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Randal Bishop Plunges Into 4-D Space: A Comment on Randal Bishop's Paper Titled "The Use of Realistic Imagery to Represent the Relationships in a Four-Dimensional Coordinate System"

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Randal Bishop Plunges into 4-D Space

A comment on Randal Bishop's paper titled "The Use of Realistic Imagery to Represent the Relationships in a Four-Dimensional Coordinate System"

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I never had the pleasure of meeting Randy Bishop personally but for the last five years we have talked by telephone and exchanged mail. This contact started shortly after he presented his paper at the meeting in Australia. When I first read it - and at that time I was in Brazil - I was surprised to see that, after all, some interest had been generated by the book that I co-authored with Professor Emeritus Steve M. Slaby of Princeton University in the academic year of 1964-1965. Since Steve Slaby and I started being interested in 4-D geometry about the same time - Slaby in Norway, around 1953, and I in Brazil in 1954 - we realized that little by little our ideas would take hold and involve more and more people. I cannot say that they are, at this time, in sufficient number to fill a bus or even a mini-van. But the point is that the message that we have being trying to get across finds a new voice in Randy Bishop. He indeed plunged into 4-D space. He breathes 4-D geometry.

In this sense his enthusiasm reminds me of my own, almost half a century ago. The difference, however, is that he chose an alternative path that neither Steve Slaby nor I - nor many other people interested in 4-D geometry - had foreseen. If the late David Brisson is the sculptor of 4-D structures, Randy Bishop is the photographer. As I see it, if not Bishop then someone else will merge the two proposals: that of Brisson and the ideas outlined by Bishop in his paper. Still, why is it so difficult to generate interest in ideas that have been dealt with for more than one century?

In my conversations with Randy Bishop and Steve Slaby I have said that it is very hard to ask others to think about what has not been thought about before. This proposal, however, has to be dealt with quite differently from that which is taken by the inventor of a not as yet existing gadget, for instance. To plunge into 4-D geometry one has to start from very basic questions, much like Randy Bishop has been doing. For instance, it is prejudicial to pre-assign a type of 4-D space with which we would want to work. By this I mean that it is not import to pre-decide if that space is Euclidean or non-Euclidean. Start with what we have and then ask some pertinent questions. For instance, I dare to assume that as Randy Bishop read for the first time my and Slaby's book what he asked himself was: if all this is correct, would it be possible to photograph a 4-D object? In other words, his interest was focused on doing something that appeared feasible and that had not been thought about before. It mattered not if the Four-Dimensional Descriptive Geometry that was proposed applied to an Euclidean space. The main thing was "to think 4-D" realizing that, as a former professor of mine, Fellipe dos Santos Reis, observed, all geometric systems can be transformed into each other: a straight line can be warped and transformed into something else but always be called a line. Thus what Randy Bishop seeks to achieve is to obtain a photograph of a 4-D object. Period. But to make things easier he will be working with an object that would be familiar to us all. Let me make additional comments about this.

If we consult Henry Poincaré writings of the beginning of the century we find that he at one time made a very curious question: How do we know if we expanded or contracted in length, width and breath from one moment to another? The answer is that we cannot, for everything else will have expanded or contracted in the same proportion. Thus we cannot perceive if our 3-D space is Euclidean or non-Euclidean. It matters not. But, if it makes things easier to work, say, with a cube constructed according to the Euclidean geometry and this at an infinitesimal scale as we relate it to the Universe as a whole, then let us do it. We can always accept the fact that that cube obeys the rules of some other geometry, rules to which we are subject. We can then talk about a "line." Never mind what kind of line it is. It might not be the Euclidean straight line, but if we start with the assumption that it is Euclidean, the next question should be addressed not to its properties but to finding out how it came about.

This observation brings me to my own reasoning some 43 years ago that led me into the world of 4-D geometry. I had read somewhere, still as a young boy, that there are no straight lines in Nature. That was a very interesting thing because, after all, I was immersed in the study of the Euclidean geometry just as boys and girls of today are. Straight lines, planes, triangles, angles, and all sort of geometry forms. It appeared strange to be studying geometric forms whose basic element had no correspondence with reality. The question then was: if we cannot find a straight line in Nature, how was the first straight ruler constructed or fabricated? Later on I came upon another curious remark: to build a geometric system using only a compass. Can it be done? Well, the answer is yes. In fact, Euclidean geometry starts exactly this way. Euclid had no ruler to draw a straight line. How did he do it then? The solution is simple. We get hold of two branches of a tree and tie together one of their ends. We have a compass. With one end mark two points on the ground. With center in these points we can draw sections of circles that intersect at points. If we vary the opening of the compass we obtain a set of points. And these points, if they are infinitesimally distant, define a straight line. We can now take a piece of some pliable material and adjust it to that line, obtaining the edge of a ruler. With this edge we can replicate the straight line.

It matters not to pre-decide if the "surface" (ground) upon which we carried on the exercise is an "Euclidean plane" or if it follows the properties of some other geometric system. What matters is that now we have some means of dealing with the intrinsic geometry of a 2-D space which we can at least see. This done we can now, yes, chose a particular geometric system to analyze what goes on that "plane." It appears that I am reasoning in circles, but in fact what I am trying to say is that we must start without pre-conceived notions. I observe that with the compass I was able to generate a geometric form that can be repeated many times but now without the use of the compass. The construction of the ruler constitutes "to think about something that had not been thought before:" how to construct a geometric form without using the same instrument applied for its definition. Next we realize that that "line" can be drawn in a totally, independent "plane" and that the "lines" might not meet even though the two "planes" do so.

If we take this line of reasoning and apply it to a "cube" we must ask the question: can I replicate that "cube" in some other, totally independent "space," a "space" with the same properties as those of the "space" into which the first "cube" was initially constructed?

The answer ought to be yes. Conceptually, I mean. And then there is no other alternative but to conceptually admit that the two "cubes" (three-dimensional things) can only coexist within a four-dimensional thing.

That is what we have to hear when Randy Bishop exclaims: "we are four-dimensional beings." He is absolutely correct. A point is "at home" in a line. A line is "at home" in a plane. We all, three-dimensional beings, are "at home" in a 4-D space. If this is not so we are all, then, Abbot beings.

What we have to hope is that sometime, someone, somewhere, will achieve Randy Bishop's proposal: snap a picture as we leisurely float within 4-D space. Then we will be able to confirm if we constantly contract and expand as time goes by, in and out of "straight" 3-D spaces, through warped Riemannian spaces or in a never ending loop within a gigantic Mobius-type 3-D space.

But before this, let us have some more of Randy Bishop's news on how does it feel to plunge into 4-D space.