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Number, Infinity and Truth: Reflections on the Spiritual in Mathematics

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1. INTRODUCTION

Mathematics, as a whole, has a reputation for precision, unambiguity, and cold reason. These are rarely considered spiritual properties which, in contrast, tend towards mystery, paradox, and transcendence. Nevertheless, mathematics has had a spiritual aspect throughout its history. Indeed, in many cultures there has been little to distinguish the mathematician from the priest.

In ancient Egypt, in Mesopotamia, and in the Maya kingdoms of Guatemala and Mexico, mathematical knowledge and spiritual knowledge converged and merged in a spectacular interplay. This mathematico-spiritual interplay existed in other cultures besides the great civilizations of antiquity and persists to the present day.

This essay looks at the nature of the interplay between mathematics and spirituality in some traditional and modern contexts.

2.0 TRADITIONS

The intersection between mathematics and spirituality appears in many religious traditions. In the western tradition, we have the case of the Pythagoreans (circa 500 B.C.E.) who assigned a definite number to everything material and spiritual. *One* was the number of reason, *four* of justice, and *five* of marriage (Burton, 1997:92). Numerology continues to attract devotees as any trip to a "new age" bookstore or a quick scan of the internet will reveal.

So numerous are the intersections of religion and number that to even mention all of the traditions in which they occur would require a book-length treatise. In this section I'll highlight a handful of examples of the interplay of mathematics and religion chosen for their diversity and ability to shine light on this interplay. The examples come from the Upanishads of India, the religious beliefs of the Oglala Sioux of North America, and the visions of a West African Dogon elder.

2.1 THE FIVE- AND SEVEN-FOLD CHANTS OF THE CHANDOGYA UPANISHAD

The intersection of mathematics and religion and spirituality in the realm of numbers is well known. One of the oldest of these intersection points is found in the Upanishads, Hindu religious texts written between 800–300 B.C.E., which may represent a tradition dating back to 4000 B.C.E. (Hume, 1977; Frawley, 1995).

In one of these texts, known as the Chandogya Upanishad, the Second Prapathaka (Chapter) is an extended discussion of the structure and meaning of the five-fold and seven-fold chants. The five-fold chant is characterized by the Hinkāra (preliminary vocalizing), the Prastāva (introductory praise), the Udgitha (loud chant), the Pratihāra (response), and the Nidhana (conclusion). An example is given in the way we may reverently understand the rainstorm:

> "In a rain-storm one should reverence the five-fold Sāman.
> The preceding wind is a Hinkāra.
> A cloud is formed—that is a Prastāva.
> It rains—that is an Udgitha.
> It lightens, it thunders—that is a Pratihāra.
> It lifts—that is a Nidhana."
> (Hume, 1977:191)

The seven-fold chant adds two new sections to the five-fold chant. The Adi (beginning) appears just before the Udgitha, and the Upadrava (approach to the end) precedes the Nidhana. The Upanishad tells us how the course of the sun through the sky parallels the seven-fold chant:

> "When it is before sunrise—that is a Hinkāra…
> Now, when it is just after sunrise—that is a Pratāva…

Now, when it is the cowgathering time—that is an Adi…
Now, when it is just at midday—that is an Udgitha…
Now, when it is past midday and before the afternoon—that is a Pratihāra…
Now, when it is past afternoon and before sunset—that is an Upadrava…
Now, when it is just after sunset—that is the Nidhana…"
(Hume, 1977:194)

Several other examples of these five- and seven-fold patterns are related in the Second Prapathaka of the Chandogya Upanishad, but the most striking mathematical passage is found in the Tenth Khanda (verse). This passage explains the mystical significance of the syllables in the categories of the seven-fold chant.

