Humanistic Mathematics Network Journal

Issue 4 Article 13

12-1-1989

An Empowering, Participatory Research Model for Humanistic Mathematics Pedagogy

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Introduction

Within the community of mathematicians and mathematics educators who identify with the term, 'humanistic mathematics,' an agreement on its meaning is still under negotiation. However, discussions in the community have been skewed toward improving the teaching and learning of mathematics, a member of the "hard sciences," by attending to its decidedly human dimensions. Therefore, a primary concern of the community is the possibility of "teaching humanistically." Abstracting from White's description of the concerns of humanistic mathematics, we have distinguished four processes involved in teaching humanistically: (1) placing students more centrally in the position of the inquirer, (2) acknowledging the emotional climate of learning mathematics, (3) having students learn from each other, and (4) making mathematics meaningful rather than arbitrary(White, 1987, p1)

Though these four processes represent a significant departure from typical concerns of the prevailing "chalk-and-chalk" pedagogy, they, nevertheless, are somewhat limited. What we propose is a broader, more inclusive vision of the third process distinguished above by including the notion of interdependent learning between students and instructors. Further, we propose that 'teaching humanistically' ought to involve an additional, or fifth, process which attends to the more general, human process of empowerment. Our concern is for the empowerment of all actors in various settings of mathematics education.

To facilitate interdependent learning and empowerment and to apply the by products of these processes to improving mathematics pedagogy is a principal function of participatory research activities. Research activities into pedagogy are participatory, and potentially empowering, when they give authority to the voices of students. For they generally feel, and are often considered, to be without power in many instructional settings. To give authority to their voices and to incorporate their perspectives in transforming mathematics pedagogy, instructors can start most profitably by listening to students.

There exists much anecdotal evidence to show that listening to students is important in improving their attitudes toward mathematics along with their performance. Reflecting on the effectiveness of instructional programs targeted at special groups of students, Lax (1988) suggested that, independent of the population of students, a common underlying spirit contributed to the success of these programs. Lax concluded that the authors of these programs "had gone to considerable lengths to find out who the students in these programs were, where they came from, what went on in their heads and hearts when they worked on math and how well and by what means they could cope with problems in their out-of-school life." This conclusion supports the research of Rosamond (1982) who, in the context of mathematics-assistance laboratory at a large, prestigious university, documented that listening to students can positively effect their learning.

In a course, one effective and efficient vehicle for "listening" to all students is journals. Certain types of journals writing activities have been shown (Countryman, 1985; Gopen and Smith, in press; Hoffman and Powell, 1989; Lopez and Powell, 1989[1]; Mett,1987; Powell, 1986) to be efficacious vehicles for a number of pedagogical imperatives. Among these imperatives, Lopez and Powell (1989) described some of what can be "heard" from students through their journals. In their case study, they identified affective and cognitive items. Affective ones included preoccupations, dispositions, and feelings; and some cognitive items were what students know, what they have yet to know, misconceptions, and discrepancies between conceptual understandings and computational and algebraic manipulations.

Their case study was a participatory investigation since Lopez was both the student whose journal writings were analyzed and a co-investigator analyzing those writings. In addition to discovering that journal writing improved Lopez's affective and cognitive functionings in mathematics, the results of the study indicated that the dynamics of the student-instructor collaboration con-

[1]This paper was originally presented at the writing and mathematics session of the 1988 annual joint mathematics meeting, Atlanta, Georgia.

tributed to the overall empowering effect of the case study (Lopez and Powell, 1989). This effect raises some important questions concerning components of the student-instructor collaboration that might have contributed to these findings.

That study also generated a number of questions about writing as a vehicle to learn mathematics. The present investigation examined one of these questions: In what ways can personal, reflective journal writings best support the enhancement of mathematical thinking? Two related sub-questions also guided the investigation.

1. Do students' writings display their attempts to specialize and generalize as well as to make conjectures and to provide justifications for them?
2. How does writing help students to construct or negotiate meaning?

