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Applied Mathematics as Social Contract

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Abstract

The author takes the position that mathematical education must redefine its goals so as to create a citizenry with sufficient knowledge to provide social backpressure on future mathematizations. This can be accomplished by increasing the part of mathematical education that is devoted to the description and interpretation of the processes of mathematization and by allowing the technicalities of the formal operations within mathematics itself to be deemphasized or automated out by computer.

This Mathematized World

As compared to the medieval world or the world of antiquity, today's world is characterized as being scientific, technological, rational and mathematized. By "rational" I mean that by an application of reason or of the formalized versions of reason found in mathematics, one attempts to understand the world and control the world. By "mathematized", I shall mean the employment of mathematical ideas or constructs, either in their theoretical form or in computer manifestations, to organize, to describe, to regulate and to foster our human activities. By adding the suffix "ized", I want to emphasize that it is humans who, consciously or unconsciously, are putting the mathematizations into place and who are affected by them. It is of vital importance to give some account of mathematics as a human institution, to arrive at an understanding of its operation and at a philosophy consonant with our experience with it, and on this basis to make recommendations for future mathematical education.

The pace of mathematization of the world has been accelerating. It makes an interesting exercise for young students to count how many numbers are found on the front page of the daily paper. The mere number of numbers is surprising, as well as the diversity and depth of the mathematics that underlies the numbers; and if one turns to the financial pages or the sports pages, one sees there natural language overwhelmed by digits and statistics. Computerization represents the effective means for the realization of current mathematizations . as well as an independent driving force toward the installation of an increasing number of mathematizations.

Philosophies of Mathematics

Take any statement of mathematics such as 'two plus two equals four', or any more advanced statement. The common view is that such a statement is perfect in its precision and in its truth, is absolute in its objectivity, is universally interpretable, is eternally valid and expresses something that must be true in this world and in all possible worlds. What is mathematical is certain. This view, as it relates, for example, to the history of art and the utilization of mathematical perspective has been expressed by Sir Kenneth Clark: ("Landscape into Art"): "The Florentines demanded more than an empirical or intuitive rendering of space. They demanded that art should be concerned with *certezza*, not with *opinioni*. Certezza can be established by mathematics".

The view that mathematics represents a *timeless* ideal of absolute truth and objectivity and is even of nearly divine origin is often called Platonist. It conflicts with the obvious fact that we humans have invented or discovered mathematics, that we have installed mathematics in a variety of places both in the arrangements of our daily lives and in our attempts to understand the physical world. In most cases, we can point to the individuals who did the inventing or made the discovery or the installation, citing names and dates. Platonism conflicts with the fact that mathematical applications are often *conventional* in the sense that mathematizations other than the ones installed are quite feasible (e.g., the decimal system). The applications are often *gratuitous*, in the sense that humans can and have lived out their lives without them (e.g., insurance or gambling schemes). They are *provisional* in the sense that alternative schemes are often installed which are claimed to do a better job (Examples range all the way from tax legislation to Newtonian mechanics). Opposed to the Platonic view is the view that a mathematical experience combines the external world with our interpretation of it, via the particular structure of our brains and senses, and through our interaction with one another as communicating, reasoning beings organized into social groups.

The perception of mathematics as quasi-divine prevents us from seeing that we are surrounded by mathematics because we have extracted it out of unintellectualized space, quantity, pattern, arrangement, sequential order, change, and that as a consequence, mathematics has become a major modality by which we express our ideas about these matters. The conflicting views, as to whether mathematics exists independently of humans or whether it is a human phenomenon, and the emphasis that tradition has placed on the former view, leads us to shy away from studying the processes of mathematization, to shy away from asking embarrassing questions about this process: how do we install the mathematizations, why do we install them, what are they doing for us or to us, do we need them, do we want them, on what basis do we justify them. But the discussion of such questions is becoming increasingly important as the mathematical vision transforms our world, often in unforeseen ways, as it both sustains and binds us in its steady and unconscious operation. Mathematics creates a reality that characterizes our age.

