitable choices in the gauge fixing terms before compactification, while there is no such possibility for the Landau gauge, thus to avoid any problem we will work always in the Feynman gauge.

Notice that the aim of the orbifold topology is to remove from the low-energy spectrum the zero mode of the fifth component of the gauge field, $A_5^{(0)}$, but as we will see, its presence would offer an interesting possibility. It could be present as a *massless* scalar if we compactify in a circumference S^1. The extra dimensional gauge symmetry forbids a mass term for it. Nevertheless, we have seen that the compactification breaks this symmetry, therefore radiative corrections will provide it mass. When these corrections are studied they are found to be finite. The UV divergences are not present because at very high energies, very small distances (compared to the compactification radius), all the dimensions are equivalent and $A_5^{(0)}$ is just one component of a gauge field and therefore it can not receive any contribution to its mass because of gauge symmetry. All the contributions to its mass came from the low energy. This situation is very similar to the way in which SUSY protects the mass of Higgs. In this case the Higgs is associated via the SUSY symmetry with a fermionic field and the chiral symmetry protects the mass terms for both superpartners. Basically, this is the idea behind the work done in [6, 7, 8, 9] . More specifically, the Higgs field, a scalar field, can be associated to a gauge field A_α and with this, its mass can be protected by gauge symmetry. Despite the interest of these kind of models we will not follow them. Instead we will continue using the orbifold topology without the $A_5^{(0)}$ field in our spectrum.

The vector fields appear in general in gauge theories. As an example of a five dimensional gauge theory we develop briefly in the next lines a theory that reduces to a version of QED after compactification. Its low energy consists in one massless chiral fermion coupled with a massless photon via a gauge interaction, a more detailed derivation is given in Ref.[5]. The Lagrangian is written as

$$\mathcal{L}^5 = \mathcal{L}_G^5 + \overline{\Psi} i \not{D} \Psi, \tag{2.42}$$

where $\mathcal{L}_G^5$ is defined in Eq. (2.34) and the five dimensional covariant derivative is defined as $D_\alpha = \partial_\alpha - i\tilde{e}A_\alpha$. After compactification the Lagrangian can be decomposed in the sum of three terms $\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_I$, where $\mathcal{L}_G$ and $\mathcal{L}_F$ are defined in Eq. (2.39) and Eq. (2.31) respectively, and the interaction term can be divided in three pieces $\mathcal{L}_I = \mathcal{L}_I^0 + \mathcal{L}_I^{0K} + \mathcal{L}_I^K$

$$\mathcal{L}_I^0 = e\,\overline{\psi}_L^{(0)} \not{A}^0 \psi_L^{(0)} \tag{2.43}$$

$$\mathcal{L}_I^{0K} = e\sum_{n=1}^{\infty} \overline{\psi}^{(n)} \not{A}^{(0)} \psi^{(n)} + e\sum_{n=1}^{\infty}\left[\overline{\psi}_L^{(0)} \not{A}^{(n)} \psi_L^{(n)} + i\overline{\psi}_L^{(0)} A_5^{(n)} \psi_R^{(n)} + \text{h.c.}\right] \tag{2.44}$$

$$\mathcal{L}^K = \frac{e}{\sqrt{2}} \sum_{n,m=1}^{\infty} \left[\overline{\psi}^{(n+m)} \not{A}^{(m)} \psi^{(n)} - i\overline{\psi}^{(n+m)} A_5^{(m)} \psi^{(n)} + \text{h.c.}\right] \tag{2.45}$$

$$\frac{e}{\sqrt{2}} \sum_{n,m=1}^{\infty} \left[\overline{\psi}^{(m)} \not{A}^{(n+m)} \gamma^5 \psi^{(n)} + i\overline{\psi}^{(m)} A_5^{(n+m)} \gamma^5 \psi^{(n)}\right], \tag{2.46}$$

where we have rewritten the five-dimensional coupling $\tilde{e}$ in terms of the four dimensional e as $e = \tilde{e}/\sqrt{\pi R}$. Notice that, as mentioned, the low-energy (below R^{-1}) effective theory corresponds exactly to QED with one chiral fermion. In addition, all the vertices conserve the KK number as it happened in the previous examples, hence KK modes must be created in pairs from the fundamental modes what reduces its impact on the low-energy phenomenology because the effective Lagrangian only receives contributions at the one-loop level. The Feynman rules can be easily read from the above formulae.

Chapter 3

SM with one universal extra dimension

Until now we have constructed a number of toy models that have helped us to study some features of the theories with one extra dimension. In particular we have shown that the fundamental modes of the different fields remain in the low-energy spectrum after compactification. If the topology of the extra dimension is taken to be an orbifold then the fundamental modes of the spinor fields are chiral and the ones of the vector fields have only four components. We exploit these results here to build a five dimensional model, which was initially proposed in Ref. [10], that after compactification reduces to the SM. We study the phenomenology and use the results to set bounds on the compactification scale, R, or what is the same, to the mass of the first KK mode, to be denoted by $M = R^{-1}$. We concentrate on the observables with a strong dependence on the mass of the top-quark, m_t, for which the corrections to SM will be enhanced. Although the process $b \to s\gamma$ has no m_t enhancement, the relative impact of the new physics is also important because it is one-loop suppressed in the SM due to gauge invariance, hence we will also study it.

3.1 The model

We use the above considerations to simplify the Lagrangian of the model, and in what follows all mass scales below m_t will be neglected. On the other hand, considering that we are not interested in strong processes, we concentrate only on the gauge group $SU(2)_L \times U(1)_Y$. The Lagrangian is separated in four pieces

$$\mathcal{L}^{\text{UED}} = \int_0^{L=\pi R} dy(\mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y). \tag{3.1}$$

The gauge piece is defined to be

$$\mathcal{L}_G = -\frac{1}{4}F^a \cdot F^a - \frac{1}{4}F \cdot F \tag{3.2}$$

where $F^a_{\alpha\beta}$ is the five dimensional gauge field strength associated with the $SU(2)_L$ gauge group and $F_{\alpha\beta}$ is the one of the $U(1)_Y$ group

$$\begin{aligned} F^a_{\alpha\beta} &= \partial_\alpha W^a_\beta - \partial_\beta W^a_\alpha + \tilde{g}\epsilon^{abc}W^b_\alpha W^c_\beta && (3.3)\\ F_{\alpha\beta} &= \partial_\alpha B_\beta - \partial_\beta B_\alpha && (3.4)\end{aligned}$$

The Higgs piece is

$$\mathcal{L}_H = (D_\alpha H)^\dagger D^\alpha H - V(H), \tag{3.5}$$

and the covariant derivative is defined as $D_\alpha \equiv \partial_\alpha - i\widetilde{g}W^a_\alpha T^a - i\widetilde{g}' B_\alpha Y$, where $\widetilde{g}$ and $\widetilde{g}'$ are the (dimension-full) gauge coupling constants of $SU(2)_L$ and $U(1)_Y$ respectively in the five dimensional theory, T^a and Y are the generators of these groups.

The fermionic piece is

$$\mathcal{L}_F = \overline{Q}(i\Gamma^\alpha D_\alpha)Q + \overline{U}(i\Gamma^\alpha D_\alpha)U + \overline{D}(i\Gamma^\alpha D_\alpha)D, \tag{3.6}$$

where Γ_α are the five dimensional gamma matrices. The fields Q, D and U carry a generational index that is not explicitly written here.

Finally, the Yukawa piece reads

$$\mathcal{L}_Y = -\overline{Q}\widetilde{Y}_u H^c U - \overline{Q}\widetilde{Y}_d H D + \text{h.c.} \tag{3.7}$$

where $H^c = i\sigma^2 H^*$ is the usual charge conjugated field and the $\widetilde{Y}_u$ are the Yukawa matrices in the five dimensional theory, which, as usual, mix different generations.

We use the topology of the extra dimension, assumed to be an orbifold S^1/Z_2, and expand the fields in a Fourier series

$$\begin{aligned}
G_\mu &= \frac{1}{\sqrt{\pi R}} G_\mu^{(0)} + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G_\mu^{(n)} \cos\left(\frac{ny}{R}\right) && (3.8)\\
G_5 &= \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G_5^{(n)} \sin\left(\frac{ny}{R}\right) && (3.9)\\
Q &= \frac{1}{\sqrt{\pi R}} Q_L^{(0)} + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \left[Q_L^{(n)} \cos\left(\frac{ny}{R}\right) + Q_R^{(n)} \sin\left(\frac{ny}{R}\right)\right] && (3.10)\\
U &= \frac{1}{\sqrt{\pi R}} U_R^{(0)} + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \left[U_R^{(n)} \cos\left(\frac{ny}{R}\right) + U_L^{(n)} \sin\left(\frac{ny}{R}\right)\right] && (3.11)
\end{aligned}$$

where the expansion for G_μ is valid for each component of the gauge fields as well as for the Higgs doublet, the one for G_5 is valid for the fifth component of the gauge fields. Analogously, the expansion for U is valid also for D. We have included different normalization for the modes to obtain canonical kinetic terms after compactification.

3.1.1 The spectrum of the model

To make any calculation we require the spectrum of the model. In order to extract it, the Higgs sector has to be studied. It will undergo SSB, hence it will contribute to the masses of the different particles. The Higgs doublet is parametrized as

$$H = \begin{bmatrix} \Phi^+ \\ \Phi^0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -i(\phi_1 - i\phi_2) \\ \phi_0 + i\phi_3 \end{bmatrix}, \tag{3.12}$$

where ϕ_i are real fields. As a Higgs potential it is chosen

$$V(H) = -\mu^2 H^\dagger H + \frac{\widetilde{\lambda}}{4!}(H^\dagger H)^2, \tag{3.13}$$

where μ^2 is a real positive parameter with mass dimensions and $\widetilde{\lambda}$ is a real parameter with dimension E^{-1}. After dimensional reduction the potential contains a number of couplings between the different KK modes. Here we show only those that are relevant

$$\begin{aligned} V &= -\mu^2 H^{(0)\dagger} H^{(0)} + \frac{\lambda}{4!}(H^{(0)\dagger}H^{(0)})^2 + \sum_{n=1}^{\infty}(-\mu^2 + m_n^2)H^{(n)\dagger}H^{(n)} + & (3.14) \\ &+ \sum_{n=1}^{\infty}\frac{\lambda}{4!}\left[(H^{(0)\dagger}H^{(0)})(H^{(n)\dagger}H^{(n)}) + (H^{(0)\dagger}H^{(n)})^2 + (H^{(0)\dagger}H^{(n)})(H^{(n)\dagger}H^{(0)}) + \text{h.c.}\right], & (3.15) \end{aligned}$$

where $\lambda \equiv \widetilde{\lambda}/(\pi R)$. The first thing to notice is that the potential for the fundamental mode induces SSB only for the zeroth mode, $H^{(0)}$, since for $n > 0$ we expect reasonably $m_n^2 > \mu^2$. This is consistent with the association of the fundamental mode of H to the SM Higgs doublet. Following with this idea, the neutral component of the doublet will get a VEV, specifically, $\langle\phi_0^{(0)}\rangle_0 = v$, what implies

$$\phi_0^{(0)}(x^\mu) = v + h(x^\mu), \qquad \langle H^{(0)}\rangle_0 = \frac{v}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{3.16}$$

On the contrary, the modes of the KK tower ($n > 0$) do not undergo SSB because their masses before SSB are positive. Nevertheless, the terms in Eq. (3.15) will modify their masses after SSB. At the end, the masses of the Higgs field and its associated KK tower are

$$m^2(h) = 2\mu^2 \equiv m_h^2, \qquad m^2(\phi_0^{(n)}) = m_h^2 + m_n^2, \qquad n \geq 1. \tag{3.17}$$

Recall that the fields $\phi_0^{(n)}$, for $n \geq 1$ do not get a VEV. For the rest of the fields the masses are

$$m^2(\Phi^{\pm(n)}) = m^2(\phi_3^{(n)}) = m_n^2, \qquad n \geq 0. \tag{3.18}$$

If our interpretation is correct, the fields $\Phi^{\pm(0)}$ and $\phi_3^{(0)}$ will be precisely the SM Goldstone bosons absorbed by $W^\pm$ and Z, the fact that they are massless is also consistent with the identification of $H^{(0)}$ with the SM Higgs doublet.

In the gauge sector, this SSB is also relevant. In addition, it is easy to convince oneself that this model coincides exactly with the SM when only zero modes are taken into account, what accounts for taking the limit $m_n \to \infty$. Thus, retaining only the zero modes, all goes much in the same way as in SM: the neutral component of the Higgs doublet gets a VEV and induces mixing between $W_{\mu 3}^{(0)}$ and $B_\mu^{(0)}$ to give a massless gauge boson, the photon $A_\mu^{(0)}$, and a massive one, the Z boson $Z_\mu^{(0)}$, the mixing being parametrized by the weak mixing angle

$$\begin{aligned} Z_\mu^{(0)} &= \cos\theta_w\, W_{\mu 3}^{(0)} - \sin\theta_w\, B_\mu^{(0)} & (3.19) \\ A_\mu^{(0)} &= \sin\theta_w\, W_{\mu 3}^{(0)} + \cos\theta_w\, B_\mu^{(0)} & (3.20) \end{aligned}$$

and the same mixing is generated for $A_\mu^{(n)}$ and $Z_\mu^{(n)}$. But we will concentrate on the charged gauge bosons because they will appear in our calculations. After the dimensional reduction, the bilinear terms relevant for the gauge sector are

$$-\frac{1}{4}F^a_{\alpha\beta}F^{a\alpha\beta}+(D^\alpha H)^\dagger(D_\alpha H)\xrightarrow{D.R.} - \frac{1}{4}F^{(n)}_{\mu\nu}F^{\mu\nu(n)}+(m_n^2+M_W^2)W^{+(n)}_\mu W^{\mu-(n)} \quad (3.21)$$
$$+ \partial_\mu W_5^{+(n)}\partial^\mu W_5^{-(n)}-M_W^2W_5^{+(n)}W_5^{-(n)} \quad (3.22)$$
$$+ \partial_\mu\Phi^{+(n)}\partial^\mu\Phi^{-(n)}-m_n^2\Phi^{+(n)}\Phi^{-(n)} \quad (3.23)$$
$$+ W^{\mu(n)-}\partial_\mu(iM_W\Phi^{+(n)}+m_nW_5^{+(n)})+\text{h.c.} \quad (3.24)$$
$$+ iM_Wm_nW_5^{-(n)}\Phi^{+(n)}+\text{h.c.}, \quad (3.25)$$

where the sum on n is implicit and we have used the tree level relation $gv/2 = M_W$. The previous equations can be understood as follows: the first two show that the vector bosons $W_\mu^{(n)}$ are now massive with mass $\sqrt{M_W^2+m_n^2}$, while the fifth components KK modes, $W_5^{(n)}$, have became massive charged scalars with masses M_W. Nevertheless, Eq. (3.24) and Eq. (3.25) show that $W_5^{(n)}$ have not diagonal mass terms, because they mix with the modes of the charged component of the Higgs doublet, $\Phi^{+(n)}$. In particular, Eq. (3.24) points out the combination that defines the Goldstone field, Φ_G^+, that is been absorbed by the $W_\mu^{+(n)}$ fields to get masses. The orthogonal combination, $\Phi_P^{+(n)}$, is a physical scalar. The expressions that relate those fields are

$$\Phi_G^{+(n)} = \frac{m_nW_5^{+(n)}+iM_W\Phi^{+(n)}}{\sqrt{m_n^2+M_W^2}} \overset{M_W\to 0}{\longrightarrow} W_5^{+(n)}, \quad (3.26)$$

$$\Phi_P^{+(n)} = \frac{iM_WW_5^{+(n)}+m_n\Phi^{+(n)}}{\sqrt{m_n^2+M_W^2}} \overset{M_W\to 0}{\longrightarrow} \Phi^{+(n)}. \quad (3.27)$$

These formulae are valid only for $n \geq 1$. In the limit of neglecting all mass scales below m_t, the mixing is not important and $W_5^{+(n)}$ can be identified with the Goldstone bosons, $\Phi_G^{+(n)}$, and $\Phi^{+(n)}$ with the physical scalars, $\Phi_P^{+(n)}$.

We pass now to study the quark sector, in particular, the third generation because it contains the top. To distinguish between the up and down components of the five-dimensional doublet Q in Eq. (3.10) we will use subindices that will carry also information about the generation, *e.g.* when we write

$$Q = \begin{bmatrix} Q_t \\ Q_b \end{bmatrix} \quad (3.28)$$

we are denoting by Q_t (Q_b) the up (down) component of the weak doublet in the *third* generation. Analogously by U_t we denote the weak singlet of the third generation. After dimensional reduction $Q_t^{(n)}$ acquires a mass m_n and $U_t^{(n)}$ a mass $-m_n$, $n > 0$. These are Dirac fermions defined as $Q_t^{(n)} = Q_{tR}^{(n)} + Q_{tL}^{(n)}$. The masses receive also contributions coming from the couplings with $H^{(0)}$, which are contained in the Yukawa sector, Eq. (3.7). Here we extract the relevant terms

$$\mathcal{L}_Y = -Y_u\overline{Q}_t^{(0)}H^{(0)c}U_t^{(0)} - Y_u\sum_{n=1}^{\infty}\overline{Q}_t^{(n)}H^{(0)c}U_t^{(n)}+\text{h.c.}+\dots \quad (3.29)$$

$$= -\frac{Y_uv}{\sqrt{2}}\overline{Q}_t^{(0)}U_t^{(0)} - \frac{Y_uv}{\sqrt{2}}\sum_{n=1}^{\infty}\overline{Q}_t^{(n)}U_t^{(n)}+\text{h.c.}+\dots, \quad (3.30)$$

where $Y_u \equiv \widetilde{Y}_u/\sqrt{\pi R}$, and we have used Eq. (3.8). Since the fundamental modes are, by construction, identified with the SM fields, then Y_u must be the SM Yukawa matrix, what implies that the KK modes $Q_t^{(n)}$ and $U_t^{(n)}$ have a mixing proportional to the mass of the top-quark, $m_t = Y_u v/\sqrt{2}$. So the bilinear terms for these fields may be written as

$$\mathcal{L}_t \equiv \bar{t}(i\partial\!\!\!/ - m_t)t + \sum_{n=1}^{\infty} \overline{Q}_t^{(n)} i\partial\!\!\!/ Q_t^{(n)} + \overline{U}_t^{(n)} i\partial\!\!\!/ U_t^{(n)} - \left[\begin{array}{cc} \overline{Q}_t^{(n)} & \overline{U}_t^{(n)} \end{array}\right] \left[\begin{array}{cc} m_n & m_t \\ m_t & -m_n \end{array}\right] \left[\begin{array}{c} Q_t^{(n)} \\ U_t^{(n)} \end{array}\right], \tag{3.31}$$

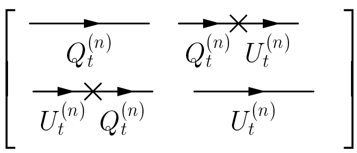
We denote by t the top-quark, $t = Q_{tL}^{(0)} + U_{tR}^{(0)}$. The mass eigenfields, denoted by a prime, are

$$\left[\begin{array}{c} Q_t^{(n)} \\ U_t^{(n)} \end{array}\right] = \left[\begin{array}{cc} \cos(\alpha_n) & -\sin(\alpha_n) \\ \sin(\alpha_n) & \cos(\alpha_n) \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 0 & \gamma^5 \end{array}\right] \left[\begin{array}{c} Q_t'^{(n)} \\ U_t'^{(n)} \end{array}\right], \tag{3.32}$$

where $\tan(2\alpha_n) \equiv m_t/m_n$ and the γ^5 is included to obtain a positive mass of the $U_t'^{(n)}$. Finally, the masses are

$$m(Q_t'^{(n)}) = m(U_t'^{(n)}) = \sqrt{m_n^2 + m_t^2} \equiv m_Q \qquad n > 0. \tag{3.33}$$

In the calculations of this work, the degrees of freedom $Q_t'^{(n)}$ and $U_t'^{(n)}$ will appear inside loops, hence it will be advantageous to work with the fields $Q_t^{(n)}$ and $U_t^{(n)}$, we call the latter the *interaction base*. Since they have not definite mass, the inverse of their quadratic forms, i.e. their propagators, are not diagonal. In this base, the expression of the couplings is simpler, but the propagators are non-canonical, and the next expressions must be used for them



$$= \left[\begin{array}{cc} i\frac{p\!\!\!/ + m_n}{p^2 - m_Q^2} & i\frac{m_t}{p^2 - m_Q^2} \\ i\frac{m_t}{p^2 - m_Q^2} & i\frac{p\!\!\!/ - m_n}{p^2 - m_Q^2} \end{array}\right]$$

3.1.2 Couplings

We will compute the dominant corrections to the modifications of the ρ parameter, which can be found by calculating the radiative corrections to the self energies of the gauge bosons $W_{\mu 1}^{(0)}$ and $W_{\mu 3}^{(0)}$. To this end, we need to extract the couplings of $W_{\mu 1(3)}^{(0)}$ with the KK modes of the Q fields because the m_t proportional contributions come exclusively from them. This is so because the mass of the top, m_t, only appears in the propagators of the Q and U fields and in the vertices proportional to the Yukawa matrices, but neither this vertices contribute at one loop in UED[1], nor do the U fields, which do not even couple directly to $W_{\mu 1(3)}^{(0)}$. Therefore, the relevant couplings are

$$\mathcal{L}_\rho = \frac{g}{2} W_{\mu 1}^{(0)} \left[\overline{Q}_t^{(n)} \gamma^\mu Q_b^{(n)} + \overline{Q}_b^{(n)} \gamma^\mu Q_t^{(n)}\right] + \frac{g}{2} W_{\mu 3}^{(0)} \left[\overline{Q}_t^{(n)} \gamma^\mu Q_t^{(n)}\right], \tag{3.34}$$

where $g = \widetilde{g}/\sqrt{\pi R}$. Notice that for simplicity we have not considered the CKM mixing matrix[2].

[1]Because in the limit of $M_W \to 0$ the Higgs doublets are not eaten by the gauge bosons, in contrast with what happens in SM.

[2]We have explicitly checked that the calculations give exactly the same result when V_{CKM} is taken into account and that this is numerically so because in our approximation all quark masses are zero except m_t.

The couplings of the physical scalar $\Phi^{+(n)}$ with the modes of the top quark are also important because they are proportional to m_t.

$$\mathcal{L}_Y = \frac{\sqrt{2}}{v} m_t V_{tj} \overline{U}_R^{(n)} Q_{j\,L}^{(0)} \Phi^{(n)+} + \text{h.c.} \tag{3.35}$$

In this case, it has been maintained the CKM mixing matrix because we will need it to cast the modifications in the $B^0 - \overline{B}^0$ mixing in a standard form, which is defined to bound any new physics affecting this mixing. Notice that although $\Phi^{(n)+}$ are physical degrees of freedom their couplings are exactly the same as the Goldstone bosons of the SM.

We are also interested in the radiative corrections to the $Z \to b\bar{b}$ decay, therefore we need to know the couplings of $Z_\mu = Z_\mu^{(0)}$.

$$\mathcal{L}_Z = \frac{g}{2c_w} Z_\mu [J_{SM}^\mu + J^{\mu(n)} + J_\Phi^{\mu(n)}], \tag{3.36}$$

where the J_{SM}^μ is the usual SM neutral current

$$J_{SM}^\mu = \overline{Q}_L \gamma^\mu 2[T^3 - s_w^2 Q] Q_L + \overline{U}_R \gamma^\mu 2[T^3 - s_w^2 Q] U_R. \tag{3.37}$$

Analogously we find for $J^{\mu(n)}$, $n \geq 1$:

$$J^{\mu(n)} = \overline{Q}^{(n)} \gamma^\mu 2[T^3 - s_w^2 Q] Q^{(n)} + \overline{U}^{(n)} \gamma^\mu 2[T^3 - s_w^2 Q] U^{(n)}. \tag{3.38}$$

If we indicate explicitly the charges

$$\begin{array}{lcl@{\qquad}lcl} T^3 Q_t^{(n)} & = & +\frac{1}{2} Q_t^{(n)} & Y Q_t^{(n)} & = & +\frac{1}{6} Q_t^{(n)} \\ T^3 Q_b^{(n)} & = & -\frac{1}{2} Q_b^{(n)} & Y Q_b^{(n)} & = & +\frac{1}{6} Q_b^{(n)} \\ T^3 U^{(n)} & = & 0 & Y U^{(n)} & = & +\frac{2}{3} U^{(n)} \end{array} \tag{3.39}$$

the current reads

$$J^{\mu(n)} = \left(+1 - \frac{4}{3} s_w^2\right) \overline{Q}_t^{(n)} \gamma^\mu Q_t^{(n)} + \left(-1 + \frac{2}{3} s_w^2\right) \overline{Q}_b^{(n)} \gamma^\mu Q_b^{(n)} + \left(-\frac{4}{3} s_w^2\right) \overline{U}^{(n)} \gamma^\mu U^{(n)}. \tag{3.40}$$

Finally, the couplings with the KK modes of the Higgs doublet charged components are

$$J_\Phi^{\mu(n)} = (-1 + 2s_w^2) \Phi^{+(n)} i\partial^\mu \Phi^{-(n)} + \text{h.c.} \tag{3.41}$$

From Eq. (3.37), Eq. (3.40) and Eq. (3.41) it is now straightforward to extract the correspondent Feynman rules for the couplings with the Z. The couplings of the photon can be derived similarly.