> "Now then, one should reverence the *Sāman* (chant), measured in itself, as leading to death.
>
> *hinkāra* has three syllables. *prastāva* has three syllables. That is the same.
>
> *adi* has two syllables, *pratihāra* has four syllables. One from there, here—that is the same.
>
> *udgitha* has three syllables. *upadrava* has four syllables. Three and three—that is the same, one syllable left over. Having three syllables—that is the same.
>
> *nidhana* has three syllables. That is the same, too. These are twenty-two syllables.
>
> With the twenty-one one obtains the sun. Verily, the sun is the twenty-first from here. With the twenty-two one wins what is beyond the sun. That is heaven."
> (Hume, 1977:194-5)

In the Tenth Khanda we see mathematics (arithmetic and algebra) at the service of spirituality. The Upanishadic calculation aims towards heaven using the syllables of the classificatory words of the chant as its data. We see two fundamental characteristics of mathematics at works here: balance and sum. The Khanda calculates the sum of the syllables to 22, while emphasizing a balance between terms based upon 3. The first paragraph affirms a balance between *hinkāra* and *prastāva*, 3 = 3. In the second paragraph, *adi* and *pratihāra* are brought into balance with some simple algebra "One from there, here." That is, take one syllable from *pratihāra* and add it to the syllables of *adi*. In modern notation, the Upanishad computes

$$4-1 = 2+1$$
$$3=3$$

Achieving balance between *udgitha* (3 syllables) and *upadrava* (4 syllables) caused the writer of this Khanda some difficulty. The writer's solution is to emphasize that both words have three syllables and that the fourth syllable of *upadrava* is "left over." Mathematically, this maneuver is suspect. Nonetheless, the writer clearly recognizes the problem and assures us that it is not a problem. Indeed, the extra syllable returns in the sum with powerful impact. We can summarize the mystical mathematics of the Tenth Khanda this way:

$$[3+3]+[(2+1)+(4-1)]+[3+3]+1+3=21+1$$

where the square brackets group balanced pairs. The writer of the Tenth Khanda employed mathematics to emphasize and perhaps justify the terms and relationships between terms of the seven-fold chant.

2.2 SACRED NUMBERS OF THE OGLALA SIOUX

The importance of numbers in the world-view of the indigenous peoples of the Americas is well documented. We find sacred or mystical numbers in the cultures of the Hopi (Young, 1988), the Maya (Freidel, et. al., 1995), the Inka (Urton, 1997), the Ojibwa (Closs, 1986), and the Oglala Sioux (Powers, 1977). I choose the Oglala for special examination here because the interplay between number and religion in their culture is representative of a large number of Indian cultures of North America. Indeed, the quadripartite view of the universe and the supernatural is prevalent in North America (Levi-Strauss, 1968; Bullchild, 1990; Young, 1988). The information about the Oglala presented in this section is from Powers (1977:48-51).

The numbers 4 and 7 are sacred to the Oglala. *Four* is of central importance because it is the number of world directions, the number of divisions of time (day, night, moon, year), and the number of periods of life (baby, child, adult, old age). The number four is central to certain aspects of human physiology, including the important observation that four is the number of fingers on each hand, the number of toes on each foot, and the total number of big toes and thumbs on a person. We thus have the "human" compound equation:

$$5-1=5-1=5-1=5-1=1+1+1+1$$

In the supernatural world, the number four is central to the structure of *wakantanka*, an Oglala concept that embodies all supernatural beings and powers. The supernatural is classified into four aspects, each of which is subdivided into four more. This hexadecimal structure looks like this:

Wakan akanta (Superior *wakan*)
- Sun
- Sky
- Earth
- Rock

Wakan kolaya (Those who *wakan* call friends)
- Moon
- Wind
- Falling Star
- Thunder-being

Wakan kuya (lower or lesser *wakan*)
- Buffalo
- Two-legged
- Four winds
- Whirlwind

Wakanlapi (those similar to *wakan*)
- Shade (Apparition)
- Life (Breath)
- Shadelike
- Potency

It must be emphasized that this 4x4 structure is not the classificatory scheme of the European anthropologist, but the Oglala themselves emphasize the "four-ness" of their cosmology. When a sweat lodge is constructed 16 willow saplings are needed, one for each aspect of *wakantanka*.

