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Struggles and Growth in Mathematics Education: Reflections by Three Generations of Mathematicians On the Creation of the Computer Game *E-Brock Bugs*

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Abstract

In the Fall of 2013 our team of three different generations of mathematicians launched the free, online *E-Brock Bugs* mathematics computer game [5] which we developed from an original probabilistic board game, *Brock Bugs*, and its digital learning object version. We constructed *E-Brock Bugs* using Devlin's [9] mathematics computer game design principles for games that prompt players' development of mathematical thinking. As we created *E-Brock Bugs* we found it necessary to go through an evolving cyclic process of design, implementation, and analysis. In this paper we reflect upon the main struggles we faced in this process and the unexpected personal growth that ensued in terms of our views and beliefs as mathematics educators.

1. Introduction

For most mathematicians the issue of the role of games in learning mathematics is a non-issue. They have succeeded in mathematics without any classroom game experiences, so why should others be unable to follow in their footsteps and become equally successful in mathematics? Schrader, Zheng, and Young [28] warn about the consequences of relying solely on one's experience: "If teachers' perceptions about educational games continue to be informed by their personal experiences the state of games in education is at risk of remaining unchanged" (page 2). Many mathematicians view games purely as a recreational activity. For some, games may be appropriate for activities in a Math Club or as a bonus for the more capable students who finish the class-assigned mathematical tasks ahead of everyone else. In large part, mathematicians see games and video games as having no role in mathematics education as presently defined in secondary and post-secondary mathematics. There is no question that some of their students follow very successfully in their footsteps. Nevertheless the number of these students is very small when compared to the total number of students in compulsory mathematics classes at the secondary and post-secondary level.

Those who have acted to counter the widespread negative attitude towards mathematics in society as a whole, and who also have worked to increase interest and performance by students in schools realise that there is no magic bullet to remedy these situations. Many different strategies have been proposed and studied. For example one finds mathematics classrooms wherein (i) activities have been set up where students can be more creative (see for instance [24]), (ii) applications and modelling have been introduced (see for instance [12]), (iii) storytelling has been initiated (see for instance [30]), (iv) activities centered on visualization have been used (see for instance [1]), (v) appropriate technologies have been introduced (see for instance [10]), and (vi) the use of computer simulation has been employed (see for instance [18]). Those who have dedicated time to popularization of the subject outside the formal classroom have created hands-on activities in science centres and museums, written books and columns in newspapers, developed *Math Trails* and activities for *Math in the Mall*, produced TV programs, and the list goes on. Through the decades these individuals have observed changes in society and have sought to take advantage of those changes that provide opportunities to engage individuals in mathematics.

At the start of the 21st century one does not have to be particularly observant to note how pervasively youth use technology and also how extensively they play video games. For example a study by the Pew Research Center concludes that 97% of American teens aged 12-17 play video games [16]. Although the integration of appropriate technologies in mathematics education continues to be an area of extensive research at the secondary and post-secondary levels, there is relatively little research on the use of video games [13]. One of the reasons is that there are only a few 'serious' mathematics video games available at this time [9], and consequently there is little information about the impact that 'serious' mathematics video games could have on student learning. It is our view that, unless much more effort is devoted to the area of mathematics video games, mathematics educators will miss out on a unique opportunity to engage a larger proportion of youth in mathematics.

Three major factors have contributed to our creation of a mathematics computer game where, we argue, a player may be prompted to unconsciously start to think like a mathematician. These are:

1. The three of us are from three different generations of mathematicians. This was important as we were pragmatically extending the boundaries of mathematics education into video games, and such an extension would most likely come from practising mathematicians, from individuals with extensive experience in the use of technology in mathematics education, and from the exuberance of youth;

2. we had all been involved in the innovative core mathematics program at Brock University, *Mathematics Integrated with Computers and Applications* (MICA), which requires students to design, program, and use interactive computer environments to explore mathematical situations of various kinds [22, 27]; and

3. in 2011, Keith Devlin authored the book *Mathematics Education for a New Era: Video Games as a Medium for Learning* [9], which focuses on video games for learning to think as a mathematician.

In [7] we described the results of this endeavour by providing details of the structure, properties, and characteristics of our free, amateur, online computer game *E-Brock Bugs* [5] as it was launched in October 2013. To develop this instructional mathematics game, we used the design principles proposed by Devlin [9]; these provided the foundations for an epistemic mathemat-

ics computer game, that is, a game in which the player becomes a (better) mathematician [23]. In other words a discipline-specific epistemic computer game is one "where players think and act like real world professionals" [2, page 36].

