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Philosophy of Mathematics, Mathematics Education, and Philosophy of Mathematics Education

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As a philosopher of mathematics, I have been thinking about, or rather, worried about the following question: is there any important relationship between the philosophy of mathematics and actual mathematical activities (including mathematical research, teaching and learning)? Or, does the philosophy of mathematics have any important influence on actual mathematical activities? I think the answer is 'yes'; and I have also tried to do some things in this direction by working in the field of methodology of mathematics (cf. Y. Zheng, 1985, 1991a, 1991b). But it is only a personal opinion and has limited influence, so when I came to the USA as a visiting scholar in 1991, this problem was still deeply rooted in my mind. However, what I have learnt in the field of mathematics education in the USA is really a great pleasure for me, as it does show clearly that there is a close relationship between the philosophy of mathematics, mathematics education and mathematics as well: it is modern research in the philosophy of mathematics which offers the necessary ideological foundation for the new reform movement of mathematics education in the USA, and then, in this way, the philosophy of mathematics can exert a great influence on the future of mathematics.

> **While the advancement of human society, is the most important external impetus, it is the theoretical studies of mathematics education during the past decade which have laid the necessary foundation for the new reform movement.**

The first and second parts of this paper will use the modern development of mathematics education in the USA as a background to make an analysis of the great influence of the philosophy of mathematics on mathematics education. The third section discusses the problem of how to develop the subject 'philosophy of mathematics education', which in fact can be regarded as an impetus from mathematics education to the further development of philosophy of mathematics and philosophy in general.

1. New Developments of Mathematics Education

Mathematics education in the USA is now undergoing a new reform movement, to which Everybody Counts, published by the National Research Council in 1990, gives the following description:

"Over the next two decades, the nation's schools, colleges, and universities will undergo major transitions in mathematics programs—transitions that will involve fundamental changes in curricular content, in modes of instruction, in teacher education, in professional development, in methods of assessment, and in public attitudes." (p. 87)

While the advancement of human society, i.e. the transition from the industrial age to the information age, is the most important external impetus, it is the theoretical studies of mathematics education during the past decade which have laid the necessary foundation for the new reform movement. In this section, we make a brief survey of the new theoretical studies. They are chiefly: the emphasis on problem solving, the psychology of mathematical learning, and the social-cultural approach to mathematics education.

(1). The emphasis on problem solving

'Problem Solving' was the main slogan for mathematics education during the eighties. "Problem solving must be the focus of school mathematics" (NCTM, 1980, p. 2); and by 'problem solving', it means 'to use a variety of mathematical knowledge and methods effectively to solve nonroutine problems, including both actual problems and those originated from mathematics itself'.

Putting forward the idea of focusing on problem solving is a giant step for mathematics education, because the idea represents a great shift in the conception of mathematics education, i.e., the idea itself is a direct negation of the traditional conception of mathematics education, especially, the teaching method based on 'transmission of information' and the trend of 'separating learning from application'. To explicate, the key points of 'focusing on problem solving' are as follows: First, students should learn mathematics by the activities of solving problems. That is, "'knowing' mathematics is 'doing' mathematics ....instruction should persistently emphasize 'doing' rather than 'knowing that'." (NCTM, 1989, p. 7) Second, by solving problems, especially those having actual meaning, students can learn to value mathematics, and become more confident in their own mathematical ability. Third, the final aim of mathematics education should be to improve students' ability of problem solving, especially help them learn to think mathematically.

Generally speaking, the idea that problem solving must be the focus of school mathematics is now widely accepted; and as this idea is directly opposite to the traditional conception of mathematics education, it is said that 'solving nonroutine problems is the central theme of the current reform movement in school mathematics.' (T. Romberg, 1991, p. 9)

(2). The emphasis on the psychology of mathematical learning

The study of the psychology of mathematical learning is itself a result of the further development of psychology: it has been beyond the level of general study and penetrated into special fields. Furthermore, where modern studies of the psychology of mathematics learning are concerned, we should pay more attention to the cognitive science approach to mathematics education and "the constructivist view of mathematics learning".

To explicate, the basic position of cognitive psychology is that the study of psychology should not (as behaviorists suggest) be limited to 'visible behavior' but penetrate into the inner information processing of the mind, including the storage, retrieval, representation, development of knowledge and so on. Also,the so-called 'constructivist view' can be regarded as a main conclusion of cognitive psychology: as far as mathematics learning is concerned, it asserts that the learning of mathematics is not a passive reception but a process of construction based on previous experience and knowledge.

