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WRITING HUMANISTIC MATHEMATICS

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SUMMARY

This article suggests some guidelines for writing mathematics texts so that students can learn by reading.

INTRODUCTION

One thing that distinguishes the teaching of mathematics from the teaching of other subjects is that the teacher rarely gives the students a list of references which they are supposed to find and study on their own. Mathematics undergraduates generally study from their notes and from handouts. Even the class text is often used only as a convenient source of exercises. The teacher is considered as a necessary interpreter between the text and the student. In a survey of first year students to investigate the attitudes to and past experiences with mathematics, Landbeck (1990) found that of the 65% who reported negative experiences, most attributed these bad experiences to bad teachers. I replicated part of this survey, asking students to describe their past experiences with mathematics and again most students reported on teachers or teaching and the overwhelming majority of experiences were negative, Hubbard and Kelly (1990).

In an essay about writing in the mathematics curriculum, Gopen and Smith characterize the prose in mathematics textbooks as follows:

"The model of good mathematical prose most available to students would seem to be their textbook; however, the writing found

there is often less than effective, and the students often avoid reading it. We trace the blame for this to generations of combined efforts from two quarters: On the one hand, authors and publishers produce textbooks that do not have to be read before doing the exercises; on the other hand, teachers acquiesce by agreeing that this is the way mathematics ought to be taught. What prose there is has tended to be introductory, apologetic, and self-justifying. It implies that the real importance lies not in the students' ability to conceptualize, but rather in their ability to function."

Another factor which affects the writing of mathematics textbooks raised by Dudley, is the need for the text to be absolutely mathematically correct and complete so that it cannot be criticized by mathematical colleagues. This requirement results in texts written for mathematicians, not students.

The way in which mathematics texts are written seems to be one reason why students do not learn mathematics by reading.

TEACHING STUDENTS TO READ MATHEMATICS

Having read the literature on teaching students to become independent learners and being acutely aware that this was not what was happening in mathematics departments I decided that as a first step I would try to teach my students to read their mathematics text, Hubbard (1990). Another reason

for trying to help students to read mathematics was the fact that at the present time university students in Australia take lectures in groups of several hundred and tutorials are overcrowded or non-existent. Students are increasingly being forced to become independent learners, not for pedagogic reasons but because of the financial constraints being placed on universities. In the process of trying to help the students to read I looked very critically at the structure of their texts and at the features that made them so difficult to read.

There is a great deal of dispute about measures of readability in ordinary English text. Should an index of readability be based on sentence length, vocabulary, number of ideas in a sentence and so on. When it comes to measuring the readability of mathematics text, Howard (1977) the problem is compounded because there is symbolic as well as ordinary text and the ordinary text is not really ordinary.

To demonstrate the problems involved in measuring the readability of mathematical text, even without considering symbols, here are some definitions of "function" from standard texts followed by an ordinary sentence of about the same length, complexity, vocabulary etc.

"A function from a set D to a set R is a rule that assigns a single element of R to each element in D ."

Thomas and Finney (1988)

"A function is a collection of pairs of numbers with the following property: if (a,b) and (a,c) are both in the collection, then $b = c$; in other words, the collection must not contain two different pairs with the same first element."
Spivak (1967)

"A function is a rule or correspondence, relating two sets in such a manner that each element in the first set corresponds to one and only one element in the second set. In

other word a functional relationship is a single-valued relationship"

Zill (1988)

"I am going along to see my grandmother this afternoon because she called me to say that she was not well so she may need some assistance around the house or a lift to the doctor's."

One obvious reason why the definitions are more difficult to read than the grandmother sentence is because they deal with abstractions. There are other reasons why mathematics textbooks are hard to read.

(a) The language is precise. Every word or phrase has an exact meaning which may or may not correspond to the meaning in everyday speech and writing.

(b) The writing is concise. Definitions use as few words as possible which means that there is no redundancy. If a single word is not understood or is overlooked the definition can be misunderstood or not understood at all.

(c) The development is highly sequential. Each new abstract concept is defined in terms of earlier abstract concepts so that layers of increasing abstraction are continually being created.

Added to this is the difficulty of reading the symbolic language and the fear that many students have of complicated formulae.

The conventional logical structure of mathematics text, definition - theorem - proof example, is not conducive to learning by reading. The following quote from Demana and Waits (1988) explains why this is so and suggests a different approach.

