Humanistic Mathematics Network Journal

Issue 15 Article 8

7-1-1997

Sand Songs: The Formal Languages of Warlpiri Iconography

James V. Rauff
Millikin University

Sand Songs: The Formal Languages of Warlpiri Iconography

James V. Rauff
Millikin University
Decatur, IL 62522
jrauff@mail.millikin.edu

This essay is an investigation into the mathematical ideas implicit, if not explicit, in the iconographic designs (sand scenes, yawalyu, site-path designs, and guruwari) from the point of view of formal language theory and provides short formal grammars that generate the languages of the designs. The essay closes with a suggestion of the pedagogical value of viewing Walpiri iconography as mathematics and whether or not the iconographic system may be properly termed "mathematics."

1. THE WARLPIRI

Historically, the Warlpiri[1] were a group of seminomadic hunters and gatherers who wandered the desert of Central Australia. Today they live in outstations and towns in their ancestral homelands. Many have claimed their traditional land and established small homesteads near larger settlements. The Warlpiri treasure their traditions and still teach their children the techniques of survival in the desert. Their sand stories and iconographical designs play an important role in keeping their nomadic traditions alive. Because these iconographical designs, their composition, and the stories they tell are my primary concerns, my description of Warlpiri culture will be of their traditional culture.

The western desert region of central Australia is a harsh place. It is a place of limited diversity and resources. Sand ridges often stretch for miles separated by flat sand plains. The most prevalent plant-life are pale green spiny grasses. Occasionally this pattern is broken by red gravel cliffs upon which various species of wattle grow. Pervading this rather desolate landscape is a scarcity of water. Rainfall is unreliable and unevenly distributed; when it comes it averages less than twenty centimeters per year.[2] Drinking water collects in waterholes while the few lakes, usually dry, hold undrinkable saltwater after heavy rains. Fred Myers describes the situation succinctly for the Pintupi, another Australian aboriginal people.

> For hunters and gatherers, the unreliability of water supplies poses the fundamental subsistence challenge. It is important to understand the nature of this resource. Although there are no permanent surface waters in the area, the Pintupi have found it possible to exploit other types of water supply. They have used large, shallow, transient pools formed by heavy rain; claypans and rock reservoirs in the hills that might be filled from lighter rainfalls; soakage wells in sandy creek beds; and 'wells' in the sand or in the rock between the sand ridges.[3]

Living off of this land is challenging, but the Warlpiri, like the Pintupi, have adapted to it, obtaining a variety of food substances from grass seeds to kangaroos as they move from water hole to water hole. This environment of the western desert, although not a determiner of Warlpiri culture, has a profound influence on their cosmology and their sand stories.

The Warlpiri trace descent from totemic ancestors who are personified environmental entities like rain, the honey ant, fire, and the yam. The wanderings of these ancestors created the present day features of the desert landscape. All of the Warlpiri ancestors traveled routes that can still be located. Indeed, major features of the landscape are the result of ancestral footprints, imprints from an ancestor sitting or lying down, or transformations of parts of an ancestor's body into a geological or topographical feature. Rock formations are metamorphosed limbs and genitals, water holes were dug by ancestral beings looking for food, and dried river beds were cut when an ancestral being dragged his tail or some object along the ground.

These geology-altering and geology-creating events took place in the time the Warlpiri call the *Jukurrpa*, or

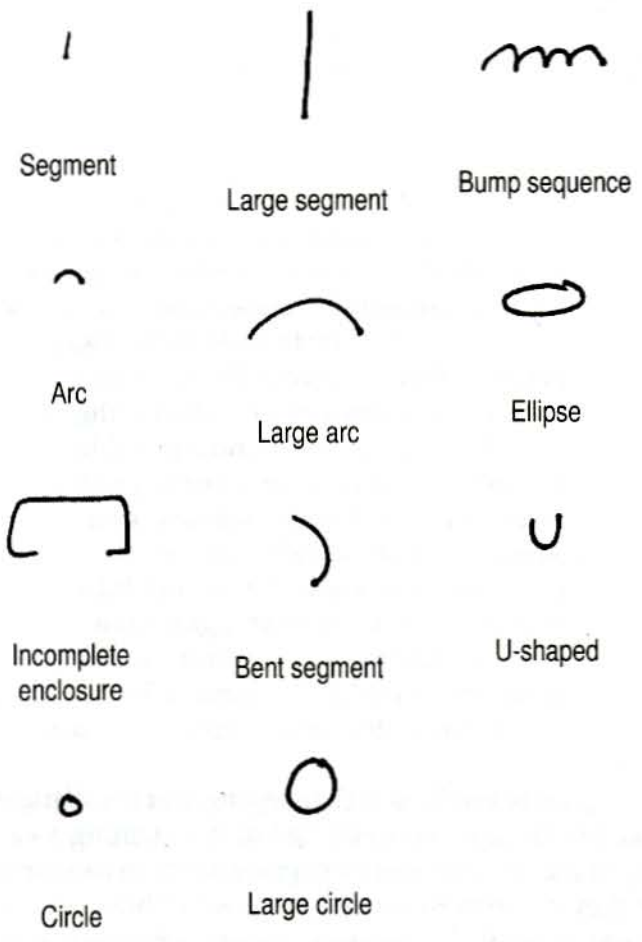


Figure 1
Sand story signs.

the Dreaming. The Dreaming, which is not unique to the Warlpiri, is arguably the most well-known and most enigmatic aspect of Australian aboriginal culture. The Dreaming refers to the time of the creation of people and of the world. Stories of the Dreaming are invariably tales of journeys. People and creatures and spiritual beings traverse the landscape, creating new landscapes, beginning traditions, defining classifications, and providing the reasons for why things in the natural world are the way they are. When a story about the Dreaming is told, it is considered a true story, a fact, a description of what is.

