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Art and Geometry: Proportion and Similarity

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A course entitled "Art and Nature" has been the first course in the art major at Maharishi University of Management for several years. The main purpose of this first course is to introduce students to the field of art and creative expression. A significant component of the course is an ongoing project in which the students choose a natural site on or near campus (for example, "around the bridge over Crow Creek") and then observe the site, document their observations, and finally create a "map" of the site. This year the course included guest speakers from physics, biology, and mathematics. This paper will describe the lesson on mathematics that I presented to the students in the course.

There are, of course, many, many ways that mathematics can be related to art. I chose proportion and similarity for a number of reasons. My time was limited, and I knew that the mathematical background of the students would be uneven. I wanted to present ideas that students could actually use in observing their sites without any further study or instruction. In addition, similarity leads naturally to a discussion of fractals, and I wanted to include something that would give a flavor of current developments in geometry. The lesson eventually covered four topics: ratios of measurements, congruence, similarity, and fractals. We then looked at paintings selected by the instructor, Dale Divoky, to see examples of these concepts as used by different artists.

RATIOS

We can compare measurements in an absolute way, as when one measurement is four inches longer than another measurement, or we can compare measurements in a relative way, when we say that this measurement is three time as large as that measurement. Ratios are used to compare measurements in this relative way. A *ratio* or *proportion* of measurements is just the fraction formed by the two measurements.

A familiar example of a ratio is π, the circumference of a circle divided by its diameter, which is the same for all circles. However, the ratio of the circumference of a circle with radius r to its area is $2/r$, which changes as r changes. For a sphere of radius r, the ratio of the surface area to the volume is

$$\frac{4\pi r^2}{\frac{4}{3}\pi r^2} = \frac{3}{r}$$

and as r increases, this ratio gets smaller. That means that a large sphere has proportionately less surface area than a smaller sphere.

If we compare the ratio of surface area to volume for a sphere of radius r, which is $3/r$, to that of a cube of side s, which is $6/s$, we see that the ratio is smaller for the sphere. In fact, it is smaller for the sphere than for any other shape enclosing the same volume. Shapes such as soap bubbles, fruit, balloons, and over-stuffed suitcases tend to be spherical because of the presence of natural forces which minimize surface area for a given volume. The ratio of volume to surface area also helps explain why cells, which are constantly transferring molecules across the cell membrane, are small and have more surface for the given volume.

On the other hand, thin flat shapes have much more surface area per volume; paper and leaves are good examples, and their surfaces are useful for writing on or catching sunlight. This one insight into measurements and their ratios can help us understand how the functions and purposes of different objects are related to their shapes.

CONGRUENCE

Two objects are *congruent* if they have the same shape and size; see Figure 1. In this case, ratios of corresponding measurements will all be 1. There are many examples of approximate congruences in nature: our two

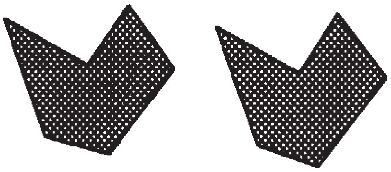

Figure 1
Two congruent figures

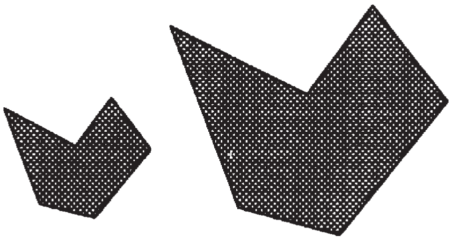

Figure 2
Two similar figures

hands, leaves on a tree, animals of the same species.

SIMILARITY

Two objects are *similar* if they have the same shape; this means that ratios of corresponding linear measurements of two similar figures will be the same. See Figure 2. Examples of similarity abound in nature. Plants and animals of the same species at different stages of growth are often similar. Crystals of the same substance are similar.

FRACTALS

A *fractal* is a figure that is similar to itself, or self-similar; this means that a part of the figure is (approximately) similar to a larger part or to the whole figure. The Koch snowflake is formed by adjoining similar equilateral triangles to the sides of an equilateral triangle; see Figure 3. The method of construction of the Koch snowflake assures that in the limit this process will give a self-similar figure. There are many examples of self-similarity in nature. Shapes like clouds or coastlines that look the same when scaled (up to certain limits, of course) are self-similar. Ferns are self-similar up to three of four levels of scaling. We see a more limited example of self-similarity in some trees, where the leaf or cone has the same overall shape as the tree itself.

IN ART

These mathematical concepts can be useful in the analysis of natural objects or works of art and can serve the artist in the design of a project. We might begin by asking questions like the following.

1. What are some of the more important or striking objects?
2. What are the overall proportions of a particular object? Why is the object shaped in this way?
3. What shapes predominate? What overall kinds of proportions do the predominant shapes have? Examples of overall proportions are large and flat, long and thin, round or spherical. How do these shapes create an effect in the observer?
4. Are any of the shapes similar or congruent? Are the objects corresponding to these shapes naturally similar or congruent?
5. Are there self-similar shapes? What feeling does this create?

STUDENT RESPONSE

Overall, I felt that the lesson was a success. The students brought out good observations from their side, and our analysis of about 20 paintings brought out all of the mathematical ideas we had discussed. Dale Divoky reported that in the analysis of their sites, several of the students were using the idea of similarity. One student kept finding similarity between more and more different objects and used it as a theme in his final "map" of his site. Dale felt that the main value of the lesson was to bring the attention of the students to the ideas of congruence, similarity, and self-similarity so that they could see them when making their observations. He said:

> Often it is the case that we only see what we know to be there or assume to be correct. Knowledge, therefore, is most valuable for observing the world around us (or within us). The knowledge of ratios, similarity, and fractals were of great value to the students in the Art and Nature course. Not only were their sensibilities heightened, but their aesthetic response became a more insightfully creative response.

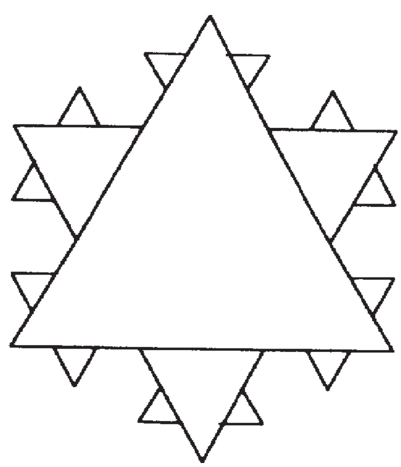

Figure 3
The first three stages in constructing the Koch snowflake.