The current version of the computer game is the result of a dynamic journey that is worth exploring, because, as in most creative endeavours, the evolution of *E-Brock Bugs* was not linear. Rather, the game development process proceeded in a cyclic fashion, with a continuous shift between stages of design, implementation, and analysis. The entire process was enriched by the composition of our design team, which offered a diverse set of perspectives and expertise. In what follows, we present our reflections on the main struggles we faced and the unexpected personal growth that ensued in terms of our views and beliefs as mathematics educators.

2. The genealogy of *E-Brock Bugs*

The journey of creating *E-Brock Bugs* unknowingly began over thirty years ago when Muller [22] conceptualized a game on a hand-carved wooden board with two dice and two different sets of tokens. The game aimed to teach young children about probability. Some fifteen years later, the game was further developed for distribution purposes. A board was printed on cardstock (see Figure 1) together with a page for players describing the game and its rules. A separate teacher document was also printed that unveiled the mathematics behind the game and proposed a classroom implementation. Funding was received for printing the game (board and rules), the teacher document, and purchasing dice and tokens in two colours. These were packaged together and distributed free of charge to mathematics teachers in Ontario.
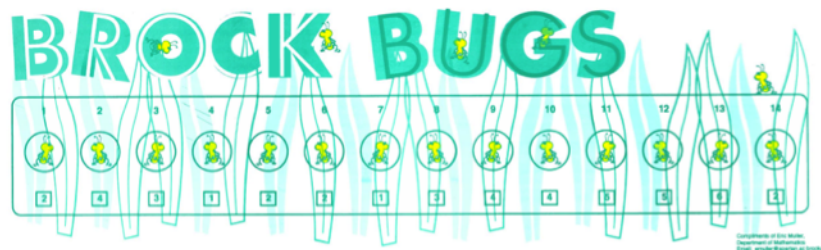

Figure 1: *Brock Bugs* board game - the cardstock version [7].

The game consists of three levels: the first two entail a two-player probabilistic game, while the final level involves a classroom experiment. To work through the first two levels, a pair of players each has a set of different coloured tokens and play with the same two dice. In the first level, only the numbers above the circles on the board are considered; these numbers represent the sum of the faces of two rolled dice. To start the game, the two players take turns in selecting what they believe is the most favorable vacant position and place a token on the corresponding circle. This is repeated until all positions are filled. The board is then set and remains unchanged for 25 rolls of the dice. Each time the two dice are rolled, the players must add the numbers on the faces and record one point for the player who has a token on the position corresponding to the sum. The player with the most points at the end of 25 rolls wins that game.

In Level 2, players continue to use the numbers above the circles for the sum of the values on the two dice. However, now they must also consider the numbers in the squares below the circles, which determine the number of points a player will win if his/her sum is rolled (e.g., a player with a chip on the circle below 12 would get 5 points when a sum of 12 is rolled; see Figure 1). The game proceeds exactly as before, and the player with the most points after 25 rolls of the dice wins the game. While Level 1 is designed to introduce the basic concept of a probability distribution, the second level aims to bring students to an understanding of expected value. Level 3 extends into the more complex use of the binomial distribution and takes the form of a guided activity that has classrooms exploring the probability of winning the Level 1 game with a particular placement of tokens and numerous but determined rolls of the pair of dice.

In 2001 the Department of Mathematics at Brock University took a new direction and implemented its MICA program which was spearheaded by Ralph [27]. This inspired Buteau and Muller to take on the project of "modernizing" *Brock Bugs* by creating an online learning object version. The new format was directly in line with the MICA program, which engages its undergraduate mathematics students in learning to design, program and use interactive computer environments either, as 'exploratory objects', to investigate mathematical concepts, theorems, conjectures or real-world situations or, as 'learning objects', to be used by teachers in their mathematics classes [22].

The goal of the *Brock Bugs* learning object was to build on the core *Brock Bugs* games (Levels 1 and 2) and exploration (Level 3), while exploiting the potentials that computer technology could provide in terms of computing power, interactivity and visualization. As in the original *Brock Bugs* board game, the student would be encouraged to develop the mathematical theory based on his/her game experience. However, manual (individual and classroom) data collection would be replaced by simulations (see Figure 2). Whenever possible the game would become more reusable, including repeated use by the same player, by incorporating randomization (e.g., the points in the Level 2 game), and the experiment, rolling two dice, would be expanded to include other objects and to introduce other probability concepts. In addition, the teacher document originally composed by Muller would be integrated into the learning object as a collection of interactive, highly visual, individualized mathematics lessons that would come after the student had played each of the games.

Figure 2: *Brock Bugs* learning object: the level 1 game in action.