3.2 Phenomenology

The detection of the first members of the KK towers would be a compelling signature in favor of extra dimensions. But until now, there is no direct detection of any member of these towers in the experiments. This means that we have to look for their contributions to observables through virtual production. Since these are expected to be small, the best places to look for them are processes that have been experimentally measured with high degree of precision or that can only proceed through radiative corrections in the SM. Among the formers we will study in the next sections the $Z \to b\bar{b}$ decay, the radiative corrections to the ρ parameter and the $B^0 - \overline{B}^0$ mixing, and among the latter the process $b \to s\gamma$.

3.2.1 Radiative corrections to the $Z \to b\bar{b}$ decay

Shifts in the $Zb\bar{b}$ coupling due to radiative corrections, either from within the SM or from new physics, affect observables such as the branching ratio $R_b = \Gamma_b/\Gamma_h$, where $\Gamma_b = \Gamma(Z \to b\bar{b})$ and $\Gamma_h = \Gamma(Z \to \text{hadrons})$, or the left right asymmetry A_b. These type of corrections can be treated uniformly by expressing them as a modification to the tree level couplings $g_{L(R)}$ defined as

$$\frac{g}{c_W}\bar{b}\gamma^\mu(g_L P_L + g_R P_R) b Z_\mu \ . \tag{3.42}$$

Z and b's are SM fields, $P_{L(R)}$ are the chirality projectors and

$$g_L = -\frac{1}{2} + \frac{1}{3}s_W^2 + \delta g_L^{\rm SM} + \delta g_L^{\rm NP} \ , \tag{3.43}$$

$$g_R = \frac{1}{3}s_W^2 + \delta g_R^{\rm SM} + \delta g_R^{\rm NP} \ , \tag{3.44}$$

where we have separated radiative corrections coming from SM contributions and from new physics, (NP). It turns out that, both within the SM as well as in most of its extensions, only g_L receives corrections proportional to m_t^2 at the one loop level, due to the difference in the couplings between the two chiralities. In particular, a shift δg_L^{NP} in the value of g_L due to new physics translates into a shift in R_b given by

$$\delta R_b = 2R_b(1-R_b)\frac{g_L}{g_L^2+g_R^2}\delta g_L^{\rm NP} \ , \tag{3.45}$$

and to a shift in the left-right asymmetry A_b given by

$$\delta A_b = \frac{4g_R^2 g_L}{(g_L^2+g_R^2)^2}\delta g_L^{\rm NP} \ . \tag{3.46}$$

These equations, when compared with experimental data, will be used to set bounds on the compactification scale.

By far the easiest way to compute the leading top-quark-mass dependent one-loop corrections to δg_L from the SM itself, $\delta g_L^{\rm SM}$, is to resort to the gaugeless limit of the SM [11], *e.g.* the limit where the gauge couplings g and g', corresponding to the gauge groups $SU(2)_L$ and $U(1)_Y$ respectively, are switched off. In that limit the gauge bosons play the role of external sources and the only propagating fields are the quarks, the Higgs field, and the charged and neutral Goldstone bosons $G^\pm$ and G^0. As explained in [12, 13] one may relate the one-loop vertex $Zb\bar{b}$ to the corresponding $G^0 b\bar{b}$ vertex by means of a Ward identity; the latter is a direct consequence of current conservation, which holds for the neutral current before and after the Higgs doublet acquires a vacuum expectation value v.

In practice, carrying out the calculation in the aforementioned limit amounts to the elementary computation of the one-loop off-shell vertex $G^0 b\bar{b}$. In the gaugeless limit and for massless b-quarks the only contribution to this vertex is depicted in Fig. 3.1, where the cross in the top-quark line represents a top-quark mass insertion needed to flip chirality (the diagram with an insertion in the other top-quark line is assumed). This diagram gives a derivative coupling of the Goldstone field to the b-quarks which can be gauged (or related to the Z vertex through the Ward identity) to recover the $Zb\bar{b}$ vertex. Then, one immediately finds

$$\delta g_L^{\rm SM} \approx \frac{\sqrt{2}G_F m_t^4}{(2\pi)^4}\int \frac{i d^4k}{(k^2-m_t^2)^2 k^2} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2} \ , \tag{3.47}$$

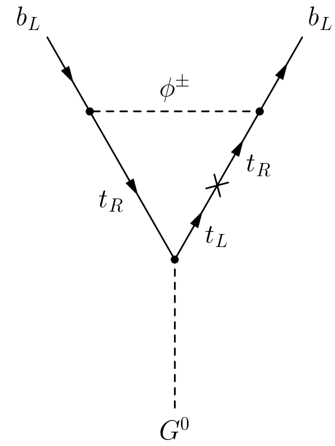


Figure 3.1: The diagram contributing to the SM $G^0 b\bar{b}$ vertex in the gaugeless limit for massless b-quarks.

where G_F is the Fermi constant. The m_t^4 dependence (coming from three Yukawa couplings and one mass insertion) is partially compensated by the $1/m_t^2$ dependence coming from the loop integral.

In the case of a single UED this argument persists: one must simply consider the analog of diagram in Fig. 3.1, where now the particles inside the loop have been replaced by their KK modes, as shown in Fig. 3.2. If we denote by δg_L^{UED} the new physics contributions in the UED model (the SM contributions are not included) the result is

$$\begin{aligned}\delta g_L^{\rm UED} &\approx \frac{\sqrt{2}G_F m_t^4}{(2\pi)^4}\sum_{n=1}^{\infty}\int\frac{id^4k}{(k^2-m_Q^2)^2(k^2-m_n^2)} \\ &= \frac{\sqrt{2}G_F m_t^4}{(4\pi)^2}\sum_{n=1}^{\infty}\int_0^1\frac{dx\,x}{xm_t^2+m_n^2}\approx\frac{\sqrt{2}G_F m_t^4}{(4\pi)^2}\frac{\pi^2R^2}{12}\ ,\end{aligned} \tag{3.48}$$

and depends *quartically* on the mass of the top quark. Notice that there are several differences with respect to the SM: (i) The cross now represents the mixing mass term between $Q_t^{(n)}$ and $U_t^{(n)}$, which is proportional to m_t; (ii) The $\Phi^{\pm(n)}$, for $n \neq 0$, are essentially the physical KK modes of the charged Higgses as shown in Eq.(3.27); (iii) From the virtual momentum integration one obtains now a factor $1/m_n^2$, instead of the factor $1/m_t^2$ of the SM case.

This simple calculation allows us to understand easily the leading corrections arising from extra dimensions. A more standard calculation of the $Zb\bar{b}$ vertex in UED yields exactly the same result. In this case the radiative corrections to the $Zb\bar{b}$ vertex stem from the diagrams of Fig. 3.3.

If we neglect the b-quark mass and take $M_Z \ll R^{-1}$, the result, for each mode, can be expressed in terms of a single function, $f(r_n)$, defined as

$$i\mathcal{M}^{(n)} = i\frac{g}{c_w}\frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}f(r_n)\overline{u}'\gamma^\mu P_L u\epsilon_\mu\ , \tag{3.49}$$

where u and u' are the spinors of the b quarks and ϵ_μ stands for the polarization vector of the Z boson. The argument of the function f is $r_n = m_t^2/m_n^2$.

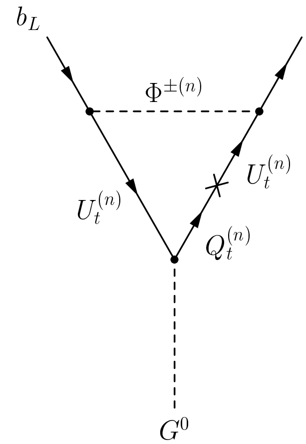


Figure 3.2: The dominant diagram contributing to the UED $G^0 b\bar{b}$ vertex in the gaugeless limit for massless b-quarks.

Although the complete result is finite, partial results are divergent and are regularized by using dimensional regularization. The contributions of the different diagrams in Fig. 3.3 are

$$\begin{aligned} f^{(a)}(r_n) &= \left(1 - \frac{4}{3}s_w^2\right)\left[\frac{r_n - \log(1+r_n)}{r_n}\right] \,, \\ f^{(b)}(r_n) &= \left(-\frac{2}{3}s_w^2\right)\left[\delta_n - 1 + \frac{2r_n + r_n^2 - 2(1+r_n^2)\log(1+r_n)}{2r_n^2}\right] \,, \\ f^{(c)}(r_n) &= \left(-\frac{1}{2} + s_w^2\right)\left[\delta_n + \frac{2r_n + 3r_n^2 - 2(1+r_n)^2\log(1+r_n)}{2r_n^2}\right] \,, \\ f^{(d)}(r_n) + f^{(e)}(r_n) &= \left(\frac{1}{2} - \frac{1}{3}s_w^2\right)\left[\delta_n + \frac{2r_n + 3r_n^2 - 2(1+r_n)^2\log(1+r_n)}{2r_n^2}\right] \,, \end{aligned} \tag{3.50}$$

where $\delta_n \equiv 2/\epsilon - \gamma + \log(4\pi) + \log(\mu^2/m_n^2)$, and μ is the 't Hooft mass scale. From Eq. (3.50) it is straightforward to verify that all terms proportional to δ_n cancel, and so do all terms proportional to s_w^2, as expected from the gaugeless limit result. Thus, finally, the only term which survives is the term in $f^{(a)}(r_n)$ not proportional to s_w^2, yielding the following (per mode) contribution to g_L:

$$\delta g_L^{(n)} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\left[\frac{r_n - \log(1+r_n)}{r_n}\right] \,, \tag{3.51}$$

which is precisely the one obtained from the gaugeless limit calculation, *e.g.* Eq. (3.48) with the elementary integration over the Feynman parameter x already carried out. Notice also that the above result is consistent with the decoupling theorem since the contribution for each mode vanishes when its mass is taken to infinity, i.e. $r_n \to 0$.

In order to compute the effect of the entire KK tower, it is more convenient to first carry out the sum and then evaluate the Feynman parameter integral; this interchange is mathematically legitimate since the final answer is convergent. Thus,

$$\delta g_L^{\rm UED} = \sum_{n=1}^{\infty} \delta g_L^{(n)} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\int_0^1 dx \sum_{n=1}^{\infty} \frac{r_n x}{1 + r_n x} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2} F_{\rm UED}(a) \,, \tag{3.52}$$

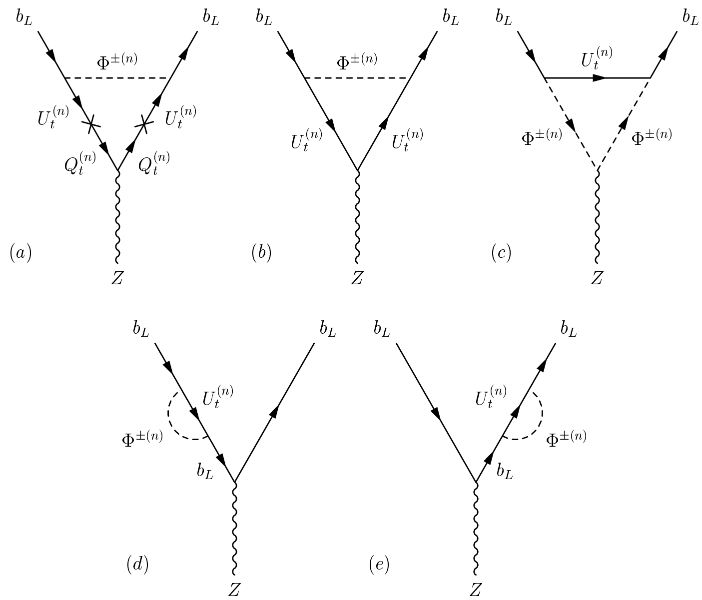


Figure 3.3: Dominant UED contributions to the $Zb\bar{b}$ vertex.

where $a = \pi R m_t$, and the function $F(a)$ is defined in general as

$$F(a) \equiv \frac{\delta g_L^{\rm NP}}{\delta g_L^{\rm SM}}. \tag{3.53}$$

In the case of UED

$$F_{\rm UED}(a) = -\frac{1}{2} + \frac{a}{2}\int_0^1 dx\,\sqrt{x}\coth(a\sqrt{x}) \approx \frac{a^2}{12} - \frac{a^4}{270} + \mathcal{O}(a^6)\ . \tag{3.54}$$

It is instructive to compare the above result with the one obtained in the context of models where the extra dimension is not universal[3]. In particular, in the model considered in Ref. [14] the fermions live in four dimensions, and only the gauge bosons and the Higgs doublet live in five [15]. In this case there is no KK tower for the fermions, and therefore, in the loop-diagrams appear only the SM quarks interacting with the KK tower of the Higgs fields. The result displays a logarithmic dependence on the parameter a, which gives rise to a relatively tight lower bound on R^{-1}, of the order of 1 TeV. Specifically, the corresponding $F(a)$ is given by[4]

$$F(a) = -1 + 2a\int_0^\infty dx \frac{x^2}{(1+x^2)^2}\coth(ax) \approx \left(\frac{2}{3}\log(\pi/a) - \frac{1}{3} - \frac{4}{\pi^2}\zeta'(2)\right) a^2\ , \tag{3.55}$$

where the expansion on the second line holds for small values of a, and ζ' is the derivative of the Riemann Zeta function. The appearance of the $\log(a)$ in $F(a)$ and its absence from $F_{\rm UED}(a)$ may be easily understood from the effective theory point of view: due to the KK-number conservation in UED models, the tree-level low energy effective Lagrangian when

[3] In Chapter 6 we study some of these models.

[4] Note that, unlike in Ref. [14], the $F(a)$ does not include the SM contribution.

all KK modes are integrated out is exactly the Standard Model; there are no additional tree-level operators suppressed by the compactification scale. Since the one-loop logarithmic contributions in the full theory can be obtained in the effective theory by computing the running of operators generated at tree level, it is clear that in the UED no $\log(a)$ can appear at one loop in low energy observables. The situation is completely different if higher dimension operators are already generated at tree level, as is the case of the model considered in Ref. [14], where the leading logarithmic corrections can be computed by using the tree-level effective operators in loops.

We next turn to the bounds on R^{-1}. We will use the SM prediction $R_b^{\rm SM} = 0.21569 \pm 0.00016$ and the experimentally measured value $R_b^{\rm exp} = 0.21664 \pm 0.00068$. Combining Eq. (3.45) and Eq. (3.53), we obtain $F(a) = -0.24 \pm 0.31$, and making a weak signal treatment [16] we arrive at the 95% CL bound $F(a) < 0.39$. The results for a single UED can be easily derived from Eq. (3.54), yielding

$$R^{-1} > 230 \text{ GeV} \qquad 95\%\text{CL}. \tag{3.56}$$

The SM prediction for the left-right asymmetry $A_b^{SM} = 0.9347 \pm 0.0001$, and the measured value $A_b^{exp} = 0.921 \pm 0.020$ gives a looser bound.

3.2.2 Radiative corrections to $b \to s\gamma$

The experimental observable is the semi-inclusive decay $\mathrm{Br}(B \to X_s\gamma)$. Using heavy quark expansion it is found that, up to small bound state corrections, this decay agrees with the parton model rates for the underlaying decays of the b quark [17, 18], $b \to s\gamma$. This flavor violating transition is a very good place to look for new physics because in the SM it is forbidden at tree level due to gauge symmetry, it can though proceed through radiative corrections. From an effective field theory point of view the transition can be understood as due to the generation via radiative corrections of the next effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i \mathcal{O}_i \tag{3.57}$$

where $\mathcal{O}_7$ operator is the one that drives the transition $b \to s\gamma$

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2} m_b \overline{s} \sigma^{\mu\nu} P_R b F_{\mu\nu}. \tag{3.58}$$

Notice that the presence of the strength tensor, $F_{\mu\nu}$, guarantees the gauge invariance of $\mathcal{O}_7$. As usual, the operators encode the low energy physics while the high energy physics is contained in the coefficients, in this case C_7.

In the SM and at the scale of the W mass, $C_7^{SM}(M_W) = -1/2\ A(m_t^2/M_W^2)$, where

$$A(x) = x\left[\frac{\frac{2}{3}x^2 + \frac{5}{12}x - \frac{7}{12}}{(x-1)^3} - \frac{\left(\frac{3}{2}x^2 - x\right)\ln x}{(x-1)^4}\right] \tag{3.59}$$

The leading logarithmic contributions of the two loop calculations are important, these come from standard QCD running from M_W to m_b. The RGE reads [19, 20]

$$C_7(m_b) \approx 0.698\ C_7(M_W) - 0.156\ C_2(M_W) + 0.086\ C_8(M_W), \tag{3.60}$$

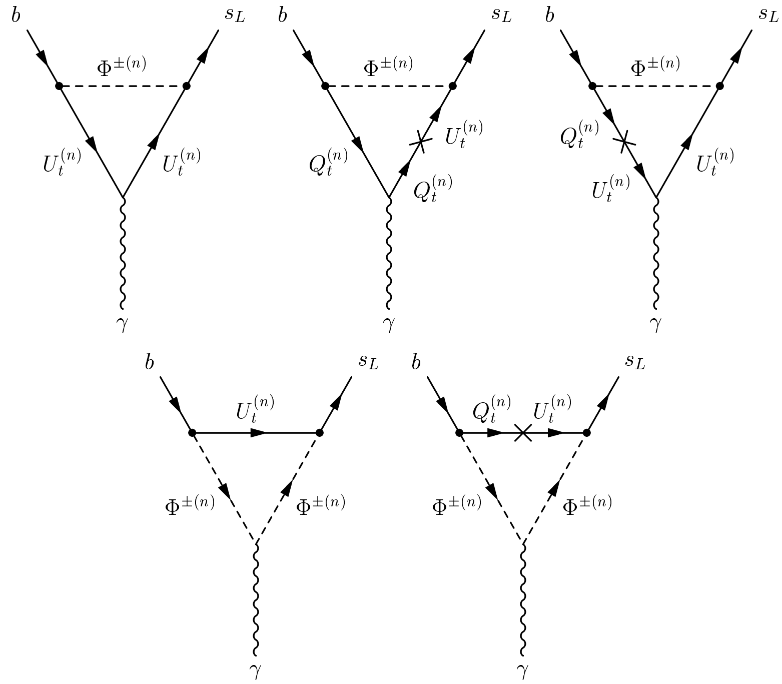


Figure 3.4: Diagrams that contribute to $\mathcal{O}_7$ in UED.

where C_2 and C_8 are the coefficients of the operators $\mathcal{O}_2$ and $\mathcal{O}_8$ respectively, which are defined as

$$\mathcal{O}_2 = [\overline{c}_{L\alpha}\gamma_\mu b_{L\alpha}][\overline{s}_{L\beta}\gamma^\mu c_{L\beta}], \tag{3.61}$$

$$\mathcal{O}_8 = \frac{g_s}{(4\pi)^2} m_b \overline{s}_{L\alpha}\sigma^{\mu\nu}T^a_{\alpha\beta}b_{R\beta}G^a_{\mu\nu}. \tag{3.62}$$

α and β are color indexes. In the case of SM, the contribution of $\mathcal{O}_8$ is negligible, $C_8(M_W) = -0.097$ [20], but the one of $\mathcal{O}_2$, $C_2(M_W) = 1$, turns out to be important.

In UED the transition also proceeds through the same effective Hamiltonian but the coefficient C_7 receives new contributions aside the ones coming from SM, and the running could be different for each case because C_2 and C_8 are also modified. The value of C_7 at the scale M_W stems from the diagrams in Fig. 3.4

There are also diagrams in which the $\Phi^{\pm(n)}$ are replaced by $W_\mu^{(n)}$ and by the non physical degrees of freedom $W_5^{(n)}$ but since the couplings of these are reduced by a factor $(M_W/m_t)^2 \approx 0.22$ we will ignore them and work at this level of precision.

Observe that we have not considered the self energies of the external legs, in opposition to what we did with $Z \to b\overline{b}$ because these diagrams do not contribute to $\mathcal{O}_7$. In addition, they are proportional to m_t and its structure is of the form $\overline{u}\gamma^\mu u$; when all the contributions with this structure are added, they cancel exactly. Hence, the $Z_\mu\overline{b}\gamma^\mu b$ vertex does not appear as it should be since it is forbidden by gauge invariance.

The contribution of the n-th mode to the C_7 coefficient can be written in the form [19]

$$C_7^{(n)} = \frac{m_t^2}{m_t^2 + m_n^2}\left[B\left(\frac{m_t^2 + m_n^2}{m_n^2}\right) - \frac{1}{6}A\left(\frac{m_t^2 + m_n^2}{m_n^2}\right)\right] \tag{3.63}$$

where $B(x)$ is given by

$$B(x) = \frac{x}{2}\left[\frac{\frac{5}{6}x - \frac{1}{2}}{(x-1)^2} - \frac{\left(x - \frac{2}{3}\right)\log x}{(x-1)^3}\right] \tag{3.64}$$

This result can be obtained by direct calculation but the more elegant reasoning given in Ref. [19] is also possible. An expansion of $C_7^{(n)}$ reveals that it is free of logarithms that relate the two different mass scales: m_n and m_t.

$$C_7^{(n)} = \frac{23}{144}\frac{m_t^2}{m_n^2} - \frac{13}{120}\frac{m_t^4}{m_n^4} + \mathcal{O}\left(\frac{m_t^6}{m_n^6}\right). \tag{3.65}$$

This result can be understood by using effective field theory ideas: when the heavy degrees of freedom are integrated out, the tree level effective Lagrangian is exactly the SM, there are no additional tree level operators suppressed by powers of m_n^{-1}. It is well known that the dominant logarithms that may appear relating the two different scales can be recovered from the running of the operators in the low energy effective Lagrangian induced by the presence of the additional operators, since in our case they are not present no logs can appear in Eq. (3.65). Finally, all contributions must be put together

$$C_7^{\rm UED}(M_W) = C_7^{\rm SM}(M_W) + \sum_{n=1}^{\infty} C_7^{(n)}(M_W), \tag{3.66}$$

where we have neglected the running between m_t and M_W, i.e. $C_7^{\rm UED}(m_t) \approx C_7^{\rm UED}(M_W)$.

Once we have determined $C_7(M_W)$ the next step is to apply the RGE given in Eq. (3.60) and compute $C_7(m_b)$. To this end we need $C_2(M_W)$ and $C_8(M_W)$, $C_2(M_W) = 1$, i.e. it takes the SM value because it is a contribution at tree level while the UED contributions are at the one-loop level. In addition, the small coefficient of C_8 in the equation (3.60) and the fact that C_8 is expected to be small allows us to neglect again this term in the running. The modifications to $b \to cl\nu$, the necessity of which will be explained later, are also negligible because UED corrects it again at the one loop level and in the SM it is already corrected at tree level.

To extract the bounds this process sets, it is used the observable

$$\widetilde{\Gamma} = \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to cl\nu)} \tag{3.67}$$

that lacks the m_b dependence and therefore presents smaller uncertainty [21]: 10% for the theoretical value and 15% for the experimental determination (both at 1σ). When compared, it is found that if a 95% CL is required the current bounds allow a modification as big as a 36% with respect to the SM value [19], i.e. $|\widetilde{\Gamma}^{total}/\widetilde{\Gamma}^{SM} - 1| \leq 0.36$. Since the process $b \to cl\nu$ is not modified by the new physics the previous equation can be easily translated into the more useful one

$$\left|\frac{|C_7^{total}(m_b)|^2}{|C_7^{SM}(m_b)|^2} - 1\right| < 0.36 \qquad 95\%\ \text{CL}, \tag{3.68}$$

and from this the bound can be found to be

$$R^{-1} \leq 300\ \text{GeV} \qquad 95\%\ \text{CL}. \tag{3.69}$$

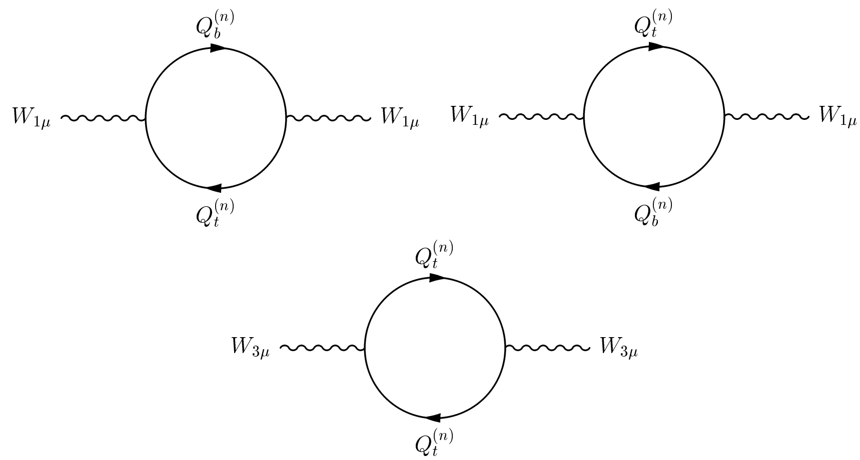


Figure 3.5: Diagrams that contribute to the ρ parameter in UED.