The traditional philosophies of mathematics: platonism, logicism, formalism, intuitionism, in any of their varieties, assert that mathematics expresses precise, eternal relationships between atemporal mental objects. These philosophies are what Thomas Tymoczko has called "private" theories. In a private theory, there is one ideal mathematician at work, isolated from the rest of humanity and from the world, who creates or discovers mathematics by his own logico-intuitive processes. As Tymoczko points out, private theories of the philosophy of mathematics provide no account either for mathematical research as it is actually carried out, for the applications of mathematics as they actually come about, or for the teaching process as it actually unfolds. When teaching goes on under the banner of conventional philosophies of mathematics, it often becomes to a formalist approach to mathematical education: "do this, do that, write this here and not there, punch this button, call in that program, apply this definition and that theorem." It stresses operations. It does not balance operations with an understanding of the nature or the consequences of the operations. It stresses syntactics at the expense of semantics, form at the expense of meaning. A fine place to read about this is in "L'age du Capitaine" by Stella Baruk, a mathematics supervisor in a French school system. Baruk writes

> "From Pythagoras in antiquity to Bourbaki in our own day, there has been maintained a tradition of instruction - religion which sacrifices

full understanding to the recitation of formal and ritual chatechisms, which create docility and which simulate sense. All this has gone on while the High Priests of the subject laugh in their corners."

How many university lecturers, discoursing on numbers, say, allow themselves to discuss where they think numbers come from, what is one's intuition about them, how number concepts have changed, what applications they have elicited, what have been the pressures exerted by applications, how we are to interpret the consequences of these applications, what is the poetry of numbers is or their drama or their mysticism, why there can be no complete or final understanding of them. How many lecturers would take time to discuss the question put by Bertrand Russell in a relaxed moment: "What is the Pythagorean power by which number holds sway above the flux?"

Opposed to "private" theories, there are "public" theories of the philosophy of mathematics in which the teaching process is of central importance. Several writers in the past half century have been constructing public theories, and I should like to add a few bricks to this growing edifice and to point out its relevance for the future of mathematical education.

Applied Mathematics as Social Contract

I shall emphasize the applications of mathematics to the social or humanistic areas though one can make a case for applications to scientific areas and indeed to pure mathematics itself. (See, e.g., Spalt).

Today's world is full of mathematizations that were not here last year or ten years ago. There are other mathematizations which have been discarded (e.g., Ptolemaic astronomy, numerological interpretations of the cosmos, last years' tax laws) which have been discarded. How do these mathematizations come about? How are they implemented, why are they accepted? Some are so new, for example, credit cards, that we can actually document their installation. Some are so ancient, e.g., numbers themselves, that the historical scenarios that have been written are largely speculative. Are mathematizations put in place by divine fiat or revelation? By a convention of Elders? By the insights of a gifted few? By an evolutionary process? By the forces of the market place or of biology? And once they are in place what keeps them there? Law? Compulsion? Inertia? Darwinian advantage? The development of a bureaucracy where sole function it is to maintain the mathematization? The development of businesses whose function it is to create and sell the mathematization? Well, all of the above, at times, and more. But, for all the lavish attention that our historians of mathematics have paid to the evolution of ideas within mathematics itself, only token attention has been paid by scholars and teachers to the interrelationship between mathematics and society. A description of mathematics as a human institution would be complex indeed, and not be easily epitomized by a catch phrase or two.

The employment of mathematics in a social context is the imposition of a certain order, a certain type of organization. Government, as well, is a certain type of organization and order. Philosophers of the 17th and 18th-century (Hobbes, Locke, Rousseau, Thomas Paine, etc.) put forward an idea, known as social contract, to explain the origin of government. Social contract is an act by which an agreed upon form of social organization is established. (Here I follow an article by Michael Levin.) Prior to the contract there was supposedly a "state of nature". This was far from ideal. The object of the contract, as Rousseau put it, was "to find a form of association which will defend the person and goods of each member with the collective force of all, and under which each individual, while counting himself with the others, obeys no one but himself, and remains as free as before". In this way, one may improve on a life which, as Hobbes put it in a famous sentence, was "solitary, poor, nasty, brutish, and short". The contract itself, whether oral or written, was almost thought of as having been entered into at a definite time and place. Old Testament history, with its covenants between God and Noah, Abraham, Moses, the Children of Israel, was clearly in the minds of contract theorists. In the United States, political thinking has often been in terms of contracts, as in the Mayflower Compact, the Constitutions of the United States and of the individual states, the Establishment of the United Nations in San Francisco in 1945, and periodic proposals for constitutional amendments and reform.