To investigate these questions, a team was assembled that included students and the instructor. In the process of the investigation, we conjectured that it would be worthwhile to examine the nature of our collaboration and its qualitative and transformative effects on the substance of both learning and teaching. In this report, we present the results of the investigation into the empowering effects of our participatory research model and suggest its relationship to a humanistic mathematics perspective.

Setting

This study was conducted, in the fall semester of 1988, in one section of a computation course, Developmental Mathematics I, at the Newark College of Arts and Sciences. The college, whose students are primarily commuters, is an urban campus of Rutgers, the State University of New Jersey. The course includes the study of some concepts of number theory, fractions, decimals, percents, and word problems as well as an introduction to elementary algebra. These topics, more or less common to such courses, were taught through a not so common pedagogical approach. It is based on an approach and course material developed by Hoffman and Powell (1988, 1987), both of which depart fundamentally from those within a "chalk-and-chalk" paradigm.

The course met three times a week for fourteen weeks and had an initial enrollment of twenty-six students out of which seventeen completed the course. Most were first-year students, and all were placed in the course on the basis of their performance on the New Jersey Test of Basic Skills or, on an in-house instrument, the Mathematics Placement Test. The content of both instruments is arithmetic computation and elementary algebra.

Based on previous scholastic experiences, many students in Developmental Mathematics I have fostered negative feelings and beliefs about mathematics and themselves as mathematics learners. A student expressed one such view as, "Mathematics is something you do, not something you understand." Like students in similar settings (Buerk, 1982) and generally (McKnight et al., 1987, pp. 42-49), most students in this course consider mathematics not only as an abstruse symbol system but also an arcane and fixed body of knowledge whose secrets almost never reveal themselves though they are expected to demonstrate a degree of mastery. They have developed an estranged relationship with academic mathematics which manifests itself in their relative high level of mathematics anxiety and phobia. This estrangement is also manifested in students' developed strategies of avoidance which include their learning passivity, inappropriate study routines, and reluctance to participate actively in class. In essence, these behaviors are manifestations of interacting sets of low expectations that students have for themselves and that most remedial programs have of them. For many students, the force of these debilitating expectations effectively have silenced and marginalized them in mathematics and related disciplines.

Method

To counter and reverse the disempowering effects of these expectations, a participatory research model was selected as the methodological process of this investigation. The process of journal writing complemented this method since writing requires an active rather than a passive involvement of learners. Focusing on these processes, this investigation aimed to empower students in the following ways:

- to promote students' awareness of and facility in the use of writing as a vehicle for learning,
- to put students at the center and in control of their own learning by engaging them in reflection and critical reflection on mathematical experiences,
- to provide opportunities for students to reflect on and transform the affective and cognitive effects of silence and marginalization, and

- to give space interdependent learning between the instructor and students by valuing their voices and so that they could affect instruction and learning.

The investigation attempted to realize these aims by involving students as co-investigators and through journal writing. Students were asked to write journals daily, or at least for each class or assignment, on any topic or issue related to their learning of or feelings about the mathematics of the course or the course itself. To help remove the chore-like conception some have of writing and to relieve anxieties many associate with the quantity to be produced, students were advised that five minutes of writing was sufficient for a journal entry. After adjusting to the idea of writing journals for a mathematics class, many found themselves spending quality time expressing their thoughts. Only to stimulate thought and reflection, a list of topics was offered (see Appendix A).

Journals were collected weekly and returned with comments on the substance of what was written. The comments were intended to be non-judgmental and, most often, took the form of questions about or suggestions on issues, ideas, and so on that students discussed to encourage them to explore further. The objective was to use journal writing as a tool for learning mathematics. Therefore, it was emphasized to students that neither their grammar nor syntax were of concern, only what they had to say. Aside from moral and other intrinsic incentives, neither penalties nor rewards, in the form of grades or otherwise, were given.

Chronologically, the participatory research model consisted of five stage: information, selection of research collaborators, background meetings, weekly meetings, and post-semester meetings. The information stage occurred during the second week of the semester. At the time, the nature and objectives of this study were discussed with the class verbally and in a letter, and research collaborators were solicited (see Appendix B). Students were asked to respond in writing, explaining whether they wished to be a research collaborator and why.

