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Architecture and Mathematics: An Introduction for Elementary and Middle School Children

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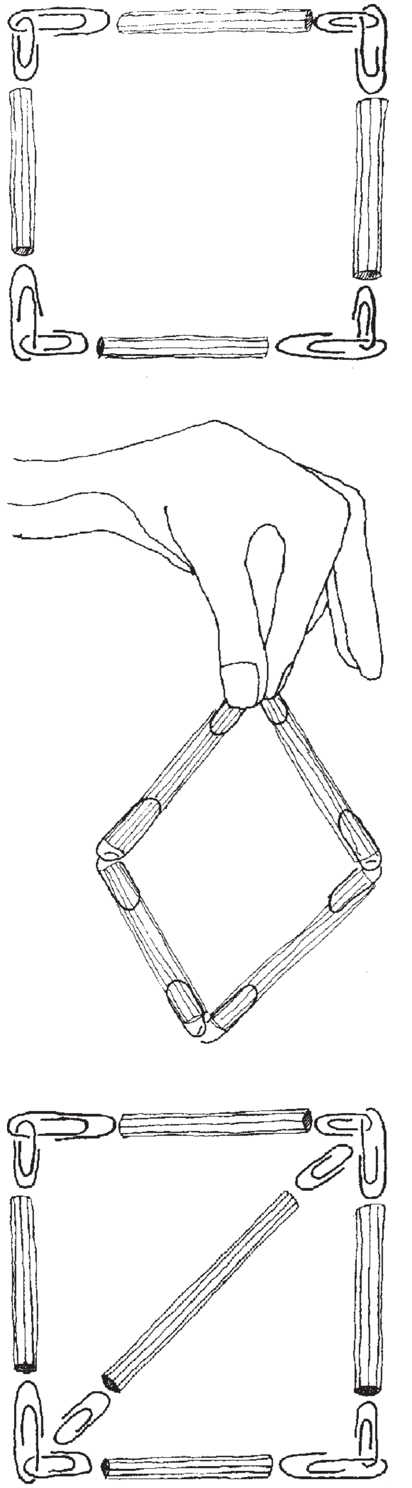
In June 1996 I directed an international conference on architecture and mathematics entitled "Nexus '96: Relationships Between Architecture and Mathematics."[1] The conference venue was my "home town," Fucecchio, a medium-sized (population 18,000) industrial town on the Arno river, about midway between Florence and Pisa (and ten minutes from Leonardo's home town, Vinci). Conferences, mathematics, architecture, professors .... all of these are almost unheard of in Fucecchio. In order that the conference not remain an "ivory tower" event in the town, I proposed a series of lessons to fourth and fifth graders in the elementary school and to sixth, seventh and eighth graders at the middle school. The idea was to show them some of the ways in which architects use mathematics. In return, I asked them to become architects, using geometrical shapes to "design" buildings. Their compositions would then be exhibited during the conference. In this article I want to share with you the brief program I put together for those lessons.

I introduced myself to the younger children by telling them how I became an architect. My family moved to Houston in 1958, and I grew up with the city. With the boom in oil prices, Houston in the 1960's grew like wildfire. I was inspired by all the new buildings in town to study architecture. I learned that architects design buildings not just using their imaginations, but also by following the tradition of architectural design that, in our western world, began as far back as ancient Egypt. The ancient Roman architect Vitruvius said that good architecture has to satisfy three conditions: firmness, commodity and delight.[2] Firmness means that a building has to stand up, supporting its own weight and protecting the people and things inside. Commodity means that the building has to serve its purpose in a convenient and appropriate way: a church or temple has to remind us that we are in the house of God; a house should feel like a home and make our daily lives comfortable and healthy. Delight means that architecture should be beautiful. Mathematics is an important tool of the architect for achieving all three of these qualities. In these lessons, we shall see some of the ways in which mathematics is used.