It was generally assumed by the contract theorists that "Human society and government are the work of man constructed according to human will even if sometimes operating under divine guidance". That "man is a free agent, rather than a being totally determined by external forces", and that society and government are based on mutual agreement rather than on force. (See Levin)

The acceptability of social contract as a historical explanation hardly lasted till the 19th century, even if political contracts continued to be entered into as instances of democratic polity. It is an instructive exercise, I believe, in order to get a grasp on the relationship between society and mathematics, to take the outline of social contract just given and replace the words "government and society" by the word "mathematization". Though it is naive to think that most mathematizations came about by formal contracts, the "contract" metaphor is a useful phrase to designate the interplay between people and their mathematics and to make the point that mathematizations are the work of man, constructed according to human will, even if operating under a guidance which may be termed divine or logical or experimental according to one's philosophic predilection.

A number of authors, some writing about theology, and others about political or economic processes, have pointed out that contracts are continuously entered into, broken, and reestablished. I believe the same is the case for mathematizations. Consider, for example, insurance. This is one of the great mathematizations currently in place, and I personally, without adequate coverage, would consider myself naked to the world. Yet I am free to throw away my insurance policies. Consider the riders that insurance companies send me, unilaterally abridging their previous agreements. Consider also that in a litigious age, with a populace abetted by eager lawyers and unthinking juries, what appears as the 'natural' stability of the averages upon which possibility of insurance is based, emerges, on deeper analysis, to incorporate the willingness of the community to adjust its affairs in such a way that the averages are maintained. The possibilities of insurance can be destroyed by our own actions.

Another example that displays the relationship between mathematics, experience, and law is the highway speed limit in the United States. Before the gas shortage in 1974, the limit was 65 miles per hour. In 1974, the speed limits were reduced to 55 miles per hour in order to conserve gasoline.

As a side effect, it was found that the number of highway accidents was reduced significantly. Now (1987) the gas shortage is over, and there is pressure to raise the legal speed limit. Society must decide what price it is willing to pay for what some see as the convenience or the thrill of higher speeds. Here is mathematical contract at work.

The process of contract maintenance, renewal or reaffirmation, in all its complexities, is open to study and description. This is a proper part of applied mathematics and I shall argue that it should be a proper part of mathematical education.

Where is Knowledge Lodged?

There is an epistmological approach to the interplay between mathematics and society and that is to look at the way society answers the question that heads this section. According to how we answer this question, we will mathematize differently and we will teach differently.

Where, then, is knowledge lodged? (Here I follow an article by Kenneth A. Bruffee.) In the pre-Cartesian age, knowledge was often thought to be lodged in the mind of God. Those who imparted knowledge authoritatively derived their authority from their closeness to the mind of God, evidence of this closeness was often taken to be the personal godliness of the authority.

In the post-Cartesian age knowledge was thought to be lodged in some loci that are above and beyond ourselves, such as sound reasoning or creative genius or in the 'object objectively known'.

A more recent view, connected perhaps with the names of Kuhn and Lakatos, is that knowledge is socially justified belief. In this view, knowledge is not located in the written word or in symbols of whatever kind. It is located in the community of practitioners. We do not create this knowledge as individuals but we do it as part of a belief community. Ordinary individuals gain knowledge by making contact with the community of experts. The teacher is a representative of the belief community.

In my view, knowledge as socially justified belief provides a fair description of how mathematical knowledge is ligitimized but we must keep clearly in mind that perceptions of what 'is', theory formation, validation, and utilization, are all part of a dynamic and iterative process. Knowledge once thought to be absolute, indubitable, is now seen as provisional or even probabilistic. Science is seen as a search for error as much as it is a search for truth. Eternally valid knowledge, may remain an ideal which we hold in our minds as a spur to inquiry. This view fits with the idea of applied mathematics as social contract, with the contractual arrangements being concluded, broken, and renegotiated in endless succession.

Another view of the locus of knowledge, not yet elevated to a philosophy, is that knowledge is located in the computer. One speaks of such things as 'artificial intelligence', 'expert systems', and more than one theoretical physicist has opined that all the essentials are now known (despite the fact that the same was asserted 100 years ago and 200 years ago) and that the computer can fill in the details and derive the consequences for the future.

