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Michael E. Goldberg
Hampton Roads Academy

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Dr. Michael E. Goldberg
Hampton Roads Academy
mgoldb@hra.org

INTRODUCTION: A NEW ANSWER TO THE OLD QUESTION

In this article, I address two works by two different authors in the May 1998 issue (#17) of *Humanistic Mathematics Network Journal*. In "Real Data, Real Math, All Classes, No Kidding," Martin Vern Bonsangue discusses the famous question so often put to math teachers: "When am I ever gonna use this stuff?" Bonsangue confesses, "I gave shop-worn answers like, 'Math teaches you how to think, so it doesn't matter,' or 'Well, we can solve word problems with math,' neither of which I believed in" (17). His solution is to find "real" examples (I explain the quotation marks in the following section), specifically data about earthquakes and life expectancy, to use in his math classes.

A few pages later, Jeffry Bohl, in "Problems that Matter: Teaching Mathematics as Critical Engagement," deals with the question again. He creates a small taxonomy of possible responses to the same age-old utilitarian question: all answers are versions of "tomorrow" (you will need the information for an upcoming chapter, course, etc.), "jobs" (you will need the information to land a good career), "general mental strength" (you will need the information to think better), or "tests" (you will need the information to pass the test, quiz, next assessment).

I would like to propose another category of response that stems from what I believe is the general principle at work behind Bonsangue's search for "real" examples. Specifically, the response to "When am I gonna need this?" involves recognizing that the question, though designed to be teleological, cannot, strictly speaking, be answered. As those of us who are a little more experienced and worldly than our students well know, *you never know when you will need to know something*.

"REAL" DATA: WHAT IS IT AND WHY DO WE LIKE IT?

One problem with mathematics textbooks is their lack of real—perhaps we should use "accurate" or "contemporary"—data. Bonsangue's math problems involve real data: earthquakes, a subject of intrinsic interest for his Californian audience; and life spans of pre-civil war women of different races, a relatively peculiar subject involving history, detective work, and heritage. However, on the face of it, neither topic would necessarily treat a student's question of "When am I going to need to know this?" By introducing data collected from "the field," as Bonsangue does, we expect relevance and a "hook" that will capture students' imaginations. In my experience, our expectations are satisfied.

Somehow, more traditional problems that also use real data are hardly so enticing. For example, "A bus departs at noon from Newport News (where I teach) averaging 42 mph. Another bus departs an hour and fifteen minutes later averaging 60 mph. If I am going to Washington, D.C., 200 miles away, which bus gets me there first?" This problem may actually be based on extant bus schedules; furthermore, D.C. is a wonderful city to visit and so knowing this information has some value. The difference is, perhaps, that these data look like a "standard" word problem, whereas Bonsangue's data are unorthodox, deriving from very *un*mathematical sources.

Indeed, math problems that seem to come from non-mathematical problems are often what we mean by "real data," and they are often very interesting to students. Even though knowing the "Distribution of 'Felt' West Coast Earthquakes January 1, 1990 through July 11, 1996" (Bonsangue 19) is not a good answer to "When am I going need to know this?", it has a good chance of engaging students because it smacks of reality. It uses information that somehow creates or demonstrates a nexus between life, math, and nature. In other words, Bonsangue's problems are truly humanistic mathematics—they genuinely engage many areas of human experience and knowledge at once—and *this* is what excites students. Math problems are often most interesting when they combine the mathematics with other areas of learning.

WHEN DO YOU NEED TO KNOW ANYTHING?

I begin with a digression: years ago, I was one of two finalists for a job teaching physics and earth science. During the second interview, I learned that my competition had more experience and expertise than I did, but she had one weakness: she was tentative about driving a school van for an after-school program. The Goldberg family car for 20 years was a huge, full-sized van (this was the pre-minivan era), so I was happy to offer my services as driver. I got the job. Never in my wildest dreams did I imagine, going into that second interview, that my competence driving vans would be critical to my teaching career. This experience confirmed one of life's great lessons, one of the most critical I have to offer to anyone, young or old: a fundamental aspect of life is that *you never know what you need to know until you need to know it*.

Intuitive understanding of this principle may well be why working with real data almost always seems so much more interesting than traditional math problems. Even if the purpose of the knowledge is not clear now, a problem-solver is gaining real, applicable knowledge that may be of utility another day. Here Bohl's "tomorrow" answer applies. Bohl believes, "we need to teach mathematics through the mathematization of real, socially relevant situations" (29). It may sound curmudgeonly, but I find "socially relevant" a problematic phrase: it too often means, "socially relevant in the eyes of a teacher and not a student." When I taught composition, I frequently covered "socially relevant" material and found I unintentionally put students in combative, uncomfortable situations. When I did not try to introduce the material, it came to light in more natural and potent ways. *Trying* to be socially relevant can be problematic.