In 2009 the *Brock Bugs* learning object project was partially completed with the implementation of the Level 1 game [21] together with the design of seven other games and their related mathematical theories. Unfortunately, by the end of that year, the design team encountered a few issues that made it difficult to continue the project, the main stumbling block being a lack of Flash programming knowledge amongst MICA majors. Flash was selected for the design of the learning object due to its animation capabilities and Internet compatibility, but the undergraduate students in MICA program are required

to learn the more mathematics-related Visual Basic.NET, Maple, and C++
programming languages. However in 2012, one MICA major, Broley, entered
the *Brock Bugs* scene with a strong programming background, digital design
skills, artistic talent, and the will to challenge herself to create a mathematics
computer game that could make a difference.

3. *E-Brock Bugs* computer game

Ultimately, elements of the board game and learning object were modified,
adapted, and embedded into a computer game environment that resulted in
the computer game *E-Brock Bugs* [5]. Our aim with *E-Brock Bugs* was to
drive players not only to be engaged in the planned mathematical activities,
but also, in accordance with Devlin's vision [9], to develop their mathematical
thinking, specifically related to basic probability. Here is the *E-Brock Bugs*
computer game synopsis:

> *Since the beginning of time, Bug City had always been a peaceful
> place to live, where even the simplest of bugs could feel right at
> home. Then, one day, the city was swarmed by an evil band of
> Bullies and their mysterious leader, the all-powerful Dr. P. Dark-*
> *ness quickly spread across all six districts that make up the city,*
> *transforming it into the wasteland it is today. But the situation*
> *is not entirely hopeless, for the player of* E-Brock Bugs *is the*
> *hero that Bug City has been waiting for! To restore the city to*
> *its original beauty, the player must journey through each district*
> *and defeat all of the Bullies at their probabilistic games. With the*
> *help of some friends made along the way, the player may finally*
> *convince the citizens of Bug City that they have nothing to fear,*
> *and that knowledge should never be used as a weapon.* [7, page 7]

In short, the task of the player in *E-Brock Bugs* is to save Bug City, which
is done by first defeating the six different Bullies at their probabilistic games
(e.g. see Figure 3), and then completing a final simulation challenge posed
by Dr. P. The player may get some help if s/he wishes; indeed, the player
is invited to visit Smarty, a brilliant bug who has managed to develop the
theory behind each Bully's game, and thereby become a better player. This
component of the game exposes the player to an explicit transition from the

game itself to the main mathematics concept(s) behind it. However, the availability of such help only comes after the player has engaged the Bully by playing her/his game in a public place in Bug City, as shown in Figure 3. For a description of all the probabilistic games and related probability concepts, see the *E-Brock Bugs* teacher document [6]; and for a narration of a player's (mathematical) adventure through *E-Brock Bugs*, see [8].

Figure 3: The *E-Brock Bugs* computer game: the District 1 game in action [7, page 6].

4. Developing *E-Brock Bugs*: Creators' Struggles and Growth

As we mentioned in Section 1 the current version of *E-Brock Bugs* is the result of a dynamic non-linear journey undertaken by three individuals representing three different generations of mathematicians with a common interest, but diverse experiences with the use of computer technology in mathematics learning and teaching. Representing the eldest of the three generations, Muller has had the unique experience of participating as an innovator in the evolution of educational technologies, from the very first possible software and hardware that came out in the 1970s to what we are discussing here, educational computer games.[1] Buteau has integrated diverse technology, e.g. Maple and programming, in her university mathematics teaching since 2004 and made this area of mathematics education one of her main

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[1]See Appendix A for an individual reflection by Muller.

research foci. However, her own mathematics education involved, up to her PhD studies, only little technology use. In contrast, Broley graduated in 2013 from the MICA program, with a specialization in mathematics education, during which she extensively used diverse digital technology in her mathematics learning/doing, including in her Honours Thesis project that resulted in the *E-Brock Bugs* game [4].[2] In what follows, we present our reflections on the main struggles we faced when developing *E-Brock Bugs*, focusing on those that ensued from our views and beliefs as mathematics educators.

The greatest challenge for all of us was definitely the transition from the learning object paradigm to that of a computer game. Before *E-Brock Bugs* was even a possibility, we had all explored and developed learning objects to various extents, and we agreed that these structured tools offered great potential for the learner of mathematics. This view combined with our experiences with these objects made it difficult to abandon certain characteristics of the *Brock Bugs* learning object (and traditional classroom practices), and to fully adopt many of Devlin's design principles for an epistemic mathematics computer game. In short, it took many discussions around (mathematics) learning before we came to fully understand what it would mean for *Brock Bugs* to become a computer game while 'naturally' integrating, in the game world, the educational potentials from the learning object and board game.