3.2.3 Radiative corrections to the ρ parameter

The ρ parameter can be defined as the ratio of the relative strength of neutral to charged current interactions at low momentum transfer. In the SM, and at tree level, it is predicted to be unity as a consequence of the custodial symmetry of the Higgs potential:

$$\rho \equiv \frac{G_{NC}(0)}{G_{CC}(0)} \approx \frac{M_W^2}{c_W^2 M_Z^2} = 1 \ . \tag{3.70}$$

Because the SM contains couplings that violate the symmetry (the Yukawa couplings and the U(1) coupling g') radiative corrections modify the tree-level value of ρ. At one loop, ρ receives corrections from vertex, box and gauge-boson self-energy diagrams; however the dominant contributions, proportional to m_t^2, come from the top-quark loops inside the gauge boson self-energies. Keeping only these contributions, one has

$$\rho = 1 + \frac{\Sigma_W(0)}{M_W^2} - \frac{\Sigma_Z(0)}{M_Z^2} \approx 1 + \frac{1}{M_W^2}\Big(\Sigma_1(0) - \Sigma_3(0)\Big) \approx 1 + N_c \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2}. \tag{3.71}$$

$\Sigma_W(0)$ and $\Sigma_Z(0)$ are co-factors of the $g^{\mu\nu}$ in the one-loop self-energies of the W and Z bosons, evaluated at $q^2 = 0$, and $\Sigma_1(0)$ and $\Sigma_3(0)$ are the equivalent functions for the W_1 and W_3 components of the SU(2) gauge bosons. In arriving at the above formula one uses the fact that the photon-Z self-energy $\Sigma_{AZ}^{\mu\nu}$ is transverse, i.e. $\Sigma_{AZ}(0) = 0$; this last property holds *only* for the subset of graphs containing fermion-loops, but is no longer true when gauge-bosons are considered inside the loops of Σ_{AZ} [22, 23]. Finally, N_c is the number of colors.

In UED the tree-level value is the same as in SM because, as we have seen, the first corrections appear at the one-loop level. The relevant diagrams are shown in Fig. 3.5.

As we are using the interaction fields, only the Q fields contribute. Since these are not the mass eigenfields there are more diagrams with the propagators that mix Q and U fields, but the latter are singlets under $SU(2)$, and, therefore, do not couple to $W_\mu^{(n)}$. No diagram with U fields gives any contribution, which is the reason why in this base the calculus is much easier. Had we chosen to work with the mass eigenstates, then we would had to consider all fields inside the loop because all of them have projection on the Q fields and the vertices would have been more complicate containing combinations of $\sin(\alpha_n)$ and $\cos(\alpha_n)$.

The self-energy corrections due to a single mode are

$$
\begin{aligned}
i\Sigma_1^{(n)}(q^2) &= i\Sigma_1^{(a)} + i\Sigma_1^{(b)} = 2g^2 N_c m_t^2 \frac{i}{(4\pi)^2}\left[\frac{1}{2\hat\epsilon} - b_1(m_n^2, m_Q^2, q^2)\right] \\
i\Sigma_3^{(n)}(q^2) &= i\Sigma_3^{(c)} = 2g^2 N_c m_t^2 \frac{i}{(4\pi)^2}\left[\frac{1}{2\hat\epsilon} - b_1(m_Q^2, m_Q^2, q^2)\right],
\end{aligned}
\tag{3.72}
$$

where

$$
m_Q^2 = m_n^2 + m_t^2 \qquad \frac{1}{\hat\epsilon} = \frac{2}{\epsilon} - \gamma + \log(4\pi) + \log(\mu^2) \tag{3.73}
$$

μ is an arbitrary mass scale introduced in dimensional regularization. The function b_1 is defined as

$$
b_1(m_1^2, m_2^2, q^2) \equiv \int_0^1 dx\, x \log\left(\frac{\Delta(m_1^2, m_2^2, q^2)}{\mu^2}\right) \tag{3.74}
$$

and

$$
\Delta(m_1^2, m_2^2, q^2) = x m_2^2 + (1-x) m_1^2 - x(1-x) q^2. \tag{3.75}
$$

The total contribution is found by summing up the whole KK tower. This can be done with a bit of care: first consider the contribution of the first N modes which can be obtained from Eq. (3.72) and Eq. (3.74). After a bit of numerics, it can be expressed in the following way

$$
\begin{aligned}
i\Sigma_1 &= \sum_{n=1}^{N} i\Sigma_1^{(n)} = g^2 N_c m_t^2 \frac{i}{(4\pi)^2}\left[\frac{N}{\hat\epsilon} - \sum_{n=1}^{N}\log(m_n^2) - \log(1+r_n) + \int_0^1 dx\, x^2 \sum_{n=1}^{N}\frac{r_n}{1+r_n x}\right] \\
i\Sigma_3 &= \sum_{n=1}^{N} i\Sigma_3^{(n)} = g^2 N_c m_t^2 \frac{i}{(4\pi)^2}\left[\frac{N}{\hat\epsilon} - \sum_{n=1}^{N}\log(m_n^2) - \log(1+r_n)\right].
\end{aligned}
$$

From the point of view of the five dimensional theory these equations can be interpreted as the regularized integrations over the five components of momentum using a *mixed* regularization scheme: dimensional regularization to render finite the integral over the four momentum and cutoff regularization for the integral over the fifth component. The second logarithm is convergent, since term by term is smaller than the general term of the harmonic series

$$
\sum_{n=1}^{\infty}\log(1+r_n) < \frac{(m_t R\pi)^2}{6} \qquad (r_n > 0) \tag{3.76}
$$

The last term in (3.76) can be also easily summed when $N \to \infty$ and then integrated by using the identity

$$
\sum_{n=1}^{\infty}\frac{1}{(n\pi)^2 + \alpha^2} = \frac{\alpha\coth\alpha - 1}{2\alpha^2}. \tag{3.77}
$$

Observe that the divergences related with the limits $d \to 4$ ($\hat\epsilon \to 0$) and $N \to \infty$ cancel when the subtraction is performed, so we can safely take these limits. It should be this way because divergences present the same symmetries than the tree level terms, and at this level $W_{\mu 1}$ and $W_{\mu 3}$ have the same mass. Thus the contribution of each mode is perfectly finite and reads

$$
\Delta\rho^{(n)} = \frac{4}{g^2 v^2}\left[\Sigma_1^{(n)}(0) - \Sigma_3^{(n)}(0)\right] = 2N_c \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2}\left[1 - \frac{2}{r_n} + \frac{2}{r_n^2}\log(1+r_n)\right], \tag{3.78}
$$

where $r_n = m_t^2/m_n^2$. The factor two of difference with respect the SM can be understood in the following way: take the limit in Eq. (3.78) $m_n \to 0$ which corresponds to gain the fundamental mode contribution, but this limit has to predict twice the actual contribution of the fundamental mode because in Eq. (3.78) each mode has left and right contributions coming from $Q^{(n)}_{t\,L(R)}$ and $Q^{(n)}_{b\,L(R)}$ running inside the loops, so the proposed limit would coincide with the SM prediction if the fundamental mode has had left and right contribution, which is not the case. In addition to this, the contribution of a diagram with a given set of fields and the diagram with all the chiralities reversed is the same.

Eq. (3.78) is consistent with the decoupling theorem [24]; if the mass of an individual mode, m_n, is taken to infinity, i.e. $r_n \to 0$, its contribution vanishes

$$\Delta\rho^{(n)} = 2N_c \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\left[\frac{2}{3}r_n + \mathcal{O}(r_n^2)\right]. \tag{3.79}$$

Finally

$$\Delta\rho^{\mathrm{UED}} = \sum_{n=1}^{\infty}\Delta\rho^{(n)} = \frac{4}{g^2 v^2}\left[\Sigma_1^{(n)} - \Sigma_3^{(n)}\right] = 2N_c\frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\int_0^1 dx\; x\left[a\sqrt{x}\coth(a\sqrt{x}) - 1\right], \tag{3.80}$$

where we have used Eq. (3.77). This can be expressed in a more compact form

$$\frac{\Delta\rho^{SM} + \Delta\rho^{UED}}{\Delta\rho^{SM}} = 2\int_0^1 dx\; x\left[a\sqrt{x}\coth(a\sqrt{x})\right], \tag{3.81}$$

where $a = m_t R\pi$.

With the previous results we can extract the bounds coming from the experimental measures. In order to discriminate between the corrections coming from SM and the ones coming exclusively from new physics we work with the T parameter as defined in PDG [25]. It contains only the corrections to ρ coming from new physics. We will adopt the PDG definitions.

The contribution to T can be extracted from Eq. (3.80), for small $m_t R$ it can be expanded as

$$T \approx 2.85(m_t R)^2\left[1 - 0.49(m_t R)^2 + 0.37(m_t R)^4\right], \tag{3.82}$$

since the contributions to the T parameter coming from new physics are bounded to be $T < 0.4$ at 95 % CL, the lower bound is

$$R^{-1}_{UED} > 450 \text{ GeV}, \tag{3.83}$$

which at the end, will be the best of all bounds. This calculation was firstly done in Ref. [10] and later on corrected in Ref. [26]. Our result is in agreement with the latter.

3.2.4 Radiative corrections to the $B^0 - \overline{B}^0$ system

The models we are studying in this paper fall within the so called *Minimal flavor violating* (MFV) models because they fulfill the next two conditions:

- Only the SM operators in the effective weak Hamiltonian are relevant
- Flavor violation is entirely governed by the CKM matrix

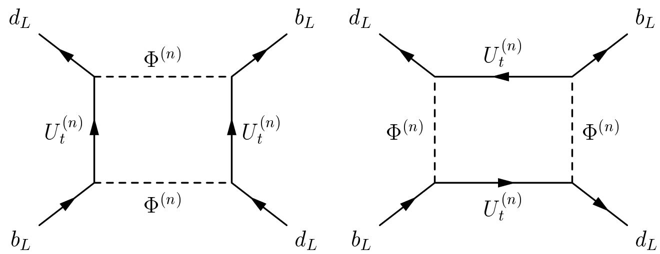


Figure 3.6: Diagrams that contribute to the $B^0 - \overline{B}^0$ mixing in UED.

The two Higgs doublet model and the MSSM at low $\tan\beta$ (and of course the SM) belong also to the MFV class of models. The interesting virtue of the MFV models is that with respect to $B^0 - \overline{B^0}$ mixings and the CP-violating parameter ε_K, they all can be parametrized by a single function $S(x_t)$ [27]. In the literature the $S(x_t)$ function appears also under the name F_{tt}, and in general ceases to be only function of $x_t = m_t^2/M_W^2$. In our case, at the level of precision we are working S is only function of x_t. $S(x_t)$ can be defined as

$$\mathcal{H}_{eff} = \frac{M_W^2 G_F^2 (V_{tb}V_{td}^*)^2}{(4\pi)^2} S(x_t)[\overline{d}\gamma^\mu(1-\gamma_5)b][\overline{d}\gamma_\mu(1-\gamma_5)b]. \tag{3.84}$$

The dominant contributions to $S(x_t)$ will be proportional to x_t, hence proportional to m_t proportional corrections. In SM, this function is dominated by the box diagrams with longitudinal W exchanges and the quark top running inside the loop

$$S_{SM}(x_t) = \frac{x_t}{4}\left[1 + \frac{9}{1-x_t} - \frac{6}{(1-x_t)^2} - \frac{6x_t^2\log(x_t)}{(1-x_t)^3}\right]. \tag{3.85}$$

The measured top quark mass $m_t = 175$ GeV implies $S_{SM}(x_t) \approx 2.5$. In UED, the radiative corrections from new physics can be encoded into a function, which we call $G(a)$ that is defined as

$$S(x_t) = S_{SM}(x_t) + \delta S(x_t), \qquad \delta S(x_t) = \frac{x_t}{4}(G(a)-1). \tag{3.86}$$

The contributions to this function come from the amplitude of the diagrams shown in Fig. 3.6. In contrast with what happens in the computation of the modifications to ρ, only the U fields are important in this case because the couplings of Q fields with the Φ are proportional to the mass of the b quark. After the usual Fierz reordering and a bit of combinatorics, done also in the SM, the result is:

$$G_{\mathrm{UED}}(a) = \int_0^1 dx\,(1-x)\left[a\sqrt{x}\coth(a\sqrt{x})+1\right] \approx 1 + \frac{a^2}{18} - \frac{a^4}{540} + \ldots, \tag{3.87}$$

where $a = m_t\pi R$. The last experimental determinations [27] agree with the SM expectations

$$1.3 \leq S(x_t) \leq 3.8 \qquad 95\%\ \mathrm{CL}. \tag{3.88}$$

The possible positive contributions have been lowered with respect to previous determinations [28] allowing better bounds

$$R_{UED}^{-1} > 40\ \mathrm{GeV} \qquad 95\%\ \mathrm{CL}. \tag{3.89}$$

	$Z \to b\bar{b}$	$b \to s + \gamma$	$\overline{B}^0 - B^0$	ρ
M(GeV)	230	300	40	450

Table 3.1: Bounds coming from the different observables.

This bound is an order of magnitude worse than the obtained using ρ. Conversely, if the bound coming from the ρ parameter is taken, contributions to $S(x_t)$ compatible with experiment are found to be relatively big.

Given the future experimental improvements on the determinations of $\sin 2\beta$ by BaBar and BELLE and in particular of the mass splitting ΔM_s for the B sector in LHC and FNAL one may use this observable to predict possible deviations from the SM predictions. It turns out that to an excellent accuracy [29] the mentioned deviations in the case of ΔM_s are governed by $G(a)$

$$G_{NP}(a) = \frac{(\Delta M_s)_{NP}}{(\Delta M_s)_{SM}} > 1 \tag{3.90}$$

The value of $G(a)$ is too small to be discriminated experimentally. This result was initially derived in Ref. [29], where it is explained that the possible existence of extra dimensions will not pollute the extraction of the CKM matrix parameters from the future improvements in the determination of the unitarity triangle.

3.3 Outlook and conclusions

In this chapter we have studied an extra dimensional extension of the SM, in which one single extra dimension is accessible to all the fields. This scenario is called universal extra dimension, or UED. As explained in the previous chapter, one consequence of the universal scenarios is the "conservation" of KK number, what implies that the corrections to the SM results are at least one-loop suppressed. Given the smallness of these corrections we have studied precision observables that display a strong dependence on the mass of the top-quark, m_t, because this dependence enhances the relative importance of the new physics with respect to the SM predictions. In particular, we have studied the radiative corrections for the $Z \to b\bar{b}$ decay, the ρ parameter and the $B^0 - \overline{B}^0$ mixing. The $b \to s\gamma$ process has been also studied. Although it is not enhanced by a large mass, the relative impact of the new physics is important for it because in the SM it is one-loop suppressed, hence the new contributions can be competitive with the SM radiative corrections.

By comparing these different observables with data, bounds on the compactification scale, R, can be set. Equivalently, these results can be translated into bounds on the mass of the first member in the KK tower, $M = R^{-1}$. The table 3.1 summarizes the results. The scale of the new physics can be as low as 450 GeV without contradicting any of the experimental determinations. This is a relatively low value for M because precision observables, in general, tend to establish the scale of any new physics around or above the TeV. The reason why the scale for the universal extra dimension can be so low without affecting too much precision observables is the above mentioned one-loop suppression due to the KK number conservation.

Chapter 4

SM with one latticized universal extra dimension

In the previous chapter we have studied models with one extra dimension and with all SM fields propagating in it. The models displayed two different energy regimes: the low energy, below the compactification scale, reduced to the SM while the high energy regime described the couplings among the modes of the KK towers. Because of the presence of the extra dimension the coupling constants are dimension full, in particular Yukawa couplings and gauge couplings, that are dimensionless quantities in the SM, have dimensions of energy raised to some negative power, which is the reason why these theories are non-renormalizable. Hence, they must be understood as effective field theories that in their low energy limit reduce to the SM. A possible ultraviolet completion was proposed in Ref. [30], which eventually treated the extra dimensions as if they were discontinuous. The idea of a discretized dimension was also simultaneously suggested in Ref. [31, 32]. The latter is not really an ultraviolet completion because the Lagrangian is described by a number of σ-models, but for this class of models there are some known possible renormalizable extensions. The aim of this chapter is to investigate how the phenomenology is modified in models with discretized, sometimes also called *latticized*, extra dimensions; specifically, we will study the latticized version of UED, called in the following LUED.

4.1 The model

The Lagrangian is divided, as usual, in four pieces

$$\mathcal{L}^{\text{LUED}} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_Y. \tag{4.1}$$

The gauge piece, $\mathcal{L}_G$, is the one associated to the gauge group $G = \Pi_{i=0}^{N-1} SU(2)_i \times U(1)_i$ and it also contains some scalars fields which will be necessary on the following, their role will be clarified later

$$\begin{aligned}
\mathcal{L}_G &= \sum_{i=0}^{N-1} -\frac{1}{4} F^a_{i\mu\nu} F_i^{\mu\nu a} - \frac{1}{4} F_{i\mu\nu} F_i^{\mu\nu} && (4.2)\\
&+ \sum_{i=1}^{N-1} \text{Tr}\{(D_\mu \Phi_i)^\dagger (D^\mu \Phi_i)\} + (D_\mu \phi_i)^\dagger (D^\mu \phi_i) - V(\Phi, \phi), && (4.3)
\end{aligned}$$

where $F^a_{i\mu\nu}$ is the strength tensor associated with the gauge field of the i-th $SU(2)_i$ and $F_{i\mu\nu}$ is the one for $U(1)_i$. Φ_i and ϕ_i are the elementary scalars that will acquire a VEV independent of i due to the potential term $V(\Phi,\phi)$. Each of them become effectively nonlinear σ model fields that can be parametrized as usual in terms of the scalar fields π_i and π_i^a

$$\phi_i = \frac{v_1}{\sqrt{2}} e^{i\pi_i/v_1} \qquad \Phi_i = v_2 e^{i\pi_i^a\sigma^a/2v_2} \tag{4.4}$$

v_1 and v_2 are the VEVs of ϕ_i and Φ_i respectively and σ^a are the Pauli matrices. In this work we will concentrate in the so called "*aliphatic model*" [31] in which the Φ_i fields are assumed to transform as $(\mathbf{2},\overline{\mathbf{2}})$ under the groups $SU(2)_i$ and $SU(2)_{i-1}$ and as singlets for the rest, they carry no $U(1)_i$ charge. On the other hand, the ϕ_i fields are singlets under all the $SU(2)$ groups and they are charged only under $U(1)_i$ and $U(1)_{(i-1)}$ with hypercharges $(Y_i, -Y_{i-1})$, later on, every Y_i will be set to $Y_i = 1/3$ [32]. With this, the covariant derivative reads

$$D_\mu \Phi_i = \partial_\mu \Phi_i - i\mathcal{W}_{\mu,i}\Phi_i + i\Phi_i \mathcal{W}_{\mu,i-1} \tag{4.5}$$

where $\mathcal{W}_{\mu,i} = \tilde{g} W^a_{\mu\, i} T^a_i$, T^a_i are the generators of the $SU_i(2)$ and $\tilde{g}$ is the dimensionless gauge coupling constant that is assumed to be the same for all the $SU(2)$ groups. The covariant derivative for ϕ_i can be constructed similarly. The gauge coupling constant for all the $U(1)$ groups will be called $\tilde{g}'$.

The next piece is the fermionic one, $\mathcal{L}_F$, it contains the following fields (generational indices assumed)

$$Q_i = \begin{bmatrix} Q_{ui} \\ Q_{di} \end{bmatrix} \qquad U_i \qquad D_i \qquad i = 0, \ldots, N-1, \tag{4.6}$$

where we have used a similar notation than in the continuous case. Q_i transforms as a doublet under $SU(2)_i$ and as a singlet for the rest of $SU(2)$ groups and among the $U(1)$ fields it is only charged under $U(1)_i$ with hypercharge $Y_Q = 1/3$. On the contrary, U_i and D_i are only charged under $U(1)_i$ with hypercharges $Y_U = 4/3$ and $Y_D = -2/3$. They are all vector fields with right and left handed chiral components except for $i = 0$. In this case they are chiral fields, Q is left-handed and U and D are right-handed, which is equivalent to impose

$$Q_{0R} = 0 \qquad U_{0L} = 0 \qquad D_{0L} = 0 \tag{4.7}$$

With this we can split the fermionic piece in: $\mathcal{L}_F = \mathcal{L}_Q + \mathcal{L}_U + \mathcal{L}_D$, where

$$\mathcal{L}_Q = \sum_{i=0}^{N-1} \left[\overline{Q}_{iL} i\not{D} Q_{iL} + \overline{Q}_{iR} i\not{D} Q_{iR}\right] - \sum_{i=0}^{N-1} M_f \overline{Q}_{iL} \left(\frac{\Phi^\dagger_{i+1}\phi^\dagger_{i+1}}{v_2(v_1/\sqrt{2})} Q_{i+1R} - Q_{iR} \right) + \text{h.c.} \tag{4.8}$$

and

$$\mathcal{L}_U = \sum_{i=0}^{N-1} \left[\overline{U}_{iR} i\not{D} U_{iR} + \overline{U}_{iL} i\not{D} U_{iL}\right] + \sum_{i=0}^{N-1} M_f \overline{U}_{iR} \left(\frac{\phi^{4\dagger}_{i+1}}{(v_1/\sqrt{2})^4} U_{i+1L} - U_{iL} \right) + \text{h.c.} \tag{4.9}$$

$\mathcal{L}_D$ can be extracted from $\mathcal{L}_U$ making the next substitutions, $U \to D$, $\phi_i \to \phi_i^\dagger$ and the exponent should be replaced $4 \to 2$. In the previous formulae $\not{D}$ is the usual covariant derivative associated with the gauge group G and M_f is a generic mass that in principle could depend on i but for simplicity it is set independent of i. The exponent of the ϕ fields must be adjusted in each case because the terms must be invariant under G by construction. In addition, when the index of a field runs out of bounds it is understood as a zero, for instance in Eq. (4.8) it must be set $Q_{NR}=0$, and so on.

The next piece in the Lagrangian, $\mathcal{L}_H$, is the one associated with the Higgs doublet [32]

$$\mathcal{L}_H = \sum_{i=0}^{N-1}(D_\mu H_i)^\dagger(D^\mu H_i) - M_0^2\left|H_{i+1} - \left(\frac{\Phi_{i+1}\phi^3_{i+1}}{(v_1/\sqrt{2})^3 v_2}\right)H_i\right|^2 - V(H_i), \tag{4.10}$$

where H_i is a doublet under $SU(2)_i$ and singlet for $SU(2)_{j\neq i}$ with hypercharges $Y_i=1$ and $Y_{j\neq i}=0$. We parametrize its components as

$$H_i = \begin{bmatrix} \Phi_i^+ \\ \Phi_i^0 \end{bmatrix} \qquad i=0,1,\dots,N-1. \tag{4.11}$$

As a potential it is chosen

$$V(H_i) = -m^2 H_i^\dagger H_i + \frac{\tilde{\lambda}}{2}(H_i^\dagger H_i)^2 \tag{4.12}$$

Finally the Yukawa sector, $\mathcal{L}_Y$, will be taken with the Yukawa matrices independent of i

$$\mathcal{L}_Y = \sum_{i=0}^{N-1}\overline{Q_i}\widetilde{Y}_u H_i^c U_i + \sum_{i=0}^{N-1}\overline{Q_i}\widetilde{Y}_d H_i D_i + \text{h.c.} \tag{4.13}$$

where $H_i^c \equiv i\tau^2 H_i^*$ is the usual Higgs doublet conjugate.

4.1.1 Relation with continuous extra dimensions

The fields and couplings proposed in the above lines are set to describe a situation in which the full SM is contained in a five dimensional space-time with four spacial dimensions but with the extra fifth dimension latticized. In LUED the length of the new dimension, L, is taken to be finite and a new variable R is defined through the relation $L=\pi R$, this will simplify the comparison of the results with the continuous situation. The extra volume is filled with 4D surfaces equally spaced by a distance a, the first one situated in $x^5=0$, see Fig. 4.1. Therefore, these magnitudes fulfill the trivial relation

$$a = \frac{\pi R}{N-1}. \tag{4.14}$$

In this picture every field that propagates in the fifth dimension will be represented by the N values that it takes in the different surfaces, i.e. $\psi_i(x^\mu)=\psi(x^\mu, x^5=ia)$. This is the reason why we have N copies for each of the SM fields.

