Humanistic Mathematics Network Journal

Issue 1 Article 6

Summer 6-1-1987

Patterns of Emotion Within Mathematics Problem-Solving

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PATTERNS OF EMOTION WITHIN MATHEMATICS PROBLEM-SOLVING

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Paper prepared for the Panel on "Mathematics as a Humanistic Discipline" presented at the Joint Meetings of the Mathematical Association of America and the American Mathematical Society, San Antonio, Texas, January 1987.

I like the clever twists of logic that turn a two page proof into a one-half page proof. There are lots of clever little insights. There's something very satisfying about a nice tight argument that no one can doubt is correct...I've worked on a research problem for over six months with no results...now I'm starting to dream about it and that's too much..the mathematics is taking too much control over me. (Angrily.) (Rosamond, 1982)

Mathematics often is viewed as the ideal discipline-pure rational thought dealing with ideal objects to produce irrefutable arguments, not coloured by any emotion. Training in mathematics is seen as producing students capable of such clear thinking in all disciplines. So why don't all mathematics teachers present mathematics in the ultimate, Bourbaki style? To mathematize is supposedly part of the human condition, so how can there be such a thing as math anxiety, when feelings should clearly not be a part of learning in mathematics?

Or does mathematic arouse emotion because it was conceived out of emotion in the first place?...What is the link between the affective and the cognitive? (CMESG Announcement, 1985)

PATTERNS OF EMOTION IN MATHEMATICS PROBLEM-SOLVING

In an effort to understand and explicate the feelings of satisfaction and anger expresssed by the mathematics graduate student in the first quotation, a Workshop on the Role of Feelings in Learning Mathematics was held during the Canadian Mathematics Education Study Group annual meetings of 1985 and 1986. We engaged in a problem-solving exercise that also was given to six mathematics education graduate students at a State college and on two occasions to six people who met in a private home.

We are all (with the exception of two people) involved in mathematics as professional mathematicians, as teachers, as graduate students or as people who use mathematics in our work. We believe that thinking, feeling and acting work together, that true understanding implies feeling the significance of an idea, and that our experiences are not far from that of our students. We decided to examine our own feelings in depth in hopes of finding outstanding commonalities that could be used to improve classroom teaching.

Studies on cognitive science (Davis, 1984. Papert, 1980), problem-solving (Silver, 1985), metacognition (Schoenfeld, 1983) and belief systems (Perry, 1970) offer some insight into the role of emotions in problem-solving but only indirectly. We are not sure we have even a vocabulary with which to describe feelings at a specific moment as a function of many variables.

To begin with, we made a list of relevant positive and negative emotion descriptors (see appendix). This list was adjusted by the results of the exercise. The exercise is a simple one. We went in pairs to different parts of the room where one person agreed to be the problem-solver and the other the observer. The rules were 1) The solver do his or her best to provide a running commentary on feelings. 2) The observer keep quiet, pay attention, take notes.

After a fixed amount of time (15 minutes, in later sessions changed to 30 minutes) all gathered and each observer reported on what the solver had done, focussing on the feelings. The solver also reported.

The roles were then switched, observer became solver. Solver became observer. Another problem was presented and the observation and reporting process repeated.

We feel many positive emotions (challenge, hope, zest, satisfaction, etc.) when doing mathematics and wish to promote these in our students. Lazarus is a noted psychologist at University of California at Berkeley who has done extensive analysis of the theory of emotions. In his paper, "Emotions: a Cognitive - Phenomenological Analysis", he describes some of the contributions positive emotions make to coping. Before describing our exercise and the implications that we found for teaching, I will briefly outline some of Lazarus' position and make some connections to mathematics.

LAZARUS ON POSITIVE EMOTIONS

Lazarus points out that negative emotions have been studied almost exclusively. Some reasons for this are that emotions have been studied as evolutionary and that negative emotions such as fear or stress influence our capacity to survive. Another reason is that emotion is studied by therapists who may view emotion as pathological. In this case happiness may be seen as hysteria, concern as paranoia and hugs as evidence of nymphomania. A third reason is that it is more difficult to measure arousal for joy, delight, and feelings of peace than it is for rage, disgust or anxiety.