One of the most fundamental building blocks of architecture is shape. Because buildings have shape, it is important that architects know geometry. Looking around the classroom, you can notice any number of differently-shaped rectangles: the door, the windows, the light fixtures, the blackboard, the desktops. What other shapes are used in architecture? We find circles, circular segments and spheres used in round windows, arches and domes. The cylinder is found in columns and in towers. Triangles are found in many roofs. Cones are found in the roofs of the towers of castles. Some shapes are important to the architect because they are stronger than others. The triangle is a very important shape for the architect, because it is "rigid." We can best understand the rigidity of a triangle through an experiment.

To experiment with the special properties of the triangle, we built squares out of pieces of drinking straws and linked paper clips.[3] The squares were constructed by linking two paper clips to form each corner, then slipping the paper clips into the ends of the drinking straws (Fig. 1a). Once we had a square shape, it was easy to see that it didn't hold its shape: we could make it into a rhombus or flatten it altogether. In technical terms, when we put pressure on one of the angles, the square "deformed under stress." Would this be a good shape for holding up a house? The students all agreed it would not. Next we added a third paper clip to two opposite corners and added a fifth drinking straw (Fig. 1c). Now we could no longer deform the square: in structural terms, it had become "rigid." What had happened to the square? We had actually transformed it into two triangles, and triangles are



Figures 1a, 1b and 1c
The construction of a square with paper clips and drinking straws. You can make a square ... but it doesn't stay a square! When a fifth drinking straw is added as a diagonal, the square is transformed into two triangles and the shape becomes rigid.

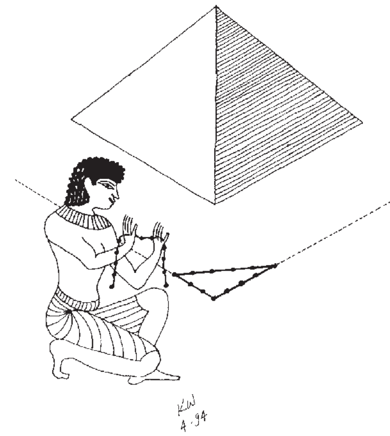


Figure 2
The triangle was used in the Pyramids of the Egyptians.

naturally rigid shapes—they do not easily deform under stress. This is the property that makes them important for architects. We find triangles in the Pyramids, for example, where four triangles lean against each other at the apex (Fig. 2). We also find them in the pediments of Greek and Roman temples, where the triangle shape was useful in supporting the roof (Fig. 3). An important use of triangles is in the construction of trusses, which are lightweight structural systems that derive their strength from triangulation. Although there are many kinds of trusses, all are based on triangles (Fig. 4). Trusses are used in bridge building and also in the construction of large buildings with open spaces where columns might get in the way of the action, such as in a basketball arena.[4]

In ancient architecture, different shapes were important because of what they symbolized. The square, for instance, represented the earth, because it has four

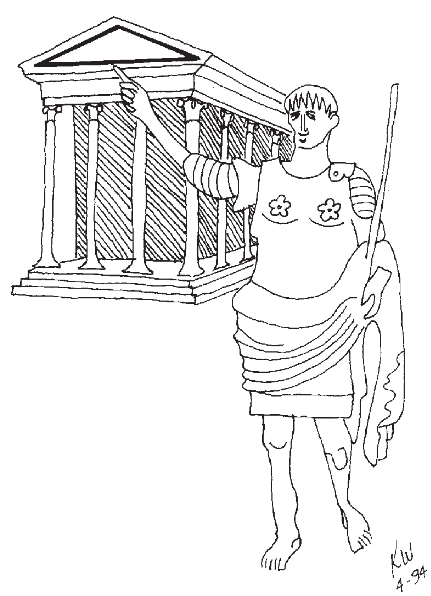


Figure 3
Greek and Roman architects used the triangle to make strong roof structures.