Seven, the other primary sacred number of the Oglala, is constructed from *four* arithmetically and spiritually. To the four directions (West, North, East, and South) are added the zenith, the nadir, and the center of the universe. Seven is explained as 4+2+1; the four directions plus sky and earth plus the universe. Prayers are smoked and sung to these seven directions when invoking the supernatural. The sociopolitical groups of the Sioux and the birth-order names of Oglala children also follow the 4+2+1 = 7 pattern.

Significantly, within the 4x4 structure of the *wakantanka* we also find the calculation 4+2+1 = 7. *Wakan akanta* and *wakan kolaya* are grouped together as *wakan kin* (the Sacred). *Wakan kuya* and *wakanlapi* form *taku wakan* (Sacred Things). The entire collection is *wakantanka*. Thus, the heptadic classification is seen simultaneously as monadic, dyadic, and tetradic.

Beyond the examples given above, all natural and cultural phenomena are classified by the Oglala into structures of 4 or 7 or 16 (4x4) or 28 (4x7). Powers (1977:51) quotes Black Elk on the importance of four and seven:

> "…the numbers four and seven are sacred; then if you add four sevens you get twenty-eight. Also, the moon lives twenty-eight days, and this is our month; each of these days of the month represents something sacred to us: two days represent the Great Spirit; two are for Mother Earth; four are for the four winds; one is for the Spotted Eagle; one for the sun; and one for the moon; one is for the Morning Star; and four for the four ages; seven are for our seven great rites; one is for the buffalo; one for fire; one for water; one for the rock; and finally one is for the two-legged people."

2.3 THE VISION OF OGOTEMMLI

Perhaps no vision of the universe is so wonderfully intertwined with numbers than that of the Dogon elder, Ogotemmli, as related by the French anthropologist Marcel Griaule (Giaule,1965). For Ogotemmli, everything in the universe has a number and the numbers themselves have qualities. The central number in Ogotemmli's vision is eight. There were eight sections to the structure of the world-system, eight primeval ancestors at the creation of the world, and eight families descended from them. There were eight seeds at the beginning of creation. There are eight joints in humans, eight *dougu* (covenant-stones) that are the repositories of the life-forces of the ancestors, eight locations of the council house pillars, eight crafts, and eight regions populated by people speaking eight languages.

For each number 1-8, Ogotemmli revealed correspondences. The correspondences expose the inherent number of the idea or thing as well as explaining the qualities of the number. As an example, consider the numbers 2 and 7. *Two* corresponds to the southwest pillar, white millet, the Toro language, the Ende region, the colors red and white, and the craft of tanning. The second ancestor was the leather-worker. *Seven* corresponds to the north pillar, rice, the Ireli language, the Ireli region, the color rose, and the crafts of weaving, music and language. The seventh ancestor was the "master of speech."

> **"**
> ***Numbers are a vehicle for beginning to understand what is perhaps not understandable.***

The number 8 also serves as a kind of culmination or unification of the preceding numbers. Ogotemmli says,

> "Seven is the rank of the Master of Speech; 1+7 = 8. The eight rank is that of Speech itself. Speech is separate from the one who teaches it, that is the seventh ancestor; it is the eighth ancestor. The eighth ancestor is the foundation of the speech which all the other ancestors used and which the seventh taught." (Griaule, 1965:48)

The number *eight* corresponds to the lingua franca of the Dogon area, all regions, and agriculture, the art that encompasses all the arts and crafts of the Dogon.

2.4 WHY NUMBERS?

In the preceding sketches of number and arithmetic in religious belief, we see that the mathematics is more than an overlay or afterthought. Indeed, the mathematics provides a framework for the organization of the spiritual world. Number and arithmetic provide a bit of precision and definiteness to concepts, beliefs, and ideas that are imprecise and indefinite. S.N. Pandey, a proponent of a school of mathematics that ties all of mathematics to the Vedas, expresses this notion succinctly.