The Dreaming is a time of the creation of the world, but it is also the present. It is always there. The Dreaming is real, an accurate description of the world and a true history of what happened. The Dreaming is symbolic, a structure of ideas, values, and social norms.

2. FORMAL MODELS OF WARLPIRI ICONOGRAPHY

The iconographic designs of the Warlpiri reflect the Dreaming metaphysic, represent the ancestral actors of the Dreaming, and relate the Dreaming stories. Here I will examine four categories of Walpiri iconography from a formal algebraic point of view.[4] My analysis sees the iconographic systems as formal languages. Formal languages are algebraic systems consisting of two sets. One set is a finite set of symbols called the *alphabet* of the language, and the other is a set of *words*. The words of the language are created by combining symbols of the alphabet in certain specified ways. (The analogy with human languages is intentional!) It is usually the case that some combinations of alphabet symbols are words of the language and some are not. For example, if we consider English to be a formal language, then *elephant* is certainly a word of the language, whereas the string of English alphabet letters *glpoki* is not.

For formal languages, strings of alphabet symbols which are words are separated from those which are not by something called a *formal grammar*. Formal grammars provide the rules for constructing words and for excluding non-word combinations. The most familiar formal grammars are those of computer programming languages which provide the legal syntax from statements. The formal grammar of the computer language BASIC will allow us to write A = C + 10, but not = C A 10 +.

As an example of a formal language and its grammar, consider the infinite set of words {01, 0011, 000111, 00001111, ...}. This language has the set {0,1} as its alphabet. Its set of words includes only those strings that begin with one or more zeros and end with a like number of ones. Its grammar can be expressed with these rules:

1. S := 01
2. S := 0A1
3. A := 01
4. A := 0A1

The letter S is a special symbol called the *start symbol*. We may generate words in the language by beginning

with any rule that has S on its left side (Rules 1 and 2 in this example). If we select Rule 1 we get the word 01. If we select Rule 2 then we get 0A1. Because A is not in the alphabet (symbols not in the alphabet are called *nonterminals*) we do not yet have a word. Thus, we must select a rule with A on its left side and replace A by the right side of that rule. For example, if we apply Rule 3, the string 0A1 becomes the word 0011. On the other hand, if we apply Rule 4, 0A1 becomes the string 00A11. We continue to apply rules until a string consisting only of alphabet elements (i.e. a word) results. It is easy to see that the four rules given above will produce only the words in the set {01, 0011, 000111, 00001111, ... }.

It is customary in formal language theory to combine rules that have the same left hand side into one rule with the options separated by pipes (|). Using this convention, our grammar becomes the two rule grammar:

1. S := 0A1 | 01
2. A := 0A1 | 01

In my discussion of Warlpiri iconography, I will suggest a formal grammar for the formation of iconographic complexes. To provide cultural context, I will also discuss the use of the iconography by the Warlpiri. It should be kept in mind that the algebraic analysis is intended as a model of the iconographic systems and is not necessarily the way the Warlpiri view their system. Whether or not the iconographic systems are actually formal algebraic systems is discussed in Section 3.

2.1 Sand Stories

Sand stories are usually told by women. A patch of ground is swept clean by hand and a story is related by drawing figures in the sand, singing corresponding songs, and providing minimal narration. The sand stories are about ancestral events and the Dreaming. Sand stories involve the patterns of daily life like gathering food, traveling, interpersonal relationships, birth, death, and ceremonies.

The nature of the sand story signs reflects the medium in which they appear. The signs are drawn in the sand with the fingers and thus consist of simple lines, and curves. Figure 1 shows the basic sand story signs.