"Formal language is kept to a minimum throughout the lessons. Definitions are not given until the conceptual base and need for the definition has been developed. Ordinary or previously used language is used until a need for more precision arises. This stems from two different factors. First, we believe that mathematics is made to appear more abstract than it really is by the use of formal definitions that do not have a proper base of meaning or need. A large number of students are lost to mathematics because it is made to seem unnecessarily abstract by the use of nonmotivated definitions. Second, stating definitions first and then progressing to the problem situations is inconsistent with our use of problems as means to the new mathematical ideas. Problems are found in familiar, known contexts described with words already in the vocabulary of the students. The ideas stem from the development of solution processes and methods. Then the ideas are refined and stated precisely. This means that definitions arise naturally."

Because of the difficulty of the conventional structure, many students who cannot understand the definitions simply omit them and focus on the examples which they try to mimic in their exercises. This often degenerates into replacing one group of numbers with another group with no attempt to understand the process involved. The definitions and proofs are ignored unless they are to be examined, in which case they are memorized.

The "improvements" produced by instructional designers, such as isolated words in the margin and shaded boxes which contain important facts, encourage the student to concentrate on these words rather than on making connections between them. These devices make "finding the formula" easier but do not make reading for understanding any easier.

CRITERIA FOR WRITING READABLE MATHEMATICS

Having achieved only limited success with teaching students to read standard texts (Hubbard, 1990), I decided that some affirmative action was required to produce mathematics text that was more readable. Reading the symbolic language of mathematics is inherently difficult but by making the ordinary language part of the text more readable I hoped to reduce the problems presented by the symbols. From my observations of students' difficulties with reading mathematics, I produced a set of guidelines which I proposed to put into practice. The guidelines address two issues, language and pedagogy.

LANGUAGE GUIDELINES

Other things being equal, short sentences are easier to read than long sentences, so most sentences should contain just one concept. If a sentence starts to get complicated, break it up into two shorter ones.

If there is a simple familiar word which means the same thing as a more erudite sounding word, choose the simple word. For example determine and evaluate can often be replaced by find and examine, and consider can be replaced by look at.

Use informal language while examples are being investigated and new concepts are being developed. Students can't be expected to understand a definition if they do not understand the technical terms involved. The concept has to come first, then when it is established it can be given a name.

Use only those technical terms that are really necessary. Often technical jargon seems to be used deliberately to create the "insider/outsider" phenomenon that Tobias describes as a factor in math anxiety.

Use the active rather than the passive voice to give the text immediacy. This effect can also be obtained by using I and you instead of the formal we. These devices help to counteract the formality and impersonality of the symbolic part of the text. It helps the student to realize that other human beings like themselves are writing the material.

Make the text redundant by explaining difficult ideas twice or even three times using different words in the hope that one of the explanations will reach each student. This is just what we do when explaining something difficult verbally.

GUIDELINES FOR PEDAGOGY

Try to find contexts for the examples which are part of the everyday experience of the students so that they can relate to them easily. Alternatively where this is not possible try to invent imaginative examples which might excite the students' curiosity or sense of humor. Most "real life" examples in standard texts are very uninteresting. Introduce new concepts by way of interesting examples so that the concept and the example become associated in the student's mind. The concept is then readily recalled by reference to the example.

Instead of dealing with a topic in its entirety in one chapter, introduce it at an elementary level the first time and return to it several times always revising and building on what has gone before.

Do not assume that the reader is an empty bucket waiting to have words of wisdom poured into it. Every student already has some mathematical concepts and possibly misconceptions in her head so it is worth trying to appeal to these in the hope of finding something to attach new concepts to.

Warn students about common misconceptions or use them as counter-examples when a new idea or formula is explained.

Resist the temptation to "tell it all", i.e. to include details which are not necessary in an elementary treatment of the subject, even at the expense of leaving a topic incomplete. Students have great difficulty recognizing main concepts from less important details. Often they learn details which are of no use without the concept to which they belong. For example, statistics students always seem to remember one-tailed and two-tailed tests even when they have no idea what is involved in testing a hypothesis. The word "tail" and the diagram that goes with it are easy to recall. The logical steps involved in testing a hypothesis are difficult to follow, so students don't learn them.

Ask questions which enable students to check their understanding as they are reading. An experienced reader constructs his own questions and if he can't answer them reads a passage again. Students need to learn to read in this critical way. So that the student does not get frustrated, when he is unsure of the appropriate response, provide the answers to these questions immediately or at the end of the chapter.