Advocates of this view have asserted that while education is now teacher oriented, in the full bloom of the computer age, education will be knowledge oriented. These two contemporary views are not necessarily antithetical, provided we accommodate the computer into the community of experts, clarify whether 'belief' can reside in a computer, and decide whether mankind exists for the sake of the computers or vice versa.

Mathematical Education at a Higher Metalevel

A mathematized and computerized world brings with it many benefits and many dangers. It opens many avenues and closes many others. I do not want to elaborate this point as I and my co-author Reuben Hersh have done so in our book "Descartes' Dream", as have numerous other authors.

The benefits and dangers both derive from the fact that the mathematical/computational way of thinking is different from other ways. Philosopher and historian Sir Isaiah Berlin called attention to this divergence when he wrote "A person who lacks common intelligence can be a physicist of genius, but not even a mediocre historian." For the mathematical way to gain ascendancy over other modes is to create an imbalance in human life.

The benefits and dangers derive also from the fact that mathematics is a kind of language, and this language creates a milieu for thought that is hard to escape. It both sustains us and confines us. As George Steiner has written of natural language (1986): "The oppressive birthright is the language, the conventions of identification and perception. It is the established but customarily subconscious unargued constraints of awareness that enslave." One can assert as much for mathematics as a language. The subconscious modalities of mathematics and of its applications must be made clear, must be taught, watched, argued. Since we are all consumers of mathematics, and since we are both beneficiaries as well as victims, all mathematizations ought to be opened up in the public forums where ideas are debated. These debates ought to begin in the secondary school.

Discussions of changes in mathematics curricula generally center around (a) the specific mathematical topics to be taught, e.g., whether to teach the square root algorithm, or continued fractions or projective geometry or Boolean algebra, and if so, in what grade, and (b) the instructional approaches to the specific topics, e.g., should they be taught with proofs or without; from the concrete to the abstract or vice versa, what emphasis should be placed on formal manipulations and what on intuitive understanding; with computers or without; with open ended problems or with "plug and chug" drilling.

Because of widespread, almost universal computerization, with handheld computers that carry out formal manipulations and computations of lower and higher mathematics rapidly and routinely, because also of the growing number of mathematizations, I should like to argue that mathematics instruction should, over the next generation, be *radically* changed. It should be moved up from subject oriented instruction to instruction in what the mathematical structures and processes mean in their own terms and what they mean when they form a basis on which civilization conducts its affairs. The emphasis in mathematics instruction ought to be moved from the syntactic-logico component to the semantic component. To use programming jargon, it ought to be "popped up" a metalevel. If, as some computer scientists believe, instruction is to move from being teacher oriented to knowledge-oriented - and I believe this would be disastrous - the way in which the role of the teacher can be preserved is for the teacher to become an interpreter and a critic of the mathematical processes and of the wce.ay these processes interact with knowledge as a database. Instruction in mathematics must enter an altogether new and revolutionary phase.

The Interpretive Component of Mathematical Education

Let me begin by asking the question: to what end do we teach mathematics? Over the millenia, answers have been given and they have differed. Some of them have been: we teach it for its own sake, because it is beautiful; we teach it because it reveals the divine; because it helps us think logically; because it is the language of science and helps us to understand and reveal the world; we teach mathematics because it helps our students to get a job either directly, in those areas of social or physical sciences that require mathematics; or indirectly, insofar as mathematics, through testing, acts as a social filter, admitting to certain professional possibilities those who can master the material. We teach it also to reproduce ourselves by producing future research mathematicians and mathematics teachers.

Ask the inverse question: what is it that we want students to learn? We may answer this by citing specific course contents. For example, we may say that we now want to emphasize discrete mathematics as opposed to continuous mathematics. Or that we want to develop a course in non-standard analysis on tape so that joggers may learn about hyperreal numbers even as they run. Or, we may decide for ourselves what the characteristic, constitutive ingredients of mathematical thought are: space, quantity, deductive structures, algorithms, abstraction, generalization, etc.., and simply assure that the student is fed these basic ingredients, like vitamins. All of these questions and answers have some validity, and tradeoffs must occur in laying out a curriculum.