4.1. Moving from the learning object to the computer game

As we described in detail in [7], *E-Brock Bugs* was designed with many principles recommended by Devlin [9]. Some of these fitted into our views and philosophical approaches to the learning of mathematics and, for these our challenge was more how, rather than why, to implement them in the design of the game. For example, we built in a constructivist approach [25] and ensured that players of *E-Brock Bugs* would "never [be] put in a position of having to 'learn something' prior to playing the game in order to play the game" [9, page 128]. This principle was already built into the design of both the board game [20] and the learning object [21], making it a natural one to implement in the computer game. Others such as "the back story is

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[2]Note that, in particular, all graphics and music in *E-Brock Bugs* are Broleys original work.

crucial to the success of a game" [9, page 134], and the selection of an avatar, though they were new aspects brought by the computer game environment, did not pose any particular concern to us (or at least at first — see Section 4.5), except maybe to understand the importance to address it right from the beginning as opposed to marginalizing it. In other words, it meant we needed to prioritize student motivation to learn equally with student learning. We also needed to carefully consider how to find a balance for the cost and reward for a player's actions as explained by Devlin [9, page 30]: "there should be sufficient 'cost' at getting something wrong to motivate correction, but not so great that it leads to the student losing heart and giving up". This balance did not stretch our personal beliefs as educators.

However, in our discussions of some of the other principles, it became clear that understanding the principle and actually choosing to implement it were two different things. In some cases, the consequences of choosing to integrate a given principle made the design of the game extremely challenging. The following sections exemplify some the struggles encountered.

4.2. Struggles prompted by the "Pre-planned, multi-route, self-paced gaming/learning experiences" Principle

As a first example of a principle that challenged our views of learning mathematics, we consider Devlin's recommendation of "Pre-planned, multi-route, self-paced gaming/learning experiences". According to this principle, though there should be an indication of where players should go next, there should not be too much control over whether or not they must actually go there; players should be able to choose between multiple routes of arriving at the end goal. Some of our own experiences of the use of technology in mathematics education came from the development and use of learning objects where the structure of learning is most often imposed on the user with much more certainty. This is also the case in traditional classroom settings, where the course, the units, and even the daily lessons often follow a rigid trail of events that are pre-planned and guided by the teacher.

The "multi-route" principle was not at all intuitive for us. However we could recall instances where, having chosen a particular instruction route, we then observed that it was successful for only some members of the class and that a different route would have been preferable for other students. We also realised that in many other computer games the players are able, within

certain parameters, to choose their own path, so we became more comfortable to design *E-Brock Bugs* with some flexibility of choice. For example, even though the Finale comes only after the defeat of all six Bullies, in the Bug City main game world players may select in which order to defeat the Bullies. Even so we constantly discussed the order of the districts in Bug City, i.e., the order of the probabilistic games and corresponding theoretical activities, in hopes of achieving the best possible progression.

The main factor we considered was consistency in a number of areas, namely, in the game dynamics; in the theoretical activities; in the order we would choose to introduce the main mathematical concepts; and in the effort required from players. The current order recommended to the players differs greatly from the original learning object design and from several subsequent versions of the computer game. We have chosen it hoping that it will best prompt the player's development of mathematical thinking as it becomes gradually more conceptually difficult. We are aware that, even after all of this planning, we should not expect all players to actually choose our preferred route. In fact, we have already observed some students who chose to ignore the proposed order of districts in favour of their own. As mathematics educators we are used to plan scaffolding of mathematics concepts, and it was our desire to build this into Smarty's theoretical explanations in a manner that it would stay close to the probabilistic game experience found in *E-Brock Bugs* and assume nothing more. For example, this guided our decision (after a few cycles of design and discussion!) to introduce non-standard theoretical explanations of the probability for not equally likely events and for the binomial distribution[3] for a fixed $n = 7$, see Figure 4. In particular, we felt that if Smarty exploited the key idea of the ratio "frequency over total number of outcomes" as often as possible, it would support the player's development of mathematical thinking in the context of basic probability.

Furthermore, implementing the multi-route principle raised the issue of what mathematics information the player would accumulate should she/he choose a different order than the one we proposed. For example, the frequency bar graph of the game play is shown after each game: on one hand, we wanted to prompt the player to start connecting the frequency distribution with the game result (i.e., supporting situated learning [15]). On the

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[3]The binomial distribution is explained in detail in *E-Brock Bugs* in the Finale.

Figure 4: Three non-consecutive screen shots of Smarty's explanations of District 4 game involving spinning two asymmetric spinners numbered 1 to 6; the two left screens explain the theoretical probability by changing the spinners to symmetric ones, and thus by using, once more, the ratio of frequency over total number of outcomes; to the right, the optional explanation using the multiplicative rule.