> "Knowledge gains perfection and unambiguity and clarity if it is expressed in terms of numbers." (Pandey, 1991:103)

Number also seems to lend power to religious practice. The author of the Chandogya Upanishad feels compelled to provide an analysis of the number of syllables in the names of the parts of the seven-fold chant. Is it to emphasize that these terms are not arbitrary, but instead incorporate some essential nature of reality?

Numbers provide unity. The pervasiveness of 4 and 7 in the natural and supernatural world provide evidence for the Oglala that the universe is not random, that there is a divine presence and a sacred unity to everything.

Numbers are a vehicle for beginning to understand what is perhaps not understandable. Ogotemmli's correspondences help the Dogon to see how things and ideas are related, interrelated, and connected to the whole. Ogotemmli 's equation 7+1 = 8 is a powerful notion of increase, culmination, and inclusiveness.

When we begin to look at mathematics beyond numbers and simple arithmetic, we can see more opportunities for its use in mystical understanding of the supernatural. In the next section, I'll examine some of the ways in which mathematical notions begin to blend with the mystical.

3.0 MATHEMATICAL NOTIONS & MYSTICAL VISION

How are mathematical notions mystical? Consider infinity. Mathematical infinity is a preponderance of seemingly contradictory and paradoxical notions. The modern study of infinity, begun by Georg Cantor in the 1870's, has shown us that there are levels of infinities, that some infinities are infinitely greater than others, that parts can be the same size as wholes, and other wonders. Here I'll only examine one of these wonders. (For a detailed non-technical discussion of the whole matter of infinity see Pickover (1995), Rucker (1995), or Vilenkin (1995). A complete mathematical discussion can be found in Fraenkel (1961), Monk (1969), or Suppes (1972).)

Cantor asks that we consider the counting numbers 1, 2, 3, 4, etc. not as a sequence of numbers, but rather as a totality, an infinite collection. It is a consideration that requires some mental wrangling and a certain amount of faith. We must behold a sequence that has no end all at once. In effect, we must see the sequence, as it were, from "outside," from a supernatural vantage point. This viewpoint, although perhaps unusual to a non-mathematician is not unique to modern mathematics. Artists wrestled with the problem geometrically in the Renaissance (Fields, 1997) and Mimica (1988) has shown that viewing the counting numbers this way is an important aspect of the culture of the Iqwaye people of Papua New Guinea.

Holding the infinite collection of counting numbers in our minds, now let us consider those that are even (2, 4, 6, …). Cantor now asks us to compare the sizes of the two collections. One way of comparing the sizes of two collections is by matching. This is a particularly efficient means of comparison if the collections are large. For example, if we want to compare the number of people with the number of seats in a large auditorium we may simply invite each person to take a seat. If we see anyone standing after everyone tries to sit down, then we know there are more people than seats. On the other hand, if everyone is seated and there are empty seats, then we know there are more seats than people. If every person has a seat and every seat has a person then we know that there were the same number of people as seats.

Now let's try to match the counting numbers with the even counting numbers. Because we are viewing the collections separately, it will be useful to distinguish numbers selected from the counting number collection from those selected from the even number collection. If a number is taken to be from the even numbers I will write it with **bold** print and if it is taken from the counting numbers I will write it in *italics*. Now we can construct a matching between the two collections by matching each counting number with its double, like this:

counting numbers: *1 2 3 4 5 6 7 8 9 10 11 …*

even numbers: **2 4 6 8 10 12 14 16 18 20 22 …**

Notice that every counting number is matched with an even number. Indeed, whatever counting number we pick, we can easily determine which even number it is matched with by doubling the number we pick. So, *1999* is matched with **3998**, *150,000,000* is matched with **300,000,000**, and so on. Furthermore, every even number is matched with a counting number. If we pick an even number we can find the counting number it is matched with by dividing the chosen even number by 2. Thus, **24** is matched with *12*, **1492** is matched with *746*, and so on.