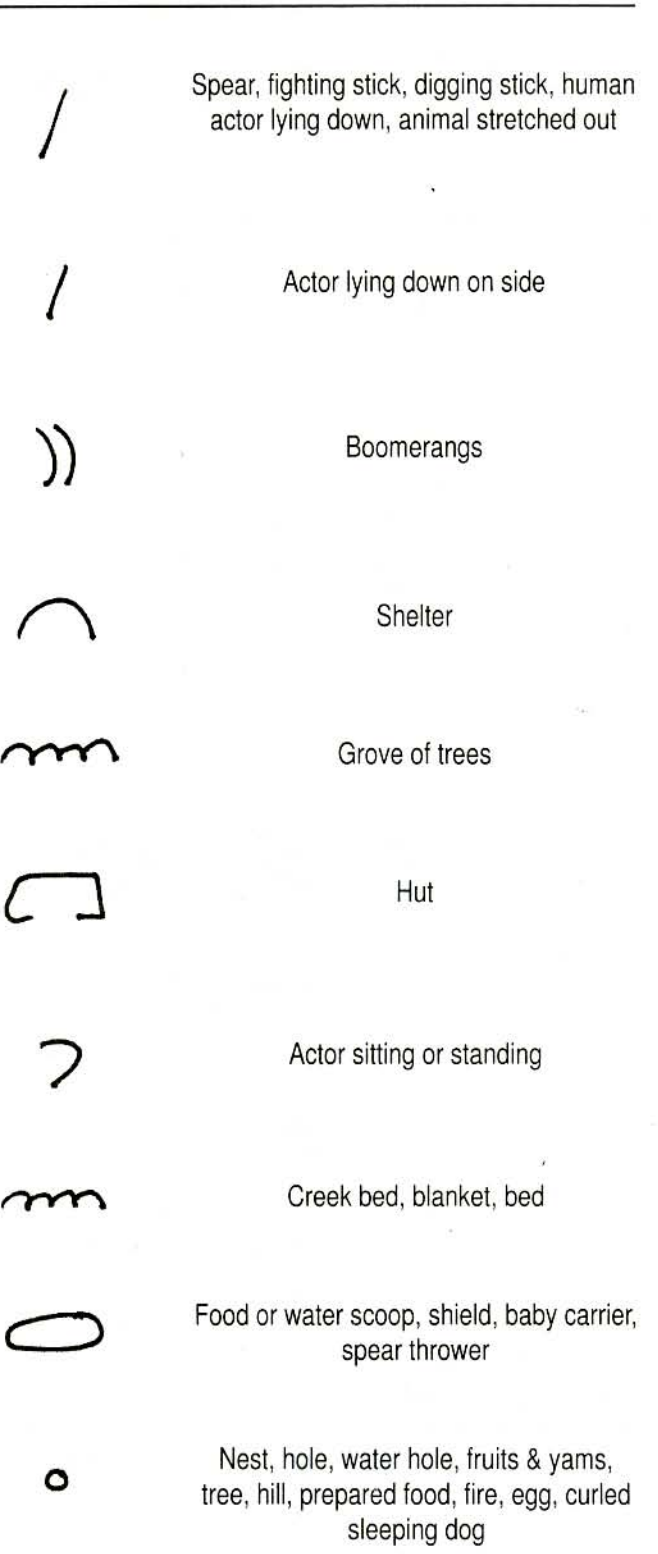
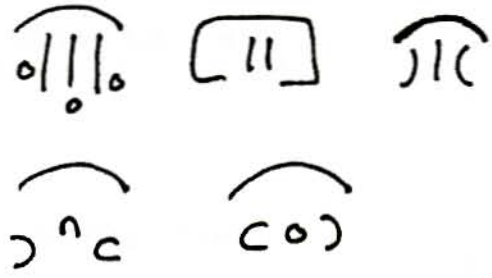


Figure 2
Range of meaning of single signs.



Camps

Foraging

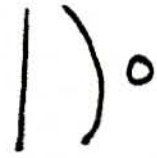
Going into the ground

Figure 3
Sand story scenes.

These signs are combined in a finite number of ways to create sand story scenes. The scenes are then sequenced to form the sand story. The ranges of meanings of the single signs are given in Figure 2. A formal language can be described that takes the basic story signs as its alphabet and produces sand story scenes as words.[5] Some typical sand story scenes are shown in Figure 3.[6]

A formal language of the sand stories, which I will call **SAND**, may be defined as follows. I use the descriptive titles from Figure 1.

SAND: The alphabet is the set {small **s**egment, long **S**egment, bent segment, **b**umps, small **c**ircle, large **C**ircle, small **a**rc, large **A**rc, **U**-shaped, **I**ncomplete enclosure, **e**llipse, [ ], { }, < >}. The **bold** letters will be used as alphabet symbols in the grammar. Thus, **stc** signifies a segment followed by a bent segment followed by a circle. Like this:



It is also necessary to introduce special symbols (like accent or punctuation marks) to specify how the combinations are to be constructed. So, I've added the following symbols to my alphabet[7]:

[ ] indicates that the enclosed string is below the previous symbol
{ } indicates that the enclosed string is inset into the previous symbol
< > indicates that the enclosed string converges on the previous symbol

For example, **A[SSS]** represents the scene:

I{ss} represents:

and **c<S[S[S]]>** represents:

The formal grammar for **SAND** is given by the following rules:

1. Start := Camp | Forage | Finale
2. Forage := **c[U]** | **c[U]** Forage
3. Camp := Regular | Ceremonial

4. Regular := **A** [P]
5. P := P1 | P2 | P3 | **b**[**b**]
6. P3 := P1 P2
7. P2 := P1 P1
8. P1 := Sleep | Sit
9. Sleep := **S** | **Sc** | **cS** | **Sc** | **S**[**c**] | **t**
10. Sit := **U** | **U**{**c**} | **U**{**Uc**}
11. Ceremonial := **A**[**b**[**b**]] | (**cccc**)[**C**{**c**}] | dance
12. dance := dancers [singers]
13. dancers := **b** | **b**[dancers]
14. singers := **U**[**U**[**U**]] | singers[**U**]
15. Finale := F<**S**[**S**[**S**]]>
16. F := **c** | **C**{**c**} | **U** | **C**{**U**}