Because we are trying to promote good problem-solving, we feel it is appropriate to focus on the positive feelings associated with our goal: on hope rather than hopelessness, challenge rather than threat, zest rather than dispair although negative emotions do need to be recognized.

Positive emotions tend to be frowned upon or viewed as "childish." Not many people exhort optimism like Ray Bradbury does: "We are matter and force turning into imagination and will! I am the center of a miracle! Out of the things I am crazy about I've made a life!...Be proud of what you're in love with. Be proud of what you're passionate about! (Bradbury, 1986) It is even hard to hear people shout gladly onto the Lord; but we were just trying to hear people shout gladly about mathematics. People who exhibit positive emotions often are accused of playing, of not being serious.

Yet playing with ideas is inherent in mathematics problem-solving. What emotions should we expect to feel when engaging in problem-solving? Lazarus answers this by saying that the essence of play is that it is highly stimulating. It is accompanied by pleasurable emotions such as joy, a sense of thrill, curiosity, surprise, wonder, emotions exploratory in nature. We recognize that we do experience these positive toned emotions when doing mathematics.

As educators we wish to know the optimum conditions that encourage problem-solving. Lazarus says, "...exploratory activity ocurs more readily in a biologically sated, comfortble and secure animal than in one greatly aroused by a homeostatic crisis. The human infant will not venture far from a parent unless it is feeling secure, at which point it will play and explore, venturing farther and farther away but returning speedily if threatened or called by the mother." As shall be discussed in more detail in the next section, mathematics problem-solving requires playing in an almost "other-world" of intense concentration. Insecurities in terms of math ability or other issues (world peace) inhibits problem-solving by interferring with the level of concentration.

USES OF POSITIVE EMOTIONS

Lazarus sees at least three ways in which a person uses positive emotions: as "breathers" from stress, to sustain coping, and to act as restorers to facilitate recovery from harm or loss. Lazarus' discussion may be interpreted with mathematics in mind.

BREATHERS OR TIMES OF INCUBATION

"Breathers" are times when positive emotion occurs as during vacations, coffee breaks or school recess. They can also be thought of as times of incubation.

Lazarus quotes the noted mathematician Poincare to suggest that it may be the good feelings themselves that allow a solution to emerge from the subconscious to the conscious.

Poincare made the surprising comment that unconscious creative mathematical ideas "are those which, directly or indirectly, affect most profoundly our emotional sensibility." By this he meant that, since creative thoughts are aesthetically pleasing, the strong, positive emotional reaction to such ideas provides an opening through which they are ushered into consciousness.

Lazarus reminds us of another relevant description of a "breather" made by the great German physicist Helmholtz:

> He (Helmholtz) said that after previous investigations of the problem "in all directions...happy ideas come unexpectedly without effort, like an inspiration. So far as I am concerned, they have never come to me when my mind was fatigued, or when I was at my working table...They came particularly readily during the slow ascent of wooded hills on a sunny day."

The acceptance of the role of a breather is reflected in the usual advice given by teachers to their students: "Concentrate long enough to get the problem firmly in your mind and to try several approaches. But then take a walk or do some pleasant activity and let your mind work on the problem for you."

SUSTAINERS OR MOTIVATORS

Positive emotions act to sustain problem-solving in the sense that good feelings build on good feelings. Mathematics and the word "challenge" often are linked together as in "The problem is a challenge." A challenge can be viewed as a threat and in our exercise, problem-solvers were momentarily worried about failure in front of an observer. However, in challenge, a person's thoughts can center on the potential for mastery or gain. This challenge is accompanied by excitement, hope, eagerness, and the "joy of battle." All these positive emotions were mentioned by problem-solvers. One solver summarized the feeling as "the joy of mental engagement and the bringing of all mental force to bear in a cohesive way." Solvers who perceived their problem as too easy felt disappointment even before they began to work on the problem. Those who felt the problem worth working felt an immediate joy even before proceeding. This joy was a signal to bring all mental force to bear on the problem, which in itself produced pleasure and therefore motivation to continue.

