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The Hermeneutics of Mathematical Modeling

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I. Introduction

Mathematics is a language. Those who speak this language frequently use it to describe the world around them. As in any language, signs (words, symbols, signifiers) are created to represent those objects of discussion in the language [20,23]. Depending on the existence of physical referents for the signs created, points of view may fall into two broad categories. There are those who believe philosophically that, physical referents are not necessary, that the only meaningful discourse in the language is through the signs and their relationships to one another. These are the "pure mathematicians" (or, one may call them "structuralists"). On the other hand, the non-structuralists, or "applied mathematicians," attempt to construct a "symbolic order" or sub-language of mathematics which, ideally, would be a perfect representation of some physical ("real world") phenomenon. This representation would be "perfect" in the sense that every change in the "real world" would be reflected by a corresponding change in the "symbolic world" and vice versa. In other words, the transformation relating the "real world" to the "symbolic world" would be explicitly known. This does not seem likely to occur. Yet, the predictive power and pragmatic application of mathematics has produced undeniable results in science and technology. The objective, then, of the applied mathematician is to minimize, in some way, the discrepancy between the behavior of the symbol used in the symbolic world and that of the object represented in the real world. In other words, applied mathematics is constantly evolving towards pure mathematics because the ultimate goal of the former is to ignore the discrepancy between sign and referent and to exist solely within the realm of the symbolic world.

This essay investigates some of the historical and philosophical background of the division between pure and applied mathematics. The "symbolic order" constructed by the pure mathematician and used by the applied mathematician to describe the "real world" is called a mathematical model. The nature, interpretation and limitations of the mathematical model are also discussed. An illustration of the means used by applied mathematicians to drive the above mentioned evolutionary process toward pure mathematics is given. This process, the "modeling cycle," is presented as a response to the "hermeneutic circle" of applied mathematics. The term "hermeneutic circle" (borrowed from the theory of literature [8,9]) refers to the dilemma that, before a model is developed, one must know the important factors contributing to the phenomenon under investigation but, in order to know these factors, one should first develop a model.

It is hoped that a better understanding of the objectives and limitations of the use of mathematical models will contribute to the increased acceptance of them as a means of providing additional information and perspectives in areas of research traditionally considered "non-quantitative." The crucial factor in this understanding is the analysis of the connection between philosophy and theory, pragmatism and application.

II. History and Theory

Plato is seen by many as one of the major figures contributing to the logocentric nature of western philosophy. Logocentrism sets forth the premise that there is a division between word and thought [21, p165ff]. Plato was consistent in maintaining this dichotomy and in the Republic, applied it to his view of mathematics:

> . . . those who deal with geometrics and calculations . . . take for granted . . . things cognate... in each field of inquiry; assuming these things to be known, they make them hypotheses, and . . . setting out from these hypotheses, they go at once through the remainder of the argument until they arrive with perfect consistency at the goal to which their inquiry was directed. . . . although they use visible figures and argue about them, they are not thinking about these figures but of those things which the figures represent.

(Strictly adhering to Platonic belief, the constructs of mathematics represent "disembodied eternal forms," or "archetypes" which are perceived only by the intellect.) For subsequent philosophers also, this was the prevailing view in mathematics — it always "represented" something. Mathematicians were constructing a language with which they could describe the world around them. This description was accomplished through what is today called a "mathematical model."

There are two general categories of models, *iconic* and *symbolic* [7]. An *iconic* model is one that is intended to resemble in some way the object modeled. A "model" train would be an example. Although the determination of "iconic" versus "symbolic" is often controversial, a road map or a schematic diagram of some electrical circuit may also be considered to be an iconic model. A *symbolic* model is one that is not iconic. This type of model often takes the form of some kind of equation whose variables represent some quantities in nature. Descartes introduced the association of iconic and symbolic models in modern mathematics. He represented algebraic relations between variables in a geometric "Cartesian coordinate system" (the type of coordinate system used in road maps). Thus an explicitly demonstrable relationship between the iconic and symbolic model contributed greatly to the facility with which later models could be built and analyzed.

The distinction "pure" and "applied" mathematics did not exist until the second half of the 19th century. Until then, the manipulation of the symbols of mathematics was simply a necessary part of the use of the "symbolic order" used to describe the non-mathematical world. Indeed, there was no need to study pure mathematics of itself until inconsistencies in certain predictions based on interpretations of the mathematics forced the consideration of the logical foundations of mathematics itself.