The **SAND** grammar can be seen to produce each of the sand story scenes in Figure 3 as well as many others. For example, the camp scene:

may be generated by the following sequence of grammar rules:

Rule 1: Start := Camp
Rule 3: Camp := Regular
Rule 4: Regular := **A**[P] **A**

Rule 5: P := P3
Rule 6: P3 := P1 P2
Rule 8: P1 := Sleep
Rule 9: Sleep := **cS** **A**[**cS**]

Rule 10: P2 := P1 P1
Rule 8: P1 := Sleep
Rule 9: Sleep := **S**[**c**] **A**[**cSS**[**c**]]

Rule 8: P1 := Sleep
Rule 9: Sleep := **Sc** **A**[**cSS**[**c**]**Sc**]

Considered as a formal language, **SAND** has several interesting features that reveal something about the iconography it models. For example, the rules pertaining to the ceremonial dances (Rules 11 -14) are infinitely recursive allowing any number of singers and dancers. Actual sand stories known to me go no larger than three rows of dancers (bump sequences) and five singers (U-shapes), but although there are certain practical limits, there appears to be no potential limit to the size of this scene.

In contrast to the dances, there is a definite upper limit on the number of people occupying a campsite enclosure. This is reflected in the grammar by the bounded derivations possible through Rule 5 which limits enclosures to three adult inhabitants.

We now turn to the female designs known as *yawalyu*. Nancy Munn[8] has shown the iconographic relationships between sand stories and yawalyu. Here we will investigate their formal properties.

2.2 Yawalyu

Yawalyu designs are generally revealed to women in dreams. A ceremony is usually performed to reveal a new design and the presentation of the design is accompanied by songs. The yawalyu dream is a story about the Dreaming. How the Warlpiri view these dreams is summed up by Nancy Munn:

> The Warlpiri view is certainly not that "life is like a dream" but more nearly the opposite: that whenever event sequences are cut off from the world of everyday life so that they seem to con-

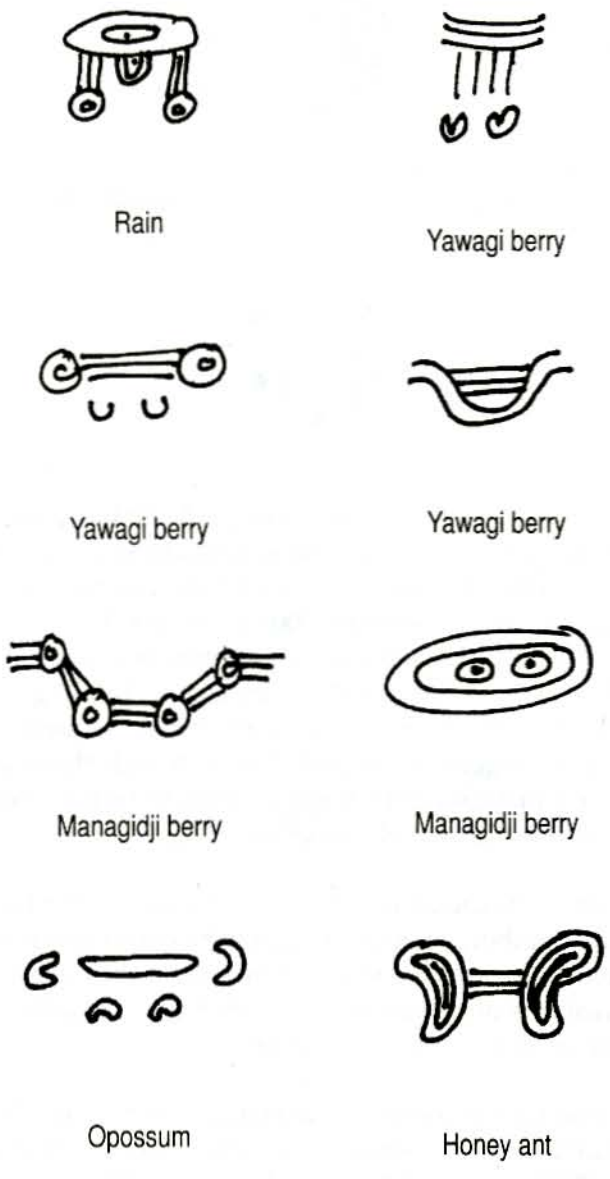


Figure 4
Yawalyu designs.

stitute a closed totality of their own and can be "talked about" but not "lived through" in the day-to-day involvement of social life, then such events are *djugurba*—stories, dreams, the ancestral past.[9]

Yawalyu designs represent specific totemic species that refer to ancestors. Some typical yawalyu designs are shown in Figure 4.

Notice that the number of signs in yawalyu designs is much less that that of the sand story scenes. (Figure 5 shows the basic yawalyu design elements.[5]) Consequently, my formal language for yawalyu designs, which I will call **YAW**, has a very small alphabet. Specifically, the formal language for yawalyu designs may be given as follows.

> **YAW**: The alphabet is {**U**-shaped, **R**ing, **C**ircle, **S**tick, **P**arallel line segments, **H**ooked segment, [ ], { }}.