Lazarus describes "flow" ,an extremely pleasant, sustaining emotion, as in the case of the basketball player who is "hot" or the inspired performance of a musician.Lazarus claims flow arises when one is totally immersed in an activity and is utilizing one's resources at peak efficiency. Mathematical problem-solving requires total immersion and we found that a comfort with notation was important in maintaining this flow. Comfort with notation will be discussed later in this paper.

The positive emotion of hope also provides motivation to keep going. Occasionally during a problem-solving episode the solver lost control of the problem. Solvers said, "I've lost control of the problem." or "This is too complicated, too many angles to label." or "I feel this is getting a little out of hand. This one and that one cancel out and I haven't used fact that it's a prime." Hope, the belief that there is even a slim chance things will work out, helps one continue. Ambibuity nurishes hope. One cannot be hopeful when the outcome is certain. We would like to know how ambiguity can serve classroom mathematics. The emotions of challenge and hope are powerful motivations in problem-solving and deserve further research.

A more obvious way in which emotions sustain actions is in terms of longer range goals. The student who has a positive feeling solving one math problem is more likely to try another. The confidence that comes from understanding mathematics empowers the student to attempt new ventures also, as in the case of a geometry student who attributes his decision to help in crime prevention directly to his success in his geometry class.

RESTORERS

Lazarus offers a third function of positively toned emotions, that of restorer. Lazarus' descriptions of recovery from depression or restorations of self-esteem might be useful to the teacher dealing with math-anxious students. Lazarus quotes Klinger:

> At some time during clinical depression patients become unusually responsive to small successes. For instance, depressed patients working on small laboratory tasks try harder after successfully completing a task than after failing one, which is a pattern opposite to that of nondepressed individuals, who try harder after failure.

It would be worthwhile for the classroom teacher to know when small successes are more likely to evoke positive emotions. Offering a small task to a math anxious student may foster optimism and incentive while the same problem may seem trivial to a non-anxious student and provoke anger or disappointment. This is an area for more research.

Much of the information on emotion in problem-solving is obtained by having students fill out questionnaires. While the information is useful, a rating on a scale from one to five of confidence in doing math, liking for math, or usefulness of math is very general. Questionnaries also are remote from the actual process of problem-solving. Recollections of feelings might not be quite the same as the feelings at the time. Also, mathematical problem-solving requires intense attention to the problem. It is likely that without some help a solver will not even be aware of his or her emotions. The above reasons together with the belief that our own feelings when doing mathematics are the same as those of our students prompted us to do an exercise utilizing a close observer and introspection.

OBSERVATIONS FROM THE EXERCISE

Altogether the exercise of observing, reporting, solving, reporting was done by 19 pairs. Problems initially were of the puzzle variety (Gardiner,1967, 1979. Mott-Smith, 1954) but in later sessions more substantial problems were chosen from Honsberger. One person kept track of time for the whole group. A group of six people (three pairs) seems the best size. We posture...laughter...intent stillness" but that description is not used in this paper.

EMBARKING ON THE PROBLEM-SOLVING

Solvers accepted their problems with curiosity and positive anticipation. These were people who did formal mathematics frequently. Two people who had not done formal math recently reported terror.

The initial reading of the problem provoked a reaction to its type followed by a sense of its difficulty. "I anticipate I will enjoy this problem but may not make much progress." or "I loathe this type of problem . It is do-able but will require a big effort. I think I will have to go through many tedius decompositions."

The word "do-able" was used often and meant either that the problem was solvable or that progress could be made in understanding the question. For one of the people who reported terror, a person who rarely uses formal mathematics rarely and who was talked into coming to the workshop, considerable time was spent blocking the reading of the problem. Emotion can be regulated by avoidance or denial. This person acknowledged feeling bad but then felt bad about feeling bad so that "Even if I could do it I couldn't." Considerable time was spent recalling past history of problem solving failures all the while avoiding (somewhat consciously) making the decision to try to do the problem. Another solver also reported "I felt unhappy and then felt unhappy about feeling unhappy." Emotions tend to feed on and reinforce each other. The math oriented solvers were predisposed to extend effort on the problem. They had much more commitment to do math.

After reading the problem, all began to develop a notation, to draw a diagram or to write some hypothesis. This was the beginning of a cycle of attention on problem - attention on self or distraction by environment - attack on problem - attention to self or environment - problem - self - problem - self, etc.