Thus, at the end of the 19th and beginning of the 20th centuries, about the time that Ferdinand de Saussure was searching to define the basic "signs" and "values" or "significations" of linguistics [20], Bertrand Russell and other philosophers and mathematicians were attempting to reduce language (and hence mathematics) to a fundamental class of irreducible objects, thus generating the entire spectrum of valid claims concerning the "language" [22, 23]. In mathematics this meant that for each area, a set of axioms was sought from which all valid theorems may be deduced. This property of a set of axioms is called "completeness." Perhaps a more desirable property of a set of axioms is that they be "consistent." This means that one must not be able to prove the validity of a proposition and its negation from the given axioms. Hilbert, Russell and Whitehead (see [22]) believed wholeheartedly in the possibility of establishing such axiomatic systems and spent a tremendous effort in attempting it. Kurt Gödel, in his article "Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I," [11, 16] finally settled the question in a most unsettling way. He first restricted his attention to the set of integers, that is, the usual whole numbers of arithmetic 0, ±1, ±2. He then proved that for this most primitive "world" the axiomatic method has inherent limitations in the following sense. If the axiomatic system is complete, then it will be inconsistent and if the system is restricted enough to be consistent, then there are propositions concerning the integers which cannot be proven from this consistent system. Since an inconsistent system is entirely unpalatable to the mathematician, consistent axiomatic systems are used at the expense of completeness. Thus the working mathematician is fully aware that there are most likely questions which may be asked but may not be resolved from within the system.

Russell had already happened upon what is now called "Russell's Paradox" [22, p124–125, p153] which foreshadowed Gödel's discovery. The most popular form of this paradox is to consider a barber in a town who shaves everyone who does not shave himself. Does the barber shave himself? Now, if the barber shaves himself, then he must be one of those who don't shave themselves. Therefore, he doesn't shave himself. On the other hand, if he doesn't shave himself, then he is one of those whom the barber shaves, so he must shave himself. Either way we answer, we arrive at a contradiction. In Set Theory, Russell's Paradox takes the following form: Let S be the set of all sets which are not elements of themselves. Is S an element of itself?

Thus, one of the most important consequences of the 19th and 20th century developments in the logical foundations of mathematics is that it is possible to prove the impossibility of proving something. Russell noted the impact of such an advance on philosophical discourse: "Those philosophers who have adopted the methods derived from logical analysis can argue with one another, not in the old aimless way, but cooperatively, so that both sides can concur as to the outcome." This, of course, refers to the "conditional" nature of modern mathematics (both pure and applied), which Russell [19] humorously expresses thus:

> We start, in pure mathematics, from certain rules of inference, by which we can infer that if one proposition is true, then so is some other proposition. These rules of inference constitute

the major part of the principles of formal logic. We then take any hypothesis that seems amusing, and deduce its consequences. If our hypothesis is about anything, and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.

Having examined some of the historical background and theoretical limitations of mathematics (and hence its applications), we address, in the next section some of the responses to the problems arising from the theory.

III. Pragmatism and Hermeneutics of Mathematical Modeling

Suppose one wishes to investigate some aspect of a particular field of inquiry and employs mathematics as a part of the analysis used to carry out the study. Variables (signs) are created which represent entities (referents). The method is to establish the behavior of the signs and make conclusions about them. Since mathematical symbols are not the objects they represent, the question then is in what way one could in applied mathematics assert that "only the behavior of the signs need be understood"? The answer is in the degree of *association* between sign and referent. In other words, a perfect association would mean that "events" taking place in the model would be perfectly reflected in the object or phenomenon being modeled and vice versa. Thus, the axioms governing the mathematical order would obtain for the model. This association and the degree of it are the two major goals (and problems) of mathematical modeling. The "pragmatic solution" to the problem of creating the association must, of course, begin with the construction of the model. One uses mathematics — or any method of analysis, for that matter — in order to understand something. So, in order to build a mathematical model, one decides first what the most influential factors governing the observed phenomenon actually are. One then represents them as variables and builds the model. But, in order to decide upon these "most influential factors" one must already understand the "observed phenomenon." The achievement of this understanding is, however, the original reason for building the mathematical model. This is a problem in general hermeneutics, the theory of interpretation which in literary terms Abrams [1] defines as "a formulation of the procedures and principles involved in getting at the meaning of all written texts." In discussing the theory of understanding texts, Dilthey labeled the problem of not understanding the whole without understanding its component parts and not understanding the parts without understanding the whole, the "hermeneutic circle" [1, p84]. Thus, anyone including mathematics as part of the process of understanding is confronted with the "mathematical hermeneutic circle" introduced above: how can the object of investigation be understood without a model, and how can a model be built without understanding the object already?