As was the case with the language **SAND**, it will be necessary to introduce special symbols (like accent or punctuation marks) to specify how the combinations are to be constructed. I use the same conventional symbols as I did with the sand story language. Thus,

[ ] indicates that the enclosed string is below the previous symbol
{ } indicates that the enclosed string is inset into the previous symbol

The grammar for **YAW** has the following rules:

1. Start := Arch | Locus | Path | Triad | **R**{**R**{EE}}
2. Arch := Cover[Attached]
3. Cover := **P** | E | Bar
4. Attached := ABA | **P**[**UU**] | **UCU**
5. A := **P**[E]
6. E := **R**{**C**}
7. Bar := E Pseg
8. B := **U**{**C**}
9. Locus := **U**[**U** Core **U**][**U**]
10. Core := **P** | **S**
11. Path := Pseg Path | **P** | Pseg Pseg **P**
12. Pseg := **P**E
13. Triad := **R**{**H**} Center **R**{**H**}
14. Center := **R** | **S** | **P**

These rules will generate the yawalyu designs shown in Figure 4 as well as many more, including some that have not been reported in the literature. An interesting test of the model would be to see if the grammar can predict yawalyu designs not yet dreamt.

A rule of particular interest is Rule 11 which generates site path designs, a fundamental and popular Warlpiri design that appears in contexts other that yawalyu designs. I turn my attention to these designs

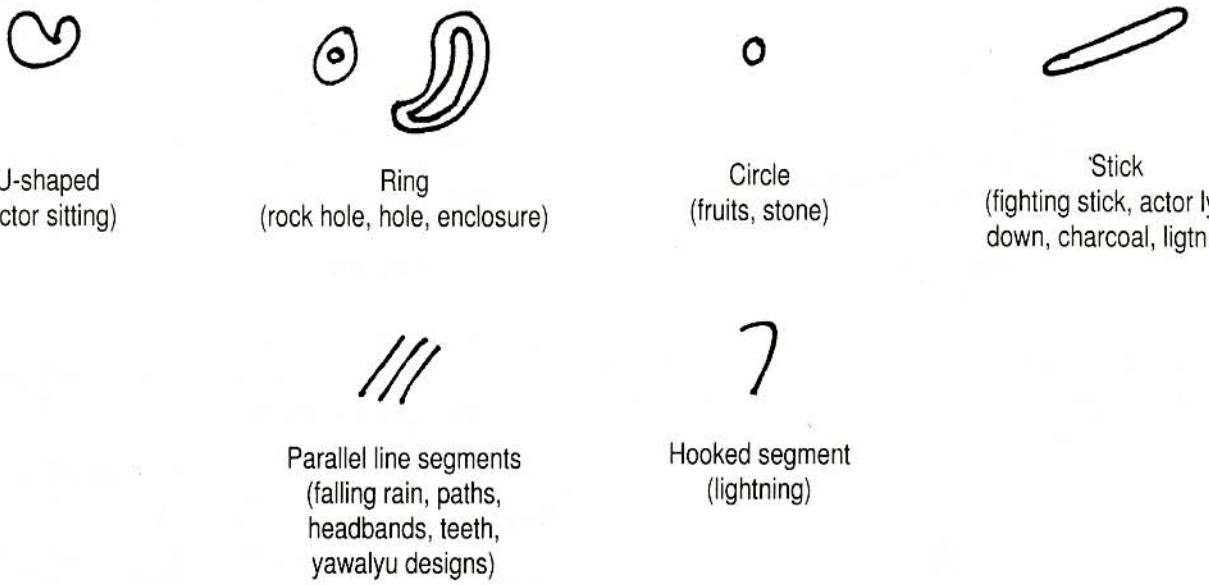


Figure 5
Basic yawalyu elements with range of meanings.

now.

2.3 Site-Path Designs

Site-path designs, usually drawn by men but sometimes occurring in yawalyu designs, depict the various routes taken by ancestral beings in the Dreaming. Consisting almost entirely of parallel lines and concentric circles, site path designs appear in Australian aborigine rock art, body painting, on shields and other artifacts, and in contemporary art for sale.[11] A typical example of a site-path design appears in Figure 6.[12]

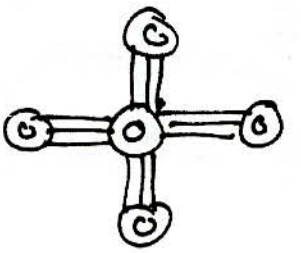
Site-path designs are clearly reminiscent of formal graphs with the concentric circles serving as vertices and the parallel line segments as edges. One significant difference between mathematical graphs and Warlpiri site-path designs is that the Warlpiri designs allow for free edges connected to only a single vertex. My formal language for site-path designs, which I will call **SITE**, is as follows.

SITE: The alphabet for **SITE** has only two Warlpiri elements, but also contains some special symbols and the natural numbers. The alphabet is {**P**arallel lines, **C**oncentric circles, :, ( ), $, 1, 2, 3, 4, 5, ...}.