However, I agree with the spirit of the advice: we are teaching not only math, but life skills (or, at least, cultural skills). Herein lies the connection between Bonsangue's "real data" and Bohl's "general mental strength." Social relevance is in the eye of the beholder, but if students understand they are learning about their world, not some abstract theoretical principle, they are more likely to file it away for use another day. If students feel that you have broadened their knowledge base, as opposed to broadening their *mathematical* knowledge base, they will often feel more fulfilled. This is not to say, "don't teach math." In fact, my point is that a general, humanistic knowledge base should be integrated with mathematics. The most exciting mathematical problems are the ones that seem to come from elsewhere besides math books.

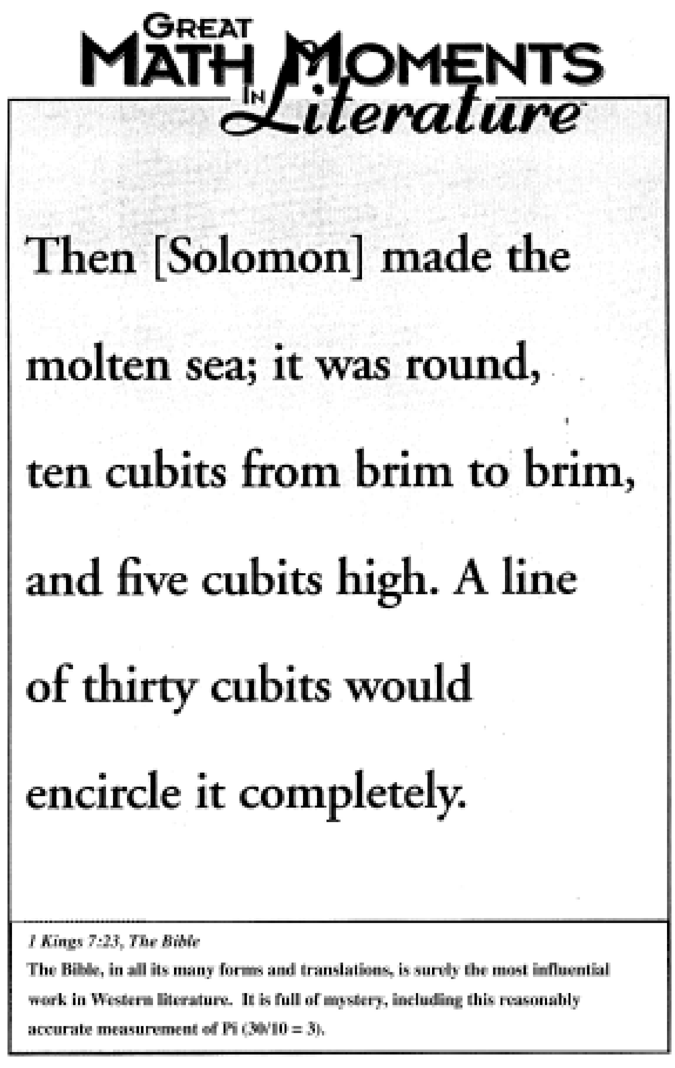
According to *A Handbook of Literature*, by Holman and Harmon, "humanism suggests a devotion to those studies supposed to promote human culture most effectively—in particular, those dealing with life, thought, language, and literature of ancient Greece and Rome" (233). It may be (unfortunately) difficult to find students eager to digest information about ancient Greece and Rome, but problems engaging other aspects of humanism tend to excite students greatly. It is, after all, far rarer that an English teacher gets asked, "When am I going to need to know this?"

Though the humanities have historically struggled for validation in America, I have found few students asking why they needed to read *Hamlet*. The presumption among students is that Shakespeare, by virtue of his importance to literature, history, philosophy, etc., is automatically worthwhile. Problems engaging more than one area of human thought (like the ones Bonsangue has constructed) fit the bill. They teach mathematics, but they also stimulate other interests, effectively displacing the "when do I need to know this" question. I, too, prefer mathematical problems that in some way ask students to engage other disciplines (art, history, literature, film, etc.), ideally a few at once. I also believe that the "social relevance" will often be contained within such problems automatically, by virtue of their complexity.