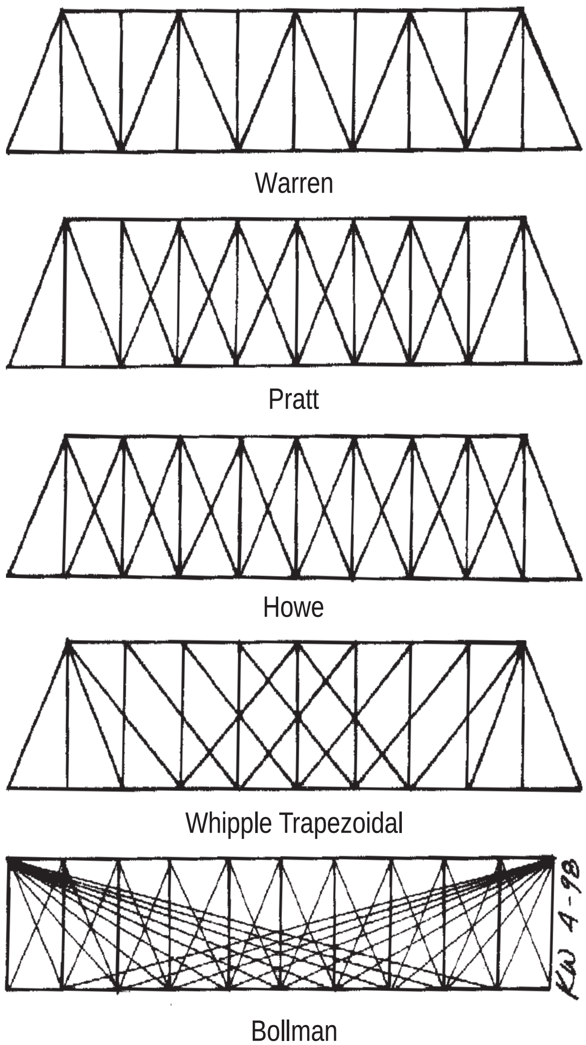


Figure 4
Some of the many kinds of trusses, all based on triangulation.

sides like the earth has the four directions, north, south, east and west. The cube also represented the earth, because it could be precisely measured. On the other hand, the circle and sphere represented heaven, because their diameters and circumferences can only be expressed using the irrational value, and not by human, rational measurements. It was believed that the irrational values belonged only to God. This is why we find the dome used over the altars in churches and temples, because the spherical shape of the dome represented heaven.

Having looked at the uses of shapes in architecture, we tried our first experiment in becoming architects. We divided the class into work groups of 3 to 4 children, and each received a package of shapes (squares, rectangles, triangles, circles, half-circles) that had been cut out from construction paper at random. They were asked to use the shapes to design their first "buildings." This was the end of the first lesson for the elemeÁtary school children.

The first lesson for the middle school children was similar to that for the elementarystudents in scope but geared for the older student. In order to introduce the idea of mathematics in architecture, I began with architecture in art, using hands-on experiments with the Moebius strip as an introduction to the art of M.C. Escher.[5] After experimenting directly with the Moebius strip, we could understand better the complexity of Escher's etchings. We had learned from the Moebius strip that while a strip of paper has two surfaces, a simple twist can make it a loop that has one surface only. Surfaces in architecture are important,