But the Φ and ϕ fields were not previously present in SM. They are related with the component of the five dimensional gauge fields polarized in the direction of the extra dimension, generically denoted by W_5. In the continuous theory these components are necessary in order to define a consistent covariant derivative. In our case the relation between these fields is [31]

$$\Phi_i(x^\mu) = \exp\left[i\tilde{g}\int_{(i-1)a}^{ia} dx^5\ \mathcal{W}_5(x^\mu, x^5)\right], \qquad i=1,\dots,N-1. \tag{4.15}$$

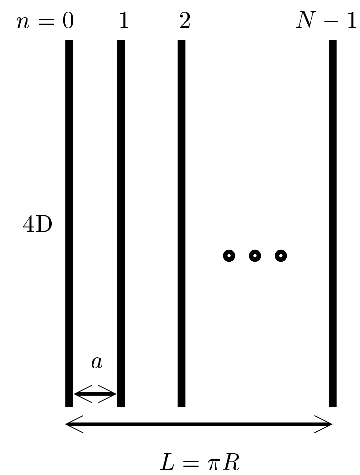


Figure 4.1: Schematic representation of the discretization in the extra dimensions.

In other words the Φ fields are Wilson lines connecting two adjacent surfaces, that is the reason why there are only $N-1$ copies of them, instead of the N that have the rest of the fields. Eq. (4.15) enables us to reinterpret the interaction terms between the different fields present in the Lagrangian with the Φ's as a lattice approximation of their covariant derivatives provided that the general masses obey $M_f = M_0 = 1/a$.

We have exploited this picture when writing the interaction terms of the fermions. Note the relative sign between the Q doublets and the U and D singlets, compare Eq. (4.8) with Eq. (4.9). Written in this form it will be easier to extract the spectrum of the theory and both possibilities are equally legitimate since they are lattice approximations, to the same order, of the fifth covariant derivative.

When studying continuous extra dimensions sometimes some of the fields are not allowed to propagate through the extra volume, there are many variations in the literature, but in particular we will study in chapter 6 the case when only fermion fields do not propagate in the extra dimension, LHG. The Lagrangian for this case can be obtained by setting in the above described one, $\psi_{iR} = \psi_{iL} = 0$ for $i \neq 0$, where ψ stands for any fermion field.

As a last remark, the assumption that the coupling constants are independent of the position translates in the latticized scenario as an independence on the index i of each group, i.e. $\tilde{g}_i = \tilde{g}(x^5 = ia) = \tilde{g}$.

4.1.2 The spectrum of the model

Before extracting the bilinear terms from $\mathcal{L}$ it is interesting to study which fields can be removed from the spectrum by exploiting the gauge freedom. Under a generic transformation of G the Φ_i transforms as

$$\Phi_i = U_i \Phi_i U_{i-1}^{\dagger} \qquad U_i \in SU(2)_i. \tag{4.16}$$

Since the vacuum configuration is by construction $\Phi_i = v_2 \mathbb{1}$, it is clear that only the gauge transformation defined by $U_0 = U_1 = \ldots = U_{N-1}$ leaves invariant the vacuum, i.e. the diagonal group $SU(2)_D$ remains unbroken after the SSB, analogously for ϕ_i. Therefore $G \to SU(2)_D \times U(1)_D$. This procedure removes the π fields from the spectrum and leaves massless only one combination of the gauge bosons. This amounts to work in the unitary gauge and it is indeed the approach followed in Ref. [31, 32], where $SU(2)_D \times U(1)_D$ is identified with the SM electroweak gauge group which is further broken by the usual SM Higgs VEV. We will not follow those steps and instead we will work in an arbitrary R_ξ-covariant gauge, in the same line as it was done in Ref. [33], therefore maintaining explicitly the π fields.

Extracting the bilinear terms is lengthy and cumbersome, but straightforward otherwise. From $\mathcal{L}_G$ it can be obtained

$$\mathcal{L}_G^{(2)} = \sum_{i=0}^{N-1} -\frac{1}{4}\widetilde{F}_i \cdot \widetilde{F}_i + M_i^2 \widetilde{W}^a_{\mu i}\widetilde{W}^{\mu a}_i + \sum_{i=1}^{N-1} \frac{1}{2}\partial_\mu \widetilde{W}^a_{5i}\partial^\mu \widetilde{W}^a_{5i} + M_i \partial^\mu \widetilde{W}^a_{5i}\widetilde{W}^a_{\mu i}. \tag{4.17}$$

Here we only show the $SU(2)$ piece but the $U(1)$ one is similar. The new tilde fields are related with the initial ones by the next change of base

$$W^a_{\mu i} \equiv \sum_{i=0}^{N-1} a_{ij}\widetilde{W}^a_{\mu j} \qquad \pi^a_i \equiv \sum_{i=1}^{N-1} b_{ij}\widetilde{W}^a_{5j} \tag{4.18}$$

where the a and b matrices are

$$a_{ij} = \begin{cases} j = 0 & \sqrt{1/N} \\ j \neq 0 & \sqrt{2/N}\cos\left(\frac{2i+1}{2}\frac{j\pi}{N}\right) \end{cases} \tag{4.19}$$

and

$$b_{ij} = \sqrt{\frac{2}{N}}\sin\left(ij\frac{\pi}{N}\right) \tag{4.20}$$

$\widetilde{F}$ is understood to be the usual kinetic term for gauge bosons expressed now in terms of $\widetilde{W}^a_{\mu i}$. Finally, the masses M_i are

$$M_i = 2\tilde{g}v_2 \sin\left(\frac{i\pi}{2N}\right) = 2\frac{N-1}{\pi R}\sin\left(\frac{i\pi}{2N}\right), \tag{4.21}$$

where we have chosen as usual $\tilde{g}v_2 = 1/a$ to guarantee that the large N limit reproduces the continuous scenario[1][31]. The massless vector bosons $\widetilde{W}^a_{\mu 0}$ and $\widetilde{B}_{\mu 0}$ are associated to the SM model gauge bosons. They are the gauge vector bosons of the unbroken diagonal group and consequently this last is identified with the SM gauge group.

On the other hand, the bilinear terms for the fermions are

$$\mathcal{L}_Q^{(2)} + \mathcal{L}_U^{(2)} = \overline{\widetilde{Q}}_{0L} i\partial\!\!\!/ \widetilde{Q}_{0L} + \overline{\widetilde{U}}_{0R} i\partial\!\!\!/ \widetilde{U}_{0R} + \sum_{i=1}^{N-1} \overline{\widetilde{Q}}_i (i\partial\!\!\!/ - M_i)\widetilde{Q}_i + \overline{\widetilde{U}}_i (i\partial\!\!\!/ + M_i)\widetilde{U}_i, \tag{4.22}$$

where the vector like fields are defined as $\widetilde{Q}_i = \widetilde{Q}_{iR} + \widetilde{Q}_{iL}$ and similarly for $\widetilde{U}_i$. The tilde fields are given by

$$\begin{array}{ll} Q_{iL} = a_{ij}\widetilde{Q}_{jL} & U_{iR} = a_{ij}\widetilde{U}_{jR} \\ Q_{iR} = b_{ij}\widetilde{Q}_{jR} & U_{iL} = b_{ij}\widetilde{U}_{jL} \end{array} \tag{4.23}$$

[1] Eq. (4.21) could also have been written in the form $M_i = 2N/\pi R \sin(i\pi/2N)$, as it is done in Ref. [31]; the large N limit is the same. This will not be a problem because the bounds that we will obtain are completely independent of which of the two definitions is taken.

the mass M_i appearing in Eq. (4.22) is the one defined in Eq. (4.21) provided $M_f = 1/a$. This will be advantageous when studying the Yukawa terms. The relative sign in the mass terms is a common feature after dimensional reduction in theories with universal (continuous) extra dimensions, where it is due to the definition of the fifth gamma matrix, $\Gamma^4 = i\gamma^5$ [10]. Here it has been explicitly introduced in Eq. (4.8) and Eq. (4.9), precisely to reproduce this feature.

Finally, in the Higgs sector one must perform the change of basis $H_i = a_{ij}\widetilde{H}_j$ and the masses are now

$$M^2(\widetilde{H}_i) = 4\frac{(N-1)^2}{\pi^2 R^2}\sin^2\left(\frac{i\pi}{2N}\right) - m^2 \tag{4.24}$$

This equation shows that $\widetilde{H}_0$ will break spontaneously symmetry, hence it is identified with the SM Higgs doublet, i.e. $\langle\widetilde{H}_0\rangle_0 = v/\sqrt{2}$ with $v = 246$ GeV. This means that new contributions to the masses came from the Yukawa piece $\mathcal{L}_Y$ and the covariant derivative of H. The Yukawa piece in terms of the tilde fields can be written as

$$\mathcal{L}_Y = \sum_{i=0}^{N-1} \overline{\widetilde{Q}}_i \frac{\widetilde{Y}_u}{\sqrt{N}} \widetilde{H}_0^c \widetilde{U}_i + \overline{\widetilde{Q}}_i \frac{\widetilde{Y}_d}{\sqrt{N}} \widetilde{H}_0 \widetilde{D}_i + \text{h.c.} \tag{4.25}$$

where we have concentrated on the terms containing the Higgs doublet. It has been used Eq. (4.23). From the first term in the sum of Eq. (4.25) is easy to convince oneself that $Y_u \equiv \widetilde{Y}_u/\sqrt{N}$ is the SM Yukawa matrix. When the Higgs doublet acquires a VEV one must diagonalize Y_u using the same field redefinitions as in SM, $\widetilde{Q}_{ui} \to U_u^\dagger \widetilde{Q}_{ui}$, $\widetilde{U}_i \to V_u^\dagger \widetilde{U}_i$. At the end the mass matrix for fermions will be[2]

$$\begin{pmatrix} \overline{\widetilde{U}}_{if} & \overline{\widetilde{Q}}_{if} \end{pmatrix} \begin{pmatrix} -M_i & m_f \\ m_f & M_i \end{pmatrix} \begin{pmatrix} \widetilde{U}_{if} \\ \widetilde{Q}_{if} \end{pmatrix} \tag{4.26}$$

where f is the index of the generation, in this case $f = u, c, t$. This mass matrix is exactly the same obtained in the continuous scenario with one extra dimension by making the substitution $m_n \to M_n$ [10], where $m_n = n/R$ is the mass of the n-th Kaluza-Klein mode of the field in the absence of Yukawa couplings. As a consequence the mixing between the $\widetilde{Q}$ and $\widetilde{U}$ is the same as in that scenario as well as the masses $M(Q'_{if}) = \sqrt{M_i^2 + m_f^2}$, prime denotes mass eigenfields, for later reference $m_Q = M(Q'_{if})$

$$\begin{pmatrix} \widetilde{U}_{if} \\ \widetilde{Q}_{if} \end{pmatrix} = \begin{pmatrix} -\gamma^5\cos\alpha_{if} & \sin\alpha_{if} \\ \gamma^5\sin\alpha_{if} & \cos\alpha_{if} \end{pmatrix} \begin{pmatrix} U'_{if} \\ Q'_{if} \end{pmatrix} \tag{4.27}$$

where $\tan(2\alpha_{if}) = m_f/M_i$. As in the continuous situation we are specially interested in the case $f = t$.

Notice that the zero-th modes have exactly the same masses as in SM, all of them coming purely from the Yukawa piece in Eq. (4.25) which, as said, for the zero-th modes coincides exactly with the SM Yukawa sector. In fact, the same happens for the rest of the pieces in the Lagrangian and one can safely identify the zero-th tilde fields, $\widetilde{Q}_{0L}$, $\widetilde{U}_{0R}$ and $\widetilde{D}_{0R}$, with the SM fields.

We will not show it explicitly but the SSB of the Higgs doublet will cause the usual mixing between $W^3_{\mu i}$ and $B_{\mu i}$ with θ_w as the weak mixing angle [32], instead we will concentrate on

[2] We do not study explicitly the term containing Y_d because it does not contain the mass of the top-quark, m_t, but its treatment would be completely similar.

the mixing of the charged bosons. Coming from $\mathcal{L}_H$ and due to the VEV of $\widetilde{H}_0$ the next terms arise

$$\mathcal{L}^{(2)}_{\widetilde{H}_0} = iM_W \widetilde{W}^-_{\mu i}\partial^\mu \widetilde{\Phi}^+_i + M^2_W \widetilde{W}^-_{\mu i}\widetilde{W}^{\mu +}_i - M^2_W \widetilde{W}^-_{5i}\widetilde{W}^+_{5i} + iM_W M_i \widetilde{W}^-_{5i}\widetilde{\Phi}^+_i + \text{h.c.} \tag{4.28}$$

In addition, due to the quartic couplings in Eq. (4.12) the masses for $\widetilde{H}_i$ are shifted from Eq. (4.24) to Eq. (4.21). This implies, jointly with Eq. (4.17), that there is a combination of fields, $\Phi^\pm_{Gi}$, that act as a Goldstone field absorbed by $\widetilde{W}^\pm_{\mu i}$ which acquires in the process a mass $M(\widetilde{W}^\pm_{\mu i}) = \sqrt{M^2_W + M^2_i}$. The orthogonal combination is a physical scalar, $\Phi^\pm_{Pi}$, with the same mass.

$$\Phi^+_{Gi} = \frac{M_i \widetilde{W}^+_{5i} + iM_W \widetilde{\Phi}^+_i}{\sqrt{M^2_i + M^2_W}} \xrightarrow{M_W \to 0} \widetilde{W}^+_{5i} \tag{4.29}$$

$$\Phi^+_{Pi} = \frac{iM_W \widetilde{W}^+_{5i} + M_i \widetilde{\Phi}^+_i}{\sqrt{M^2_i + M^2_W}} \xrightarrow{M_W \to 0} \widetilde{\Phi}^+_i \tag{4.30}$$

In the limit in which all the mass scales below m_t are neglected, the Goldstone bosons and the physical scalars can be directly identified with $\widetilde{W}^\pm_{5i}$ and $\widetilde{\Phi}^\pm_i$ respectively. There will be terms that cross the massive vector bosons $\widetilde{W}^\pm_{\mu i}$ with the derivatives of their Goldstone bosons, $\Phi^\pm_{Gi}$; these can be removed using a convenient R_ξ gauge as done in [27, 34].

With this, we have concluded the demonstration that, at least for the degrees of freedom we will be interested in, the spectrum of this model is equivalent to one continuous universal extra dimension with m_n replaced by M_n.

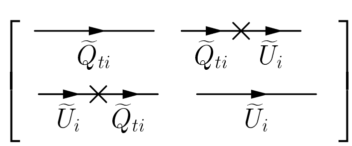
4.1.3 Couplings

Our aim will be to extract the dominant corrections to some precision observables and from them to extract bounds on the new physics. We will concentrate on the corrections proportional to the top-quark mass m_t, thus as a approximation we will identify $\Phi^\pm_{Pi} = \widetilde{\Phi}^\pm_i$ and $\Phi^\pm_{Gi} = \widetilde{W}^\pm_{5i}$, which is equivalent to neglect M_W. This will simplify greatly the calculus. In the next lines we will extract the relevant couplings under the previous approximations.

For computing the contributions to the ρ parameter we will work with the base of fields $\{\widetilde{Q}_i, \widetilde{U}_i\}$ instead of the mass eigenfields $\{Q'_i, U'_i\}$ because in the former the couplings with $\widetilde{W}^{1/3}_{\mu 0}$, the only ones needed, are rather simple

$$\mathcal{L}_\rho = \frac{g}{2}\sum_{i=1}^{N-1} \widetilde{W}^1_{\mu 0}\left[\overline{\widetilde{Q}}_{it}\gamma^\mu \widetilde{Q}_{ib} + \overline{\widetilde{Q}}_{ib}\gamma^\mu \widetilde{Q}_{it}\right] + \widetilde{W}^3_{\mu 0}\left[\overline{\widetilde{Q}}_{it}\gamma^\mu \widetilde{Q}_{it}\right], \tag{4.31}$$

where we have already used the relation $g = \widetilde{g}/\sqrt{N}$ [32]. So the couplings in this base are the same as in SM but with the difference that the propagators of the tilde fields are



$$= \begin{bmatrix} i\frac{\not p + M_i}{p^2 - m^2_Q} & i\frac{m_t}{p^2 - m^2_Q} \\ i\frac{m_t}{p^2 - m^2_Q} & i\frac{\not p - M_i}{p^2 - m^2_Q} \end{bmatrix}$$

From the Yukawa piece, the couplings proportional to m_t are

$$\mathcal{L}_Y = \sum_{i=1}^{N-1} m_t \frac{\sqrt{2}}{v} V_{tb} \overline{\widetilde{U}}_{tiR} \Phi_i^+ b_L + \text{h.c.} \tag{4.32}$$

The couplings with the Z will be required, these can be obtained from

$$\mathcal{L}_Z = \frac{g}{2c_w} Z_\mu [J^\mu_{SM} + J^\mu_F + J^\mu_\Phi] \tag{4.33}$$

where J^μ_{SM} is the usual SM current and

$$J^\mu_F = \sum_{i=1}^{N-1} \left(1 - \frac{4}{3}s_w^2\right) \overline{\widetilde{Q}}_{ti} \gamma^\mu \widetilde{Q}_{ti} - \frac{4}{3} s_w^2 \overline{\widetilde{U}}_{it} \gamma^\mu \widetilde{U}_{it} \tag{4.34}$$

$$J^\mu_\Phi = \sum_{i=1}^{N-1} (-1 + 2s_w^2) \widetilde{\Phi}_i^+ i\partial^\mu \widetilde{\Phi}_i^- + \text{h.c.} \tag{4.35}$$

The couplings with the photon can be derived similarly.

4.2 Phenomenology

In this section we establish which is the minimum value for the energy scale of the new physics allowed by experiments. We set bounds on the first mode, M_1, as defined in Eq. (4.21) and called simply M in the following. To this end, we compute the impact of the new physics on a set of standard electroweak observables. In particular, we focus on those which in SM display large radiative corrections due to their strong dependence on the top-quark mass: the decay rates $b \to s\,\gamma$ and $Z \to b\overline{b}$, the ρ parameter and the rates of $B^0 \rightleftharpoons \overline{B^0}$. We expect that they will be also very sensitive to the top mass since the structure of these kind models is basically the same of the SM replicated.

4.2.1 Radiative corrections to the $Z \to b\overline{b}$ decay

The theory developed in Sec. 3.2.1 can be straightforwardly used here, in particular the new physics is also parametrized through the modifications to g_L. Now these come from the same set of diagrams displayed in Fig. 3.3, where the tilde fields are now the fields that run inside the loop. The different contributions are parametrized in exactly the same way they were in the continuous scenario, we reproduce them here for commodity of the reader.

$$i\mathcal{M}_i = i\frac{g}{c_w}\frac{\sqrt{2}G_F m_t^2}{(4\pi)^2} f(r_i) \overline{u}' \gamma^\mu P_L u \epsilon_\mu \tag{4.36}$$

where u and u' are the spinors of the b quarks and ϵ_μ stands for the polarization vector of the Z boson. The argument of the function f is now $r_i = m_t^2/M_i^2$.

$$\begin{aligned}
f^{(a)}(r_i) &= \left(1-\frac{4}{3}s_w^2\right)\left[\frac{r_i-\log(1+r_i)}{r_i}\right] &(4.37)\\
f^{(b)}(r_i) &= \left(-\frac{2}{3}s_w^2\right)\left[\delta_i-1+\frac{2r_i+r_i^2-2(1+r_i^2)\log(1+r_i)}{2r_i^2}\right] &(4.38)\\
f^{(c)}(r_i) &= \left(-\frac{1}{2}+s_w^2\right)\left[\delta_i+\frac{2r_i+3r_i^2-2(1+r_i)^2\log(1+r_i)}{2r_i^2}\right] &(4.39)\\
f^{(d)}(r_i)+f^{(e)}(r_i) &= \left(\frac{1}{2}-\frac{1}{3}s_w^2\right)\left[\delta_i+\frac{2r_i+3r_i^2-2(1+r_i)^2\log(1+r_i)}{2r_i^2}\right] &(4.40)
\end{aligned}$$

where $\delta_i \equiv 2/\epsilon - \gamma + \log(4\pi) + \log(\mu^2/M_i^2)$, and μ is the 't Hooft mass scale. From the above equations it is straightforward to verify that all the terms proportional to δ_i cancel, and so do all terms proportional to s_w^2. Thus, finally, the only term which survives is the term in $f^{(a)}(r_i)$ not proportional to s_w^2, yielding the following contribution

$$\delta g_{Li} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\left[\frac{r_i-\log(1+r_i)}{r_i}\right] \tag{4.41}$$

The gaugeless limit leads exactly to the same conclusions as it is done in Ref. [35] to derive essentially the same calculation in UED. Notice also here the absence of logarithms in Eq. (4.41) when $r_i \to 0$. The full contribution $\delta g_L^{NP} = \sum_{i=1}^{N-1}\delta g_{Li}$ expressed in terms of $F(a)$ can be written in the form

$$F_{\rm LUED}(a) = \int_0^1 dx \sum_{i=1}^{N-1}\frac{a^2 x}{4(N-1)^2\sin^2(i\pi/2N)+a^2x} \tag{4.42}$$

This function captures the correction proportional to m_t^2, the full one loop result could be adapted from Ref. [29] by replacing $m_n \to M_i$ as explained above. We have shown in Sec. 3.2.1 that $F(a)-1 < 0.39$ at 95%C.L., from which the results displayed in Fig. 4.2 follow.

4.2.2 Radiative corrections to $b \to s\gamma$

Given the similitude with the continuous case we will not develop all the theory again, instead we will focus on the computation of the contribution to the i-th mode to the C_7 coefficient, $C_{7\,i}(M_W)$. Again, it comes from the diagrams of Fig. 3.4, when the tilde fields are the ones that run inside the loop and amounts to

$$C_{7\,i} = \frac{m_t^2}{m_t^2+M_i^2}\left[B\left(\frac{m_t^2+M_i^2}{M_i^2}\right)-\frac{1}{6}A\left(\frac{m_t^2+M_i^2}{M_i^2}\right)\right] \tag{4.43}$$

where $A(x)$ and $B(x)$ are defined in Eq. (3.59) and Eq. (3.64) respectively. Of course, an expansion of $C_{7\,i}$ is free of logarithms that relate the two different mass scales M_i and m_t, due to the same reasons as in the continuous scenario. The total result reads

$$C_7^{\rm LUED}(M_W) = C_7^{\rm SM}(M_W) + \sum_{i=1}^{N-1} C_{7\,i}(M_W), \tag{4.44}$$

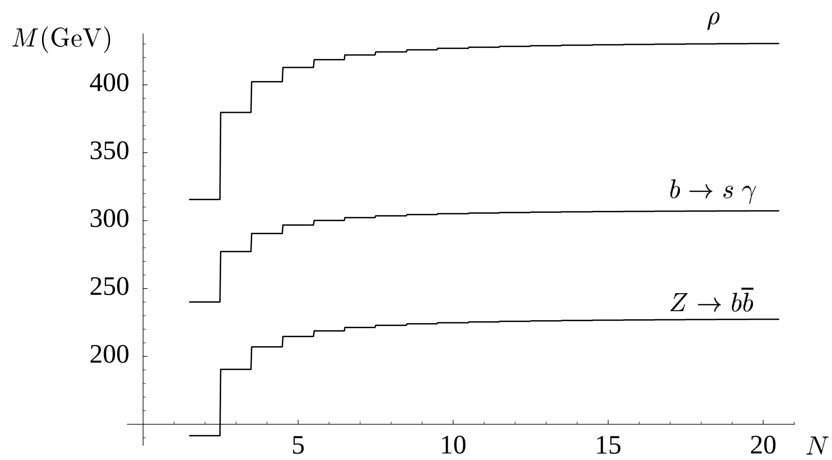


Figure 4.2: The bounds on the mass of the first KK mode, M, as a function of the number of sites N.

where we have neglected the running between m_t and M_W, i.e. $C_{7\,i}(m_t) \approx C_{7\,i}(M_W)$. To take into account the QCD running from M_W to m_b Eq. (3.60) is used. Again C_2 takes the same value as in SM model $C_2(M_W) = 1$ and the contribution of $C_8(M_W)$ is neglected. The modifications to $b \to cl\nu$ are also negligible because LUED corrects it again at the one loop level.

Using the result derived in Ref. [19]

$$\left| \frac{|C_7^{total}(m_b)|^2}{|C_7^{SM}(m_b)|^2} - 1 \right| < 0.36 \qquad 95\% \text{ CL.} \tag{4.45}$$

With this the results can be extracted, and they are shown in Fig. 4.2.

4.2.3 Radiative corrections to the ρ parameter

For this observable the same steps as in the continuous cases must be done, this leads to

$$\Delta\rho_i = \frac{4}{g^2 v^2}\left[\Sigma_{1\,i}(0) - \Sigma_{3\,i}(0)\right] = 2N_c \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2}\left[1 - \frac{2}{r_i} + \frac{2}{r_i^2}\log(1 + r_i)\right], \tag{4.46}$$

where $r_i = m_t^2/M_i^2$ and the correspondent diagrams are displayed in Fig. 3.5. The total contribution would be found summing $\Delta\rho^{\rm LUED} = \Delta\rho^{\rm SM} + \sum_{i=1}^{N-1}\Delta\rho_i$. Now, for each value of the number of sites N we can extract the correspondent bounds, these are displayed in Fig. 4.2.