To summarize this section, let us review why the bus problem described above is relatively drab to my students. (I should point out that it would not be dull to *all* students. The trip to Washington, D.C. is, for some, a rewarding concept to entertain.) There do not appear to be many dimensions or applications to the problem. We need to go to a specific place and the math helps us make a better decision about how to get there. The "better decision" is better only insofar as it saves us about half an hour. The beauty of taking real data is that these data are not math problems first: they are problems dealing with life and thought, problems dealing with humanism. It thus turns out—and this is surprising and exciting to many novices—that mathematics is part of the pursuit of humanistic (not just "human") knowledge. A friend in college was fond of pointing out that math departments are not in "arts and sciences" divisions *not* because math is a *science*, but because it is an *art*. A problem that can be put in a student's "I may need to know this someday" file is an important, artistic problem.

AN EXAMPLE: GREAT MATH MOMENTS IN LITERATURE

As should be quite obvious, I am a humanist at heart. I have a doctoral degree in English literature; wrote a dissertation on James Joyce (himself no minor humanist); and spent ten years teaching composition, film, and English literature. (Thus, I can claim with some authority that students do not question the teaching of *Hamlet*.) I also have a strong background in math and physics, but for a decade I taught very little math, save my "calculus tip of the day," when I taught a writing course geared toward college engineering majors. For years, people asked me about my diverse background. Would I ever need to know all that math? Why was it so interesting to me anyway (I was an *English* major)? I come by my devotion to interdisciplinarity honestly, and I believe that it is precisely the search for links between different modes of thought and different knowledge bases that best furthers learning.



Having mused about the intrinsic needs of the young (and old?) mind to encounter more than just math, it is time to turn to my experience and my suggestions for teaching humanistic mathematics. I will keep this section short; this Journal contains a wealth of ideas regularly, and I only want to add enough to connect theory with practice.

I am now a high school mathematics teacher and I delight in adapting *non*-mathematical situations into my classroom. I enjoy asking students to look at relatively obvious "real life" problems (some examples: altering recipes, renting cars, predicting landfall of a hurricane, buying concert tickets, carbon-dating bones) as well as those that, in talking about Bonsangue's "life expectancy" project, I call "relatively peculiar." These are particularly enjoyable in my calculus class, where I used a problem called "the perfect glass of chocolate milk" to explain saturation (and derive a formula), and used derivatives to calculate how far a professional wrestler jumped by knowing his "hang time." My point is this: both of these are fairly simple problems, but they are taken from daily experience and thus, judging by my students' responses, evoke great interest.

Unique to my classroom is a series of posters that I created (along with some friends and a graphic artist), called "Great Math Moments in Literature." With a few notable exceptions, I generally did not take examples from science fiction. Though I enjoy science fiction very much, to sample it would be to take the "obvious" choices (kind of like using the bus problem). The best posters are those that use examples from work seemingly *un*mathematical in nature. There is room for Madeleine L'Engle, Thomas Pynchon, Lewis Carroll, and Arthur C. Clarke, but what pleases me the most are the posters that come from less scientific or mathematical sources.

My favorite example is a poster I use while discussing irrational numbers with students in my Algebra I classes. We should never forget how apt the term denoting numbers like π or $\sqrt{2}$ really is: irrational numbers *do not make sense* to the novice. Only experience, it seems, eventually allows us to get a handle on these quantities that we can write as symbols, point to on a number line, but never know the exact value of. When the class arrives at this head-spinning concept, I point to a poster that contains a quotation from the Bible: "Then [Solomon] made the molten sea; it was round, ten cubits from brim to brim, and five cubits high. A line of thirty cubits would encircle it completely." The poster can (and does!) occasion a variety of wonderful discussions, including ones of metaphor ("the molten sea" is a poetic term for a gigantic chalice) or of the nature of units (a "cubit" is about 18 inches, the distance from the elbow to the fingertip). Generally, though, I prefer to point out the biblical approximation of π to 3 (circumference of 30 divided by diameter of 10). The class can begin drawing and measuring circles, research the circumference/diameter relationship other ways (on the Internet?), or merely engage in a lively discussion about how accurately we can measure anything. Whatever discussion/exercise ensues, by the time we are done, the students are satisfied that they have gained knowledge of philosophy, theology, geometry, and, by the way, Algebra I. The concept of irrational numbers becomes a part of a much bigger, more important discussion.