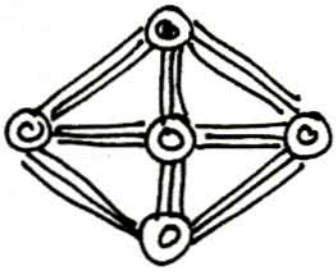
I'll use the numerals, parentheses, and colons to express complex linkages. A **PC** sequence preceded by a numeral *k* and a colon and enclosed in parentheses is attached to the *k*th concentric circle of the preceding or following sequence. Multiple numerals indicate multiple attachments. Thus, we have the following notations:

CPCPC for:

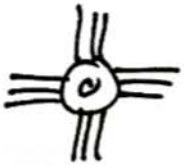
(2:**CP**)**CPCPC**(2:**PC**) for:



and (1:2:3:**CP**)**CPCPC**(1:2:3:**CP**) for:

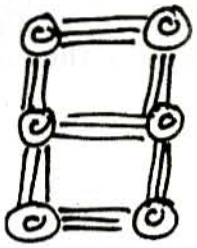


Free edges attached to the same concentric circle are denoted by repeating the **P** symbol. Thus, we have the following notation:
PPPPC for:

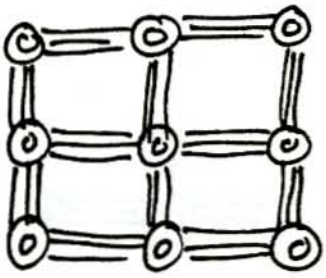


The dollar sign, $, applied to a site path sequence enclosed in parentheses attaches a copy of that sequence via corresponding concentric circles. Thus, we have the following notations:

(**CPCPC**)$ for:



and (**CPCPC**)$$ for:



We may then give the grammar rules for SITE as follows.

1. Start := Road | Star | Poly | Net | Five
2. Road := **CPCPC** | Road **PC**
3. Star := Free | Jack
4. Free := **PPC** | **P** Free
5. Jack := (2:**CP**)Road (2:**PC**)
6. Poly := (1:2:**PC**)**CPC** | (1:3:**PC**)**CPCPC**
7. Net := (Road) $ | Net$
8. Five := (1:2:3:**CP**) **CPCPC** (1:2:3:**CP**) | (1:2:3**CP**) ((1:2:**PC**)**CPC**)

Rules 6 and 8 are potentially families of rules. Larger polygons and complete graphs may be specified by using longer "Roads" and changing the numerals in the parenthetical attachments. The alternative to the family of rules is a context sensitive grammar or some sort of regulated rewriting system.[13] However, the site path designs known to me are small in size and only the "Roads" and "Nets" seem to be potentially infinitely extendable.

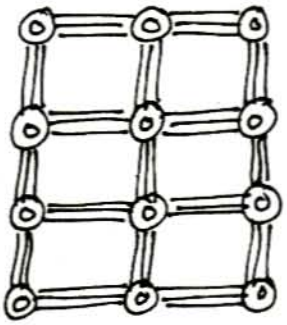


Figure 6
A Site-Path Design

2.4 Guruwari

Guruwari are men's ancestral designs. Guruwari are painted on ceremonial regalia, boards, stones, the ground, and on bodies. They tell the stories of the ancestors, their travels, the founding of the clans, history, ecology, geography, and geology of Warlpiri country. Guruwari designs are powerfully charged with dream value. They originate in dreams and tell about the Dreaming.

Figure 7 presents a sampling of men's ancestral designs. Notice that the designs, like the sand story scenes and the yawalyu, are complexes composed of a small set of basic signs. The basic signs are variables, taking on a variety of semantic values. The undulating line can represent both "snake" and "lightning" and the dots may be "eggs" or "ants."

Also, notice that the undulating line and straight line serve as base symbols which are flanked on both sides by the satellite symbols (dots, small circles, short pairs of parallel lines, etc.). These patterns make the formal language of men's ancestral designs, I'll call it **GURU**,

relatively straightforward.

GURU: The alphabet for the formal language of guruwari designs is {**S**nake line, **D**ots, **C**ircle, d**A**shes, se**G**ment, **F**ootprints, &, ( ), { }, [ ]}. Figure 8 shows each of these alphabetic elements.

I will also use the braces to denote inset symbols and the square brackets to denote positioning under a symbol as I did in the **SAND** grammar. In addition, notice that the guruwari designs often include a scattering of an arbitrary number of basic elements as in Figure 7c (honey ants). To denote this scattering, **GURU** includes the symbol & as a prefix to denote scattering. Thus, Figure 7c may be represented as &**DG**&**D**.

The grammar of **GURU** consists of the following rules:

1. Start := Adjunct Core B
2. Core := **S** | G | **c**[**p**[**c**[**p**[**c**]]]]
3. c := **C**{**C**}
4. p := **GG**
5. Adjunct := &**D** | &**C** | &**F** | &**A**
6. &**D** Core B := &**D** Core &**D**
7. &**C** Core B := &**C** Core &**C**
8. &**A** Core B := &**A** Core &**A**
9. &**F** Core B := &**F** Core &**F**

Rules 6-9 are context-sensitive rules that are necessary to insure that the left adjunct is the same symbol as the right adjunct.