Do not provide summaries. By providing summaries you encourage the student to read and memorize without understanding. Summaries are most useful when they have been constructed by the student before or after she has worked some exercises.

WRITING TO THE CRITERIA

I have put these criteria into practice in two quite different areas. One is a distance education bridging course for adults in remote areas of Queensland who wish to prepare for university studies. Distance education students do not have a teacher to act as an intermediary between them and mathematics and to a large extent have to depend on print materials. Readable text is absolutely vital in this context.

The second is a first-year undergraduate statistics course for non-mathematics majors. What you might ask, is the point of producing yet another first level statistics text? The main point is that I wanted the students to learn about those aspects of statistics that still need to be done by humans and to relegate the routine calculations to a computer. In order to learn statistics in this way the students have to use a statistical package and some of these packages are powerful tools for investigation, animation and simulation. Since a thorough search failed to find any texts which approached the subject from this viewpoint, I had to write my own and this gave me the opportunity to apply my writing criteria.

THE BRIDGING COURSE

People who undertake mathematics bridging or remedial courses come from mathematically deprived backgrounds. If in addition, they are isolated, with only the possibility of an occasional telephone conversation with a tutor, whom they have never met, providing them with mathematics text that they can read on their own is a tremendous challenge. Furthermore, the course starts with simple operations involving money and finishes with calculus and mechanics so it is enormous. A close-knit team of four*, also separated geographically by vast distances, has written the materials according to the above guidelines.

Symbolic notation is introduced slowly and carefully. The student is shown how every new symbol is pronounced and encouraged to say the words aloud. The student is continually reminded to think about the meaning of the symbols. The questions which check the students understanding of her reading are frequent and the student is given a few lines in which to answer these questions. Students are also asked to write down ideas and rules in their own words, to construct their own summaries and to express their feelings about their progress.

Because the course is for adults we try to use examples which they could find useful. The arithmetic in the first part of the course revolves around the theme of how to manage money because everyone has to do this. But there are also examples about sport, horse racing, cooking, arts and crafts, TV soaps and farming, because these are the pursuits of country Queenslanders. We have tried to avoid sex stereotyping by having men go shopping for clothes and women ordering fencing materials.

We have also tried to broaden our students horizons by setting mathematics in a historical or cultural context. Not the picture of Descartes with a few dull biographical notes underneath, which is standard in many texts and which students don't read, but something more imaginative like the following.

"You have just climbed to the top of the leaning tower of Pisa and in your excitement you drop your camera over the edge. This distance s in meters that it falls in t seconds is given by the formula

$$s = 4.9 t^2$$

This formula was derived by Newton, building on the work of Galileo, who was a Pisan and who dropped things (not cameras) from the top of the tower in order to calculate gravitational force.

You would like to know the speed with which your camera will hit the ground. The height of the leaning tower of Pisa is about 55 m. (The height keeps changing as the tower does more leaning)....."

Later problems about velocity and acceleration refer back to the Leaning Tower problem which the student is unlikely to have forgotten.

The central concept of function is introduced very early and revisited many times. Domain and range are introduced in diagrams showing the relationships of characters in TV soaps. Thereafter almost every module asks students to recall their concepts of function via these examples and develops the ideas in greater depth.

Students are given opportunities to investigate the relationships between equations and their graphs and to try their hand at generalizing from special cases using a simple graphing package. The Queensland Government has established Open Learning Centers in virtually all rural communities and these are equipped with PC's.

This is a brand new endeavor so I cannot report on the success of the project at this time.

A READABLE STATISTICS BOOK

The problem with writing a first level statistics text is that many of the main concepts and formulae cannot be explained in terms of elementary mathematics. Most authors are forced into stating formulae and giving students numbers to substitute into them. For non-mathematicians, substituting into a formula may help to memorize the formula but does not help to give it meaning. I would go so far as to say that substituting into formulae temporarily dulls the students mind and prevents her from thinking about the meaning of the formulae.

Since in this situation the formulae cannot be satisfactorily explained, I have left as many as possible out. Of course this is not possible in those parts of mathematics where it is necessary to operate on formulae. It is for these purposes that formulae were invented. However, contrary to popular opinion and (I suspect) the opinions of many mathematics teachers, mathematics deals with abstract concepts and the symbols are merely a convenient way to represent the concepts. If it is possible to discuss the concepts without the symbols, no harm is done.