I close with an imagined example, using another poster in my classroom. The poster cites a later Sherlock Holmes story ("The Final Problem"), in which the great detective explains Moriarty's evil genius. It seems,

> [Moriarty's] career has been an extraordinary one. He is a man of good birth and mathematical faculty. At the age of twenty-one he wrote a treatise upon the Binomial Theorem which has had a European vogue. But the man had hereditary tendencies of the most diabolical kind.

Great Math Moments in Literature

[Professor Moriarty's] career has been an extraordinary one. He is a man of good birth and excellent education, endowed by Nature with a phenomenal mathematical faculty. At the age of twenty-one he wrote a treatise upon the Binomial Theorem which has had a European vogue.... But the man had hereditary tendencies of the most diabolical kind.

"The Final Problem" in The Memoirs of Sherlock Holmes (1894)

Arthur Conan Doyle (1859-1930) was a prolific writer, but he is primarily remembered for his creation of the world's greatest detective, Sherlock Holmes.

This would be an enjoyable starting point for a discussion of the Binomial Theorem (given a great deal of time or, perhaps, a graduate-level classroom, it might even be fun to let students do some research and take a crack at writing Moriarty's treatise). It seems to me also a good place to investigate the nature of proofs and theorems. I say this because most students accept math as "truth." How, they might wonder, could someone write a "treatise" on an incontrovertible truth? How could such a paper enjoy "a vogue"? Isn't math either true or false? The point, then, would be to use a seemingly innocuous quotation as an introduction into the complexities and unknowns of mathematics, thence an introduction into the complexities and unknowns of our world.

Sherlock Holmes' genius lies in two abilities: he is remarkably observant and he has a huge knowledge base. Just to take one example, in "The Five Orange Pips," he deduces that the *Lone Star* must be a boat from America because he knows it borrows the nickname for Texas. This is impressive knowledge for a proper English gentleman in 1892, for whom knowing state nicknames would appear to be utterly useless information. It *is* utterly useless, until Holmes needs it one day to solve this case; armed with this information, the case is solved quickly and effectively. Holmes is therefore one of the great, if not the greatest of all exemplars of the "you do not know what you need to know until you need to know it" principle. Citing him as a model is attractive: he uses the seemingly "mathematical" science of deduction to help with the most human of issues, including broken relationships, jealousy, greed, and evil. If math problems were like Sherlock Holmes stories, students would be endlessly engaged, entertained, and educated. Diversity, interdisciplinarity, and real data make math problems that much more like Sherlock Holmes stories.

CONCLUSION

I am not suggesting we teach literature in math classes. I am suggesting we break down the barriers between disciplines and I have chosen Sherlock Holmes as a metaphor. In my own teaching I try (though, of course, I do not always succeed) to apply the framework I have outlined here. Any good teacher—not only one who handles math—must have his or her own answers to "When am I going to need to know this?" However, the best answer is not always a time, date, or place. The best answer is that no one quite knows. We may, in part, teach math so that students can be ready for tomorrow, do well in jobs, improve their critical thinking, and pass the next test. In fact, I have some support for all of these reasons. However, we also have an imperative to teach humanism, to mathematize the world in ways such that our students are ready for whatever life offers them. After all, in "real life," people get jobs teaching physics just because they know how to drive a van.

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Book Review: *The Teaching Gap* by James W. Stigler and James Hiebert Review Part II: Contrasting U.S. and Japanese Beliefs about Mathematics Teaching

Michael L. Brown
Simmons College
Boston, MA 02115-5898
michael.brown@simmons.edu

The Teaching Gap. James W. Stigler and James Hiebert. The Free Press (Div. Simon & Schuster Inc.): New York, 1999. ISBN 0-684-85274-8

This is the second of three articles that together form an in-depth book review of The Teaching Gap. After a brief summary of the first article[1], we explore the contrasting cultural beliefs that support mathematics teaching in the U.S. and Japan, and in doing so, find several surprises that are relevant to college teaching.

SUMMARY OF THE FIRST ARTICLE

As our first book review article discusses, *The Teaching Gap* addresses critical questions about how mathematics teaching is actually done in the U.S., Japan, and Germany, based on uniquely valuable data: the first videotaped national random samples, in each country, of eighth-grade mathematics classroom lessons. Remarkably, teaching varied greatly from one culture to the next, and comparatively little within each culture, giving an empirical foundation to the pivotal claim in the book that "teaching is a cultural activity."[2] The authors claim, and we are persuaded, that their findings of cultural differences go far beyond the eighth grade. Indeed, we believe they have much of importance in common with what we en-