4.2.4 Radiative corrections to the $B^0 - \overline{B}^0$ system

The new physics modifies the value of $S(x_t)$ defined in Eq. (3.84) and again we parametrize this modification through the function $G(a)$. The diagrams are the ones in Fig. 3.6 when the tilde fields run inside the loop and the result is

$$G_{\rm LUED}(a) = \frac{S_{NP}}{S_{SM}} = 1 + 2\int_0^1 \sum_{n=1}^{N-1} \frac{a^2 x(1-x)\,dx}{4(N-1)^2 \sin^2(n\pi/2N) + a^2 x} \tag{4.47}$$

The last experimental determinations [27] agree with the SM expectations

$$1.3 \le S(x_t) \le 3.8 \qquad 95\% \text{ CL.} \tag{4.48}$$

The possible positive contributions have been lowered with respect to previous determinations [28] allowing better bounds. Despite the enhancement on the upper bound in Eq. (4.48) the bound on M is still rather weak. At the end, the bounds one can set on the masses of the new modes are below the W mass, and are therefore irrelevant compared to previously discussed bounds. Given the future experimental improvements on the determinations of $\sin 2\beta$ by BaBar and BELLE and in particular of the mass splitting ΔM_s for the B sector in LHC and FNAL one may use this observable to predict possible deviations from the SM predictions. It turns out that to an excellent accuracy [29] the mentioned deviations in the case of ΔM_s are governed by $G(a)$

$$G_{\rm NP}(a) = \frac{(\Delta M_s)_{NP}}{(\Delta M_s)_{SM}} > 1 \tag{4.49}$$

The greater values of $G(a)$ occur for small N but they are at most $G(a) \le 1.14$ that happens to be a too small deviation to be discriminated experimentally, in fact it is of the same order as the deviation studied in Ref. [29] and therefore the same reasonings given there apply here. The virtue of this results is that the possible existence of extra latticized dimensions will not pollute the extraction of the CKM matrix parameters from the future improvements in the determination of the unitarity triangle.

4.3 Outlook and conclusions

We have studied a five-dimensional extension of the SM in which the extra spatial dimension is latticized, and all SM fields propagate in it. The model has the property that there are no tree-level effects below the threshold of production of new particles. Therefore, to set a lower bound on the scale of the new physics one should consider one-loop processes. We considered a number of well-measured observables, and which depend strongly on the top-quark mass: the ρ parameter, $b \to s\gamma$, $Z \to b\bar{b}$, and the $B^0 \rightleftharpoons \overline{B}^0$ mixing. The dominant corrections, i.e. those proportional to the top-quark mass, have been computed, and compared with the ones obtained when only the SM is considered. It is found that the known bounds for the continuous version (UED) are rapidly reached when the extra dimension is latticized by only about 10 to 20 (four dimensional) sites. However, when a smaller number of sites is considered, the bounds on the scale of new physics is lowered by roughly a factor of 10%–25%, as can be seen in Fig. 4.2. This suggests that the phenomenology of latticized scenarios can be more accessible than in the continuous cases. Then, the limits on new particles are about 320–380 GeV. The bounds shown in Fig. 4.2 correspond to the mass of the lightest modes, defined in Eq. (4.21).

Chapter 5

Power corrections in models with extra dimensions

We revisit the issue of power-law running in models with extra dimensions[36]. The analysis is carried out in the context of a higher-dimensional extension of QED, with the extra dimensions compactified on a torus. It is shown that a naive β function, which simply counts the number of modes, depends crucially on the way the thresholds of the Kaluza-Klein modes are crossed. To solve these ambiguities we turn to the vacuum polarization, which, due to its special unitarity properties, guarantees the physical decoupling of the heavy modes. This latter quantity, calculated in the context of dimensional regularization, is used for connecting the low energy gauge coupling with the coupling of the D-dimensional effective field theory. We find that the resulting relation contains only logarithms of the relevant scales, and no power corrections. If, instead, hard cutoffs are used to regularize the theory, one finds power corrections, which could be interpreted as an additional matching between the effective higher-dimensional model and some unknown, more complete theory. The possibility of estimating this matching is examined in the context of a toy model. The general conclusion is that, in the absence of any additional physical principle, the power corrections depend strongly on the details of the underlying theory. Possible consequences of this analysis for gauge coupling unification in theories with extra dimensions are briefly discussed.

5.1 Introduction

The study of models with extra dimensions has received a great deal of attention recently [37, 38, 39, 40], mainly because of the plethora of theoretical and phenomenological ideas associated with them, and the flexibility they offer for realizing new, previously impossible, field-theoretic constructions. One of the most characteristic features of such models is that of the "early unification": the running of gauge couplings is supposed to be modified so strongly by the presence of the tower of KK modes, that instead of logarithmic it becomes linear, quadratic, etc, depending on the number of extra dimensions [41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57]. Specifically, it has been widely argued that the gauge couplings run as μ^{δ}, where the δ is the number of compact extra dimensions. Thus, if the extra dimensions are sufficiently large, such a behavior of the couplings could allow for their unification at accessible energies, of the order of a few TeV, clearly an exciting possibility.

The assertion that gauge-couplings display power-law running is based on rather intuitive arguments: In $\overline{\rm MS}$ schemes the QED β function is proportional to the number of "active" flavors, namely the number of particles lighter than the renormalization scale. Using this argument, and just counting the number of modes lighter than μ, one easily finds that the "β function" of QED in models with extra dimensions grows as μ^δ. This behavior is also justified by explicit calculations of the vacuum polarization of the photon using hard cutoffs; since the cutoff cannot be removed, due to the non-renormalizability of the theory, it is finally identified with the renormalization scale, a procedure which eventually leads to a similar conclusion [42, 43] (but with the final coefficient adjusted by hand in order to match the naive expectation in $\overline{\rm MS}$).

Even though these arguments are plausible, the importance of their consequences requires that they should be scrutinized more carefully [58]. In particular, the argument based on $\overline{\rm MS}$ running is rather tricky. As it is well known, the $\overline{\rm MS}$ scheme, because of its mass independence, does not satisfy decoupling, already at the level of four-dimensional theories. Instead, decoupling has to be *imposed* by hand every time a threshold is passed: one builds an effective theory below the threshold, m, and matches it to the theory above the threshold. This matching is carried out by requiring that some physical amplitude or Green's function (i.e. the effective charge) is the same when calculated using either theory, at energies where *both* theories are reliable, namely at Q^2 much below the threshold. Then, since the renormalization scale, μ, is still a free parameter, one chooses μ around m, in order to avoid large logarithms in the matching equations. In the case of gauge couplings and $\overline{\rm MS}$ schemes with $\mathrm{Tr}\left\{I_{\rm Dirac}\right\} = 4$ one finds (at one loop) that gauge couplings are continuous at $\mu = m$. This statement is, however, extremely scheme dependent: just by choosing $\mathrm{Tr}\left\{I_{\rm Dirac}\right\} = 2^{D/2}$ it gets completely modified (see for instance [59]) . In addition to these standard ambiguities, a new complication arises in the context of higher-dimensional models. In particular, the aforementioned procedure requires that the different scales be widely separated in order to avoid that higher dimension operators, generated in the process of matching, become important. However, the condition of having well-separated thresholds is rather marginally satisfied in the case of an infinite tower of KK modes with $M_n = nM_c$ (M_c is the compactification scale). In fact, as we will see in detail later, the results obtained for a β function that just counts the number of active modes depend very strongly on the prescription chosen for the way the various thresholds are crossed.

As has been hinted above, the deeper reason behind these additional type of ambiguities is the fact that, gauge theories in more than 4 dimensions, compactified or not, are not renormalizable. At the level of the 4-dimensional theory with an infinite number of KK modes the non-renormalizability manifests itself by the appearance of extra divergences, encountered when summing over all the modes. If the theory is not compactified the non-renormalizability is even more evident, since gauge couplings in theories with δ extra dimensions have dimension $1/M^{\delta/2}$. Therefore, gauge theories in extra dimensions should be treated as effective field theories (EFT). Working with such theories presents several difficulties, but, as we have learned in recent years, they can also be very useful. In the case of quantum field theories in extra dimensions, there is no alternative: basic questions, such as the calculation of observables or the unification of couplings, can only be addressed in the framework of the EFT's. However, before attempting to answer specific questions related to the running of couplings in the extra-dimensional theories, one should first clarify the type of EFT one is going to use, since there are, at least, two types of EFT [60]: In one type, known as "Wilsonian EFT" (WEFT) [61], one keeps only momenta below some scale Λ, while all the effects of higher momenta or heavy particles are encoded in the couplings of the effective theory. This method is very intuitive and leads, by definition, to finite results at each step; however, the presence of the cutoff in all expressions makes the method cumbersome to use, and in the particular case of gauge theories difficult to reconcile with gauge-invariance. The WEFT approach has already been applied to the problem of running of couplings in theories with compact extra dimensions, but only for the case of scalar theories [57]. Within the context of another type of EFT, often termed "continuum effective field theories" (CEFT) (see for instance [62, 63, 64, 60, 65, 66, 67]), one allows the momenta of particles to vary up to infinity, but heavy particles are removed from the spectrum at low energies. As in the WEFT case the effects of heavier particles are absorbed into the coefficients of higher dimension operators. Since the momenta are allowed to be infinite, divergences appear, and therefore the CEFT need to undergo both: regularization and renormalization. In choosing the specific scheme for carrying out the above procedures particular care is needed. Whereas in principle one could use any scheme, experience has shown that the most natural scheme for studying the CEFT is dimensional regularization with minimal subtraction [62, 63, 64, 60, 65, 66, 67]. CEFT are widely used in Physics: for example, when in the context of QCD one talks about 3, 4 or 5 active flavors, one is implicitly using this latter type of effective theories [68, 63]. Moreover, most of the analyses of Grand Unification [69, 64] resort to CEFT-type of constructions: one has a full theory at the GUT scale, then an effective field theory below the GUT scale (SM or MSSM) is built, and then yet another effective field theory below the Fermi scale (just QED+QCD). In these cases the complete theory is known, and the CEFT language is used only in order to simplify the calculations at low energies and to control the large logarithms which appear when there are widely separated scales. Nevertheless, CEFT's are useful even when the complete theory is not known, or when the connection with the complete theory cannot be worked out; this is the case of Chiral Perturbation theory (χPT) [70, 71, 72, 73] (for more recent reviews see also [65, 74, 75]).

It is important to maintain a sharp distinction between the two types of EFT mentioned above, i.e. Wilsonian or continuum, because conceptually they are quite different. However, perhaps due to the fact that the language is in part common to both types of theories, it seems that they are often used interchangeably in the literature, especially when employing cutoffs within the CEFT framework. In particular, since the couplings α_i have dimensions $[\alpha_i] = M^{-n}$, when computing loops one generally obtains effects which grow as $(\Lambda^n \alpha_i)^m$, where Λ is the formal CEFT cutoff, and as such is void of physics. As a consequence, physical observables should be made as independent of these cutoffs as possible by introducing as many counterterms as needed to renormalize the answer. Not performing these renormalizations correctly, or identifying naively formal cutoffs with the physical cutoffs of the effective theory, can lead to completely non-sensible results (see for instance [76, 77]). This type of pitfalls may be avoided by simply using dimensional regularization, since the latter has the special property of not mixing operators with different dimensionalities.

The usual way to treat theories with compactified extra dimensions is to define them as a 4-dimensional theory with a truncated tower of KK modes at some large but otherwise arbitrary N_s, a procedure which effectively amounts to using a hard cutoff in the momenta of the extra dimensions. Thus, physical quantities calculated in this scheme depend explicitly on the cutoff N_s, which is subsequently identified with some physical cutoff. However, as already commented, N_s plays the role of a formal cutoff, and is therefore plagued with all the aforementioned ambiguities. Identification of this formal cutoff with a universal physical cutoff can give the illusion of predictability, making us forget that we are dealing with a non-renormalizable theory with infinite number of parameters, which can be predictive only at low energies, where higher dimension operators may be neglected.

In this paper we want to analyze the question of the running of gauge couplings in theories with compact dimensions from the CEFT "canonical" point of view. We hasten to emphasize that even the CEFT presents conceptual problems in theories with compactified dimensions. Specifically, as mentioned above, in the CEFT approach the (virtual) momenta are allowed to vary up to infinity; however, momenta related to the compactified extra dimensions turn out to be KK masses in the 4-dimensional compactified theory, where it is supposed that one only keeps particles lighter than the relevant scale. Thus, truncating the KK series amounts to cutting off the momenta of the compactified dimensions. Therefore, in order to define a true "non-cutoff" CEFT scheme we are forced to keep all KK modes. Our main motivation is to seriously explore this approach, and investigate both its virtues and its limitations for the problem at hand. We hope that this study will help us identify more clearly which quantities can and which cannot be computed in effective extra-dimensional theories.

In section 5.2 we discuss the usual arguments in favor of power-law running of gauge couplings and show that they depend crucially on the way KK thresholds are crossed. In particular we show that, a one-loop β function which simply counts the number of modes, diverges for more than 5 dimensions, if the physical way of passing thresholds dictated by the vacuum polarization function (VPF) is imposed.

In section 5.3 we introduce a theory with one fermion and one photon in $4+\delta$ dimensions, with the extra δ ones compactified. This theory, which is essentially QED in $4+\delta$ dimensions, serves as toy model for studying the issue of power corrections and the running of the coupling in a definite framework.

In section 5.4 we study the question of decoupling KK modes in the aforementioned theory by analyzing the behavior of the VPF of the (zero-mode) photon. Since, as commented above, decoupling the KK modes one by one is problematic, we study the question of how to decouple all of them at once. To accomplish this we consider the VPF of the photon with all KK modes included, and study how it reduces at $Q^2 \ll M_c$ to the standard QED VPF with only one light mode. Since the entire KK tower is kept untruncated, the theory is of course non-renormalizable; therefore, to compute the VPF we have to regularize and renormalize it in the spirit of the CEFT, in a similar way that observables are defined in χPT. As in χPT, it is most convenient to use dimensional regularization with minimal subtraction, in order to maintain a better control on the mixing among different operators. However, at the level of the 4-dimensional theory the non-renormalizability manifest itself through the appearance of divergent sums over the infinite KK modes, and dimensional regularization does no seem to help in regularizing them. The dimensional regularization of the VPF is eventually accomplished by exploiting the fact that its UV behavior coincides to that found when the δ extra dimensions have not been compactified[1]. To explore this point we first resort to the standard unitarity relation (optical theorem), which relates the imaginary part of the VPF to the total cross section in the presence of the KK modes; the latter is finite because the phase-space truncates the series. For $Q^2 \gg M_c^2$ the uncompactified result for the imaginary part of the VPF is rapidly reached, i.e. after passing a few thresholds. We then compute the real part of the one-loop VPF in the non-compact theory in $4+\delta$ dimensions, where, of course we can use directly dimensional regularization to regularize it (since no KK reduction

[1] This is in a way expected, since for very large $Q^2 \gg M_c = 1/R_c$ the compactification effects should be negligible. Note, however, that this is not always the case; a known exception is provided by the orbifold compactification [58].

has taken place). For later use we also present results in which the same quantity is evaluated by using hard cutoffs. Finally, we show that the UV divergences of the one-loop VPF are indeed the same in both the (torus)-compactified and uncompactified theories. Therefore, in order to regularize the VPF in the compactified theory with an infinite number of KK modes it is sufficient to split the VPF into two pieces, an "uncompactified" piece, corresponding to the case where the extra dimensions are treated at the same footing as the four usual ones, and a piece which contains all compactification effects. We show that this latter piece is UV and IR finite and proceed to evaluate it, while all UV divergences remain in the former, which we evaluate using dimensional regularization.

The results of previous sections are used in section 5.5 to define an effective charge $\alpha_{\rm eff}(Q)$ which can be continuously extrapolated from $Q^2 \ll M_c$ to $Q^2 \gg M_c$. We use this effective charge to study the matching of couplings in the low energy effective theory (QED) to the couplings of the theory containing an infinite of KK modes. In the context of dimensional regularization we find that this matching contains only the standard logarithmic running from m_Z to the compactification scale M_c, with no power corrections. On the other hand, if hard cutoffs are used to regularize the VPF in the non-compact space, one does find power corrections, which may be interpreted as an additional matching between the effective $D = 4+\delta$ dimensional field theory and some more complete theory. We discuss the possibility of estimating this matching in the EFT without knowing the details of the full theory. This point is studied in a simple extension of our original toy-model, by endowing the theory considered (QED in $4+\delta$ compact dimensions) with an additional fermion with mass $M_f \gg M_c$, which is eventually integrated out.

5.2 Crossing thresholds

The simplest argument (apart from the purely dimensional ones) in favor of power-law running in theories with extra dimensions is based on the fact that in $\overline{\rm MS}$-like schemes the β function is proportional to the number of active modes. Theories with δ extra compact dimensions contain, in general, a tower of KK modes. In particular, if we embed QED in extra dimensions we find that electrons (also photons) have a tower of KK modes with masses $M_n^2 = \left(n_1^2 + n_2^2 + \cdots + n_\delta^2\right) M_c^2$ with n_i integer values and $M_c = 1/R_c$ the compactification scale. The exact multiplicity of the spectrum depends on the details of the compactification procedure (torus, orbifold, etc). As soon as we cross the compactification scale, the KK modes begin to contribute, and therefore one expects that the β function of this theory will start to receive additional contributions from them. In a general renormalization scheme satisfying decoupling one can naively write

$$\beta = \sum_n \beta_0 f\left(\frac{\mu}{M_n}\right) \; , \tag{5.1}$$

where μ is the renormalization scale, β_0 is the contribution of a single mode, and $f(\mu/M)$ is a general step-function that decouples the modes as μ crosses the different thresholds, namely $f(\mu/M) \to 0 \quad \mu \ll M$ and $f(\mu/M) \to 1 \quad \mu \gg M$. For instance in $\overline{\rm MS}$ schemes $f(\mu/M) \equiv \theta(\mu/M - 1)$ where $\theta(x)$ is the step-function. Then one finds $\left(\Omega_\delta = 2\pi^{\delta/2}/\Gamma(\delta/2)\right)$

$$\beta = \sum_{n<\mu/M_c} \beta_0 \approx \beta_0 \int d\Omega_\delta n^{\delta-1} dn = \beta_0 \frac{\Omega_\delta}{\delta} \left(\frac{\mu^2}{M_c^2}\right)^{\delta/2} \; . \tag{5.2}$$

This argument, simple and compelling as it may seem, cannot be trusted completely because in $\overline{\mathrm{MS}}$ schemes the decoupling is put in by hand. Therefore, other types of schemes, in which decoupling seems natural, have been studied in the literature. For instance, in Ref. [43] the VPF of the photon at $Q^2 = 0$ was calculated in the presence of the infinite tower of KK modes by using a hard cutoff in proper time, and the result was used to compute the β function; in that case the modes decouple smoothly. In addition, after adjusting the cutoff by hand one can reproduce the aforementioned result obtained in $\overline{\mathrm{MS}}$. One can easily see that this procedure is equivalent to the use of the function $f(\Lambda/M) \equiv e^{-\frac{M_n^2}{\Lambda^2}}$ to decouple the KK modes

$$\beta = \sum_n \beta_0 e^{-\frac{M_n^2}{\Lambda^2}} \approx \beta_0 \left(\pi \frac{\Lambda^2}{M_c^2}\right)^{\delta/2} . \tag{5.3}$$

If one chooses by hand $\mu^\delta = \Gamma(1+\delta/2)\Lambda^\delta$, the sum in Eq. (5.3) agrees exactly with the sum obtained if one uses a sharp step-function. Even though this particular way of decoupling KK modes appears naturally in some string scenarios [78, 79, 80, 40], it hardly appears compelling from the field theory point of view; this procedure is not any better conceptually than the sharp step-function decoupling of modes: one obtains a smooth β function because one uses a smooth function to decouple the KK modes.

These two ways of decoupling KK modes, due to the very sharp step-like behavior they impose, lead to a finite result in (5.1) for any number of extra dimensions. One is tempted to ask, however, what would happen if one were to use a more physical way of passing thresholds. In fact, heavy particles decouple naturally and smoothly in the VPF, because they cannot be produced physically. Specifically, in QED in 4-dimensions at the one-loop level, the imaginary part, $\Im m\Pi(q^2)$, of the VPF $\Pi(q^2)$ is directly related, via the optical theorem, to the tree level cross sections σ for the physical processes $e^+e^- \to f^+f^-$, by

$$\Im m\Pi(s) = \frac{s}{e^2}\,\sigma(e^+e^- \to f^+f^-) . \tag{5.4}$$

Given a particular contribution to the spectral function $\Im m\Pi(s)$, the corresponding contribution to the renormalized vacuum polarization function $\Pi_R(q^2)$ can be reconstructed via a once–subtracted dispersion relation. For example, for the one–loop contribution of the fermion f, choosing the on–shell renormalization scheme, one finds (if q is the physical momentum transfer with $q^2 < 0$, as usual we define $Q^2 \equiv -q^2$):

$$\begin{aligned}\Pi_R(Q) &= Q^2 \int_{4m_f^2}^{\infty} ds \frac{1}{s(s+Q^2)} \frac{1}{\pi} \Im m\Pi(s) \\ &= \frac{\alpha}{\pi} \times \begin{cases} \dfrac{1}{15}\dfrac{Q^2}{m_f^2} + \mathcal{O}\left(\dfrac{Q^4}{m_f^4}\right) & Q^2/m_f^2 \to 0 \\ \frac{1}{3}\ln\left(\frac{Q^2}{m_f^2}\right) - \frac{5}{9} + \mathcal{O}\left(\frac{m_f^2}{Q^2}\right) & Q^2/m_f^2 \to \infty , \end{cases}\end{aligned} \tag{5.5}$$

where $\alpha \equiv e^2/(4\pi)$. The above properties can be extended to the QCD effective charge [81], with the appropriate modifications to take into account the non-Abelian nature of the theory, and provide a physical way for computing the matching equations between couplings in QCD at quark mass thresholds. One computes the VPF of QCD with n_f flavors and that of QCD with $n_f - 1$ flavors, and requires that the effective charge is the same for $Q^2 \ll$

m_f in the two theories. This procedure gives the correct relation between the couplings in the two theories [82, 59, 83]. However, one can easily see that this cannot work for more than one extra dimension. To see that, let us consider the decoupling function $f(\mu/M)$ provided by the one-loop VPF, which, as explained, captures correctly the physical thresholds. The corresponding $f(\mu/M)$ may be obtained by differentiating $\Pi_R(Q)$ once with respect to Q^2; it is known [84] that the answer can be well-approximated by a simpler function of the form $f(\mu/M) = \mu^2/(\mu^2 + 5M^2)$. We see immediately that if we insert this last function in Eq. (5.1) and perform the sum over all KK modes the result is convergent only for one extra dimension (with a coefficient which is different from the one obtained with the renormalization schemes mentioned earlier), while it becomes highly divergent for several extra dimensions. We conclude therefore that the physical way of decoupling thresholds provided by the VPF seems to lead to a divergent β function in more than one extra dimension. As we will see, this is due to the fact that, in order to define properly the one-loop VPF for $\delta > 1$, more than one subtraction is needed.

5.3 A toy model

To be definite we will consider a theory with one fermion and one photon in $4+\delta$ dimensions, in which the δ extra dimensions are compactified on a torus of equal radii $R_c \equiv 1/M_c$. The Lagrangian is given by

$$\mathcal{L}_\delta = -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + i\bar{\psi}\gamma^\alpha D_\alpha\psi + \mathcal{L}_{\rm ct} \ , \tag{5.6}$$

where $\alpha = 0, \cdots, 3, \cdots, 3+\delta$. We will also use Greek letters to denote four-dimensional indices $\mu = 0, \cdots 3$. $D_\alpha = \partial_\alpha - ie_D A_\alpha$ is the covariant derivative with e_D the coupling in $4+\delta$ dimensions which has dimension $[e_D] = 1/M^{\delta/2}$. After compactification, the dimensionless gauge coupling in four-dimensions, e_4, and the dimensionfull $4+\delta$ coupling are related by the compactification scale [2]

$$e_4 = e_D\left(\frac{M_c}{2\pi}\right)^{\delta/2} . \tag{5.7}$$

Evidently e_D is determined from the four-dimensional gauge coupling and the compactification scale, but in the uncompactified space we can regard it as a free parameter (as f_π in χPT). Finally $\mathcal{L}_{\rm ct}$ represents possible gauge invariant operators with dimension $2+D$ or higher, which are in general needed for renormalizing the theory; they can be computed only if a more complete theory, from which our effective theory originates, is given. For instance, by computing the VPF we will see that a $\mathcal{L}_{\rm ct}$ of the form

$$\mathcal{L}_{\rm ct} = \frac{c_1}{M_s^2}D_\alpha F^{\alpha\beta}D^\lambda F_{\lambda\beta} + \cdots \tag{5.8}$$

is needed to make it finite.