In the selection of research collaborators stage, students were chosen from among those who responded affirmatively to the letter. Three students (Selby, Sheridan, and Walker) did so and were accepted as collaborators. In the fifth week of the semester, another student (Jeffries) was encouraged to and did join the research team. Each student either held a part-time job or was involved in a College-sponsored sports team. Before the end of the semester, one student withdrew from the College to accommodate his need to work full-time, while the demands of work and other course work lead another to drop out of the project. Along with the instructor, the two student collaborators who remained are the co-authors of this paper and whose work and interactions in the team are the bases of this investigation.

Before the weekly meeting stage, Powell met to discuss with each student their history with mathematics. By the third week of the semester, weekly meetings of the research team were held. At these meetings, which were approximately an hour and twenty minutes each, collaborators distributed among themselves copies of their journals, instructor's comments, from the previous week. During these meetings, student collaborators reflected on written evidence of mathematical thinking and any other striking feature of another's journal and wrote their reflections. Afterward, Powell and the students read and discussed their comments as well as raised questions concerning the course. Finally, from the journal writing and class discussions, the team identified twenty-eight processes that it found and determined were involved in thinking mathematically (see Appendix C).

During post-semester meetings, the research reviewed, discussed, analyzed its data. These consisted of the following:

- the weekly journals of the student collaborators,
- their and the instructor's comments and analysis of each journal entry, and
- tape recording of discussions among team members on the nature of the participatory research activity and its effects on both students' learning and the instructor's teaching.

Results

The two student authors became research participants in different ways and for different reasons. Selby responded to Powell's letter immediately and perceived the project as an opportunity to confront her fears of mathematics. The following are excerpts from Selby's reflections on why she accepted the invitation to become a research collaborator.

> I found the goal of the research project intriguing because it presented me with new way of learning. The goal of the project was also interesting. Because I have had negative experiences learning mathematics, I immedi-

ately jumped at the opportunity to collaborate on this project.

Another thing that attracted me to this project was the idea of working with a professor as well as with other students. This was appealing. I was never offered the opportunity to work closely with a professor. I believe that working with a group can have its strong points. In the past, I found that working with a group was rewarding and allowed me to benefit from the opinions and views of others.

Most importantly, I decided to accept the invitation to eliminate the fear I had for mathematics. I hope to learn how to think mathematically. Being able to think mathematically seems to be essential in learning mathematical concepts.

The needs that Selby recognized motivated her to join the project. In addition to overcoming her fears of mathematics, she wanted to improve her ability to think mathematically and to collaborate with instructor and other students. The latter motivation indicates that Selby wished to have a voice and to be heard as well as to gain the benefits of perspectives other than her own.

Unlike Selby, Jeffries did not volunteer first. He contended that his involvement on the College's fencing team precluded his participation in the project. Though Powell felt that potentially Jeffries and the project could benefit from his involvement, Powell did not attempt to persuade Jeffries to reconsider his initial decision. It was not until the fifth week of the semester that Powell urged Jeffries to join the project. Some time later, Jeffries disclosed that there were reasons other than sports that prevented him from volunteering, even though he was encouraged to join after reading Powell's letter:

When the invitation was extended to me, I initially rejected the idea. I felt that I was not competent enough. I also was afraid that I would be successful. I knew that if I was successful in this endeavor I would be expected to repeat that success. I didn't know if I was ready to fulfill those expectations, because all my life I had been a poor math student. Why change? I had been labeled a poor math student and I had long ago since accepted the label, and what's worse is that I believed it. I was a lazy student when it came to math. I had no confidence in my mathematical ability because I was never given the opportunity to take risks in math, it was always a subject that I loathed and feared, and I was happy with poor grades as long as I passed the course.

After the semester started, after my confidence grew, after my professor pushed, I was finally persuaded to join the research project.

Jeffries' performance had been predicted by his primary and secondary school authorities, and he had accepted their low expectations of his mathematical abilities. In any case, he argued, his poor grades bore out these expectations. As one can well understand, he developed both a fear and a loathing for the subject as a way of justifying it all. This accounts for his initial reluctance to join the project.