other hand, we could now no longer assume that the player had heard about *frequency* and *frequency distribution*. We ended up addressing this by keeping track of the player's first visit to Smarty (in any one of the six districts); see Figures 5 and 6 where we display four non-consecutive screen shots in the game.
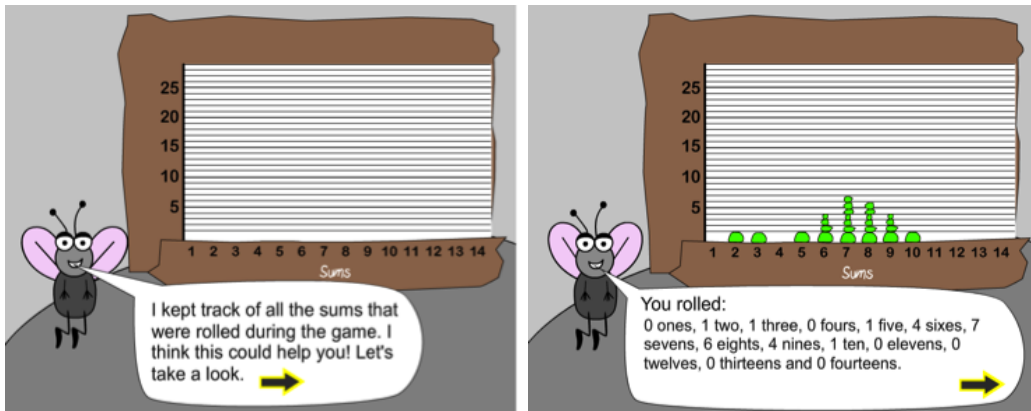

Figure 5: The bar graph of the player's game before having visited Smarty, and assuming the player has not heard of *frequency* and *frequency distribution*.

Although this lack of control was difficult for us to accept, it raised the possibility of trusting students to personalize their learning according to their own strengths, methods and interests.
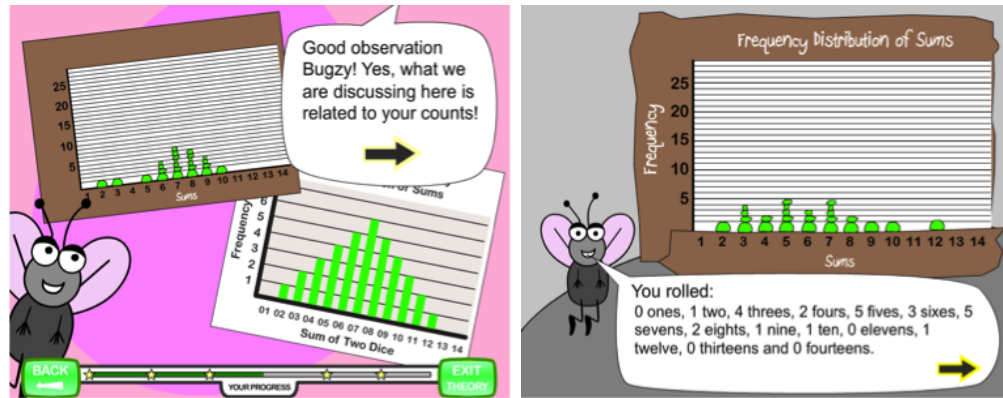
Figure 6: Left, Smarty replies to Bugzy and starts to explain the connection of the player's game bar graph and the theoretical distribution bar graph she just constructed with the player; right, the bar graph of the player's game with *frequency* and *frequency distribution* labels, that will appear from now on.

4.3. Struggles prompted by the "Mathematics knowledge only on-demand and just-in-time" Principle

Loss of control came up again and again in our discussion as it was a common theme in the implementation of other principles as well. The notion of "mathematics knowledge only on-demand and just-in-time" [9, page 99], for example, implied that any theoretical activities be kept entirely optional to the player. This meant that the player must be given the power to choose not only when they need access to the theory, but also which parts they would like to see, an idea that pushes the learning object paradigm beyond its natural limits and is almost entirely absent from the classroom context. Similar to our extensive discussions on the order of the Bug City districts, our explorations of the separation of the mathematical knowledge from the games developed at first erratically and then more progressively. For instance, there was the difficult decision of allowing players to decide whether or not they would like to visit Smarty's Shack, with the implication that some players would never formally meet Smarty (and thus never explicitly see the mathematical explanations behind the probabilistic games). To ease this decision (and follow Devlin's principles), Smarty briefly appears to tease the player to think about his/her game's bar graph, which could then bring the player to wanting to visit Smarty:

> *The higher the bar on the graph, the more frequently the sum was rolled. There is a reason why certain bars tend to be higher than others!*

In another situation, after some debate, we decided in favour of progress bars that would give players the option of jumping to specific parts of the theoretical activities. A posterior we realize that this parallels the non-linear approach that mathematicians use when working on a problem. If our aim is to develop the players' mathematical identity, we need to provide them with opportunities to act as mathematicians. Also, the above "just-in-time" principle implied that the mathematics explained by Smarty needed to be narrowly focused on strategies to defeat the Bullies. As a result, we eventually realized that Smarty had to contain herself (and so had we!) to present only mathematical concepts and skills needed in that part of the game. We decided on a compromise where, in a few instances, Smarty would tease the player indicating she had neat extra interesting things to share:

> *I still have some really neat mathematics left to share with you! But, I know that you may be itching to get back to playing Fitz [District 2 Bully]. Can I keep going, or would you like to move on to practice your game strategy?*

In other words, we accepted having neither the school curriculum nor the teacher's interest driving the structure, but rather only the player's need to defeat the Bullies and save Bug City. This took us a while to accept and then to implement accordingly.