The spectrum after compactification contains a photon (the zero mode of the four-dimensional components of the gauge boson), the δ extra components of the gauge boson remain in the spectrum as δ massless real scalars, a tower of massive vector bosons with masses $M_n^2 = \left(n_1^2 + n_2^2 + \cdots + n_\delta^2\right) M_c^2$, $n_i \in \mathbb{Z}$, $n_i \neq 0$, $2^{[\delta/2]}$ massless Dirac fermions (here the symbol $[x]$ represents the closest integer to x smaller or equal than x), and a tower of massive Dirac

[2] Note that the factors 2π depend on the exact way the extra dimensions are compactified (on a circle, orbifold, etc).

fermions with masses given also by the above mass formula. Note that this theory does not lead to normal QED at low energies, first because the δ extra components of the gauge boson remain in the spectrum, and second because in $4+\delta$ dimensions the fermions have $4 \cdot 2^{[\delta/2]}$ components, which remain as zero modes, leading at low energy to a theory with $2^{[\delta/2]}$ Dirac fermions. In the $D = 4+\delta$ theory these will arise from the trace of the identity of the γ matrices, which just counts the number of components of the spinors. To obtain QED as a low energy one should project out the correct degrees of freedom by using some more appropriate compactification (for instance, orbifold compactifications can remove the extra components of the photon from the low energy spectrum, and leave just one Dirac fermion). However this is not important for our discussion of the VPF, we just have to remember to drop the additional factors $2^{[\delta/2]}$ to make contact with usual QED with only one fermion. Theories of this type, with all particles living in extra dimensions are called theories with "universal extra dimensions" [10] and have the characteristic that all the effects of the KK modes below the compactification scale cancel at tree level due to the conservation of the KK number. In particular, and contrary to what happens in theories where gauge and scalar fields live in the bulk and fermions in the brane [15, 85], no divergences associated to summations over KK towers appear at tree level. Finally, the couplings of the electron KK modes to the standard zero-mode photon are universal and dictated by gauge invariance. The couplings among the KK modes can be found elsewhere [5, 34]; they will not be important for our discussion of the VPF that we present here.

5.4 The vacuum polarization in the presence of KK modes

In this section we will study in detail the behavior of the one-loop VPF in the theory defined above for general values of the number δ of extra dimensions. The main problems we want to address are: i) the general divergence structure of the VPF, ii) demonstrate that it is possible to regulate the UV divergences using dimensional regularization, iii) the appearance of non-logarithmic (power) corrections, and, iv) their comparison to the analogous terms obtained when resorting to a hard-cutoff regularization.

5.4.1 The imaginary part of the vacuum polarization

One can try to compute directly the VPF of the zero-mode photon in a theory with infinite KK fermionic modes. However, one immediately sees that, in addition to the logarithmic divergences that one finds in QED, new divergences are encountered when summing over the infinite number of KK modes. One can understand the physical origin of these divergences more clearly by resorting to the unitarity relation (here s denotes the center-of-mass energy available for the production process):

$$\begin{aligned}\Im m \Pi^{(\delta)}(s) &= \frac{s}{e_4^2} \sum_n \sigma(e^+e^- \to f_n^+ f_n^-) \\ &= \frac{\alpha_4}{3} \sum_{n<n_{\rm th}} \left(1 + \frac{2M_n^2}{s}\right) \sqrt{1 - 4M_n^2/s}\ , \end{aligned} \tag{5.9}$$

where $n < n_{\rm th}$ represents the sum over all the electron KK modes that fulfill the relation $4\left(n_1^2 + n_2^2 + \cdots n_\delta^2\right) M_c^2 < s$, and $\alpha_4 = e_4^2/(4\pi)$. This sum can be evaluated approximately for

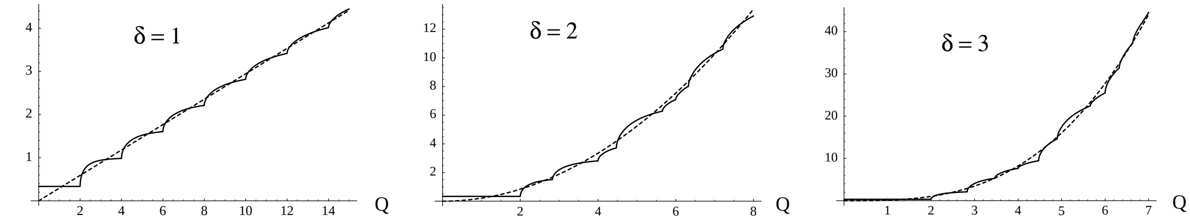


Figure 5.1: $\Im m\Pi^{(\delta)}(Q)$ as compared with the asymptotic value ($\delta = 1, 2, 3$). Q is given in units of M_c.

$s \gg M_c^2$ by replacing it by an integral; then we obtain

$$\Im m\Pi^{(\delta)}(s) \approx \frac{\alpha_4}{2^{3+\delta}} \frac{(\delta+2)\,\pi^{(\delta+1)/2}}{\Gamma\left((\delta+5)/2\right)} \left(\frac{s}{M_c^2}\right)^{\delta/2} . \tag{5.10}$$

It turns out that this last result captures the behavior of the same quantity when the extra dimensions are not compact; this is so because, at high energies, the effects of the compactification can be neglected. In fact, this result may be deduced on simple dimensional grounds: as commented, the gauge coupling in $4 + \delta$ dimensions has dimension $1/M^{\delta/2}$; therefore one expects that $\Im m\Pi^{(\delta)}(s)$ will grow with s as $\left(s/M^2\right)^{\delta/2}$, which is what we obtained from the explicit calculation. To see how rapidly one reaches this regime we can plot the exact result of $\Im m\Pi^{(\delta)}(s)$ together with the asymptotic value. As we can see in Fig.5.1, the asymptotic limit is reached very fast, especially for higher dimensions. For practical purposes one can reliably use the asymptotic value soon after passing the first threshold, $Q > 2M_c$, incurring errors which are below 10%.

Now we can try to obtain the real part by using a dispersion relation as the one used in 4-dimensional QED, i.e. Eq.(5.5). However, one immediately sees that it will need a number of subtractions which depends on the value of δ. Thus, for just one extra dimension, as in 4-dimensional QED, one subtraction is enough, for $\delta = 2$ and $\delta = 3$ two subtractions are needed, see Eq. (5.10), and so on. This just manifests the non-renormalizability of the theory, and in the effective field theory language, the need for higher dimension operators acting as counterterms. Even though this "absorptive" approach is perfectly acceptable, it would be preferable to have a way of computing the real part directly at the Lagrangian level (by computing loops, for instance). As commented in the introduction, to accomplish this we will use dimensional regularization.

5.4.2 The vacuum polarization in uncompactified $4 + \delta$ dimensions

When using dimensional regularization to compute the VPF in uncompactified space, to be denoted $\Pi_{\rm uc}$, simple dimensional arguments suggest that one should typically obtain contributions of the form

$$\Pi_{\rm uc}(Q) \propto e_4^2 \left(2\pi\frac{Q}{M_c}\right)^{\delta} ,$$

since the two vertices in the loop provide a factor e_D^2, whose dimensions must be compensated by the only available scale in the problem, namely Q^2. In the above formula we have used the relation of Eq.(5.7) in order to trade off e_D for e_4. The omitted coefficient in front will be generally divergent, and will be regularized by letting $\delta \to \delta - \epsilon$.

Let us compute the VPF $\Pi^{\alpha\beta}_{\rm uc}(q)$ in uncompactified space, assuming that, if necessary, the dimensions will be continued to complex values. We have that

$$\Pi^{\alpha\beta}_{\rm uc}(q) = ie_D^2 \int \frac{d^{4+\delta}k}{(2\pi)^{4+\delta}} {\rm Tr}\left\{\gamma^\alpha \frac{1}{\not k}\gamma^\beta \frac{1}{\not k + \not q}\right\} \; , \tag{5.11}$$

which, by gauge-invariance assumes the standard form

$$\Pi^{\alpha\beta}_{\rm uc}(q) = \left(q^2 g^{\alpha\beta} - q^\alpha q^\beta\right) \Pi_{\rm uc}(q) \; .$$

If we now were to use that, in D-dimensions, ${\rm Tr}[\gamma^\alpha\gamma^\beta] = 2^{[D/2]} g^{\alpha\beta}$, we would find that the low energy limit has an extra $2^{[\delta/2]}$ factor, which, as commented, is an artifact of the torus compactification: there are $2^{[\delta/2]}$ too many fermions in the theory. Therefore we simply drop this factor by hand. Moreover, we use Eq.(5.7) and employ the proper-time parametrization in intermediate steps, thus arriving at:

$$\begin{aligned} \Pi_{\rm uc}(Q) &= \frac{e_4^2}{2\pi^2}\left(\frac{\pi}{M_c^2}\right)^{\delta/2} \int_0^1 dx x(1-x) \int_0^\infty \frac{d\tau}{\tau^{1+\frac{\delta}{2}}} \exp\left\{-\tau\, x(1-x) Q^2\right\} \\ &= \frac{e_4^2}{2\pi^2} \frac{\pi^{\delta/2}\,\Gamma^2(2+\frac{\delta}{2})}{\Gamma(4+\delta)} \Gamma\left(-\frac{\delta}{2}\right)\left(\frac{Q^2}{M_c^2}\right)^{\delta/2} \; . \end{aligned} \tag{5.12}$$

A simple check of this result may be obtained by computing its imaginary part. To that end we let $Q^2 \to -q^2 - i\epsilon$ with $q^2 > 0$. Then

$$\Im m \left\{-q^2 - i\epsilon\right\}^{\delta/2} = -\left(q^2\right)^{\delta/2} \sin\frac{\delta\pi}{2} \, .$$

Now we can use that $\Gamma(-\delta/2)\Gamma(1+\delta/2) = -\pi/\sin(\delta\pi/2)$ to write

$$\Im m \left\{\Pi_{\rm uc}(q)\right\} = \alpha_4 \frac{2\pi^{\delta/2}\,\Gamma^2(2+\frac{\delta}{2})}{\Gamma(4+\delta)\Gamma(1+\delta/2)}\left(\frac{q^2}{M_c^2}\right)^{\delta/2} = \frac{\alpha_4}{2^{3+\delta}} \frac{(\delta+2)\,\pi^{(\delta+1)/2}}{\Gamma\left((\delta+5)/2\right)}\left(\frac{q^2}{M_c^2}\right)^{\delta/2} \; ,$$

which agrees with our previous result of Eq.(5.10).

For odd values of δ, the one-loop $\Pi_{\rm uc}(Q)$ computed above is finite, since the $\Gamma(-\frac{\delta}{2})$ can be calculated by analytic continuation. This result is in a way expected, since in odd number of dimensions, by Lorentz invariance, there are no appropriate gauge invariant operators able to absorb any possible infinities generated in the one-loop VPF; this would require operators which give contributions that go like Q^δ. Notice, however, that at higher orders $\Pi_{\rm uc}(Q)$ will eventually become divergent. For instance, in five dimensions at two loops, the VPF should go as Q^2, since there are four elementary vertices. The divergences generated by these contributions could be absorbed in an operator such as the one considered in the previous section, namely $D_\alpha F^{\alpha\beta} D^\lambda F_{\lambda\beta}$. On the other hand, when δ is even, $\Gamma(-\frac{\delta}{2})$ has a pole, and subtractions are needed already at one loop. To compute the divergent and finite parts in a well-defined way we will use dimensional regularization, i.e. we will assume that $\delta \to \delta - \epsilon$. Notice however that, unlike in 4-dimensions, we do not need to introduce an additional scale at this point, i.e. the equivalent of the 't Hooft mass scale μ: M_c plays the role of μ, and can be used to keep e_4 dimensionless. After expanding in ϵ we find a simple pole accompanied by the usual logarithm

$$\Pi_{\rm uc}(Q) \propto \left(\frac{Q^2}{M_c^2}\right)^{\delta/2} \left\{-\frac{2}{\epsilon} + \ln(Q^2/M_c^2) + \cdots\right\} . \tag{5.13}$$

Here the ellipses represent a finite constant. Now, to renormalize this result we must introduce higher dimension operators (for instance, if $\delta = 2$ the operator $D_\alpha F^{\alpha\beta} D^\lambda F_{\lambda\beta}$ will do the job) which could absorb the divergent piece. The downside of this, however, is that we also have to introduce an arbitrary counterterm, κ, corresponding to the contribution of the higher dimension operator; thus we obtain a finite quantity proportional to $\log(Q^2/M_c^2) + \kappa$. Note that, since κ is arbitrary, we can always introduce back a renormalization scale and write $\log(Q^2/M_c^2) + \kappa = \log(Q^2/\mu^2) + \kappa(\mu)$ with $\kappa(\mu) = \kappa + \log(\mu^2/M_c^2)$. It is also important to remark that, in the case of odd number of dimensions, although at one loop we do not need any counterterm to make the VPF finite, higher dimensional operators could still be present and affect its value.

In the case of uncompactified space, it is interesting to compare the above result with that obtained by regularizing the integral using a hard cutoff. To study this it is enough to carry out the integral of Eq. (5.12), with a cutoff in $\tau_0 = 1/\Lambda^2$:

$$\Pi_{\rm uc}(Q) = \frac{e_4^2}{2\pi^2}\left(\frac{\pi}{M_c^2}\right)^{\delta/2} \int_0^1 dx\, x(1-x) \int_{\tau_0}^\infty \frac{d\tau}{\tau^{1+\delta/2}} \exp\left\{-\tau\, x(1-x) Q^2\right\} . \tag{5.14}$$

Then, for $\delta = 1, 2, 3$ we obtain

$$\Pi^{(1)}_{\rm uc}(Q) = \frac{e_4^2}{2\pi^2}\left(-\frac{3\pi^2 Q}{64 M_c} + \frac{\sqrt{\pi} Q^2}{15 M_c \Lambda} + \frac{\sqrt{\pi}\Lambda}{3 M_c}\right) \quad , \tag{5.15}$$

$$\Pi^{(2)}_{\rm uc}(Q) = \frac{e_4^2}{2\pi^2}\left(\frac{\pi\Lambda^2}{6 M_c^2} + \frac{\pi Q^2}{30 M_c^2}\left(\log(Q^2/\Lambda^2) + \gamma - \frac{77}{30}\right)\right) \quad , \tag{5.16}$$

$$\Pi^{(3)}_{\rm uc}(Q) = \frac{e_4^2}{2\pi^2}\left(\frac{5\pi^3 Q^3}{768 M_c^3} - \frac{\pi^{3/2} Q^2 \Lambda}{15 M_c^3} + \frac{\pi^{3/2}\Lambda^3}{9 M_c^3}\right) \quad . \tag{5.17}$$

As we can see, the pieces which are independent of the cutoff are exactly the same ones we obtained using dimensional regularization. But, in addition, we obtain a series of contributions which depend explicitly on the cutoff. For instance we find corrections to the gauge coupling which behave as Λ^δ, and just redefine the gauge coupling we started with [86]. In the case of five dimensions we also generate a term linear in Q^2; however it is suppressed by $1/\Lambda$, and therefore it approaches zero for large Λ. In the case of six dimensions we obtain the same logarithmic behavior we found with dimensional regularization, and the result can be cast in identical form, if the cutoff is absorbed in the appropriate counterterm. For seven dimensions we also find divergent contributions which go as Q^2. This means that, when using cutoffs, higher dimension operators in the derivative expansion (e.g. operators giving contributions as Q^2 or higher) are necessary to renormalize the theory and must be included. In the case of dimensional regularization this type of operators is not strictly needed at one loop; however, nothing forbids them in the Lagrangian, and they could appear as "finite counterterms". If one were to identify the Λ in the above expressions with a physical cutoff, one might get the impression that, contrary to the dimensional regularization approach where arbitrary counterterms are needed, one could now obtain all types of contributions with only one additional parameter, namely Λ. This is however not true: the regulator function is arbitrary, we simply have chosen one among an infinity of possibilities. By changing the regulator function we can change the coefficients of the different contributions at will, except for those few contributions which are independent of Λ. These latter are precisely the ones we have obtained by using dimensional regularization. Thus, even when using cutoffs one has to add counterterms from higher dimension operators, absorb the cutoff, and express the result in terms of a series of unknown coefficients. The lesson is that with dimensional regularization we obtain all calculable pieces, while the non-calculable pieces are related to higher dimensional terms in the Lagrangian.

What we will demonstrate next is that the one-loop VPF in the compactified theory on a torus can be renormalized exactly as the VPF in the uncompactified theory; this will allow us to compute it for any number of dimensions, and examine its behavior for large and for small values of the Q^2.

5.4.3 The vacuum polarization in δ compact dimensions

From the four-dimensional point of view the vacuum polarization tensor in the compactified theory is

$$\Pi^{\mu\nu}(q^2) = \sum_n ie_4^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left\{\gamma^\mu \frac{1}{\not{k} - m_n} \gamma^\nu \frac{1}{\not{k} + \not{q} - m_n}\right\}$$

with $m_n^2 = \left(n_1^2 + n_2^2 + \cdots + n_\delta^2\right) M_c^2$; for simplicity we have assumed a common compactification radius $R = 1/M_c$ for all the extra dimensions. The sum over n denotes collectively the sum over all the modes $n_i = -\infty, \cdots, +\infty$. The contribution of each mode to this quantity seems quadratically divergent, like in ordinary QED; however, we know that gauge invariance converts it to only logarithmically divergent. But, in addition, the sum over all the modes makes the above expressions highly divergent. Instead of attempting to compute it directly, we will add and subtract the contribution of the vacuum polarization function of the uncompactified theory in $4+\delta$ dimensions:

$$\Pi^{\mu\nu}(q) = [\Pi^{\mu\nu}(q) - \Pi^{\mu\nu}_{\mathrm{uc}}(q)] + \Pi^{\mu\nu}_{\mathrm{uc}}(q) = \Pi^{\mu\nu}_{\mathrm{fin}}(q) + \Pi^{\mu\nu}_{\mathrm{uc}}(q)\ . \tag{5.18}$$

Here we have taken already into account the relation between the coupling in $4+\delta$ dimensions and the four-dimensional coupling and have restricted the external Lorentz indices to the 4-dimensional ones. Depending on the value of δ the vacuum polarization can be highly divergent (naively as $\Lambda^{\delta+2}$, and after taking into account gauge invariance as Λ^δ). However, we can use dimensional regularization (or any other regularization scheme) to make it finite. The important point is that the quantity $\Pi^{\mu\nu}_{\mathrm{fin}}(Q)$ is UV and IR finite and can unambiguously computed.

Instead of doing the two calculations from scratch, we will do the following:

i) We will first compute the compactified expression by using Schwinger's proper time, τ, to regularize the UV divergences.

ii) We will show that the UV behavior of the compactified theory, $\tau \to 0$, is just the behavior of the uncompactified theory.

iii) Therefore, to compute $\Pi^{\mu\nu}_{\mathrm{fin}}(Q)$ it is sufficient to compute $\Pi^{\mu\nu}(Q)$ and then subtract its most divergent contribution when $\tau \to 0$. We will see that it is sufficient to make it finite.

After a few manipulations $\Pi^{\mu\nu}(Q)$ can be written as

$$\Pi^{\mu\nu}(q) = \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) \Pi(q) \ ,$$

where [43]

$$\Pi(Q) = \frac{e_4^2}{2\pi^2} \sum_n \int_0^1 dx x(1-x) \int_0^\infty \frac{d\tau}{\tau} \exp\left\{-\tau\left(x(1-x)Q^2 + m_n^2\right)\right\} \ .$$

$\Pi(Q)$ can be written in terms of the function

$$\bar\theta_3(\tau) \equiv \sum_{n=-\infty}^{+\infty} e^{-n^2\tau} = \sqrt{\frac{\pi}{\tau}}\,\bar\theta_3\left(\frac{\pi^2}{\tau}\right)$$

as

$$\Pi(Q) = \frac{e_4^2}{2\pi^2} \int_0^1 dx x(1-x) \int_0^\infty \frac{d\tau}{\tau} \exp\left\{-\tau x(1-x)\frac{Q^2}{M_c^2}\right\} \bar\theta_3^\delta(\tau) \ ,$$

where we have rescaled τ in order to remove M_c from the $\bar\theta_3(\tau)$ function. This last expression for $\Pi(Q)$ is highly divergent in the UV ($\tau \to 0$), because in that limit the $\bar\theta_3(\tau)$ function goes as $\sqrt{\pi/\tau}$. Then, if we define, as in Eq. (5.18), $\Pi_{fin} = \Pi - \Pi_{\rm uc}$, we have

$$\Pi_{\rm fin}(Q) = \frac{e_4^2}{2\pi^2} \int_0^1 dx x(1-x) \int_0^\infty \frac{d\tau}{\tau} \exp\left\{-\tau x(1-x)\frac{Q^2}{M_c^2}\right\} \left(\bar\theta_3^\delta(\tau) - \left(\frac{\pi}{\tau}\right)^{\delta/2}\right) \ ,$$

which is completely finite for any number of dimensions. In fact, the last term provides a factor

$$F_\delta(\tau) \equiv \bar\theta_3^\delta(\tau) - \left(\frac{\pi}{\tau}\right)^{\delta/2} \stackrel{\tau\to 0}{\longrightarrow} 2\delta\left(\frac{\pi}{\tau}\right)^{\delta/2} \exp\left\{-\frac{\pi^2}{\tau}\right\} \ ,$$

that makes the integral convergent in the UV, while for large τ this function goes to 1 quite fast. In this region the integral is cut off by the exponential of momenta; so we can think of the exponential $\exp\left\{-\tau x(1-x)\frac{Q^2}{M_c^2}\right\}$ as providing a cutoff for $\tau > 4M_c^2/Q^2$, and $F_\delta(\tau)$ as providing a cutoff for $\tau < \pi^2$. With this in mind, we can estimate $\Pi_{\rm fin}(Q)$ as

$$\Pi_{\rm fin}(Q) \approx \frac{e_4^2}{2\pi^2} \sum_n \int_0^1 dx x(1-x) \int_{\pi^2}^{4M_c^2/Q^2} \frac{d\tau}{\tau} = -\frac{e^2}{2\pi^2}\frac{1}{6}\log\frac{Q^2\pi^2}{4M_c^2}, \qquad Q^2 < 4M_c^2/\pi^2; \tag{5.19}$$

it is just the ordinary running of the zero mode. As Q^2 grows, the upper limit of integration is smaller than the lower limit, and then we expect that $\Pi_{\rm fin}(Q)$ should vanish. In that region $\Pi(Q)$ will be dominated completely by $\Pi_{\rm uc}(Q)$.

Let us evaluate $\Pi_{\rm fin}(Q)$ for any number of extra dimensions. To this end we will approximate the function $F_\delta(\tau)$ as follows

$$F_\delta(\tau) = \begin{cases} 2\delta\left(\frac{\pi}{\tau}\right)^{\delta/2} \exp\left\{-\frac{\pi^2}{\tau}\right\} & \tau < \pi \\ 1 + 2\delta\exp\{-\tau\} - \left(\frac{\pi}{\tau}\right)^{\delta/2} & \tau > \pi \end{cases} \ . \tag{5.20}$$

The matching point in $\tau = \pi$ makes the function continuous. In Fig. 5.2 we display the exact function $F_\delta(\tau)$ (solid) and the approximation above (dashed) for $\delta = 1, 2, 3$. The approximation is very good except at a small region around the matching point $\tau = \pi$. This can be further improved by adding more terms from the expansions of the $\bar\theta(\tau)$ functions.

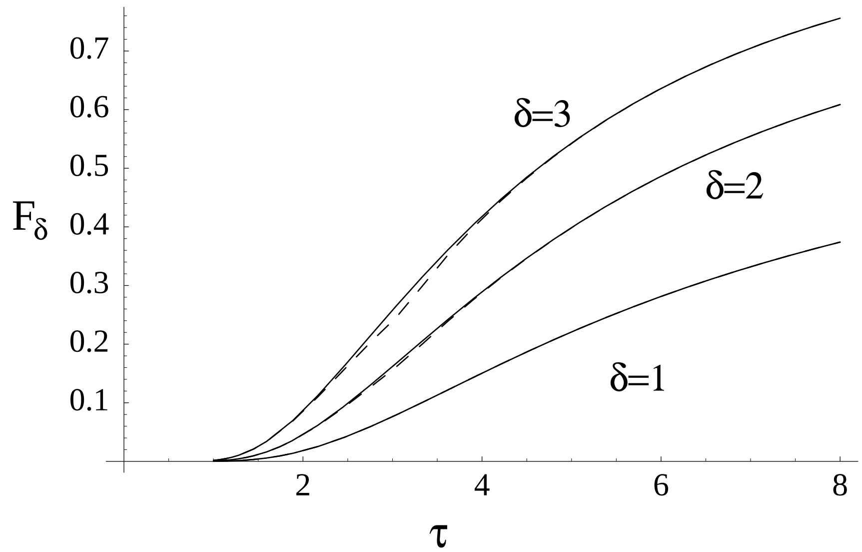


Figure 5.2: Exact values of $F_\delta(\tau)$ (solid) as compared with the approximation discussed in the text (dashed) for $\delta = 1, 2, 3$.