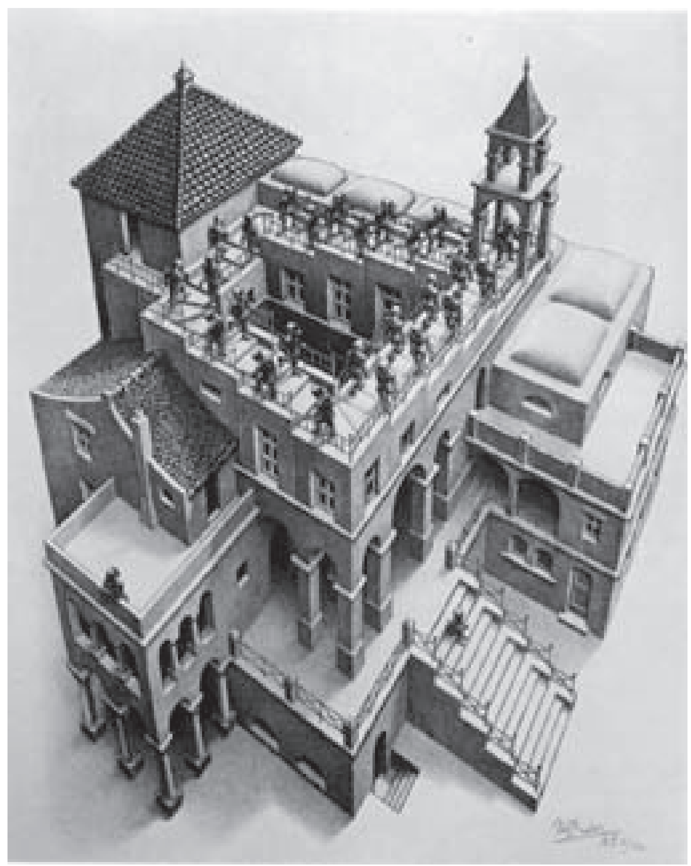
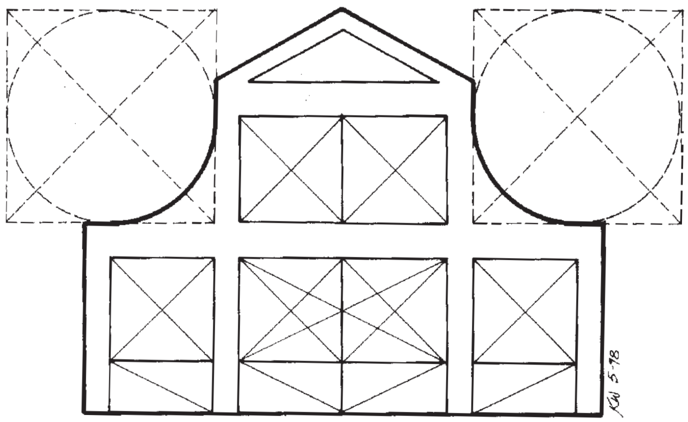


Figure 5
M.C. Escher's impossible stairs in the etching *Ascending and Descending*. All M.C.Escher's works copyright Cordon Art B.V. - Baarn - the Netherlands. Reproduced by permission.

too, as they enclose the architectural space. Could the students imagine what might happen if we distorted the surfaces in architecture? Escher did just that in some of his other etchings. At the beginning of his career Escher studied architecture in Holland, and he remained a very careful observer of architecture, often depicting it in his work. We looked at some of his etchings of "impossible" architecture, in which, for instance, he depicts stairs that lead both up and down at once (Fig. 5). He creates this illusion by depicting surfaces and shapes that appear at first to be normal. Upon closer examination, however, it can be seen that it is by distorting the shapes that he creates his illusions. This then led to the discussion of the importance of shape in architecture, much as I had discussed it with the younger students.

As a first exercise for the middle school children, we examined some of the buildings in our town to see if we could identify shapes in them (Fig. 6a and Fig. 6b).[6] Afterwards, the students were given the random collection of shapes and asked to design their first building.

The subject of the second lesson was proportion. Proportion is a comparative relation between sizes of elements. We used their first compositions as a starting point to discuss what proportion is- how big is a window in relation to the door, and how big is the door in relation to the whole wall? Proportion determines the relationships between parts of a building. Sometimes proportions are important in making sure that a building has, as Vitruvius says, firmness. If a building is many stories tall, its columns must be heavier to support its greater weight. If it is only one story high, the columns can be proportionately lighter. Proper proportions can help satisfy Vitruvius' requirement that architecture have "commodity:" the wall must be bigger than a door, but a door can be either bigger or smaller than a window.