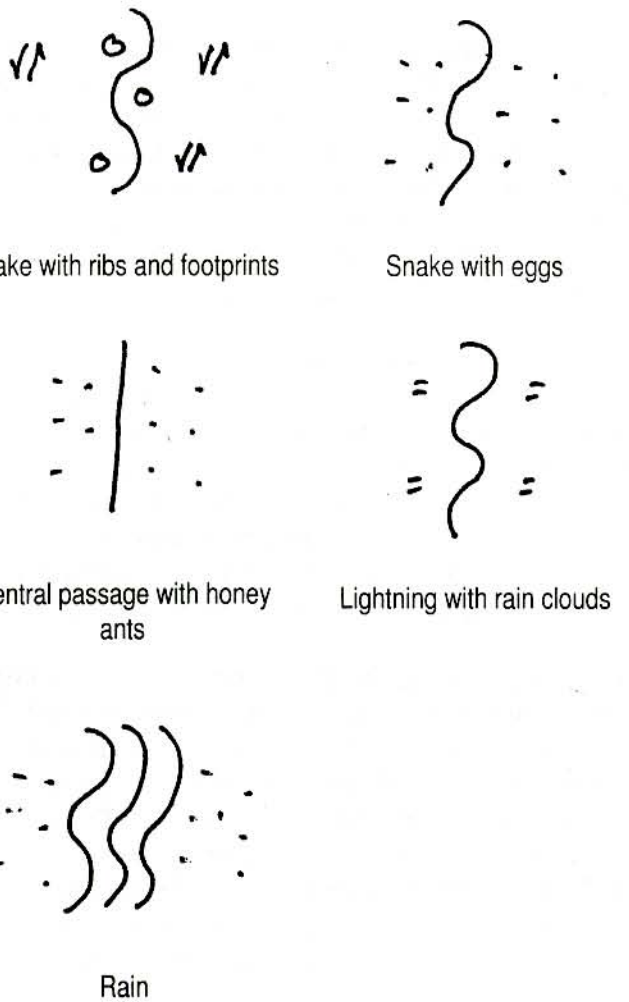


Figure 7
Guruwari.

We have seen how various subsets of Warlpiri iconography may be modeled as formal languages with context-free or context-sensitive rules. It is natural to ask at this point if this is but an empty exercise or is the Warlpiri iconography a mathematical system of some sort. A beginning of an answer to that question is the topic of the next section.

3. IS WARLPIRI ICONOGRAPHY MATHEMATICS?

Originally, I began to think of Warlpiri iconography as a formal language in an attempt to provide an interesting, but manageable, formal language modeling examples for my computer science and mathematics students. The Warlpiri designs are appealing in their own right, just unusual enough to engage students, much more interesting than arbitrary sequences of letters, and smaller in scope than natural languages or computer languages. Additionally, Warlpiri designs are a real set of objects with an uncoverable formal algebraic structure. As such they provide an entry to the mathematical modeling of human artifacts.

My students were successful in producing formal languages with grammars for selected collections of Warlpiri designs.[14] However, the question of whether or not the iconographic system of the Warlpiri is a Warlpiri mathematics arose frequently in our discussions. That is, can we say that the iconographic system of the Warlpiri is a mathematical system?

A short answer to the question, in my opinion, is a tentative yes, the Warlpiri iconographic system is mathematics. The Warlpiri iconographic system has the components that we expect of an algebra interacting in a way that is only slightly different from the

abstract algebra we learned.

The visual algebraic components of Warlpiri iconography are obvious. We have a finite set of symbols and the symbols may be combined to make more complex structures according to a finite set of rules. Compare this to the formation of equations in college algebra where the letters x and y, the integers 2 and 3, and the symbols =, + ,*, and ^ combine to make the complex notion of a quadratic function: $y = x\text{^}2 + 3x$.

Beyond what we see in the iconography is its power to model the real world of the Warlpiri. The iconographic designs are, to a large extent, models of the Dreaming, the fundamental reality of the Warlpiri. The Warlpiri iconographic system can, in this way, be seen as a mathematical model of aspects of reality parallel in form and function to the mathematical models of trajectories we study in college algebra.

The Warlpiri recognize the components and rules of their iconography and they recognize its modeling functions. What they don't seem to have is a "theory of iconography" that abstracts general patterns from the sand scenes, yawalyu, and guruwari. Here we may question whether or not the iconography is mathematics. Perhaps we are safer to say, as Marcia Ascher has suggested, that the Warlpiri iconography be called "mathematical ideas" rather than mathematics.[15]

I prefer to leave the question open at this time. Obviously, that we can model the iconography with mathematics does not imply that the iconography is mathematics, but neither does it imply that it isn't. The quandary may be resolved by researchers working in the field of ethnomathematics.