What does the result of this matching process suggest? There are no unmatched numbers from either collection, therefore the collections are the same size! Yet, clearly the odd numbers are not included in the collection of even numbers. So, shouldn't the collection of counting numbers be greater than a collection which is part of it? Normal intuitions fail when we consider the infinite.

So then, we may ask, are all infinite collections ultimately the same size? To this Cantor's surprising answer is "no." To explain his answer, I'll need the convenient mathematical notions of a set and subset. A set of things is simply a collection of those things. A subset of a set is a collection containing items from the set. Sets and subsets are denoted mathematically with braces surrounding the names or descriptions of the items in the set. The subset of the counting numbers that contains the numbers 2, 4, and 6 is written {2, 4, 6}. Thus, for the set of counting numbers we have many subsets, including

{2},
{1,2,3,4,5},
{1492,1776,1812,1864,1969,1998}
{2,4,6,8,...} the even numbers,
{1,3,5,7} the odd numbers,
{2,3,5,7,11,13,...} the prime numbers,
{4,7} the sacred numbers of the Oglala,
{1,2,3,4,5,6,7,...} the counting numbers,
etc.

There are an infinite number of subsets of the set of counting numbers. This is easily seen because each counting number can be form a one-item set {1}, {2}, {3}, and so on. Is it possible to match the collection of subsets of the counting numbers with the counting numbers? Cantor says no, and here's why.

Suppose that some very bright person came up with what he or she thought was a matching scheme between the subsets of the counting numbers and the counting numbers. The matching would look like this:

$1 \rightarrow S_1$
$2 \rightarrow S_2$

$3 \rightarrow S_3$
…
$1200 \rightarrow S_{1200}$
$1201 \rightarrow S_{1201}$
…

The subscripted S indicates the set that is matched with that number. So, S_{24} is the set that is matched with counting number 24. We don't know what S_{24} is, but our bright person should be able to tell us if we ask. Similarly, if we ask our bright person what number the set {4,7} is matched with, he or she should be able to tell us.

Now we'll identify a particular subset of the counting numbers and ask what number with which it is matched. In the matching, some sets will contain the number they are matched with and some will not. That is, 24 may be an element of S_{24} or it may not. Consider the set containing only those numbers which are not elements of the set with which they are matched. Call this set C. Now if 24 is not in S_{24}, then 24 is in C. On the other hand, if 24 is in S_{24} then 24 is not in C. Our bright person's matching claims to account for all the subsets of the counting numbers, so C ought to be matched with some number, call it *k*. Thus, $C = S_k$.

Is *k* in C? If *k* is in C, then *k* is not in S_k. But S_k=C, so *k* is not in C. On the other hand, if *k* isn't in C, then *k* is in S_k which means *k* is in C. Either possibility is contradictory. Logically, then, some earlier assumption in the argument must be false. C is a reasonably defined set. Thus, we are left with the conclusion that C must not be matched with any counting number. The set of subsets of the counting numbers is larger than the set of counting numbers. We have a collection that is infinite, yet is larger than infinity. Cantor went on to show that there are infinite levels of infinitely larger infinities.

Mathematical infinity demands considerable contemplation and is arguably similar to paradoxical mystical language. Cantor, of course, did not suggest that his infinities were a path to the mystical understanding that everything is interconnected and has a single source. Nor did he offer them as triggers of mystical experience. Nevertheless, contemplation of Cantor's infinite hierarchy of infinities, each of which is infinitely greater than its predecessor, can affect one in a fashion reminiscent of the Zen *koans*.

Borchert, in describing the mystical experience, may come close to describing the experience one has when contemplating the paradoxes of infinity:

> "These opposites seem to exclude one another; to destroy—to recreate; miserly—openhanded; terrifying—attractive. Because of the tension between opposites, a narrow opening comes into existence, and it is through this channel that the mystic sees something which cannot be set down in *one* word. What it is cannot be expressed, but can only be suggested." (Borchert, 1994:19)

Infinity is only one of many mathematical notions that suggest mystical visions. The mathematical field of complex dynamical systems brings before us the infinitely self-replicating patterns of the Mandelbrot set and engaging fractal portraits (Peitgen & Richter, 1986). Computer generated fractal images seem to be not far removed from the meditative art forms of mandalas or yantras used in Eastern mysticism. And, in the mathematics of quantum computing we see parallel universes, timelessness, and a host of other metaphysical constructs. (Deutsch, 1997).