During the weekly meetings, he and other research collaborators commented on features of each other's journal entries that they considered striking. In most cases, entries were considered striking if they revealed the presence or absence of one's affective or cognitive struggles with some aspect of the course. In particular, evidence of mathematical thinking was especially looked for. At first, borrowing from Mason, Burton, and Stacey (1985), we distinguished four processes, or habits of the mind, involved in thinking mathematically: generalizing, specializing, conjecturing, and justifying. Later, through the course of the semester, we identified twenty-four other processes of mathematical thinking (see Appendix C). These were abstracted both from considering journal entries and from analyzing what the students involved in the research team did as they worked on mathematical problems.

The extent to which the writing that students do supports and reveals their mathematical thinking depends on attributes of their writing. Hoffman and Powell (1989) conjectured that journal writing is more useful for learning and best supports mathematical thinking when it is personal and reflective. Journal entries are personal to the extent that they represent the subjective understanding and feeling of the writer as opposed to the writer's perception the viewpoints or feelings of others. Reflective writing goes beyond the mere description and approaches of analysis. In reflective writing, the writer is inquisitive and contemplative and searches for meaning.

Attributes of the writing that the student researchers produced were not immediately personal and reflective. These attributes were encouraged through comments that Powell made on the substance of the journal entries. For example, during the third week of the semester, Selby wrote the following journal entry:

> I have to finally admit to myself that for once in my life I truly enjoy doing math. I feel good inside when I can take something learned in class one day and apply to something new on a different day. The homework assignment in Chapter 2/Section 2, was a combination of what I learned in class two or three days ago. When or while I was completing the assignment, I was surprised that I was able to do each problem without some kind of struggle. It was very unusual for me. One of the reasons why I am able to understand the class & homework is because first, in class it is explained to me in a very simple & understandable fashion. Another reason is that the worksheets also break up each step in a simple way which is easy to understand & follow. I have never had math taught to me in the manner & methods that I am now learning from. I love it!!

In the above entry, Selby wrote a personal, nonreflective, and general summary. She neither stated specifically what she learned nor what she did not learn. It appears, as she later verified, that she wrote down what she thought the instructor wanted to hear.

Through the process of weekly meetings the student authors became aware of the attributes of personal and reflective writing and included an evaluation of these attributes in their commentary on each other's journal entries. After Jeffries joined the research team, he read and commented on above Selby's entry.

> This journal appears to me to just fill the page. I think that was Aleshia's goal. She doesn't give specific examples of her problems, she instead gives blanket statements concerning her work. I think this is so because Aleshia didn't know what to write so she simply filled the page.

Jeffries recognized that the blanket statements, given without examples or context, were attempts to fill the page. He too produced a similar entry the week he joined the research team.

> In my problem solving course this summer, I got to use to signed numbers but I found that I confused myself. It's one thing to see something as an equation, but it is another thing when that equation is embedded in a word problem. Why is that? I thought that if I mastered an equation, I could do it if I saw it in a word problem. To my surprise, I found I couldn't. Why? Maybe you know.

This journal entry was written during the sixth week of class, a week after Jeffries joined the research team. He shows little attachment to the writing and gives the reader little context in which to interpret his questions. Reflecting on this entry some weeks later, he states that his motives were simply to fill up a page while hoping that the instructor would not read the entry. He also stated that he did not fully understand the purpose of the journals or what he was expected to do. This was true, Jeffries claimed, although Powell had written comments on previous journals suggesting ways that he might use them more profitably.

Suggestions on profitable ways to use journal writing were discussed during each team meeting. In fact, the participatory nature of the research project affected teaching as well as affective and cognitive features of learning. During each meeting, students read and commented on each others journal entries. The comments that students made were similar to those made by the instructor and, at times, were in a language that they could easily comprehend. As the semester progressed, the interactions between student investigators grew more substantive and lively; their observations about learning became increasingly more insightful and elaborate. In addition, as we will show in the journal excerpts below, the movement toward personal, reflective writing was facilitated by the interactions that occurred among the students.