Figures 6a and 6b
The facade of the church of S. Maria delle Vedute in Fucecchio and the analysis of its shapes.

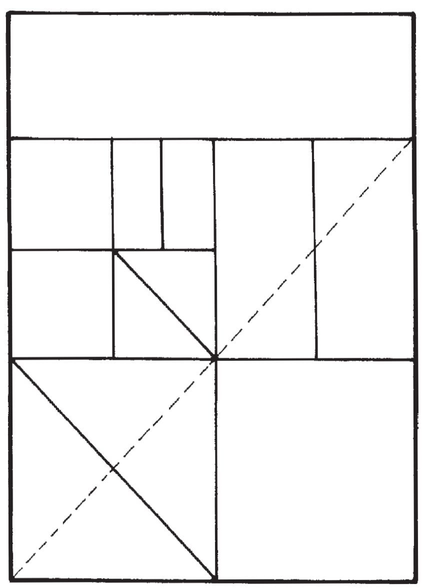


Figure 7
The proportional system of shapes created from paper folding.

Proportion is also important because some shapes are believed to be naturally more beautiful than others. How were these shapes discovered? Ancient architects used to compare the proportions of architecture to those of the human body. Just as we can tell in our own drawings of a person when we have made the head too big or the arms too long, the architect could tell when the columns were too tall or the

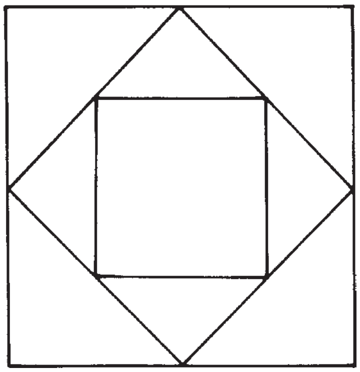


Figure 8
The *ad quadratum* proportional system is based on ever-larger squares created from the diagonals of smaller squares.

doorways too narrow. Over time, architects analyzed what they considered to be the most beautiful buildings in order to be able to record in numbers what the perfect proportions were. These special proportions were used to satisfy Vitruvius' essential requirement that architecture be beautiful.

Finally, if the architect designs his whole building using a proportional system, he can also make sure that all the parts fit together. Having the parts fit means that the building will work well structurally and functionally and also be beautiful.

One way to create a proportional system is by coordinating all the shapes we use in the composition. This provides us with a "vocabulary of shapes." Just as we use a vocabulary of words to make up a sentence, we can use a vocabulary of shapes to design a building. As an exercise, we created a proportional system through paper folding. Starting with a regular piece of construction paper, we folded one comer down to create a perfect square, then cut or carefully tore the remaining rectangle away, (With standard European paper, it isn't necessary to discard the remaining rectangle, because A4-format paper has the shape of a root-2 rectangle). We folded this "reference square" into four smaller squares; these were then subdivided into either two triangles, two rectangles, or four smaller squares. Finally, we said that circles could be cut out from any square as needed (Fig.7). These coordinated shapes became our "vocabulary of forms." By examining our shapes, we discovered that some of them could be added together to make a shape we already had (such a using two rectangles to make a square) but also that some of them could be added to make new shapes (as when we added a square to a rectangle to make a longer, narrower rectangle). With the older students, I reintroduced the idea of the irrational quantities, looking at the diagonals of the rectangles as a way of creating new forms. We discovered that by dividing one of the squares along its diagonals into 4 triangles and rearranging them, we could create larger squares that were not related to the rational lengths of the original reference square, a system often used by Roman architects known as *ad quadratum* (Fig. 8). We could also create rectangles that had one rational side and one irrational side.

At the end of the second lesson, the students were invited to make new compositions with the new "vocabulary of shapes." These compositions were more refined than the first, due to the more sophisticated set of forms as well as to the confidence the children had gained from their first compositions.