Within an overcrowded mathematical arena with many new ideas competing for inclusion in a curriculum, I am asking for a substantial elevation in the awareness of the applications of mathematics that affect society and of the consequences of these applications. If formal computations and manipulations can be learned rapidly and performed routinely be computer, what purpose would be served by tedious drilling either by hand or by computer? On-the-job training is certainly called for, whether at the supermarket checkout counter or on the blackboards of a hi-tech development company. If mathematics is a language, it is time to put an end to overconcentration on its grammar and to study the "literature" that mathematics has created and to interpret that literature. If mathematics is a logico-mechanism of a sort, then just as only a very few of us learn how to construct an automobile carburetor, but all of us take instruction in driving, so we must teach how to "drive" mathematically and *to interpret what it means when we have been driven mathematically in a certain manner.*

What does it mean when we are asked to create sex-free insurance pools? What are the consequences when people are admitted or excluded from a program on the basis of numerical criteria? How does one assess a statement that procedure A is usually effective in dealing with medical condition B? What does it mean when a mathematical criterion is employed to judge the quality of prose or the comprehensibility of a poem or to create music in a programmatic way? What are the consequences of a computer program whose output is automatic military retaliation? The list of questions that need discussion is endless. Each mathematization-computerization requires explanation and interpretation and assessment. None of these things are now discussed in mathematics courses in the concrete form that confronts the public. If a teacher were inclined to do so, the reaction from his colleagues would probably be "Well, that is not mathematics. That is applied mathematics or that is psychology or economics or social-anthropology or law or whatever." My answer would be: I am trying, little by little, to bring in discussions of this sort into my teaching. It is difficult but important.

If the claim were made, with justice, that these matters cannot be discussed intelligently without deep knowledge both of mathematics and of the particular area of the real world, then I would agree, and point out that this claim forces into the open the conflict between democracy and "expertocracy". (See e.g., Prewitt). This conflict has received considerable attention in areas such as medicine, defense, and technological pollution, but has hardly been discussed at the level of an underlying mathematical language. The tension between the two claims, that of democracy and of expertocracy, could be made more socially productive by an education which enables a wide public to arrive at deeper assessments, moving from daily experience toward the details of the particular mathematizations. While we must keep in mind certain basic mathematical material, we must also learn to develop mathematical 'street smarts' which enable us to form judgements in the absence of technical expertise. (Cf. Prewitt).

A philosophy of mathematics which is "public" and not "private", lends support to introducting this kind of material into the curriculum. The discussion of such curriculum changes will be assisted by the perception of the mathematical enterprise as a human experience with contractual elements; and by the realization that every civilized person practices and utilizes mathematics at some level, and thereby enters a certain knowledge and belief community.

Again, following Kenneth Bruffees' article (with additions and modifications), I would like to suggest several lines of inquiry.

(a) Identify and describe the mathematical beliefs, constructions, practices that are now in place. Where and how is mathematics employed in real life?

(b) Describe the mathematical beliefs, constructs, and practices that have been justified by the community. What are justifiable and unjustifiable? What are the modes of justification?

(c) Describe the social dimensions of mathematical practice. What constitutes a knowledge community? What does the community of mathematicians think are the best examples that the past has to offer?

As part both of (a) and (b) one should add: describe the nature of the various methods of *prediction* and the bases upon which prediction can become prescription (i.e., policy).

This type of inquiry is rarely carried out for mathematics. For example, the concrete question of where such and such a piece of university mathematics is used in practical life and how widespread is its use, is seldom answered. Many textbook claims are made in textbooks, but show me the real bottom line. It is important to know. How, in fact, would you define the bottom line?

The technical term for inquiries such as the above is 'hermeneutics'. This word is well established in theology, and in the last generation has been commonly employed in literary criticism. It means the principles or the lines along which explanation and interpretation are carried out. It is time that this word be given a mathematical context. Instruction in mathematics must enter a hermeneutic phase. This is the price that must be paid for the sudden, massive and revolutionary intrusion of mathematizations - computerizations into our daily lives.

Conclusion

Mathematics is a social practice. This practice must be made the object of description and interpretation. It is ill-advised to allow the practice to proceed blindly by "mindless market forces" or as the result of the private decisions of a cadre of experts. Mathematical education must find a proper vocabulary of description and interpretation so that we are enabled to live in a mathematized world and to contribute to this world with intelligence.

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