For instance, Jeffries transformed the nature of his journal writing with the help of the substantive comments he received from the other student investigators during the team meetings. Consider the following journal entry written during the ninth week of the semester.

On page 44 of Chapter 4, Section 5 problem number 2 gave me some difficulties. It reads as follows:

$$\left\{\frac{\left[5(x+1)^{\frac{1}{3}}-1\right]}{3}+7\right\}^{-2}=\frac{1}{1000}$$

Now when I went to solve this as a circle equation[2], a problem occurred.

When I saw the fraction 1/1000, I made some sort of mental error. I felt that 1/1000 meant that I had to divide something in the equation. Instead of taking the reciprocal, I attempted to incorporate division into the equation.

My question is why does the fraction bar in some cases mean division and in other cases the fraction bar does not? More importantly, what is a fraction? The only thing that I am sure of if that when the number underneath the fraction bar is 1, you accept the fraction to be an integer such as 10/1 = 10. In what I refer to as regular fractions, such as 2/3, what does this expression mean? Does it mean that 2 parts of 3 are being spoken for. Perhaps it means 2 divided by 3 or vice versa.

The above carefully written entry is characteristic of others that Jeffries wrote that week and, more or less, throughout the rest of the semester. Like this one, they were both personal and reflective and reveal his ability to identify what confuses him.

In the above entry, Jeffries states an example and, thereby, provides the reader with a context for the questions he later poses. He understands that to solve the equation he must begin by reversing the action of the given exponent. He also demonstrates awareness of two interpretations of the division bar. His question is which interpretation should he act on. Jeffries is puzzled by the choices before him. Should he divide 100 into 1? If he chooses this operational interpretation, then he would have a representation of the number, a decimal, which would make it difficult for *him* to reverse the action of the given exponent. It appears that Jeffries is comfortable with raising a fraction to a negative exponent; as such, one senses that he would prefer to interpret the division bar of the fraction, 1/1000, according to its non-operational meaning. However, through the process of writing, it appears that Jeffries stumbles upon another question: What meaning should he attach to those fractions he calls regular?

In the latter part of the semester, this process of discovery and negotiation of meaning, illustrated above in Jeffries' journal entry, was evidenced more frequently in his writing and the writings of other team members, as well. Selby, for example, wrote the following entry during the seventh week.

I have found a way to solve the problems that seems easiest to me. I have no problems adding integers, however, I had problems subtracting. Now, I found that by changing all of my subtraction problems to addition problems that they are easier to solve.

Ex: $3 - {}^{-}2 = 3 + 2 = 5$ $\qquad {}^{-}5 - {}^{-}2 = {}^{-}5 + 2 = -3$

Also, I was confused about making connections to problems, transforming them into other problems, and about how to link them to a problem that would give me the same result. I believe what confused me was for example, making 5 - 3 look differently, yet having the same result. After or should I say during class, I realized how simple it was to convert or transform 5 - 3 to make it look like ¯3 - ¯5. What helped me understand the procedure of transforming the two was the commutative property and the concept of additive opposite. The concept of additive opposites seems like the same thing I did when I changed subtraction problems to find the result

This journal entry is personal and reflective and gives evidence of mathematical thinking. In the above entry, Selby describes and analyzes insights that lead her to create a generalized procedure, one which she finds easier for subtracting signed numbers. In the first part of the entry, she articulates two concepts that she synthesized to devise her procedure. The procedure involves transforming subtraction expressions into equivalent ad-

[2]Circle equations are a technique for solving a certain class of equations. For an elaboration of this technique see, Hoffman and Powell (1988).

ditions. Moreover, in the second part of the entry, using the technical language meaningfully, she discusses her struggle to see and create links between subtraction and equivalent addition problems. Making connections between equivalent expressions and using these specialized equivalences to devise and conjecture a generalized procedure for transforming a given problem into an easier, equivalent one, these are complex processes in which Selby engaged her mind and are powerful manifestations of mathematical thinking.