The approximate expression of Eq.(5.20) can be used to obtain semi-analytical expansions for $\Pi^\delta_{\rm fin}(Q)$ for small Q^2 (we define $w \equiv Q^2/M_c^2$)

$$\Pi^{(1)}_{\rm fin}(Q) = \frac{e_4^2}{2\pi^2}\left(-0.335 - 0.167\log(w) + 0.463\sqrt{w} - 0.110w + \cdots\right) , \tag{5.21}$$

$$\Pi^{(2)}_{\rm fin}(Q) = \frac{e_4^2}{2\pi^2}\left(-0.159 - 0.167\log(w) - 0.105w\left(\log(w) - 1.75\right) + \cdots\right) , \tag{5.22}$$

$$\Pi^{(3)}_{\rm fin}(Q) = \frac{e_4^2}{2\pi^2}\left(-0.0937 - 0.167\log(w) + 0.298w - 0.202\sqrt{w^3} + \cdots\right) . \tag{5.23}$$

To see how good these approximate results are, we can compare with the exact results that can be obtained when $\delta = 1$. In this case we have

$$\begin{aligned}
\Pi^{(1)}_{\rm fin}(Q) &= \frac{e_4^2}{2\pi^2}\int_0^1 dx x(1-x)\sum_{n=1}^{\infty}\int_0^\infty \frac{d\tau}{\tau}\exp\left\{-\tau x(1-x)w\right\} 2\left(\frac{\pi}{\tau}\right)^{1/2}\exp\left\{-\frac{n^2\pi^2}{\tau}\right\} \\
&= \frac{e_4^2}{2\pi^2}\int_0^1 dx x(1-x)\sum_{n=1}^{\infty}\frac{2}{n}\exp\left\{-2\pi n\sqrt{x(1-x)w}\right\} \\
&= \frac{e_4^2}{2\pi^2}\int_0^1 dx x(1-x)\left(-2\log\left(1-\exp\left(-2\pi\sqrt{x(1-x)w}\right)\right)\right) .
\end{aligned} \tag{5.24}$$

This last expression can be expanded for $w \ll 1$, and the integral over x can then be performed analytically, yielding

$$\Pi^{(1)}_{\rm fin}(Q) \approx \frac{e_4^2}{2\pi^2}\left(\frac{1}{18}\left(5 - 6\log(2\pi)\right) - \frac{1}{6}\log w + \frac{3\pi^2}{64}\sqrt{w} - \frac{\pi^2}{90}w + \cdots\right) , \tag{5.25}$$

which is in excellent agreement with our approximation (see Eq. (5.21)).

Eq. (5.24) can also be used to obtain the behavior of $\Pi^{(1)}_{\rm fin}(Q)$ for $w \gg 1$. In this limit we obtain

$$\Pi^{(1)}_{\rm fin}(Q) \approx \frac{e_4^2}{2\pi^2}\frac{3\zeta(5)}{\pi^4}\frac{M_c^4}{Q^4}\,, \qquad Q^2 \gg M_c^2\,.$$

For higher dimensions things are more complicated, but the behavior is the same, and we find

$$\Pi^{(\delta)}_{\rm fin}(Q) \approx \frac{e_4^2}{2\pi^2}\frac{4\delta\Gamma(2+\delta/2)}{\pi^{4+\delta/2}}K_\delta\frac{M_c^4}{Q^4}\,, \qquad Q^2 \gg M_c^2\,,$$

where K_δ is of the order of unity and is determined numerically ($K_1 = \zeta(5) = 1.037$, $K_2 = 1.165$, $K_3 = 1.244$). However, since the uncompactified contribution grows as $\left(Q^2/M_c^2\right)^{\delta/2}$ it is obvious that the contributions to $\Pi^{(\delta)}(Q)$ from $\Pi^{(\delta)}_{\rm fin}(Q)$ will be completely irrelevant for $Q^2 \gg M_c^2$.

Adding the finite and the uncompactified contributions we find that for $Q^2 \ll M_c^2$ the uncompactified contribution exactly cancels the corresponding piece obtained from the expansion of $\Pi^{(\delta)}_{\rm fin}(Q)$ (the $\sqrt{w}$ piece for $\delta = 1$, the $w\log(w)$ piece for $\delta = 2$, or the $\sqrt{w^3}$ for $\delta = 3$). Then, for $Q^2 \ll M_c^2$ and choosing $\mu = M_c$ we finally find:

$$\Pi^{(\delta)}(Q) = \frac{e_4^2}{2\pi^2}\left(a_0^{(\delta)} - \frac{1}{6}\log\left(\frac{Q^2}{M_c^2}\right) + a_1^{(\delta)}\frac{Q^2}{M_c^2} + \cdots\right)\,, \qquad Q^2 \ll M_c^2 \tag{5.26}$$

with the coefficients for 1,2 and 3 extra dimensions given by

δ	1	2	3
$a_0^{(\delta)}$	-0.335	-0.159	-0.0937
$a_1^{(\delta)}$	-0.110	0.183	0.298

As we will see below, in general the coefficients $a_1^{(\delta)}$ can be affected by non-calculable contributions from higher dimension operators in the effective Lagrangian, which we have not included.

The following comments related to Eq. (5.26), which is only valid in the dimensional regularization scheme we are using, are now in order:

(**i**) From Eq. (5.26) we see that for small Q^2, as expected, we recover the standard logarithm with the correct coefficient, independently of the number of extra dimensions. In addition, interestingly enough, we can compute also the constant term. Thus, although the full theory in $4+\delta$ dimensions is non-renormalizable and highly divergent, the low energy limit of the VPF calculated in our dimensional regularization scheme is actually finite: when seen from low energies the compactified extra dimensions seem to act as an ultraviolet regulator for the theory.

(**ii**) When the energy begins to grow, we start seeing effects suppressed by Q^2/M_c^2, which are finite, at one loop, for any number of dimensions except for $\delta = 2$. This is so because the gauge couplings have dimensions $1/M^{\delta/2}$, and therefore, the one-loop VPF goes like $1/M^\delta$.

(**iii**) For $\delta = 1$ one finds that, because of gauge and Lorentz invariance, there are no possible counterterms of this dimension. The VPF must be finite, and that is precisely the result one obtains with dimensional regularization. This of course changes if higher loops are considered: for instance, two-loop diagrams go like $1/M^2$, and, in general, we expect that they will have divergences, which, in turn, should be absorbed in the appropriate counterterms. In principle the presence of these counterterms could pollute our result; however, the natural size of these counterterms, arising at two loops, should be suppressed compared to the finite contributions we have computed.

(**iv**) For $\delta = 2$ one finds that the VPF goes as $1/M^2$, already at one loop, and that the result is divergent. The divergences have to be absorbed in the appropriate counterterm coming from higher dimension operators in the higher dimensional theory. The immediate effect of this, is that the coefficient of the Q^2 term in $\Pi^{(2)}(Q)$ becomes arbitrary, its value depending on the underlying physics beyond the compactification scale.

(**v**) For $\delta > 2$ all loop contributions to the Q^2 term are finite, simply because of the dimensionality of the couplings. This, however, does not preclude the existence of finite counterterms, which could be generated by physics beyond the compactification scale, that is, contributions from operators suppressed by two powers of the new physics scale like the operator in Eq. (5.8).

For $Q^2 \gg M_c^2$ the full VPF is completely dominated by the uncompactified contribution:

$$\begin{aligned}
\Pi^{(1)}(Q) &= -\frac{e_4^2}{2\pi^2}\frac{3\pi^2}{64}\sqrt{\frac{Q^2}{M_c^2}}\,, \\
\Pi^{(2)}(Q) &= \frac{e_4^2}{2\pi^2}\frac{\pi}{30}\frac{Q^2}{M_c^2}\log\left(\frac{Q^2}{M_c^2}\right) \qquad Q^2 \gg M_c^2\,, \\
\Pi^{(3)}(Q) &= \frac{e_4^2}{2\pi^2}\frac{5\pi^3}{768}\left(\frac{Q^2}{M_c^2}\right)^{\frac{3}{2}}\,.
\end{aligned}$$

As before, the VPF could also receive non-calculable contributions from higher dimension operators which we have not included; in fact, for $\delta = 2$, these are needed to renormalize the VPF. How large can these non-calculable contributions be? Since our D-dimensional theory is an effective theory valid only for $Q^2 \ll M_s^2$, even above M_c the results will be dominated by the lowest power of Q^2. In the case of $\delta = 1$, the first operator that one can write down goes as Q^2; therefore we expect that the one-loop contribution, of order $\sqrt{Q^2}$, that we have computed, will dominate completely the result, as long as we do not stretch it beyond the applicability of the effective Lagrangian approach. For $\delta = 2$, counterterms are certainly needed at order Q^2; still one can hope that the result will be dominated by the logarithm (as happens with chiral logarithms in χPT). For $\delta = 3$ (and higher), the one loop result grows as $\left(Q^2\right)^{\delta/2}$; however there could be operators giving contributions of order Q^2 with unknown coefficients (in fact although in dimensional regularization those are not needed, they must be included if cutoffs are used to regularize the theory). Therefore, unless for some reason they are absent from the theory, the result will be dominated by those operators.

5.5 Matching of gauge couplings

Using the VPF constructed in the previous section we can define a higher dimensional analogue of the conventional QED effective charge [87, 88], which will enter in any process involving off-shell photons, e.g.

$$\frac{1}{\alpha_{\text{eff}}(Q)} \equiv \frac{1}{\alpha_4}\left(1+\Pi^{(\delta)}(Q)\right)\Big|_{\overline{\text{MS}}_\delta}\,, \tag{5.27}$$

where $\alpha_4 = e_4^2/(4\pi)$. We remind the reader that e_4 denotes the (dimensionless) coupling of the four-dimensional theory including all KK modes; it is directly related to the gauge coupling in the theory with δ extra dimensions by Eq.(5.7). The subscript $\overline{\mathrm{MS}}_\delta$ means that the VPF has been regularized using dimensional regularization in $D = 4 + \delta - \epsilon$ dimensions, and that divergences, when present, are subtracted according to the $\overline{\mathrm{MS}}$ procedure.

To determine the relation between α_4 and the low energy coupling in QED, we have to identify the effective charge computed in the compactified theory with the low energy effective charge, at some low energy scale (for instance $Q^2 = m_Z^2 \ll M_c^2$), where both theories are valid. In that limit we can trust our approximate results of Eq. (5.26), and write

$$\frac{1}{\alpha_{\rm eff}(m_Z)} = \frac{1}{\alpha_4} + \frac{2}{\pi} a_0^{(\delta)} - \frac{2}{3\pi} \log\left(\frac{m_Z}{M_c}\right) . \tag{5.28}$$

This equation connects the low energy QED coupling with the coupling in the compactified D-dimensional theory, regularized by dimensional regularization. Note that this equation is completely independent of the way subtractions are performed to remove the poles in $1/\epsilon$. These poles only appear (and only for even number of dimensions) in the contributions proportional to Q^δ, which vanish for $Q \to 0$. Eq. (5.28) contains, apart from a finite constant, the standard logarithmic running from m_Z to the compactification scale M_c. It is interesting to notice that, in this approach, the logarithm comes from the finite piece, and should therefore be considered as an infrared (IR) logarithm. When seen from scales smaller than M_c, these logarithms appear to have an UV origin, while, when seen from scales above M_c, appear as having an IR nature.

It is important to emphasize that, in this scheme, the gauge coupling does not run any more above the compactification scale. This seems counter-intuitive, but it is precisely what happens in χPT when using dimensional regularization: f_π does not run, it just renormalizes higher dimensional operators [62].

Now we can use Eq. (5.28) to write the effective charge at all energies in terms of the coupling measured at low energies:

$$\frac{1}{\alpha_{\rm eff}(Q)} \equiv \frac{1}{\alpha_{\rm eff}(m_Z)} + \frac{1}{\alpha_4}\left(\Pi^{(\delta)}(Q) - \Pi^{(\delta)}(m_Z)\right)\Big|_{\overline{\mathrm{MS}}_\delta} . \tag{5.29}$$

Note that the last term is independent of α_4 due to the implicit dependence of $\Pi^{(\delta)}$ on it. Eq. (5.29) has the form of a momentum-subtracted definition of the coupling; in fact, in four dimensions it is just the definition of the momentum-subtracted running coupling. For $\delta = 1$ and at one loop, $\Pi^{(\delta)}(Q) - \Pi^{(\delta)}(m_Z)$ is finite, and $\alpha_{\rm eff}(Q)$ can still be interpreted as a momentum-subtracted definition of the coupling. For $\delta > 1$, however, Eq. (5.29) involves additional subtractions, a fact which thwarts such an interpretation.

For $Q^2 \ll M_c^2$ we can expand $\Pi^{(\delta)}(Q)$ and obtain

$$\frac{1}{\alpha_{\rm eff}(Q)} \equiv \frac{1}{\alpha_{\rm eff}(m_Z)} - \frac{2}{3\pi}\log\left(\frac{Q}{m_Z}\right) + \mathcal{O}\left(\frac{Q^2}{M_c^2}\right) ,$$

which is nothing but the standard expression of the effective charge in QED, slightly modified by small corrections of order Q^2/M_c^2. However, as soon as Q^2/M_c^2 approaches unity, the effects of the compactification scale start to appear in $\alpha_{\rm eff}(Q)$, forcing it to deviate dramatically from the logarithmic behavior, as shown in Fig. 5.3.

The crucial point, however, is that this effective charge *cannot* be interpreted anymore as the running coupling (as can be done in four dimensions) since it may receive contributions from higher dimension operators; in fact some of them are needed to define this quantity

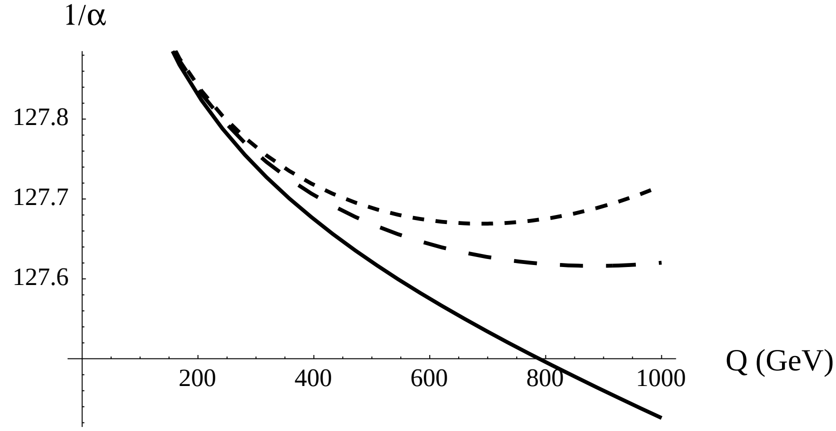


Figure 5.3: The "effective charge" against the energy scale for $\delta = 1$ (solid), $\delta = 2$ (short dash), $\delta = 3$ (long dash). Contributions from counterterms have not been considered.

properly. These contributions have nothing to do with the gauge coupling which is defined as the coefficient of the operator F^2. In particular, one should not use this quantity to study gauge coupling unification. Instead, one could use Eq. (5.28), which relates the coupling measured at low energies with the one appearing in the D-dimensional Lagrangian valid at energies $M_c < Q < M_s$. This relation involves a logarithmic correction, which is the only contribution that can be reliably computed without knowing the physics beyond M_s.

It is instructive to see what happens if instead of dimensional regularization we use hard cutoffs to regularize the uncompactified part of the VPF as in Eqs. (5.14)–(5.17). Then, when using cutoffs, one can define an "effective charge" as in Eq. (5.27)

$$\frac{1}{\alpha_{\rm eff}(Q)} \equiv \frac{1}{\alpha_4(\Lambda)} \left(1 + \Pi^{(\delta)}(Q)\right)\Big|_\Lambda , \tag{5.30}$$

where $\alpha_4(\Lambda)$ now is the coupling constant in the theory regularized with cutoffs and the subscript Λ indicates that the VPF has been regularized with cutoffs. The use of a Λ dependent coupling obviously implies the WEFT formulation, in which the cutoff is not removed from the theory. On the other hand, in the CEFT formulation one should renormalize the coupling constant by adding the appropriate counterterms and then take the limit $\Lambda \to \infty$. This usually brings in a new scale at which the coupling is defined, and which effectively replaces Λ in the previous equation. Notice also that for $\delta = 2$ in Eq. (5.16) there are logarithmic contributions proportional to Q^2, which cannot be removed when $\Lambda \to \infty$. The same is true for $\delta > 2$, but with dependencies which are proportional to $\Lambda^{(\delta-2)}$. This just manifests the need of higher dimensional operators, as was already clear in the dispersive approach, to define properly the effective charge. As one can see, the full VPF contains a term that goes as Λ^δ and is independent of Q. This piece survives when $Q \to 0$, and thus we obtain (we assume $m_Z^2 \ll Q^2$)

$$\frac{1}{\alpha_{\rm eff}(m_Z)} = \frac{1}{\alpha_4(\Lambda)} + \frac{2}{3\pi\delta}\left(\sqrt{\pi}\frac{\Lambda}{M_c}\right)^\delta + \frac{2}{\pi}a_0^{(\delta)} - \frac{2}{3\pi}\log\left(\frac{m_Z}{M_c}\right) . \tag{5.31}$$

Since $\alpha_{\rm eff}(m_Z)$ should be the same in the two schemes, we find the following relation between

α_4 and $\alpha_4(\Lambda)$

$$\frac{1}{\alpha_4} = \frac{1}{\alpha_4(\Lambda)} + \frac{2}{3\pi\delta}\left(\sqrt{\pi}\frac{\Lambda}{M_c}\right)^\delta . \tag{5.32}$$

If one identifies Λ with the onset of a more complete theory beyond the compactification scale, but at which the EFT treatment is still valid, i.e. if one assumes that $\Lambda \sim M_G \ll M_s$, M_G being this new scale, Eq.(5.32) could be reinterpreted as a matching equation between the coupling α_4 of our effective theory and the coupling of the theory at scales M_G, $\alpha_4(M_G)$. Eq. (5.32) generically tells us that one expects corrections which go as $(M_G/M_c)^\delta$. However, without knowledge of the full theory beyond M_G, the meaning of M_G (or even $\alpha_4(M_G)$) is unclear. In particular, if the new theory is some Grand Unified Theory in extra dimensions, M_G will be, in general, not just one single mass, but several masses of the same order of magnitude, related by different coefficients. In the case of logarithmic running those coefficients can be neglected, because they give small logarithms next to the large logarithms containing the common scale. However, in the case of contributions which depend on powers of the new physics scale the situation is completely different, and the presence of several masses could change completely the picture of unification. Cutoffs can give an indication of the presence of power corrections, but the coefficients of these corrections cannot be computed without knowing the details of the full theory.

To see this point more clearly, we add to our $4+\delta$ dimensional theory an additional fermion with mass M_f satisfying $M_s \gg M_f \gg M_c$, such that compactification corrections may be neglected, and compute its effects on the coupling constant for $M_c^2 \ll Q^2 \ll M_f^2$, using dimensional regularization. We have

$$\Pi_f^{(\delta)}(Q) = \frac{e_4^2}{2\pi^2}\left(\frac{\pi}{M_c}\right)^{\delta/2}\Gamma(-\delta/2)\int_0^1 dx x(1-x)\left(M_f^2 + x(1-x)Q^2\right)^{\delta/2} . \tag{5.33}$$

By expanding for $Q^2 \ll M_f$ and integrating over x we obtain

$$\Pi_f^{(\delta)}(Q) = \frac{e_4^2}{2\pi^2}\left(\sqrt{\pi}\frac{M_f}{M_c}\right)^\delta \Gamma\left(-\frac{\delta}{2}\right)\left(\frac{1}{6} + \frac{\delta}{60}\frac{Q^2}{M_f^2}\right) . \tag{5.34}$$

For odd values of δ we can use the analytic continuation of the Γ function to obtain a finite result. For even values of δ we will allow a slight departure of the integer value in order to dimensionally regularize the integral. Clearly, integrating out the heavy fermion gives power corrections to the gauge coupling. In addition, it also generates contributions to the higher dimension operators, e.g. contributions proportional Q^2 and higher powers. As can be seen by comparing with Eqs. (5.14)–(5.17) these power corrections are qualitatively similar to those calculated using a hard cutoff. Evidently, in the context of a more complete theory (in this case, given the existence of a heavy fermion), power corrections may be encountered even if the dimensional regularization is employed. However, as one can easily see by setting, for example, $\delta = 1$ in Eq.(5.34), the coefficients of the power corrections obtained knowing the full theory are in general different from those obtained using a hard cutoff, e.g. Eq.(5.15). In fact, no choice of Λ in Eq.(5.15) can reproduce all the coefficients appearing in Eq.(5.34).

The situation is somewhat similar to what happens when χPT with $SU(2)\otimes SU(2)$ is matched to χPT with $SU(3)\otimes SU(3)$. In the $SU(2)\otimes SU(2)$ theory, just by dimensional arguments, one can expect corrections like m_K^2/f_π^2. But, can one compute them reliably without even knowing that there are kaons?

5.6 Conclusions

We have attempted a critical discussion of the arguments in favor of power-law running of coupling constants in models with extra dimensions. We have shown that the naive arguments lead to an arbitrary β function depending on the particular way chosen to cross KK thresholds. In particular, if one chooses the physical way of passing thresholds provided by the vacuum polarization function of the photon, a β function that counts the number of modes is divergent for more than 5 dimensions.

We have studied the question of decoupling of KK modes in QED with $4+\ \delta$ (compact) dimensions by analyzing the behavior of the VPF of the photon. We have computed first the imaginary part of the VPF by using unitarity arguments, and found that it rapidly reaches the value obtained in a non-compact theory (only a few modes are necessary). We also showed that it grows as $(s/M_c^2)^{\delta/2}$, exhibiting clearly the non-renormalizability of theories in extra dimensions. To obtain the full VPF, one can use an appropriately subtracted dispersion relation. Instead, we use the full quantum effective field theory, with the expectation, suggested by the calculation of the imaginary part of the VPF, that the bad UV behavior of the theory is captured by the behavior of the uncompactified theory. To check this idea, we have computed the VPF in the uncompactified theory, regularized by dimensional regularization ($\delta \to \delta - \varepsilon$). We have found that, after analytical continuation, the one loop VPF is finite, and proportional to Q^δ for odd number of dimensions, and has a simple pole, proportional to Q^δ, for even number of dimensions. This result can be understood easily, because there are no possible Lorentz and gauge invariant operators in the Lagrangian able to absorb a term like Q^δ for odd δ. For δ even it shows that higher dimension operators are needed to regularize the theory. As a check we also recovered the imaginary part of the VPF in the limit of infinite compactification radius.

For comparison with other approaches, we have also obtained the VPF in the case that a hard cutoff is used to regularize it. We found that the pieces that do not depend on the cutoff are exactly the same as those obtained by dimensional regularization, while the cutoff dependent pieces are arbitrary, and can be changed at will by changing the cutoff procedure.

Next we have computed the VPF in the compactified theory, and showed that it can be separated into a UV and IR finite contribution and the VPF calculated in the uncompactified theory; as was shown previously, the latter can be controlled using dimensional regularization. The finite part is more complicated, but can be computed numerically for any number of dimensions. Also, some analytical approximations have been obtained for the low and the high energy limits ($Q \ll M_c$ and $Q \gg M_c$ respectively). Adding these two pieces, together with the counterterms coming from higher dimension operators, we obtain a finite expression for an effective charge which can be extrapolated continuously from $Q \ll M_c$ to $Q \gg M_c$; however, its value does depend on higher dimension operator couplings.

Decoupling of all KK modes in this effective charge is smooth and physically meaningful, and the low energy logarithmic running is recovered. We use this effective charge to connect the low energy couplings (i.e. $\alpha_{\text{eff}}(m_Z)$) with the coupling of the theory including all KK modes, regularized by dimensional regularization. We find that this matching only involves the standard logarithmic running from m_Z to the compactification scale M_c. In particular, no power corrections appear in this matching. However, if cutoffs are used to regularize the VPF in the non-compact space, one does find power corrections, exactly as expected from naive dimensional analysis. In the EFT language one could interpret these corrections as an additional matching between the effective D dimensional field theory and some more complete theory. The question is how reliably can this matching be estimated without knowing the complete theory. By adding to our theory an additional fermion with $M_f \gg M_c$, and integrating it out, we argue that power corrections cannot be computed without knowing the details of the complete theory, in which the D dimensional theory is embedded. Some examples in which this matching can, in principle, be computed are some 5D GUT's and string models [44, 50, 89, 56, 90, 91], and the recently proposed de-constructed extra dimensions [30, 32, 92, 93, 54]. For the question of unification of couplings this result seems rather negative, at least when compared with standard Grand Unified Theories, where gauge coupling unification can be tested without knowing their details. Alternatively, one can approach this result from a more optimistic point of view, and regard the requirement of low-energy unification of couplings as a stringent constraint on the possible extra-dimensional extensions of the SM.