When preparing to choose a method of attack, there was considerable emotion tied in with "not cheating." Each person placed the problem in a certain context and at a certain level of difficulty and felt it would be cheating, bad sport, to use a technique that was too powerful. One solver says, "Can I use fancy stuff?... Then I'll use Jordan Curve Theorem....laughs". Backtracks. "Maybe an easier way." Another solver resisted but finally made a grudging commitment to using calculus for a problem entitled, "An Obvious Maximization."

Using brute force was considered almost as bad as using a too powerful method. "I'm annoyed because I can't see any other way than brute force and that would not yield for me any understanding of the problem...there must be an easier way." Solvers wanted to find solutions that were generalizable. Using a too powerful method, brute force, or an "obvious method" brought forth comments of feeling embarrassed or annoyed.

A less conscious resistance to cheating was the seen in the imposition of ridiculous restrictions on oneself. For example, one solver had Honsberger's book in hand and was to "Use the 'Method of Reflection' to...". (Honsberger, p.70). The solver's reaction was, "I understand the problem but don't know this method...I wish I could read the chapter..." . Instead of simply reading the chapter, the solver tries to invent a plausible 'Method of Reflection'.

Another solver spent long moments seemingly aimless. "I'm feeling a little out of control of the problem...lots of parameters...seems to be a lot of ways to define this problem...I'd like to clarify the problem by asking whoever wrote it." Finally with a forced air, "I could break it up into cases myself and come to grips on my own terms and get partial solutions...got control back."

Self imposed restrictions would slow a solver down until there were reports of, "I'm squandering time. I really haven't done anything." Then there would be a squaring of the shoulders and a businesslike assertion to "...take a stand and try to prove it ..." even though this might mean grinding out a meaningless, albeit correct, solution.

INVOLVEMENT WITH THE PROBLEM

Once commitment was made to attempt the problem, there was a lorelai seductiveness about it, a delicious slipping off into another world. Solver became oblivious to self, observer, or environment. This total immersion was a wonderful release from daily life. Poland (CMESG, 1985) used mathematics to help him ignore the pain of an illness. Some people use the other-world quality of doing mathematics to avoid interaction with peers. Mathematics can help with depression as the famous mathematician Kovalevskaya said in a letter: "I am too depressed...in such moments, mathematics comes in handy, and one enjoys the existence of a world completely outside of oneself." (Knopp, 1985).

Mingled with the charm of seduction there was a dangerous quality, a frightening isolation if one stayed immersed too long. Rosamond (1982) gives examples in which the solver feels consumed by a too dominating mathematics. As one mathematics graduate student said with tears in his eyes, "What do you do if you are 80 - 90% mathematics? If you've let yourself become consumed by mathematics so that that is what you are. And then you want to let someone get to know you. What do you do when you can't explain that much of yourself to them?" The presence of the observer comforted the solver and lessened the dangerous quality in the isolation.

There was a letdown feeling of disapointment if the solution came so easily that little emotion needed to be invested in in the problem. Typical is the remark, "The problem must have been too easy, I got it. So what's the big deal? I feel let down."

or "It was fun but not intense because not a challenge. I feel let down because I didn't spend a lot of emotion." The complexity of the problem came like a revelation to one solver who then responded with a BIG smile. Overall, the amount of satisfaction with the problem correlated directly with the intensity of concentration. The perceived level of difficulty of the problem also influenced satisfaction and this will be discussed later.

However, one cannot maintain a constant level of intensity throughout the solving of a problem. The use of notation in a ritualistic manner provided a "breather" or moments of relaxation while allowing the solver to remain in the "other-world". When no progress was being made on a problem, the solver remained in the intense state by writing out some formal routine. Some solvers would rewrite the definition of the variable. One solver began, "There are two cases: a) the problem is solveable and b) the problem is not solveable." Almost everyone used x's and y's at one time and then decided to switch to a's and b's (or vice versa). Some would say, "I'm going to try induction." and then write out the induction hypotheses. The rote writing out of hypotheses or the rote switching of variables afforded a lull within the other-world state and continued the flow. The importance of these rituals was to help focus on the problem. To sit too long without progress or a ritual meant the solver would think about self again.