The fifty compositions that resulted from the time spent with all the children were exhibited in the restored medieval palace where the conference was held (Figs. 9 and 10). Some days before the conference opened we had an "art opening" for the students and their parents and teachers. The experiment was deemed a success from all points of view. The students were very interested in architecture and the fact

Figure 9
An example of the students' work.

that mathematics isn't just something that has to be studied in the classroom, but is of great value as a creative tool as well. We also found the lessons provided a new way for the children to understand the built environment, using mathematical tools that are part of their normal curriculum. This is especially important because there is little or no architectural education at the levels of elementary and middle schools. The teachers were very pleased with a demonstration of geometric principles applied to a "real" activity. I was more than gratified by the enthusiasm of the students, not only for this activity, but for all facets of my work as an architect. The drawings remained on display during the conference, and were very much appreciated by all conference participants. Who could help but admire such "mathematical" architecture?

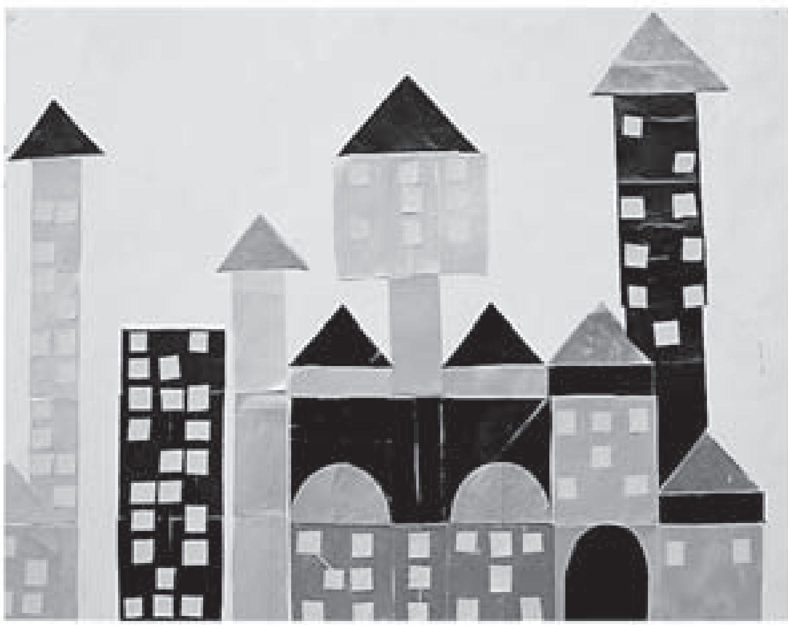


Figure 10
Another example of the students' work.

NOTES

1. For a review of the collection of essays that resulted from this conference, cf. Joseph Malkevitch "Book Review: Nexus: Architecture and Mathematics," Humanistic Mathematics Network Journal, 16 (November 1997), 52-53.

2. Cf. Vitruvius, *The Ten Books on Architecture*, Morris Hicky Morgan, trans. (New York: Dover Publications, Inc., 1960) book I, chap. III, sect. 2, 17. In this translation, "firmness, commodity and delight" are rendered "durability, convenience and beauty."

3. This experiment was described in Kim Williams, "How Buildings Take Shape," *Highlights for Children*. For the technique of building with drinking straws and paper clips, I am indebted to Howard Jacobs, *Mathematics: A Human Endeavor*, 2nd ed. (New York: W.H. Freeman, 1982) 267-269.

4. There are other ways of demonstrating structural rigidity through form through experiments with simple pieces of paper. Cf. Mario Salvadori, *The Art of Construction*, 3rd ed. (Chicago: Chicago Review Press, 1990), 109-118.

5. Some hands-on experiments with the Moebius strips are described in Jacobs, *Mathematics: A Human Endeavor*, 605-606.

6. This experiment can be done without leaving the classroom, by showing slides or xeroxed photographs of buildings and asking the students to analyze the facades in terms of shape.