USING THE COMPUTER TO COMPUTE

For a long time statistics textbooks concentrated on the computational aspects of the subject because without doing the calculations it is not possible to produce any results. But even though computer packages have been available for many years this has had very little effect on statistics texts, even recent ones. Some authors acknowledge that the packages exist by appending printouts at the ends of chapters, hoping this will help to sell their books. All the formulae are still there and the examples show how to substitute the numbers into the formulae. The student is still expected to do all the substitution exercises. When he is finished, he is asked to type the numbers once more into a computer package and admire the printout.

By using a statistical package which already contains suitable data sets, the student is freed from the burden of entering the data and doing the calculations and is able to concentrate on the more important aspects of the subject.

USING THE COMPUTER TO EXPLORE

Statistics is concerned with the analysis of data. The first step in this analysis is investigating the data to find out what interesting information it contains. This aspect of statistical education was almost completely neglected in the past because without a computer to produce graphs and tables it was too time consuming to carry out. In all the standard texts students are given lists of numbers and told to do this or that with them. They are not asked whether the result of doing this or that has any meaning or whether it might have been better to try doing something else.

Using a computer, students learn to choose appropriate methods of investigating data and to think about the outcomes. They can observe the effects of changing scales, changing class intervals, detecting and removing outliers, transforming data

to produce linear relationships. They can also relate the results of statistical analysis to the impressions they formed from studying the data.

USING COMPUTER ANIMATION

There are a number of packages, for example those produced by Bowman and Robinson, which contain animated displays. They show the building up of probability distributions from repeated sampling and the generation of the various sums of squares in regression and the analysis of variance. These are powerful visual aids to understanding complicated abstract processes. One particular procedure in this package which allows the user to move the regression line about on the screen and gives the residual sum of squares at each stage is not only a valuable learning experience but also an interesting game.

USING THE COMPUTER TO SIMULATE

An alternative to stating formulae without proof is to use simulation to show that what the formula states actually does happen. As I stated earlier, the key results such as the central limit theorem cannot be proved in elementary statistics courses. Many innovative ways of simulating these theoretical results to make them plausible appear in the Minitab Handbook but are not referred to in the standard texts so I assume they are not widely used.

Simulations of the distributions of sample statistics help to convince students that sample means and proportions do appear to have normal distributions with standard errors that approximate the theoretical values. Simulations can show that confidence intervals do contain the parameter about the right proportion of the time and that the level of significance does lead to the rejection of the null hypothesis in about the correct proportion of cases.

Simulation is also useful in teaching probability. Because the idea of probability is a part of our culture, most students already have some concepts and misconceptions about probability. The games

from which they developed their probability concepts can easily be simulated to ensure that their intuitive ideas are consistent with the mathematical theory of probability. Textbooks which introduce probability through set notation help to ensure that the connections between "real life" probability and textbook probability are not made.

USING COMPUTER OUTPUT FOR DISCUSSION AND WRITING

The interpretation of computer output provides an excellent topic for discussion among students and tutors. I have students working in pairs in tutorial classes because this encourages informal discussion. By describing their results and comparing them with the results of others, students get practice in using the technical vocabulary. This is reinforced if they also submit written reports on their findings. Drawing conclusions from their investigations and analyses gives students opportunities to clarify their ideas by writing about them.

CONCLUSION

I have digressed somewhat from my writing guidelines in order to establish two points about my writing in statistics. The first point is that my approach to statistics is sufficiently different from the standard approach to warrant a different kind of text. The second point is that I wanted to demonstrate that by using a computer, the learning of statistics can become a truly creative human activity.

Returning to the writing guidelines, it is interesting to note that in response to surveys, almost all students found my writing very readable although they were not asked for specific reasons why this was so. The only guidelines the students were given for their writing were that they must use sentences and that they should take care with spelling and grammar. Nevertheless many students reverted in their writing to the standard textbook style. For example they would frequently preface their comments with, "It can be stated that" or "It can therefore

be concluded that" instead of just saying what they think. This result is hardly surprising, as these students would have been exposed to the language of mathematics textbooks for many years. It is going to take a long time to change the way mathematics is written.

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* The members of the team are, M. Fuller, I. Putt, P. Surman and R. Hubbard.