Chapter 6

A model with a non-universal extra dimension

Before the universal extra dimensions were studied many other scenarios had been proposed. These models were justified in some string scenarios[37, 94, 39, 40, 95, 96, 97, 4]. Initially, they were proposed to solve the hierarchy problem by introducing a new scale near the electroweak scale, but their consequences extended to many other fields: the value of the cosmological constant [98, 99], supersymmetry breaking [100], fermion masses [101, 102], etc... These models are radically different with respect to the universal ones because they do not conserve the fifth component of the momentum, hence the KK number is no longer conserved. As a consequence, the bound on this new physics is set above the TeV. In what follows we will study a model in which only the Higgs and gauge bosons propagate through one continuous extra dimension; the latticized version is also considered. We show briefly the main features as well as how to study the phenomenology by using the formalism developed in the preceding chapters.

6.1 One continuous extra dimension

6.1.1 The model

This case can be straightforwardly studied by retracing the steps done in Chapter 3. The Lagrangian has the same pieces, Eq. (3.1), and the Eqs. (3.2 - 3.7) remain the same. The topology of the extra dimension is an orbifold, S^1/Z_2, but now only the Higgs and gauge fields propagate through the bulk, this is the main feature of this model, that is why we will refer to it as HG. The expansions in Eq. (3.8) and Eq. (3.9) are still valid while Q and U do not present now any mode.

Upon compactification, we find that the spectrum for the fermions is exactly the same as in SM. On the other hand, there are again two towers of scalars, $\Phi_G^{(n)}$ and $\Phi_P^{(n)}$, one unphysical that is eaten to give mass to the tower of gauge bosons and the other physical with mass m_n. In particular the conclusion about the dominant contributions coming from $\Phi_P^{(n)}$ running inside the loop is again valid.

The absence of extra modes for the fermions has as a first consequence that the coupling between the fundamental modes of the gauge bosons and the fermionic part of the theory, $\mathcal{L}_\rho$, Eq. (3.34) is exactly the same as in SM. Important differences appear, though, in the Yukawa piece of the Lagrangian

$$\mathcal{L}_Y = \int_0^{\pi R} dx^5 (-\overline{Q}\widetilde{Y}_u H^c U - \overline{Q}\widetilde{Y}_d H D + \text{h.c.})\delta(x^5), \tag{6.1}$$

where the delta stands to force the interactions only in the brane defined by the condition $x^5 = 0$, this brane is taken to be a rigid one, therefore it breaks translation invariance in the fifth dimension. Due to this, the fifth component of the momentum is not conserved and the KK-number conservation (as well as all its implications) no longer applies. Plugging the expansion of the Higgs doublet, Eq. (3.8), in Eq. (6.1), one obtains the expression of the Yukawa couplings in this scenario

$$\mathcal{L}_Y = \frac{2}{v} m_t V_{tj} \bar{t} P_L d_j \Phi^{+(n)} + \text{h.c.}, \tag{6.2}$$

where we have already particularized for the m_t proportional couplings, t and d_j are the usual spinor fields of the top and down-like quarks. Note the presence of the extra $\sqrt{2}$ factor with respect to the UED and SM case Eq. (3.35), it is present because in the expansion for H the zero-th mode and the rest have a $\sqrt{2}$ factor of difference. In UED it was absorbed by the tower of modes of U in Eq. (3.35) that here are substituted by the delta function.

All the part that has to do with SSB and the zero gauge modes, ranging from Eq. (3.13) to Eq. (3.18), goes exactly the same way. The couplings with the Z boson can be found by using the same procedure as in the UED case and the outcome shows that the current that couples to Z is the same as in UED, i.e. Eq. (3.36), with the sole difference of the lacking of the $J^{\mu(n)}$ piece

$$\mathcal{L}_Z = \sum_{n=1}^{\infty} \frac{g}{2c_w} Z_\mu^{(0)} [J^{\mu(0)} + J_\Phi^{\mu(n)}]. \tag{6.3}$$

$J_\Phi^{\mu(n)}$ is the same as in UED, given by Eq. (3.41), it does not get any extra factor due the presence of two Higgs fields in the current.

6.1.2 Bounds

To know which is the minimum possible value of the scale of this model we will use experimental determinations of some observables, some of them are the same as in the universal scenarios but as we will show the tightest restrictions come from the modification of the value of G_F, that is modified already at tree level. Modifications at the tree-level are now possible because of the non-conservation of the KK number. As usual we will define $a = m_t \pi R$ and denote the lightest KK mode by $M = R^{-1}$.

6.1.2.1 The ρ parameter

The corrections to the ρ parameter proportional to the m_t mass are the same as in SM, essentially because the Higgs field propagates in the fifth dimension while the fermions do not. Since the top quark has no KK tower associated, the only possible source of terms proportional to m_t are the Yukawa couplings, but these do not contribute at one-loop to the self-energies of the Z and W masses.

6.1.2.2 Radiative corrections to the $Z \to b\bar{b}$ decay

The contributions to this process will be parametrized using the $F(a)$ function defined in Eq. (3.53). The diagrams that contribute can be obtained from the diagrams in Fig. 3.3 replacing the fermionic modes inside the loops by the lines of the SM fermions

$$Q_t^{(n)} \to t_L \qquad U^{(n)} \to t_R. \tag{6.4}$$

The result turns out to be [14]

$$F(a) = -1 + 2a \int_0^\infty dx \frac{x^2}{(1+x^2)^2} \coth(ax) \approx \left(\frac{2}{3} \log(\pi/a) - \frac{1}{3} - \frac{4}{\pi^2} \zeta'(2) \right) a^2 \tag{6.5}$$

In the expansion can be found a logarithm that relates the two scales: m_t, the only mass we maintain in the SM and M, the mass of the first KK mode. It is because the KK-number is not conserved, what permits in this kind of model the appearance of effective operators at the tree level that modify the decay at the one-loop level in the effective theory[1]. Using the bound $F(a) \leq 0.39$ at 95% CL derived in Sec. 3.2.1, one can find the next bound to M

$$M = R_{HG}^{-1} > 1 \text{ TeV} \qquad 95\% \text{ CL}. \tag{6.6}$$

6.1.2.3 Radiative corrections to $b \to s\gamma$

The contributions in this scenario came from diagrams that can be obtained from the ones in Fig. 3.4 by performing the change of Eq. (6.4). In addition, each vertex between the fermions and the Higgs modes (the ones dotted in the figure) get an extra $\sqrt{2}$ factor, and as a consequence all diagrams get an overall factor of 2. The contribution per mode to the value of the C_7 coefficient defined in Eq. (3.57) when also the new spectrum is taken into account is

$$C_7^n = 2 \left[B\left(\frac{m_t^2}{m_n^2} \right) - \frac{1}{6} A \left(\frac{m_t^2}{m_n^2} \right) \right] \tag{6.7}$$

and the value of C_7 in this model is therefore

$$C_7^{\text{HG}}(M_W) = C_7^{\text{SM}}(M_W) + \sum_{n=1}^{\infty} C_7^n(M_W) \approx -0.195 + 0.265a^2 - \frac{2}{9} a^2 \ln(a). \tag{6.8}$$

It is easy to check that C_7^n vanishes for each mode when its mass is taken to infinity, as it should be because of the decoupling theorem. Notice again the presence of the logarithm relating the two scales. But what we need, is $C_7^{\text{HG}}(m_b)$, that can be obtained by using Eq. (3.60) provided we know $C_2^{\text{HG}}(M_W)$. This receives contributions at tree level from the virtual exchange of W modes that could in principle modify appreciably its value, see Fig. 6.1. Although the full tower must be considered, the modification to the SM result happens to be finite and small

$$C_2^{HG}(M_W) = 1 - \frac{M_W^2 R^2 \pi^2}{3}, \tag{6.9}$$

which represents a modification at the level of 2 %, much lower than the error we are committing ignoring the contribution of the $W_\mu^{(n)}$ and $W_5^{(n)}$ fields inside the loops. The process

[1] For a more detailed explanation see Ref. [14].

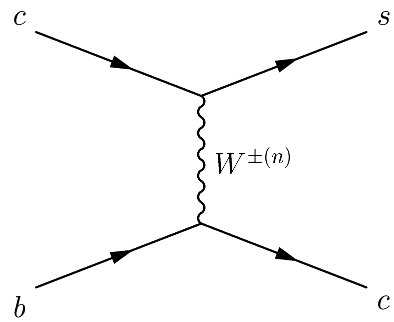


Figure 6.1: The virtual exchange of the KK modes $W^{\pm(n)}$ modifies the value of C_2 with respect to the SM value.

$b \to cl\nu$ will also affected at the tree level by an amount of the same order, therefore we will again ignore it. In addition, $C_8^{HG}(M_W)$ will be of the same order and its contribution will be neglected. When experimental bound on the value of $C_7(m_b)$ is used, Eq. (3.68), one can find the next bound for M, Ref. [103]

$$R_{HG}^{-1} > 1.2 \text{ TeV} \qquad 95\% \text{ CL.} \tag{6.10}$$

6.1.2.4 Radiative corrections to the $B^0 - \overline{B}^0$ system

This case is fully studied in Ref. [103] were all the details are given. The diagrams can be obtained from the ones in Fig. 3.6 with the same modifications as in the previous sections and also the correspondent factors have to be considered. The contributions to $S(x_t)$ can be parametrized in $G(a)$, defined in Eq. (3.86)

$$G_{\text{HG}}(a) = 2a^2 \int_0^\infty dx \frac{x^3}{(1+x^2)^2} \coth^2(ax) \approx 1 - 1.143a^2 - \frac{4}{3}a^2 \log(a) + 2a^2 \ln(n_s), \tag{6.11}$$

where $a = m_t \pi R$ and $n_s = \Lambda_s/M$. Λ_s comes from the cut-off that has been performed in the sum of the modes, see Ref. [14]. Λ_s is the scale where the scale where the more complete theory starts to be important.

Again the effective field theory point of view helps us to understand the results: the reason for the presence of $\ln(a)$ is the same as in the previous observables. There is now a new $\ln(n_s)$ term, that cames from cutting the summation on the KK modes, which at the end is just cutting the integration of the fifth component of the momentum from the point of view of the five dimensional theory. The presence of this logarithm implies that a full calculation in the theory in which HG is supposedly embedded contains these logarithmic pieces. But now, we cannot directly check this because the details of the corresponding full theory are unknown.

Finally, using the last experimental determinations of Ref. [27, 28] the bounds can be extracted. The one coming from this observable is

$$R_{HG}^{-1} > [560, 900] \text{ GeV} \qquad 95\% \text{ CL,} \tag{6.12}$$

the variations are present because we have to choose a value for n_s. We have taken $n_s = 10$ and $n_s = 100$ respectively.

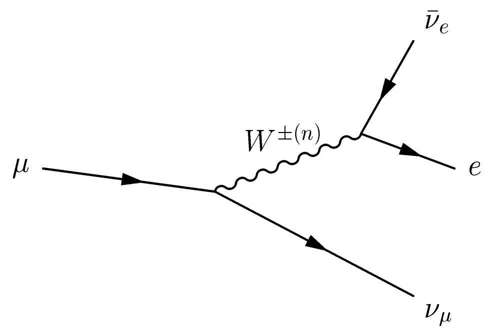


Figure 6.2: The decay of the muon through virtual KK modes modifies the value of G_F.

6.1.2.5 $\Gamma(Z \to \ell\bar{\ell})$ restrictions. G_F modifications.

In UED and LUED this observable receives contributions at the one loop level and therefore it will be dominated by the SM tree level contributions, that is why it was not studied in those cases. On the contrary in HG (and later on in LHG) it receives contributions already a the tree level and could be substantially modified, see Ref. [104, 105]. In the presence of KK modes, the relation between the coupling constant g, M_W and the Fermi constant as extracted from the muon lifetime, G_F, is modified at tree level due to the virtual exchange of the KK tower of $W^{(n)}$, Fig. 6.2:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} + \sum_{n=1}^{\infty} \frac{(\sqrt{2}g)^2}{8m_n^2} = \frac{g^2}{8M_W^2}\left[1 + \frac{(M_W\pi R)^2}{3}\right], \tag{6.13}$$

while for SM the relation can be obtained by setting $R = 0$ in the previous formula. The fact that this relation is different for HG and SM has as a consequence that the predictions for $\Gamma(Z \to \ell\bar{\ell})$, when expressed in terms of G_F, are different[2]

$$\Gamma^{\rm HG} = \Gamma^{\rm SM}\left[1 - \frac{(M_W\pi R)^2}{3}\right]. \tag{6.15}$$

Using the experimental bound [25] $\left|\frac{\Gamma^{\rm exp}}{\Gamma^{SM}} - 1\right| < 0.0028$ at 95% CL one gets

$$R_{HG}^{-1} > 2.8 \text{ TeV} \qquad\qquad 95\% \text{ CL}. \tag{6.16}$$

Including radiative corrections reduces a bit this number, see Ref. [104].

Summarizing, the results of all the observables are collected in table 6.1, where it is clearly shown that the scale of the new physics in this scenario is above the TeV.

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[2]The prediction for $\Gamma(Z \to \ell\bar{\ell})$ when written in terms of g is exactly the same for HG and SM

$$\Gamma(Z \to \ell\bar{\ell}) = \frac{M_Z}{24\pi}\frac{g^2}{c_W^2}(g_R^2 + g_L^2). \tag{6.14}$$

It is the different relationship between g and the low energy parameter G_F what causes the discrepancy.

	$Z \to b\bar{b}$	$b \to s + \gamma$	$\overline{B}^0 - B^0$	G_F
M(TeV)	1.0	1.2	[0.5,0.9]	2.8

Table 6.1: Bounds coming from the different observables to the mass of the first KK mode M.

6.2 One latticized extra dimension

6.2.1 The model

The steps are nearly the same as in the previous section. It can be found that the spectrum is the same provided one changes $m_n \to M_i$, defined in Eq. (4.21), and considers the finiteness of the spectrum, i.e. $i = 0, 1, \ldots, N-1$.

Again for the ρ parameter the m_t proportional corrections are the same as in SM. The Yukawa sector is the usual one, but with the fermions coupled only to the zero-th copy of the Higgs doublet

$$\mathcal{L}_Y = -\overline{Q_L}\widetilde{Y}_u H_0^c u_R + \text{h.c.}, \tag{6.17}$$

and by resorting to the mass fields we find the important part for us

$$\mathcal{L}_Y = \sum_{n=1}^{N-1} \frac{2m_t}{v} V_{tj} \cos\left(\frac{n\pi}{2N}\right) \bar{t}\widetilde{\Phi}_n^+ P_L d_j + \text{h.c.} \tag{6.18}$$

The $\widetilde{\Phi}_n^+$ are the physical degrees of freedom. Notice that the coupling is the SM one with the addition of an extra $\sqrt{2}$ factor and the presence of a cosine that vanishes in the limit of large N. The cosine appears because there is only one Higgs field. Finally the couplings with the Z boson can be obtained straightforwardly

$$\mathcal{L}_Z = \frac{g}{2c_w} Z_\mu \sum_{i=1}^{N-1} (-1 + 2s_w^2)[\widetilde{\Phi}_i^+ i\partial^\mu \widetilde{\Phi}_i^-] + \text{h.c.} \tag{6.19}$$

The absence of any cosine is because there are two Higgs fields. Thus the couplings of the Z are the same as for HG and the differences are encoded in the spectrum. With this we are prepared for extracting the different contributions.

6.2.2 Bounds

To extract the bounds, Ref. [103], we can use the results derived for HG. The corresponding results can be found from the ones in HG by adding an extra factor of $\cos^2(n\pi/2N)$. The cosines appear only in the Yukawa vertices, and in every diagram that we consider there are always two of these vertices. In the case of the the radiative corrections to the $Z \to b\bar{b}$ decay the result is

$$F_{\text{LHG}}(a) = 1 + \int_0^1 dx \sum_{n=1}^{N-1} \frac{2(1-x) r_n \cos^2(n\pi/2N)}{(1-x) r_n + x} \tag{6.20}$$

$$= 1 + \int_0^1 dx \sum_{n=1}^{N-1} \frac{2(1-x) a^2 \cos^2(n\pi/2N)}{(1-x) a^2 + 4(N-1)^2 x \sin^2(n\pi/2N)} \tag{6.21}$$

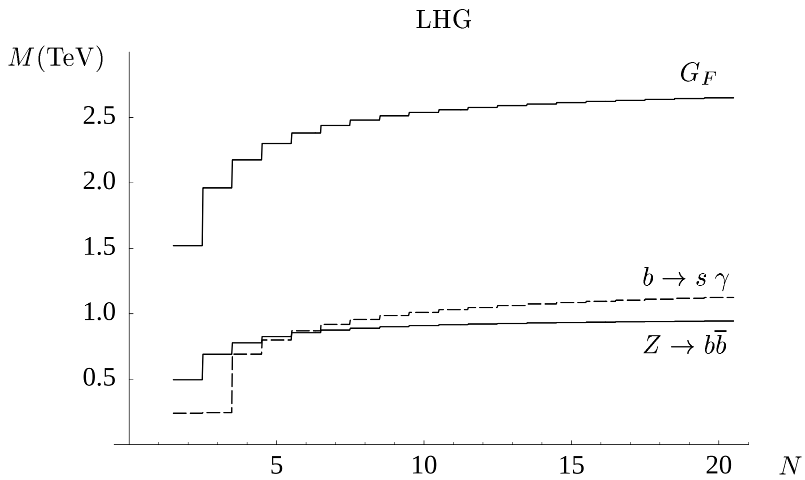

Figure 6.3: The bounds on M as a function of the number of sites, N, in the LHG scenario.

from the bound $F(a) < 0.39$ the bounds on M can be extracted as a function of N.

The radiative corrections to $b \to s\gamma$ can be easily found from the HG ones

$$C_7^{\rm LHG}(M_W) = C_7^{\rm SM} + 2\sum_{n=1}^{N-1} \cos^2(n\pi/2N) C_7^n, \tag{6.22}$$

where C_7^n is defined in Eq. (6.7). The function that parametrizes the corrections to the $B^0 - \overline{B}^0$ system is now

$$G_{\rm LHG}(a) = 1 + 2\int_0^1 \sum_{n=1}^{N-1} \frac{2a^2 x(1-x)\cos^2(n\pi/2N)\, dx}{4(N-1)^2 \sin^2(n\pi/2N)(1-x) + a^2 x}. \tag{6.23}$$

Finally, the branching ratio $\Gamma(Z \to \ell\bar{\ell})$ is modified to

$$\Gamma^{\rm LHG} = \Gamma^{\rm SM}\left[1 - \frac{1}{2}\left(\frac{M_W \pi R}{N-1}\right)^2 \sum_{n=1}^{N-1} \frac{\cos^2(n\pi/2N)}{\sin^2(n\pi/2N)}\right]. \tag{6.24}$$

With these expressions it is possible to extract bounds to the first mode as a function of the number of sites N. These bounds are displayed in Fig. 6.3, where it is shown that the continuous limit is rapidly reached. If the number of sites is small, the bound can be reduced by a factor of about 20%-40%, thus allowing new modes ranging between $1.5 - 2.0$ TeV.

Chapter 7

Outlook and conclusions

The possibility of the existence of more dimensions different than those we can experience directly with our senses has been for a long time a very active field of interest for physicists. They were initially introduced to achieve a more elegant formulation of the equations of nature, soon the extra dimensions became a key piece in important theories like supergravity or string theory, which tried to give a quantum treatment to gravity. It was realized in the latter that, under some assumptions, the extra dimensions could be of the size as low as the TeV without contradicting any of the experimental results. Extra dimensions have been also studied using the quantum field theory formalism, and many extensions of the SM have been proposed. In general, the scale associated with the extra dimensions in these extensions is again of the order of TeV, what offers a relatively accessible, and rich, phenomenology for future experiments. Placing a new scale at the TeV has important consequences: it could solve the hierarchy problem, scenarios of grand unification are modified because the running of the coupling constants is greatly modified above this scale, the possible localization of the wave functions for the different fermions in the space of the extra dimensions could be the origin of the fermion masses, even the value of the cosmological constant could be modified, just to cite some. This variety of phenomena explains the interest in extra dimensional theories in the last years.

Among the above mentioned extensions of the SM, there is one of special interest in which a single extra dimension is considered and all the fields propagate through it, a fact that gives it the name "*universal extra dimension*". In this scenario there are no tree-level modifications to the SM results, therefore all the corrections are, at least, one-loop suppressed. This outcome is a consequence of the *local* extra-dimensional Lorentz symmetry of the theory, which implies the existence of a "conserved" number called Kaluza-Klein number. Due to this suppression in the SM modifications, the scale of the new physics can be as low as hundreds of GeV, clearly a challenging situation for the next generation of accelerators.

In the first part of this work we have studied in detail the phenomenology of this scenario. In particular, we have concentrated on observables that display a strong dependence on the mass of the top-quark, m_t, because this enhances the corrections coming from the new physics: $Z \to b\bar{b}$, ρ parameter and $B^0 - \overline{B}^0$ mixing. We have also studied the process $b \to s\gamma$; although it does not present a strong dependence with m_t, the relative importance of the new physics is also enhanced because the SM contribution to this transition is suppressed. This is so because this process is forbidden at tree level due to gauge invariance and is only allowed through radiative corrections. The details of the study are given in Chapter 3 and the bounds on the size of the compactification scale or, equivalently, on the mass of the first Kaluza-Klein mode are given in table 3.1.

We have also studied a non-universal scenario, in which only the Higgs and gauge bosons of the SM are allowed to propagate through the extra dimension, we have given this scenario the name of HG. We have calculated the new physics contributions to several one loop processes which depend strongly on the top-quark mass and used them to set bounds on the mass of the lightest KK mode, M. The results are summarized in table 6.1 and compared with other relevant bounds found in the literature.

In the HG case the new physics scale should be above 1 TeV but surprisingly in the UED case it can be well below the TeV. One might think that the scale for UED should be bigger than for HG since UED represents a bigger modification to SM than HG (because in HG only Higgs and gauge bosons propagate in the extra dimension). Nevertheless, the contributions of the fermionic KK modes in the UED case are such that produce several cancellations in the amplitudes. The reason is the above mentioned KK number conservation due to the local Lorentz symmetry. The lack of this symmetry in the HG case permits some observables to be modified already at the tree level; as a consequence the most restrictive bound comes from a very well measured quantity, G_F.

These extra dimensional models are non-renormalizable because the coupling constants have canonical dimensions of mass to some negative power. This does not mean at all that these models should be wrong, on the contrary, this indicates that they must be regarded as effective field theories that are valid only below some scale at which new physics should enter to correct the bad behaviour of the theory. There is a class of string theories which could provide such a new physics, however, this is only one among the possible ultraviolet completions. Another kind of completion, proposed initially in Ref. [30, 31, 32], used a discretization or *lattization* of the fifth dimension. In these models, the number of modes is finite and their masses are modified with respect to the continuous cases. As a possible extension of the SM, it is also interesting to study the phenomenology that stems from these scenarios. In this work we have studied the latticized versions of UED and HG, called LUED and LHG. Among the results we obtained, we found that the predictions are the same as in the respective continuous cases when the extra dimension is latticized in a relatively small number of (four dimensional) sites. For a very small number, the results show that in all cases the natural scale could be further lowered with respect to the continuous scenarios what makes more accessible the new physics for future experiments. The scale can be reduced by factor of about 10%-25%, thus allowing new modes ranging between $320 - 380$ GeV in the case of UED and of about 20%-40%, with the new modes ranging between $1.5 - 2.0$ TeV in the case of LHG. The details can be found in Fig. 4.2 and Fig. 6.3.

On the other hand, it has been suggested that the existence of extra dimensions could make the gauge couplings to run as a power of the energy scale instead of the usual logarithmic running. This is a very interesting property because Grand Unification Theories in extra dimensions could achieve unification at much lower energies (few TEV) than the usual four dimensional GUT's. This would allow us to test unification at the planned accelerators, clearly a very exciting possibility. This scenario is also challenging from the theoretical point of view because gauge couplings in extra dimensions are dimensionful and gauge theories in extra dimensions are not renormalizable. Therefore, the behaviour of couplings can depend on the approach to renormalization of non-renormalizable theories: Wilsonian effective field theory or continuum effective theory. We have attempted a pure continuum effective field theory approach based in dimensional regularization. In this approach the running of couplings, at most, logarithmic and power corrections are recovered as matching conditions at the (extra-dimensional) GUT scale. Therefore, to compute them one needs to know the complete details (full spectrum and pattern of symmetry breaking) of the GUT model. This is quite different from the usual four-dimensional GUT's that can be tested to good accuracy without knowing the details of the GUT model.

To conclude, we have studied the phenomenology of two extra-dimensional models with all or only part of the SM particles propagating in the extra dimensions. We have also studied their latticized versions. We have focused on one-loop effects that display a strong dependence on the top-quark mass which contribute to observables which are measured with good precision. For the models considered, those effects provide the best limits on the compactification radius. We have also studied the question of power-law running in extra dimensions by using the continuum effective field theory framework with dimensional regularization and discussed its impact in (extra-dimensional) GUT's. We found that, at difference from the four-dimensional case, unification can depend strongly on the details of the GUT model if it is dominated by power corrections.

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