Dilthey [5,6] and, more specifically, Gadamer [8,9] advocated an approach to overcome the problem of the hermeneutic circle and, interestingly, mathematicians have arrived at the analogous "solution" in their own context. Gadamer's solution was that one establishes a "dialogue" between the "pre-understanding" brought to the text being read and the ideas expressed by the text itself. The reader must then modify the "preunderstanding" using a synthesis of the new ideas of the text and the "old understanding." Thus, one builds to an understanding of a given text through the spiraling process of reading, dialogue (comparison) and synthesis of ideas.

The analogous situation in applied mathematics is the "modeling cycle" [13]. The investigator observes the "natural phenomenon" and conjectures what the most significant factors affecting the observed behavior and the relationships among these variables are. A model is then developed, which can include virtually any object or technique considered to be within the realm of mathematics. For the sake of argument, we may assume the model takes the form of some kind of equation (this is usually the case, but not always). Then the model must be analyzed mathematically. This is the "pure" or "abstract" stage of the process. After a "solution" to the equation is obtained or the mathematical analysis has been otherwise completed the results are compared to the actual observations. This is part of the measurement of the association between sign and referent. If this comparison reveals that the representation is not adequate, then more observations must be made and also more conjectures as to the missing "important factors." Then the existing model is usually modified to improve the approximation to (association with) the observations. The process then continues until the researcher is satisfied that the representation is sufficient for the intended purpose. Ideally, the association between sign and referent would become "perfect," consistent with the rigors of the axiomatics of pure mathematics. However, if such a perfect correlation were possible, we would still be faced with the problem of the incompleteness of the axiomatic system. That is to say, it is possible that questions may be asked that cannot be answered from within the system itself. While the modeling cycle is a pragmatic answer to the problem of obtaining functional representations of physical systems, the problem of incompleteness is a limitation of the use of mathematics.

Despite the limitations, there are many attractive features of the mathematical modeling method. Since mathematicians have chosen to forego completeness in favor of consistency, results derived from the mathematics itself cannot be contradictory. The variables and relations of mathematics, i.e. the vocabulary, is free of connotation. Nagel and Newman [16, p12] attribute this to the fact that "the validity of mathematical demonstrations is grounded in the structure of statements, rather than in the nature of a particular subject matter." Implicit in this view is that the variables, their relations and the means of analysis are clearly and unambiguously defined. In other words, with a mathematical model one may create an idealized world in which all variables and factors influencing them are known and fully controlled. The point of view taken then is that if some particular behavior is observed in the idealized world, then one cannot exclude the possibility of it occurring in the "real" world and possibly for the same reasons. This can and will be expressed more strongly depending on the degree of the sign-referent association. In many cases, there is a way of measuring the extent of this association. This measurement is based on the simple fact that models have a certain predictive power. Thus, in many instances one may compare the predictions of the model with subsequent occurrences in the "world of referents" and formulate a sense of confidence or no confidence in the ability of the idealized world to reflect this behavior. Besides being predictive, models may indicate further areas of research, reveal fundamentals of the underlying dynamical processes observed (subject to the degree of sign/referent association), or, in some cases, discover previously unknown relationships between variables. Finally, a great advantage of mathematics is that its results are reproducible. That is to say, if two investigators accept the same axiomatic system and the same hypotheses concerning the phenomenon in question, both will obtain the same results. This is Russell's observation about "philosophers" arguing from "methods derived from logical analysis." The modelers or philosophers may argue about axiomatic systems or hypotheses but once these are fixed, so are the results.