Other pauses also bump one out of concentration. When the solver paused overlong in appreciation of some success, then attention tended to turn to self or environment. The jolt of finding a counterexample to a hoped-for truth caused one to notice the ticking of the clock or the coldness of the room. Extended frustration of method caused recall of poor geometric visualization in the past and then embarrassment. Attention was diverted from the problem to the self. This usually was for a brief amount of time, less than a minute. Solvers would look around, sigh, stroke the pen, scratch, talk a little and then go back into the problem.

Most solvers were engrossed in the problem when time was called and these people were irritated at being interrupted. They almost all mumbled "I'll continue later." Solvers who were in an attention-outward part of the problem-solving cycle just prior to time being called generally sat back and waited out the time. They did not work on the problem further while waiting but mentioned that they would return to it later. There was reluctance to allow oneself to get lost in a train of thought and then yanked out of it.

IMPLICATIONS FOR THE CLASSROOM: VARIABLES THAT INFLUENCE ENGAGEMENT

The primary goal of our exercise is to improve classroom teaching. It would be useful for a teacher to know what a particular emotion looks like. For example, a teacher who knows that yawning is a release of nervous tension and not an indication of boredom have an immediate and obvious clue that a student needs help. (And the teacher knows not to get personally insulted by the yawn.) In the opposite direction, the teacher who wants to indicate positive emotions to the students would know how to do it because he or she would know what they look like.

To this end we took notice of some physiological indications of emotive arousal (flushed face, sweaty palms, muscle tension, etc.) and of body movement (twitching, sighing, laughing, etc.) but more work should be done here and these indications are not elaborated on in this paper.

We found that overall satisfaction in problem solving is directly related to the intensity of engagement with the problem. The engagement is influenced by several variables: the nature of the problem, the perceived usefulness of mathematics, the role of the observer, the use of mathematics rituals, and the testing situation. Each of these variables will be discussed along with their implications for the classroom.

NATURE OF THE PROBLEM

All solvers were more encouraged by harder problems than by ones marked "obvious" or ones perceived as easy. There had to be a sense of value of the problem, not that it must be directly applicable to daily life, but rather that one needed to think in order to understand the problem. If one could get the answer just by asking someone else or by looking it up then that made the problem artificial and was almost an affront to the solver.

Surprisingly, solvers felt threatened whenever they saw the words, "Clearly", "It is easy.", or "Obviously". Most felt that teachers should not say, "This is easy." and that textbooks should not indicate the easy exercises. Solvers sometimes worried that the problem looked so simple. They felt they were missing the point and that their solution was not elegant enough. One solver found three solutions by varying the constraints and then felt less humiliated.

One solver exhibited obvious arousal with eyes wide open, clear face and a slight laugh. "Hey, there's an infinite process..." Exploration didn't bear out infinite process and then there was "That was neat. What was the problem?" together with a clear drop of interest and rather emotionless settling again into the problem. The challange of the infinite process stimulated playing around in the "math-world."

The math-world is a mental out-of-body arena of intense concentration in which a person can play with ideas. Trivial problems do not make good play-mates. One solver's most satisfactory experience of problem-solving came after having spent a week on a problem only to have the professor tell the class that the problem was not solveable.

Solvers felt initial relief at seeing an easy problem but were quickly bored, disappointed or insulted. The classroom teacher must pay careful attention to the quality of problems offered and should not label them easy or difficult.

USEFULNESS OF MATHEMATICS

Doing mathematics is seductive but one must allow oneself to be seduced. Three different participants at three different sessions (all women) felt that going off and doing mathematics was a luxury. A teacher of older women said she had to convince her students that they were not squandering time while problem solving. Women are always productive. They even knit while watching TV. She got around her students' hesitancy by saying, "I'm going to show you some games to teach your kids and improve their math."

The notion of usefulness was mentioned by only three women but it is a construct that has been singled out as the most important attitudinal factor in decisions to take math classes (Sherman and Fennema, 1977.)