Conclusions

As we have defined it, a humanistic mathematic perspective includes the notion that students and instructors can learn together. Such interdependent learning is unlikely to occur through a "chalk-and-chalk" instructional method; for it presupposes the instructor as the only authority on matters of content and form and monologue as the discursive mode. Students and instructor infrequently engage in dialogue about either the nature of mathematics or approaches to learning and teaching mathematics. When dialogue does occur, rarely is its purpose to transform, more than in a superficial manner, the nature of instruction and learning. Within the perspective of humanistic mathematics, to realize interdependent learning and to transform instruction require new pedagogical and research methods.

The methodological approach of our study is offered as a first attempt to develop a new research model consistent with and facilitative of the following five processes which we have suggested are involved in teaching mathematics humanistically:

- placing students more centrally in the position of inquirer,
- acknowledging the emotional climate of learning mathematics,
- interdependent learning among students as well as between instructors and students,
- making mathematics meaningful rather than arbitrary, and
- empowering instructors and students.

These processes are best catalyzed by participatory investigations. We recognize that all investigative initiatives manipulate and transform reality and, therefore, posit that the structure of a participatory model should skew change in the direction of improved teaching and learning. Furthermore, we posit that the structure of research model should contribute to empowering both instructors and students. This imperative implies that all actors participate in the research as investigators. Students are transformed from objects of educational research into active subjects or co-investigators. That is, students participate in and are integral to the interpretation of data collected from their work and the analysis of pedagogical techniques and approaches under which they are taught. There are three important reasons for including students as co-investigators. They are (a) to ensure the ethical quality, (b) to include multiple perspectives so as to ensure the validity of research findings, and (c) to empower students intellectually.

We observed that ways in which our participatory research project affected the learning and teaching as well as contributed to empowering students and the instructor can be located in one of the following ten categories:

1. becoming an independent mathematics learner
2. learning how to learning mathematics
3. gaining insights into teaching
4. expressing ideas using mathematical terms
5. becoming an mathematics autonomous learner
6. quality and quantity of involvement lead to
   a) enjoyment
   b) diffusion of fears
   c) finding mathematics interesting, and
   d) vicarious learning
7. gaining confidence
8. gaining a sense of responsibility
9. communicating clearly
10. gaining authority

Space does not allow us to elaborate on each of these categories. Here we will discuss aspects of how our project influenced the ability of students to communicate mathematics clearly and the instructor to listen to students and have that affect his teaching. For the student researchers, participating in the project promoted a sense of community and increased their quantitatively and qualitatively writing and thinking about the mathematics of the course. In turn, these features of their involvement led to a number of by products. First, each collaborator felt committed to writing and had a sense that others depended on her or his written contributions. This commitment encouraged more writing, more often. Second, reading, analyzing, and discussing their journal entries during project meetings simply increased the number of reflections students made on the mathematics of the course. In addition to more writing, project meetings also increased the opportunities for students to do and talk about mathematics. Over time, we observed a corresponding increase in the range, depth, and clarity of the mathematical talk and writing.

Finally, in addition to contributing to the empowerment and learning of students, the participatory nature of this research project ensured that the instructor listened to students. Opportunities to listen occurred in project meetings when students read and commented on each other's journals. Their verbal and written commentary were insightful, rich, and honest. The comment that a student made about another's journal entries in one project meeting positively affected that student's subsequent writings. The powerful and efficacious nature of these interactions stimulated Powell to think of ways to incorporate aspects of the project meeting as regular features of instruction. Through the course of the semester, it became clear that the verbal and written critiques that students made of each other's journals contributed significantly to promoting personal, reflective writing. To reproduce this type interactions among students requires that instruction be transformed to give value to group work. Since cooperative, small group work is already a feature of the course, students within a group could become an interacting community reflecting and commenting on each other's journal writings. This would make widespread the empowering intellectual experience that the student authors had.

Appendix A

Professor Arthur Powell
Developmental Mathematics I

ABOUT JOURNALS

You are asked to keep a journal on 8 1/2" x 11" sheets of loose-leaf paper. Generally, one or two sheets will be sufficient for a week's worth of journal writing. Neither your syntax nor grammar will be a concern or checked; my only concern and interest is what you say, not how you say it. You are asked to make, at least, one journal entry for each meeting that we have, and, as a rule of thumb, you need not spend more than five to ten minutes writing each entry. Each week, the latest journal entries will be collected and returned with comments.