IV. Illustration

To make the ideas discussed in the previous sections more concrete, consider the following examples from epidemiology. Among the first models of this type are those constructed by Kermack and McKendrick [15]. The book by Bailey [3] treats such equations but is also a comprehensive introduction to the subject matter with an extensive bibliography. The models presented here are from Hethcote [12]. They were selected for several reasons. First, they are from a field of investigation in which the analytic tool of mathematics has not yet been fully accepted. Second, they do involve typical modeling techniques. Third, they are qualitative in the sense that, while they involve parameters which cannot be measured (or have not yet been measured), they nevertheless may indicate significant characteristics of epidemics.

The objective of mathematical modeling in epidemiology is understanding better the dynamic factors influencing the spread and/or maintenance of a communicable disease throughout a population. This information may be useful in designing strategies for reducing the incidence of the disease or eliminating it altogether. Indeed, much mathematical research is now being done to understand the dynamics of AIDS. (See Jacquez. et. al. [14], and the references there.)

It is easy to posit many factors which could contribute to the transmission of a disease. For example, some diseases are incurable, some the body will eventually overcome; some confer immunity, some don't; some are preventable by immunization, others must run their course. Many diseases are transmitted from person to person, some from animal to person or vice versa and some even travel from person to animal to person. Sometimes it is possible for a person to be a carrier of the disease, i.e. to transmit the disease without demonstrating the symptoms. The population dynamics may also play a role. Individuals may enter or leave a population through birth and death or through emigration or immigration. The age of individuals in the population could be important as well as the presence and interference of other diseases. Sexual promiscuity could be important (even outside the context of epidemiology). Geographic location and spread among numerous other factors may affect the disease.

To "break into" the modeling cycle, many simplifying assumptions must be made. It may be validly argued that these assumptions are too restrictive to provide a realistic representation of the transmission of disease, but it must be kept in mind that this is simply the first step in the process. The intent is to improve the initial models. At the outset, the following definitions and assumptions will then be made:

1) A *susceptible* is an individual who does not have the disease in question but is capable of contracting it. The set of all susceptibles is the *susceptible class*. The fraction of the population that is susceptible is called the *susceptible fraction* and at time t will be denoted by S(t);

2) An *infective* is an individual who has and is actively transmitting the disease or at least contacting other individuals sufficiently to transmit the disease. Definitions for *infective class*, *infective fraction* and I(t) are analogous to those above;

3) A *removed* is an individual who, by any means (immunity, inoculation, isolation) is not involved in the susceptible-infective interaction. The removed class and fraction and R(t) are also defined as above.

4) Each individual in the population must be in one of the three classes described above. Thus, S(t) + I(t) + R(t) = 1 for all t.

5) Diseases will be classified by the epidemiological states through which an individual passes in the course of the disease. Thus, an SI disease is one in which the susceptible becomes infective and never recovers. Herpes simplex is an example. An SIS disease is one that can be cured, but confers no immunity. An example is gonorrhea. A disease that confers permanent immunity is an SIR disease. Measles is such a disease.

6) A *contact* is any interaction between an infective and any other individual in the population that is sufficient to transmit the disease if the other individual is susceptible. The *contact rate*, λ, is the average number of contacts per unit time per infective. We will assume that the contact rate is constant.

7) The population size, N, will be assumed to be large and constant. This assumption is largely mathematically motivated. It allows a tractable model to be developed. It is, however, biologically defensible if the disease is to be studied over a relatively short period of time.

8) The population is assumed to be *homogeneously mixing*. This means that the probability of any two individuals coming in contact with one another is the same. This is admittedly restrictive, but again, a tractable model is then possible. This restriction can then be removed by considering the population to be composed of several homogeneously mixing subpopulations, so the difficulty may be overcome.

9) The susceptible-infective interaction is assumed to follow the "law of mass action" from physics. This means that the rate of loss from the susceptible class (gain in the infective class) is proportional to the product of the susceptible and infective fractions. This is perhaps made clearer by the following development: NS(t) is the actual number of susceptibles. $(NS(t))'$ is simply the mathematical notation for the rate of change per unit time of the number of susceptibles. Each infective contacts λ individuals per unit time and there are NI(t) infectives, so a total of $\lambda NI(t)$ individuals are being contacted per unit time. However, not all those contacted are susceptible. In fact, only S(t) (the susceptible fraction) are susceptible, so the rate of loss from the susceptible class due to the susceptible-infective interaction is given by $-\lambda NI(t)S(t)$.