4.4. Struggles prompted by the "Conceptual understanding not guaranteed" Principle

We discussed at length and repeatedly Devlin's counsel that "while conceptual understanding is a goal that educators should definitely strive for, we need to accept that it cannot be guaranteed, and accordingly we should allow for the learner to make progress without fully understanding the concepts" [9, page 115]. For us, this position brought up a myriad of concerns about 1) giving up control to the players, 2) offering instantaneous feedback, 3) providing, in case of failure, sufficient incentives to continue, and 4) encouraging access to Smarty, the virtual tutor in *E-Brock Bugs*.

By this time we had principally accepted giving majority control to the player and we had also addressed the second concern in other contexts, by ensuring that the feedback was instantaneous at the end of each game. We dealt with the concerns 3) and 4) by implementing "playing mathematically leads to faster game progression" as a way to draw the players to the mathematics. This determinant required us to develop a game structure in which players who use mathematics proceed much more quickly. It also acknowledged that we allowed for success to occur without understanding explicitly the mathematics generated within the games. This differed from what we had originally envisaged in a learning object, where users are often forced (or strongly encouraged) to display their understanding in some way (e.g., by solving some problem) before they are able to proceed. In mathematics education in general, students are almost always required to show an explicit use of certain mathematical concepts, with the goal of verifying whether they have achieved the desired level of understanding or if they need some additional help. It was therefore difficult for us to allow players to proceed in the game simply by chance, but that was a key characteristic needed to convert *Brock Bugs* into a computer game, and also an important aspect of understanding probability — one can win a game even if the odds are stacked against us!

4.5. Struggles in the design of the Finale of the game adventure: the "crucial back story" Principle

In addition to handling these contentious principles, we were particularly challenged during the design of the Finale, which involved bringing classroom experimentation into a computer game environment. This situation occurred because Level 3 of the *Brock Bugs* board game involves a classroom of students exploring the probability of winning the regular two dice game, with a particular placement of tokens, and analyzing the effects of changing the number of rolls required to complete the game (i.e. a generalization of the District 1 game in *E-Brock Bugs*). The implementation of the Bullies' games within *E-Brock Bugs* had been more or less straightforward since they had already been designed in the learning object and they mainly involved modifying the probability experiment. In contrast, the third level of *Brock Bugs* board game [20] had not been completed within the context of the learning object, which meant that we had a much smaller foundation on which to build our design. The greatest challenge was to transform the

experimentation into a sort of probabilistic game, and to ensure consistency and enrichment of the storyline. We planned this activity to be the *E-Brock Bugs* "grand finale" both for the probabilistic game (which ended up to be a simulation challenge) and for the storyline.

This is an example of the deliberations we had as we were developing the simulation challenge. At that point in the story, the player would have saved all the districts. Now what if all the defeated bullies were captured by Dr. P as a punishment for their inability to win their individual game and for their rebellion against their dictator Dr. P? Should some of the ex-bullies escape capture to help the player in this most complex final game? As the simulation did not involve expected values, from Dr. P's viewpoint, ex-Bully Wicked the Wasp from District 6 would not be of much help to the player. As designers, we realized that Mac the Mosquito ex-Bully from District 1 needed to be free because the optional theoretical explanations by Smarty required the use of his game as an example to build the complex theory; see Figure 7. This is just one example of how constructing a storyline, as a unique aspect of a computer game environment, called for an extra amount of effort that is usually not required in learning objects or in classroom activities. In a sense, this brought up again the importance of the shift to prioritize equally both student motivation to learn and student learning. Concretely, as indicated in Section 4.1, this aligns with Devlin's view that game designers must think carefully about their storyline because it "is crucial to the success of a game" [9, page 134], and consequently to the player's learning experience.

Figure 7: Consecutive screenshots in the "grand finale": the storyline had to take into account the game mechanism limitations and have Wicked (character to the left) and Mac (character to the right), ex-Bullies, excluded from Dr. P's kidnapping.

4.6. An issue of visual representation and conceptual understanding

We end by briefly recalling how a computer game design principle brought us to identify an issue of visual representation that failed to support conceptual understanding. This means that in this case, the principle was not the source of our struggle, but rather served as a guardian for the instructional quality of the computer game.