Ethnomathematics includes "all practices of a mathematical nature, such as sorting, classifying, counting, and measuring, which are performed in different cultural settings, through the use of practices acquired, developed, and transmitted through generations."[16]

The Austrian mathematician Roland Fischer provides a way of viewing mathematics that is helpful in understanding "ethnomathematics." Fischer writes,

> Mathematics provides a *means* for individuals to explain and control complex situations of the natural and of the artificial environment and to communicate about those situations. On the other hand, mathematics is a *system* of concepts, algorithms and rules, *embodied in us*, in our thinking and doing; we are subject to this system, it determines parts of our identity.[17]

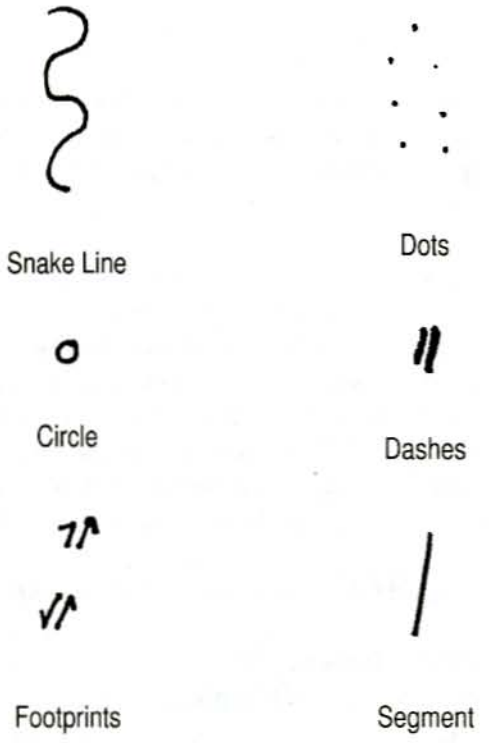


Figure 8
Guruwari alphabet.

When mathematics is viewed as a means and as a system embedded within a culture, our understanding of what mathematics enlarges to encompass much more than formal school mathematics. Instead, mathematics includes a multitude of practices that are characterized by algorithms, formal processes, and abstraction. In this context, the Warlpiri iconography emerges as an ethnomathematical system.

Whatever our position on the mathematical nature of Warlpiri iconography, one thing, however, is clear. The Warlpiri have developed a sophisticated symbolic system for describing their world.

A pervasive myth in the history of mathematics is that

the Australian aborigines are one of the least competent mathematical thinkers in the world. These arguments arise from early anthropological reports of the simplicity and lack of power of aboriginal counting systems. These reports were misguided at best, reflections of a cultural superiority complex at worst. It has been shown that the aboriginal people can count perfectly well if they want to. However, traditional aboriginal culture had no need for counting because it did not value possessions. If they counted at all, aborigines counted for purposes of sharing or sorting.[18] The case of Warlpiri iconography suggests that the simplicity may have been on the part of the European anthropologists. They were looking for counting and arithmetic in Aboriginal culture, but they missed abstract algebra!

NOTES

[1]Ethnographic information on the Warlpiri is from Nancy Munn's *Walbiri Iconography* (Chicago: University of Chicago Press, 1986).

[2]See *Aboriginal Man in Australia*, edited by D. Mulvaney and J. Golson (Canberra: Australian National University Press, 1971).

[3]See p.26 of Fred Myers ethnography of the Pintupi, *Pintupi Country, Pintupi Self* (Berkeley: University of California Press, 1991).

[4]The categories of Warlpiri iconographic designs and the typical examples are from Nancy Munn's *Warlpiri Iconography* (Chicago: University of Chicago Press, 1986).

[5]The sand stories themselves may be considered a subset of the nth order Cartesian product on the set of sand story scenes or as a formal language in their own right.

[6]Adapted from *Warlpiri Iconography* by Nancy Munn (Chicago: University of Chicago Press, 1986), pp.70-71.

[7]The sand stories are supplemented by finger movements showing direction of action. These non-pictorial signs, although important aspects of the sand story are not dealt with in my grammar which focuses on the static aspects of Warlpiri iconography.

[8]Op. Cit. pp. 89-118.

[9]Ibid. p.117.

[10]Nancy Munn (Ibid. p.104) sees five basic elements, but in my opinion her data clearly show six.

[11]See Rockman, Peggy and Napaljarri Cataldi, *Warlpiri Dreamings and Histories* (San Francisco: HarperCollins, 1994); Layton, Robert, *Australian Rock Art: A New Synthesis* (Cambridge: Cambridge University Press, 1992); and Morphy, Howard, *Ancestral Connections* (Chicago: University of Chicago Press, 1991).

[12]Adapted from Rockman and Cataldi, Op. Cit., Plate 4.

[13]See Dassow, J. and G. Paun, *Regulated Rewriting in Formal Language Theory* (Berlin: Springer-Verlag, 1989).

[14] Also successful in this endeavor were a group of junior high students faced with the same task. The results of this little experiment leads me to believe that similar formal language writing tasks may offer an earlier entry into modeling with abstract algebras.

[15]Personal communication, February 1996.

[16]D'Ambrosio, Ubiratan "Ethnomathematics: A Research Program on the History and Philosophy of Mathematics with Pedagogical Implications," *Notices of the American Mathematical Society*, Volume 39, No. 10, pp.1183-1185 (1992).

[17]Fischer, Roland. "Mathematics as a means and as a system". In Restivo, Sal; van Bendegem, Jean Paul; and Roland Fischer, Eds. *Math Worlds: Philosophical and Social Studies of Mathematics and Mathematics Education* (Albany: State University of New York Press, 1993), pp.113-133.

[18]Several detailed discussions of Australian Aboriginal counting systems and practices may be found in the *Work Papers of SIL-AAB, Series B, Volume 8, Language and Culture*, edited by S. Hargrave (Darwin: Summer Institute of Linguistics, Australian aborigine Branch, 1982).