10) Recovery from the disease will be assumed to follow the "law of exponential growth and decay." That is, the rate of loss from the infective class due to recovery is proportional to the size of the class. This is the same assumption made in radioactive decay or, in another context, the calculation of interest compounded continuously (at 5.5% interest compounded continuously, the rate of change of the amount of money is .055 x Amount, or $A' = .055A$). The principle involved is that the rate of growth or decay is proportional to the amount present. Thus, the rate of loss from the infective class due to recovery is given by $-\gamma NI(t)$. γ is called the *recovery rate* (it is analogous to the .055 above).

Assumptions 6 through 10 are debatable. They do, however, allow a model to be created. Noting that $(NS(t))'$, $(NI(t))'$, $(NR(t))'$ represent the rates of change per unit time of the numbers of susceptibles, infectives and removeds respectively, the model becomes the following system of equations:

$$\begin{aligned} (NS(t))' &= -\lambda NI(t)S(t) \\ (NI(t))' &= \lambda NI(t)S(t) - \gamma NI(t) \\ (NR(t))' &= \gamma NI(t) \\ \text{and} \quad NS(0) + NI(0) + NR(0) &= N, \quad NI(0) > 0 \end{aligned}$$

We will not actually do a detailed analysis of this system of equations; it is presented only for the sake of discussion and illustration. Notice that the individuals leaving the susceptible class (first equation) go into the infective class (second equation) and those leaving the infective class (second equation) go into the removed class. The last equation simply says that initially (t=0) everyone in the population falls into one of the three categories and that we do have some infectives (NI(0)>0). Notice also that i) all variables are explicitly defined, ii) all relationships between the variables are demonstrated, and iii) the only dynamic factors involved are the susceptible/infective interaction and recovery. Although the mathematical analysis is not pertinent to the present study, we can see from the following mathematical calculations that the model is inadequate for certain diseases, and hence by refining the model, we will illustrate the modeling cycle. Notice that if the disease is in an endemic equilibrium, that is to say, has stabilized at some persistent level in the community, then the rate of change of NS(t) and NI(t) must be zero. This yields, from the second equation, that

$$0 = \lambda IS - \gamma I$$

Factoring out the common factor of I, we obtain the equation $0 = I(\lambda S - \gamma)$. So, either $I = 0$ and the disease dies out (no infectives), or $I \neq 0$ in which case $\lambda S - \gamma$ must be zero, so $S = \gamma/\lambda$. If this is the case, then $-\lambda NIS$ cannot be zero, so the rate of change of the number of susceptibles cannot be zero and we would not be at an equilibrium. This is a contradiction. Therefore, the only possibility is that at equilibrium $I = 0$. The disease dies out. This is unsatisfactory based on physical observations. Measles is an SIR disease and therefore should have these dynamic characteristics, but has shown no tendency to die out. To follow the modeling cycle then, we must make new conjectures as to the important dynamical factors determining the spread of the disease. Since a disease following explicitly the old assumptions would eventually "run its course" and die out, perhaps the introduction of new individuals into the population would replenish the depleted pool of susceptibles. Following through on this conjecture we make the assumptions:

11) Births and deaths occur at the same rate, α (exponential growth and decay as above). Note that the assumption of exponential growth in all cases in which it is assumed is also subject of "verification" through the comparison phase of the modeling cycle.

12) There are no disease-related deaths. So, deaths occur at the same rate in each class.

13) Birthrate equals deathrate. This is a mathematical assumption to guarantee that the population remains constant; and

14) All newborns are susceptible. Since maternal antibodies confer temporary immunity, a "newborn" is defined to be a child of 12–15 months.

Building on the old model, the new model becomes

$$\begin{aligned}(NS(t))' &= -\lambda NI(t)S(t) + \alpha N - \alpha NS(t)\\ (NI(t))' &= \lambda(t)S(t) - \gamma NI(t) - \alpha NI(t)\\ (NR(t))' &= \gamma NI(t) - \alpha NR(t)\\ NS(0) + NI(0) + NR(0) &= N\end{aligned}$$

The *infectious contact number*, σ, is defined to be the average number of contacts per infective per infectious period. The analysis of the new model yields that $\sigma = \lambda/(\gamma + \alpha)$ and the result that if $\sigma > 1$, then the disease remains in the population. This seems to give a more realistic prediction of the behavior of the disease than the original model. One conclusion that may be drawn from this is that the "vital dynamics" (births and deaths) are important in creating the behavior in the model that is actually observed in human populations. It then may be the case that the introduction of new susceptibles into the population is essential in the transmission characteristics of some diseases. There are, of course, many questions and objections that may be raised, among which are:

1) Why should the contact rate be constant? In schools, for example, winter contact rates should be much higher than summer contact rates.