The District 5 game concerns the binomial distribution for a fixed n: $n = 7$. For the experiment instrument, we naturally thought of coins (who wouldn't?). The initial idea was to flip seven similarly "tricked coins" where the probability of getting a "Bug head" would be indicated as a ratio, e.g. $\frac{1}{4}$, beside the coin. The "bug coins" fitted well in the Bug City context. This visual was however not clear without an explanation of what that $\frac{1}{4}$ was about. Furthermore, such an explanation for players without experience of formal probability could be particularly challenging and could introduce an unnecessary layer of complexity. We realized that this visual representation of the binomial experiment did not adequately support its conceptual understanding. In other words, the principle of "never put [the player] in a position of having to 'learn something' prior to playing the game in order to play the game" [9, page 128] — here what $\frac{1}{4}$ means — prompted us to identify an issue of poor visual representation. This eventually led to what we call bi-spinners, a visual representation that required no explanation; see Figure 8.

Figure 8: On the left, the initial instrument we developed for the binomial experiment: it required explanation to the player of what the fraction (for probability p) meant, and thus was not supporting well students' learning of the concept. In the centre, the revised instrument implemented: a bi-spinner flower first pops visually showing the probability p (centre), then spread (right) to fill the instrument for a total of $n = 7$ copies of the same bi-spinner, providing a good visual representation of the experiment and requiring no verbal explanation.

5. Concluding remarks

In the end, the creation of *E-Brock Bugs* was a humbling and often shocking experience that encouraged all three of us to challenge, often without resolution, some of the norms that we held of the mathematics classroom. The transition of *Brock Bugs* from a board game to an online learning object had been developed with relative ease. The same cannot be said with the transformation from a learning object to a computer game. Our journey started, naively, by reading Devlin's book on mathematics computer games [9]. From it we acquired some ideas and computer game design principles which led us to epistemic mathematics computer games, and which turned out to be new and surprising to us. In some cases, the proposed ideas and principles seemed at first straightforward, but in the process of implementing them in a computer game environment, they became a source of challenge and enrichment to our views as educators.

Our reflections have taken us beyond *E-Brock Bugs* as we now wish to explore the implementation of certain principles beyond the game environment. For instance, what would the mathematics classroom look like if we were to strictly have individual students driving their own learning? What if we introduced concepts to students only as they were needed to accomplish goals that are seen as worthwhile to them? And also, would the creation of a consistent theme (or "storyline") in the classroom help to encourage students to want to learn more mathematics? Gee [11] claims that, in formal education, the main impediment to implementing good principles of learning that are built into successful games, is cost. He explains: "This, however, is not only (or even so much) monetary cost. It is, importantly, the cost of changing minds about how and where learning is done and of changing one of our most profoundly change-resistant institutions: the school" [11, page 15]. Our own experience has shown us how challenging, though also enriching, it is to embrace these good principles, both in creating a computer game and reflecting on their classroom implementation.

As for *E-Brock Bugs*, our focus was to develop a good mathematical computer game and to provide an environment where students engage as learners of mathematics. We hope that this differentiates *E-Brock Bugs* from other mathematics games where students are engaged as users of mathematics, as contrasted by Hoyles and Noss [14]:

> *. . . it is important to distinguish the needs of mathematical learn-*
> *ers from the needs of mathematical users — learners need to*
> *search for and appreciate generality and structure, while users*
> *want simply to get a particular job done or a problem solved"* [14,
> page 325].

Clearly some players will only focus on winning the game; hopefully others will build and learn from the common generality of some concepts, situations, and explanations. In reality our vision went beyond seeing the player as a learner because our aim was to build an epistemic mathematics computer game in which "[t]he player becomes a mathematician and problem solver within the context of the game" [23, page 45]. Or, as by described by Devlin: "Thinking mathematically should simply be part of what [the] character does in that world" [9, page 127].

A. A mathematician's journey from manipulatives to e-games

Below are some personal reflections of the third author about his journey.

In many years of university teaching I have integrated external representations/applications of mathematics into my lectures, in computer laboratory student activities, and in assignments, but also, outside my classroom, as a means to popularize mathematics. This desire to link mathematical concepts to tactile and digital objects most likely comes from my own university education when, in the late 1950s and early 1960s, I was a student of both theoretical and experimental sciences. In the end I decided to focus on the mathematical sciences and, after my PhD, joined a mathematics department. However my background in experimental sciences, my continued interest in how mechanical and other systems work, and my explorations into the role of technology in undergraduate mathematics education, all helped to persuade me that for some students working with external representations/applications of mathematics provide memory links to, and instances of mathematical concepts.

In the following, I briefly reflect on my journey as a university mathematics educator from the use in the classroom of manipulatives to e-games. For the purpose of this appendix let me briefly describe how I classify *manipulatives*, *learning objects*, and *games*, in mathematics education.