Usefulness was elaborated on at length by one solver waho was able to solve the assigned problem in a short time and with no intense engagement. The solver was disappointed and felt letdown. It was not clear if the following remarks would have been made had the solver been given a more engaging problem. I asked at the time but the solver was very agitated and insisted that another problem would have made no difference.

> "What would have beeen a meaningful problem? How come I'm not satisfied? I had an expectation about solving that problem that did not get fulfilled. It didn't make me happy. There were some moments of tension and some of excitement but not intense. It was entertaining like a grade C movie.

"Math has no social relevance to me...I am willing to solve math problems. even ready but it feels completely disjoint from what interests me. I still love it (This solver has a Masters degree in math and is an active MD.) but its importance seems miniscule compared to world problems...beautiful but frivolous to use my mind in this way." (It would be useful for other people to do math but there were more pressing issues for this particular solver.)

Usefulness of mathematics in terms of careers or its sometimes theraputic value as a means of escape is an affective variable that may be easy for teachers to influence. Teachers can present information about the mathematics required by various careers as well as the mathematics courses that should be taken to keep options open in the future.

THE USE OF RITUALS

The use of formal routines that keep one's attention on task while providing a sort of restful interlude speaks directly to the classroom teacher. Students must have a comfort with notation not only becausd the notation itself sometimes points to the solution but because that comfort sustains concentration.

ROLE OF OBSERVER

Contrary to almost everyone's expectation, having someone observe while working the mathematics was positive. At first, some solvers felt less inclined to free associate with ideas in front of an observer who might have the problem already all figured out or the solver sometimes felt that the observer must be bored. Some solvers wanted to talk things over with their observer or would look up at the observer hoping for confirmation.

It turned out that the presence of the observer was an impetus to persistence in doing the problem. This is a very important point. Liking the problem was directly and positively related to the amount of time spent working on it. Almost everyone liked their problem more the longer they worked. Those that did not like their problem initially began to like it after all and to get interested in it. Without an observer, those solvers might have quit.

Being observed evoked other feelings. As noted earlier, the presence of an observer reduced the feeling of danger in isolation that lengthy immersion in the problem sometimes brought. There was a feeling of honor. "I felt honored that another peson was taking the time to observe me." Another feeling was intimacy. "It felt intimate to have someone commited to watch the workings of my mind."

While more emotion seemed to come from being watched, it was also important to be the watcher. Watching seemed to take away some of the secret charge of the observer's own problem-solving anxieties. The observer could recognize his or her own feelings in the other person and see how the feelings influenced their actions. Watching another person struggle with anxieties made the solver think, "Why don't they just get on with it."

One participant reported, "The most poignant part of the exercise was hearing the observer say what I'd done. I did not feel intimidated. I didn't get any of the bad response I expected. The observer demystified my emotional and intellectual engagement by simply listing what I did: 1, 2, 3, 4. This cut it down to size, gave it true proportion."

This exercise of being observer then reporter, then switching to being solver then recipient of report should be explored as a means of eliminating math anxieties in our students. The real key is the switching. This exposes and throws out the power of negative feelings while encouraging positive ones.

It should be noted that no one argued with their observer. A few points of clarification were made but there were no misinterpretations. It is possible that finer gradations or other categories of feelings can be made, but there was good correspondence within our vocabulary.

THE TESTING SITUATION

Concern about the nature of the problem carries over into the testing situation. One solver commented on the problems found on math tests. "A test is an almost random set of narrow problems where one thing must trigger another. It is not about figuring things out. Test questions do not show that math is a process." This solver had as a partner a professional research mathematician. The solver was not intimidated by being observed even though the problem was not solved because "The observer could hear that I have math training. He could see how my math mind works, how I assimilate information, manipulate, and use an asrsenal of strategies. This is so much different from taking a math test where I am not tested on how my mind works. On a math test, I could expect not to be able to show what I know. I would feel shame."

Part of almost any testing situation is a time constraint. Having only 15 or 30 minutes annoyed and inhibited these solvers. Some reported feeling "hemmed in...I do best by playing around...ordinarily would draw pictures and really understand...build up a pattern." Another felt pressure to categorize a solution method quickly. "Without a time constraint I probably would have been more impulsive...would have guessed and then worked backward. I felt forced to be more systematic, meticulous, more step-by-step and mechanical. I think I could have solved this in a shorter amount of time if there had been no time limit."