The focus of your journal entries should be on your *learning* of mathematics or on the *mathematics* of the course. That is, your reflections should be on what <u>you</u> do, feel, discover, or invent. Within this context, you may write on any topic or issue you choose. To stimulate your thoughts and reflections, here are some questions and suggestions.

- What did <u>you</u> learn from the class activity and discussion or the assignment?
- What questions do <u>you</u> have about the work <u>you</u> are doing or not able to do?
- Describe any discoveries <u>you</u> make about mathematics (patterns, relationships, procedures, and so on) or yourself.
- Describe the process <u>you</u> undertook to solve a problem.
- What attributes, patterns, or relationships have <u>you</u> found?
- How do <u>you</u> feel about <u>your</u> work, discoveries, the class or the assignment?
- What confused <u>you</u> today? What did <u>you</u> especially like? What did <u>you</u> not especially like?
- Describe any computational procedure <u>you</u> invent?

APPENDIX B

19 September 1988

Dear Developmental Mathematics I Student:

This semester, I will conduct a research project for which I am looking for student collaborators. The goal of the research project is to discover whether writing about the mathematics that one is learning and doing can be helpful in learning mathematics. Let me tell what the project is about.

In this course, I am asking each of you to keep a journal about your learning and to do other types of short writing assignments related to the course. Most of the writings that you do I will collect and analyze, and to some writings I will respond. Those who collaborate with me may be asked to do a bit more writing than others. Each week, collaborators and I will meet as a research team to analyze their writings.

The central research questions that I hope to answer by the end of this research project is: In what ways can personal, reflective journal writings best support the enhancement of mathematical thinking? In addition, there are also two sub-questions that I will be asking about the writing that you do.

- Do students' writings display their attempts to specialize and generalize as well as to make conjectures and to provide justifications for them?
- How does writing help students to construct or negotiate meaning?

Why do I ask students to write in a mathematics class? Last year, a Developmental Mathematics I student and I collaborated on a research project to determine whether journal and free writing were useful vehicles to learn mathematics. Based on that study, which will be published soon, we have concluded that writing can be a powerful tool in learning mathematics. Now I wish to examine more closely how writing can support the development of mathematical thinking.

This close examination of your writing will, I believe, benefit you in two ways. First, the writing that you do will improve your learning. Second, what you choose to write about will inform my teaching and, thereby, improve the lessons I conduct.

I intend to co-author a paper, with those who collaborated with me, on the finding of this project. Let me know by letter whether you would like to work with me on this project. If you would like to collaborate with me and have the time,in your letter, discuss why you are interested and what you wish discover about yourself as a learner of mathematics. I will collect these letters on Wednesday 21, September.

Sincerely,

Arthur B. Powell

Appendix C
Processes Involved in Thinking Mathematically (or Habits of the Mind)

- ☐ posing problems and questions
- ☐ exploring a question systematically
- ☐ generating examples
- ☐ specializing
- ☐ generalizing
- ☐ devising symbols and notations
- ☐ making observations
- ☐ recording observations
- ☐ identifying patterns, relationships, and attributes
- ☐ formulating conjectures (inductively and deductively)
- ☐ testing conjectures
- ☐ justifying conjectures
- ☐ communicating with an audience
- ☐ writing to explore one's thoughts
- ☐ writing to inform an audience
- ☐ using appropriate techniques to solve a problem
- ☐ using technical language meaningfully
- ☐ devising methods, ways of solving problems
- ☐ struggling to be clear
- ☐ revising one's views
- ☐ making connections between equivalent statements or expressions, transformations
- ☐ making comparisons
- ☐ being skeptical, searching for counter examples
- ☐ reflecting on experiences
- ☐ suspending judgement
- ☐ sleeping on a problem
- ☐ suspending temporarily work on a problem and returning to it later
- ☐ listening actively to peers

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