- For me the term *manipulative* is self-explanatory, that is, it is applicable to whatever one can manipulate or operate, and this includes both tactile and digital (sometimes called "virtual") items. For the latter, teachers can find many school activities in The National Library of Virtual Manipulatives [29], while at the university level the *MIT Mathlets* [19] provide rich and varied digital resources. Manipulatives in the digital realm are, for me, programs that have been developed by experts in which learners can manipulate the parameters, operate graphic representations, etc. What can be manipulated has been pre-selected by the experts while the student, having no access to the code, can "use" the manipulative but is unable to modify it.

- I see *learning objects* as more comprehensive digital environments that mainly focus on one area or concept in mathematics. Most developers make their environments visually attractive with the activities imbedded in interesting situations. Customarily they include an expert's sequence of instructions and experimentations, followed by practice for the learner and often incorporate some form of assessment. The expert developer controls the sequence within the learning object; in other words, s/he determines the order in which activities are introduced, based on his/her pedagogy and understanding of the learner's mathematical knowledge and skills. In 2002, funding enabled William Ralph and myself to assemble a team of school teachers, mathematics students, and computer science programmers to produce mathematics learning objects. The results of this work are located on our department's site [3].

- My view of tactile *games* and *e-games* is that they involve play with manipulatives, play that is delineated by their limitations (i.e., rules) and structures. Games also have a defined objective, which can be as simple as a single goal, or as complex as involving many optimal systems. In a mathematical game, reaching the specified goals is facilitated by the exploitation of mathematical ideas. In addition, games often involve competition, sometimes against oneself, other times against opponents that arise within the game environment (as they do in *E-Brock Bugs*), or through the introduction of other human players. However, competition is not a necessity for everyone to enjoy a game. Pleasure can be achieved in other ways, for example, by achieving a set of goals.

Examples of my use of manipulatives both tactile and virtual — teaching mathematics

In the undergraduate mathematics courses I taught, I used many tactile items. For instance I used manipulatives that arise naturally within the activities contained in Mason, Burton and Stacey's book *Thinking Mathematically* [17]. It was not difficult for me to integrate the use of digital manipulatives into my teaching because, by the time these were available, I had already introduced my students to technologies as computational, algebraic, and graphical tools. When one introduces the use of manipulatives in one's teaching, I believe that the challenge is to motivate and ensure that the learner makes the transition from the manipulative to the mathematics. In other words, the student needs to build on the wealth of examples provided by the technology to connect to the abstract, theory, conjecture, etc. that is the mathematics. Even though students operate so freely and successfully in the digital world, I retain the view that they need to also work with tactile manipulatives to engage other senses such as touch and experience the transition to mathematics from tactile situations.

Examples of my use of mathematics learning objects — teaching mathematics

Within the core MICA program at Brock, students develop their own exploratory and/or learning objects. We have described the learning objects in this program as:

> *A Learning Object is an interactive and dynamic computer-based environment that engages a learner through a game or activity and that guides him/her in a stepwise development towards an understanding of a mathematical concept. [22, page 64]*

Therefore future teachers who graduate from the MICA program are able to develop and program their own learning objects to meet the demands of a teaching situation. My transition from using manipulatives to using Learning Objects in my teaching was facilitated because I had integrated in my undergraduate Calculus courses Ralph's software *Journey Through Calculus* [26]. It became evident to me that students could learn mathematics in a fun but challenging "Learning Object" type of computer environment and furthermore that they could gain independence of action in mathematics within the MICA program as they became proficient in programming.

Examples of my use of mathematical games — popularization

My experience with mathematical games has been limited to activities for the popularization of mathematics. For example I developed the *Brock Bugs* board game [20] and widely disseminated it to mathematics teachers to help students engage with mathematics, more specifically probability. Other board games have been developed by my colleagues and me at the Department of Mathematics and Statistics and given away to mathematics teachers. My venture into e-games has been limited to research around the development of *E-Brock Bugs*, and this research has convinced me that in educationally sound digital mathematical games, the player is engaged by taking on the personality of the digital hero and thereby more likely to raise questions and to try different strategies. In other words the player engages in a self-assessment of his/her progress in a situation that is essentially of a mathematical nature, and he/she is more likely to do this in these kinds of circumstances than in traditional top-down teaching situations.

In conclusion, my view is that digital learning objects provide a mathematical learning environment with a pre-selected pedagogical approach to learning mathematics. On the other hand, mathematical e-games such as *E-Brock Bugs* that are designed according to Devlin's principals offer additional benefits, including the ones listed in this paper as well as a very different engagement into mathematics. Such games may even engage those who have difficulty progressing in the subject. We are only starting to venture into the use of (epistemic) e-games for learning mathematics and more research is needed for us to gain confidence on how best to integrate them in our teaching. I look forward to seeing how things will progress!

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