What all of these mathematical notions share is their ability to articulate with some precision notions that transcend everyday experience. The mathematics allows us to peer into aspects of reality that go beyond our senses and often beyond our commonsense notions of rationality. And this transcendence leads us to questions of truth and reality.

4.0 MATHEMATICAL TRUTH AND SPIRITUAL TRUTH

A reasonable question concerning this mind-bending infinity of infinities is that of its connection to what we commonly consider to be reality. We might wonder if these arguments are no more than mental recreation. Have we learned a truth about something? If so, what? The philosopher Michael Resnick (Resnick, 1998) offers some notions of truth that I think are useful in assessing the truth of Cantor's visions. Resnick sees at least two aspects of mathematical truth: immanent truth and transcendent truth.

Immanent truth applies only to statements within its own language. The truth of the statements about infinity that I have offered is established entirely within the realm of the infinities of sets. Sets, numbers, and logic provide the basis for the truth of the hierarchy of infinities. These truths do not rely on things, relationships, or observations outside of the realm of mathematics. As a mathematician I am prepared to learns truths about mathematical objects because I have been trained to understand claims about mathematical objects.

Immanent mathematical truth may be contrasted to transcendent mathematical truth, which seeks support in reference to physical objects or correspondence between mathematical objects and non-mathematical objects. I learn transcendent mathematical truth through experiment as well as through proof.

For example, as a transcendent mathematical truth, 2 + 1 = 3 makes claims about the number of people in my car after my son and I meet and pick up my wife. The truth of the equation is confirmed by its correspondence to my experienced world. The same equation is viewed as an immanent truth when I am persuaded by this mathematical proof about sets:

"Debates about the existence of numbers and other mathematical objects parallel those about the existence of gods and spirits.

1. The set { } is a number. Call it 0. (Definition of a number.)
2. If x is a number, then S_x = { x, {x}}. (Definition of S_x)
3. If x is a number, then S_x is a number. (Definition of number.)
4. Only things satisfying Statements 1 or 3 are numbers. (Definition of number)
5. Let 1 denote S_O = {0, {0}}. (Shorthand notation for S_0)
6. Let 2 denote S_1 = {1, {1}} = {{0,{0}}, {{0,{0}}}}. (Shorthand notation for S_1)
7. Let 3 denote S_2 = { {{0,{0}}, {{0,{0}}}}, { {{0,{0}}, {{0,{0}}}} } }. (Shorthand notation for S_2)
8. If x is a number, then either x=0 or there is a number y such that S_y = x. (Follows from Statement 4.)
9. If x is a number, then x + 1 = S_x. (Definition of + 1)
10. 2 + 1 = S_2 (Replace x by 2 in statement 9)
11. S_2 = 3 (Statement 7)
12. Therefore, 2 + 1 = 3. (From statements 10 & 11)

I have learned that 2 + 1 = 3 is true in this case because I understand the notion of a set and some simple principles of logic. It is a truth in the world of sets and logic and will be true for anyone prepared to receive it.

I think that spiritual truths also have this immanent/transcendent Janus face. On the one hand they are truths within their own language. If one is taught to understand the spiritual concepts, then one can learn truths about them. Further, it is legitimate to say that without adequate preparation, one may well be incapable of understanding spiritual truths. On the other hand, many spiritual truths aspire to transcend their own language and claim the status of truth in other realms.

The Islamic teacher Ayatollah Khalkhalli said that "Reality will always prevail." (Naipaul 1998:210) From this we are to understand that reality means truth and that truth stands against falsity. For Khalkhalli, the spiritual truths of his faith have transcended the language of Islam to become truths of all languages, everywhere.

