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In addition to the inclusion of humanistic concepts in the classroom presentation of mathematics and the assignments/projects that students are asked to complete, I have found it valuable to structure the courses I've developed in terms of literary and musical forms.

Mathematical knowledge, as that of many disciplines, is appropriately regarded as a web of ideas, having a great many interconnections. This can make it very difficult to imagine presenting material in a linear fashion—the richness of the discipline can be lost.

Upon reflection, it is clear that structural considerations are not and should not be limited to the development of new courses. Whenever we consider a course that we will teach in the next term, we have the opportunity of seeking an overarching structure for the subject.

One way to develop a path through the web of knowledge is to look to other arts for suggestions of ways to organize material. Two that I have found to be particularly helpful are literary and musical forms.

As a first example (and one which encouraged me to think of curriculum this way) a geometry course, based for example on Martin Jay Greenberg's *Euclidean and Non-Euclidean Geometries*[1], could have the form of a novel, with its plot the independence of the Euclidean parallel postulate. To this end, one would first introduce the setting—namely, the origins of geometry, the axiomatic method, Euclid's first four postulates, and the parallel postulate. This would be followed by ideas from logic that establish the context in which independence can be demonstrated or defeated: theorems, proofs, and models. The non-human characters, namely Hilbert's Axioms, would then appear and their consequences and interactions would be explored, constructing a portion of neutral (absolute) geometry. Demonstrating that a number of statements are equivalent to the parallel postulate would then show the students what is at stake in its proof or refutation. Tension would increase through a chapter on the history of the parallel postulate, introducing additional human characters and leading to the discovery of non-Euclidean geometry. Finally the independence of the parallel postulate would be established through models of hyperbolic geometry; by this time, the students welcome this as the culmination of the semester's story! After this denouement, the philosophical implications of the independence of the parallel postulate could be discussed, providing an additional reward for the completion of the plot of this novel.

Compare this description of a course based on Greenberg's text with the typical section-by-section, chapter-by-chapter presentation in most courses. Highlighting the difference: in a course with the plot sketched above, it would be unconscionable to run out of time before finishing the story!

It is worthwhile to let the students know what role the day's topic plays in the story of the independence of the parallel postulate—this helps establish a context for them, again exhibiting a significant difference from a course without a plot. In fact, when I asked students on the in-class final to sketch the plot of the course, all but one were quite successful.

A traditional first-semester calculus course adapts readily to this idea of structure via a novel. Exactly what the plot line is and who the major characters are will depend on the text adopted for the course. In a traditional calculus course, very often the Fundamental Theorem of Calculus is the goal of the plot. Almost every topic preceding this can either be related to the development of this theorem, or presented as subplots and asides. In the CCH reformed approach to calculus based on modeling[2], the fundamental theorem may have become an obvious consequence of the emphasis on derivatives as rates of change and integrals as total change. Now instead, the introductory chapter introduces the various families of functions as the

characters in a logically developed sequence of mystery tales and rags-to-riches stories with unlikely heroes. Which function is responsible for modelling some situation, and what evidence can one present for this? How well can an unlikely hero, a linear function, perform as an adequate substitute for a more complicated function in some particular situation?

Another example of the use I've made of a form from the arts is in developing a portion of a history of mathematics course offered for middle grades education majors through mathematics masters candidates, and since adapted for inclusion in other survey courses. The approach taken was that of a novella or tone poem; within an overall story line, themes reappear. This story, based on number and numeration systems, traces ideas from pre-history through the second half of the twentieth century. Ideas from Eudoxus recur with Dedekind; prehistoric counting reappears as the basis of Cantor's cardinality of sets. The discomfort caused earlier mathematicians by irrational, negative, and complex numbers reappears in my students with an introduction to the infinitesimals of the hyperreal numbers of Abraham Robinson.

In contrast to a structure modeled after a novella/musical tone poem, a unit on "Shape" based on the chapter in *On the Shoulders of Giants*[3] took the form of a theme and variations with some fugal entrances. Ideas of similarity, dimension, symmetry, dissections, and combinatorial geometry all appeared and were interwoven.

A book explicitly constructed in sonata-allegro form is John McCleary's *Geometry from a Differentiable Viewpoint*[4]; a summary of his comments in the introduction indicate the structure of his text (p.*ix-xi*). His first section of five chapters opens with a prelude of spherical geometry, then introduces some of the main themes, including Euclid's parallel postulate. The eight chapters in the development section establish what will be required to provide a rigorous model of non-Euclidean geometry, and introduce a new theme of an intrinsic feature of a surface, the Gaussian curvature. In the last three chapters, the recapitulation and coda, McCleary finishes the development, provides the climax with the construction of models of non-Euclidean geometry, and then provides a coda on the theme of the intrinsic.

In addition to providing an over-arching structure, there are additional advantages to thinking about courses in terms of literary or musical forms. Making the structure apparent to students, providing a conceptual narrative, and reminding them from time to time where they are in the course helps address the needs of students who are not in Sheila Tobias's[5] "first tier" (pp.31, 38, 46, 89). Such students often feel discomfort with an unmotivated section-by-section presentation.

Thinking of the course in these terms also can suggest potential projects and assignments for students that take a more humanistic approach. For example, one could ask students to select the most important theorem from a chapter containing major theorems of calculus leading up to the fundamental theorem, and explaining the reason for the choice. As another example, one could encourage students to write a poem after the students have seen hyperreal numbers, reacting as a Pythagorean might to the discovery of incommensurable magnitudes; a graduate student of education felt this was the best activity of a two-semester survey course.

Finally, considering over-arching themes for a course also encourages thought and discussion among faculty regarding strands in the mathematics curriculum. As institutional (or external) pressures grow to shorten (or at least not lengthen) majors, this will become increasingly important to the integrity of an undergraduate major in mathematics.

REFERENCES

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[5]Tobias, Sheila, *They're Not Dumb. They're Different: Stalking the Second Tier* (Tucson: Research Corporation, 1990).