When the timing in itself counts, it is as though what the problem means in itself is not enough. Perhaps the discomfort of a time constraint forces one's attention to be divided between the math-world and present time. Not only are different methods of solution chosen at the onset of the process, but also the total immersion into the problem-world is not as possible or as deep.

CONCLUSION

It is important to state that a basic assumption of this experiment is that we professional teachers and mathematicians have at least the same feelings that students have. We may experience a difference in intensity (less anxiety, more confidence) or have other feelings in addition (sense of commitment) but overall how we respond gives an indication of how our students respond. A mathematics educator refused to participate in our exercise saying that it might be worthwhile for "personal growth" but that it would give no insight into how students feel. He believes that teacher feelings are completely different from student feelings.

But imagine your feelings if the Chair of your Math Department suddenly announced that you must take a test. If you have not taught a particular course in the past two years you must pass a test before you can teach it. What course are you scheduled to teach that you have not taught recently? What is your reaction to your Chair's announcement? You are not being tested on how well you review the materal during the semester or on how carefully you prepare your lessons. You are not being asked to share ideas with a colleague. You are being evaluated on questions someone else has chosen and already knows the answers to. I think your reaction to this thought-experiment may show that seasoned teachers can feel anxiety in a test situation similar to what their students feel in their test situations.

The act of knowing is not antiseptic; rather it is wrapped in feelings. It is the engagement of feelings. The primary goal of our work is to improve classroom teaching. This paper indicated only a few of the emotions inseparately connected within mathematical activity and specifically calls the classroom teacher's attention to the nature of the problems, the perceived usefulness of mathematics, the role of observer, the use of mathematics rituals and the testing situation.

REFERENCES

Canadian Mathematics Education Study Group Announcement for the 1985 Proceedings. Universite Laval, June 1985.

Davis, R. B. Learning Mathematics: The Cognitive Science Approach to Mathematics Education. Norwood, NJ: Ablex, 1984.

Gardner, M. Mathematical Circus. New York: Alfred Knopf, 1979.

Gardner, M. The Scientific American Book of Mathematical Puzzles and Diversions. New York: Simon and Schuster, 1967.

Honsberger, R. Mathematical Gems: Dolciani Mathematical Expositions. The Mathematics Association of America, 1973.

Honsberger, R. Ingenuity in Mathematics: New Mathematical Library. The Mathematics Association of America, 1970.

Knopp, K. "Mathematics as a Cultural Activity." The Mathematical Intelligencer Vol. 7, No.1, 1985, p.7.

Lazarus, R.S., Kanner, A.D., and Folkman,S. "Emotions: a Cognitive-Phenomenological Analysis." In R. Plutchik and H. Kellerman (eds.), Emotion: Theory, Research and Experience, Vol. 1 New York: Academic Press, 1980.

Mott-Smith, G. Mathematical Puzzles for Beginners and Enthusiasts. New York: Dover Publications, 1954.

Papert, S. Mindstorms: Children, Computers, and Powerful Ideas. New York: Basic Books, 1980.

Perry, W.G., Forms of Intellectual and Ethical Development in the College Years: A Scheme. New York: Holt, Rinehart, Winston,

Rosamond, F. and Poland, J. "The Role of Feelings in Learning Mathematics." CMESG/GCEDM Proceedings: Universite Laval, June 1985.

Rosamond, F. "Listening to Our Students." in David Wheeler (ed.) For the Learning of Mathematics 3, (1) Montreal: FLM Publishing Association, July 1982.

Schoenfeld, A.H. "Beyond the Purely Cognitive: Belief Systems, Social Cognitions, and Metacognitions as Driving Forces in Intellectual Performance." Cognitive Science 7, 1983.

Sherman, J. and Fennema, E. "The Study of Mathematics by High School Girls and Boys: Related Variables." American Educational Research Journal 14, 1977.

Silver, E.A.(Ed) Teaching and Learning Mathematical Problem-Solving: Multiple Research Perspectives. NJ: Lawrence Erlbaum Associates, 1985.