The parallel between mathematical truth and spiritual truth is important because of the success each has had in transcending its own language. Mathematics has provided numerous truths to science and spirituality has done the same for human culture.

Debates about the existence of numbers and other mathematical objects parallel those about the existence of gods and spirits. How are these entities to be identified? Can they be discovered in any objective way? Can only those who believe really understand? Mathematical and spiritual truths have survived outside of their home realms despite, or perhaps because, they are truths about objects with ambiguous existential status.

5.0 CONCLUSION

I doubt that mathematics will ever become a theology (although stranger things *have* happened), but its value in understanding reality is undeniable, and it has the power to bring its practitioner to a meditative state. Indeed, early Islamic scholars saw mathematics as religiously legitimate and as a way to Holy knowledge (Høyrup, 1994:111). Whether we experience God or the Tao or the ground of being when deeply involved in mathematical thought I will leave to the theologians. Nevertheless, the realms of mathematics and spirituality do intersect.

That intersection may be beautifully summarized by the following passage from Chapter 21 of the *Tao Te Ching*.

"The Tao is elusive and intangible.
Oh, it is intangible and elusive, and yet within is image.
Oh, it is elusive and intangible, and yet within is form.
Oh, it is dim and dark, and yet within is essence.
This essence is very real, and therein lies faith."
(Feng & English, 1974)

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Skolem's Paradox and Contradictory Popular Songs

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Skolem's paradox (named after the logician Thoralf Skolem) essentially points out that logic is relative: it depends on where you sit. More specifically, it is a paradox in set theory. It states that set theory has a countable model, which nevertheless contains uncountable sets. Formal set theory implies that there exists a set which is infinite, but no function exists which will map this set one-to-one onto the natural numbers: it is uncountable. Hence any model of set theory mirrors this "uncountable" set. But, according to the well-known Lowenheim-Skolem theorem, set theory has a model with only a countable number of objects in it. How can this be? The answer often given is "it depends on where you put the emphasis:" Do you emphasize the metamathematical countability or the formalized uncountability?

So now we turn to contradictory popular songs. Whether they are contradictory or not depends on where you put the emphasis. With some mental effort they might even be consistent. In these love songs we are supposed to imagine hopeful lovers: clearly, the emphasis is on "yes" rather than "no."

1. LET'S CALL THE WHOLE THING OFF

In this song the lovers are debating whether or not to call off their relationship (or a planned rendezvous). It seems that they disagree on the pronunciation of words such as "oyster," "pajamas," "either" and such (I would like to add "quark"). The debate continues until the last two lines, which are "so let's call the calling off off' and "let's call the whole thing off." These last two lines contradict each other, and I for one do not know whether it was called off or not.

2. BEGIN THE BEGUINE

According to the literature the Beguine was said by Cole Porter to be a romantic dance among certain natives, but he denied it later. Apparently the issue is whether or not to begin this memorable love dance or song. In one line you hear, "So don't let them begin the Beguine!...Let the love that was once a fire remain an ember," to be soon followed by "Oh yes, let them begin the Beguine, make them play!" This contradictory behavior can be understood by allowing for the emotional state of the singer. It seems to me that the emphasis is on the "yes" here, rather than the "no." Artie Shaw circumvented having to make the decision by producing a strictly instrumental version of the song (which is presently in top place on a popular radio station).

3. I'M IN THE MOOD FOR LOVE

The song begins with the words, "I'm in the mood for love." The singer then proceeds to explain why he or she is in the mood for love. This goes on until you hear the words "If it should rain, well let it; but for tonight forget it; I'm in the mood for love." This last sentence doesn't seem to make sense to a sensitive listener who is startled by "forget it" only to hear again "I'm in the mood for love." Louis Prima and Keely Smith avoid this paradox by substituting the phrase "if it should rain, well let it; but for tonight well let it; I'm in the mood for love."

It is interesting to speculate how a Turing machine would decide these "decision problems."