2) The disease may be affected by spatial (geographic) spread.

3) What happens if the population size is allowed to vary?

4) How might the effect of immunization programs be studied?

5) Is the assumption of homogeneous mixing too severe?

6) What about the effects of immigration and emigration?

Most of these questions can and have been addressed by researchers in this area. Many interesting possibilities for explanations of the occurrence and transmission of communicable diseases have been suggested along with indications of areas of investigation not considered before the introduction of the modeling method — another benefit of the modeling approach.

V. Conclusion

If the language of mathematics is considered as a symbolic order with a very precisely defined syntax (axiomatic system), the distinction between pure and applied mathematics may be drawn through the treatment of sign versus referent. Pure mathematics considers the symbolic order itself as the subject of investigation, thus referents are not necessary. Applied mathematics, on the other hand, must deal with referents, and through an association which always seems to be imperfect.

The goal of applied mathematics is to become "pure" in the sense of working toward a perfect association between sign and referent so that the syntax of pure mathematics may be applied and only the signs need to be analyzed. The problem in the realm of pure mathematics is that it cannot solve all problems that may arise. It is incomplete. Accepting this, the applied mathematician nevertheless works toward the goal of the perfect association through the modeling cycle — a process designed to understand the phenomenon to which the mathematics is being applied. The interpretation of the results obtained through a mathematical model must be taken in the sense that, if a certain behavior of the model is observed, one cannot exclude the possibility of it occurring in the observed phenomenon and possibly for the same reasons.

The illustration of the modeling cycle in epidemiology showed how conjectures about the dynamical factors affecting the spread of infectious disease could be represented by equations. Thus, a system of signs was developed to analyze the behavior of physical referents. First the equations proved to be inadequate, but upon improvement, "behaved" well while revealing an additional factor (vital dynamics) not initially considered.

Now, one may think that modeling is very fruitful in those areas of inquiry to which it is applicable but leaves the question of identification of these areas open. The identification question may also be approached through the philosophy presented here. Are there phenomena so complex that they cannot be analyzed through the mathematical method? This must be rephrased (generalized) to ask whether there are phenomena so complex that they cannot be understood by human beings. The answer is "probably." However, the mere fact that a subject is being investigated at all is admission that those carrying out the investigation believe that some understanding may be achieved. The most pertinent response to the question posed above is that the "sufficient" complexity of the phenomenon in question cannot be determined a priori. In this sense, the modeling cycle could actually result in the conclusion that the mathematical method is inadequate for the problem at hand. But this in itself would be a significant contribution to the understanding of the phenomenon (if only to understanding its complexity). The major result of our study is that mathematical modeling may be considered as a particular form of philosophical discourse and as such should not be discounted as an approach to understanding.

There are special cases in which the "validity" of a particular model in science has been "proven." The connotation of the word "proven" in this case means that the model has "pragmatic validity." For example, predictions of chemical reactions based on present atomic theory are very consistently correct. The question of whether matter actually *is* made up of atoms then becomes irrelevant. We have a model and a high degree of *association* between model and observations. The first atomic theories however were not entirely adequate. The model has undergone many changes in recent decades. In one sense, the modeling cycle assumes that models are accepted only until they are "refined" or "replaced." As situations arise in which a model has been reformulated, the "new" model replaces the old, thus guaranteeing the evolution of the field in a constructive direction.

In those areas under mathematical investigation where the "pragmatic validity" has yet to be proven or where a controversy exists concerning mathematical applications at all, modeling must be considered to be the type of philosophical discourse mentioned above. In this light, the boundaries between those traditional "sciences" and the "non-quantitative" subjects have been identified and they are vague. If the argument against the mathematical method is that it cannot provide us with "truth," then we must reject any means of discourse, since none yet have succeeded in providing "truth." On the other hand, as a form of discourse, the conclusions from the argumentation are always subject to human interpretation, acceptance or rejection.

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