If the idea "focusing on problem-solving" is a direct negation of the traditional concept of mathematics education, then the cognitive studies of mathematics learning, especially the constructivist view of mathematics learning, have offered further arguments for this fundamental transition from the microscopic view. And just for this reason, the constructivist view on mathematics learning has recently attracted great attentions in the field of mathematics education. For example, as R. Davis, C. Maher and N. Noddings say in Constructivist Views on the Teaching and Learning of Mathematics:

we should take as a background the whole culture of human society in the study of mathematics education. This is to say, mathematics education should represent clearly the features of the time.

"The idea of "constructivism"—hardly mentioned a few years ago—nowadays attracts a lot of attention in the world of mathematics education. A great many people now think and write about it, and the people who do so do not agree with one another ...Still, beneath the theoretical argumentation, there is a substantial agreement about the nature of learners, the nature of mathematics, and the appropriate form of pedagogy." (R. Davis, C. Maher and N. Noddings 1991, p. 187)

(3). The social-cultural approach to mathematics education

The first implication of the social-cultural study of mathematics is that we should take as a background the whole culture of human society in the study of mathematics education. This is to say, mathematics education should represent clearly the features of the time. In fact, it is exactly the most important feature of the new reform movement of mathematics education in the USA: it is the transition from the industrial society to the information society which offers the most important impetus to the movement, and the final aim of the movement is to create the kind of mathematics education that not only meets the need of the time but also uses fully the new technology; in a word, we should create the mathematics education of the information age.

Secondly, the social-cultural approach to mathematics education has also made clear the social nature of mathematics learning and teaching. Although the construction of mathematics knowledge should be carried out relatively independently by all individuals, such activities are carried on in some 'social environment', and must include the processes of expressing, communicating, comparing, criticizing, improving and so on, so that it is in fact a 'social construction'. Besides, the social nature of mathematics teaching can be seen clearly by the fact that the role played by teachers is just the 'intermediate' between the whole system of education and the objects of education. In other words, teachers' duty is to carry out the overall aim of mathematics education in facing the concrete students and the concrete situation of teaching in general.

First, every mathematics teacher is (consciously or unconsciously) doing his work under the influence of some conception of mathematics and mathematics education, and the latter are in fact manifestations of the social nature of mathematics education.

Finally, one more important implication of the social-cultural approach to mathematics education is the importance of conception both to mathematics teaching and learning: First, every mathematics teacher is (consciously or unconsciously) doing his work under the influence of some conception of mathematics and mathematics education, and the latter are in fact manifestations of the social nature of mathematics education. Secondly, as far as students are concerned, the importance of conception lies in the fact that mathematics learning is a process in which not only mathematical knowledge is constructed but also some conception, belief and attitude of mathematics are formed, and the latter in turn will exert great influence on the learners' further study of mathematics and even for their whole life (as a part of their whole ideology). For example, it is just by such consideration that <u>Curriculum and Evaluation Standards for School Mathematics</u>, which is one of the most important documents shaping the new reform movement, lists 'learning to value mathematics' and 'becoming confident of one's own ability' as the first two goals for mathematics education.(pp. 5-6)

Obviously, if the psychology of mathematics learning is the study on the microscopic level, then the social-cultural study belongs to the macroscopic level; and just as J. Kilpatrick points out in his <u>A History of Research in Mathematics Education</u>, 'Researchers were taking the social and cultural dimensions of mathematics education more seriously.' (1992, p. 30)

2. From the philosophy of mathematics to mathematics education

Research in the above three directions as a whole represents a new conception of mathematics education, whose kernel is new ideas about the questions 'what is mathematics' and 'what it means to know mathematics'. At just these points, we can see clearly the important influence exerted by philosophy of mathematics on mathematics education.

To explicate, philosophy of mathematics had for a long time been under the tradition of 'foundational studies'. The common position for all the main schools in the study of mathematics foundations, i.e. logicism, intuitionism and formalism, took mathematics as a body of mathematical knowledge, and it was hoped that, by the logical analysis of the inner structures of mathematical knowledge, they could lay a firm foundation for mathematics so that the problem of the soundness of mathematics could be solved forever.

The researchers of the above three schools had produced many important results. As far as their final aims were concerned, however, they all failed, and as time passed, a big deficiency of the foundational studies has become clear, i.e, it deviates terribly from actual mathematical activities. So, after the period of the 'the golden age' (about 1890-1940), the study of the philosophy of mathematics stagnated.

In the sixties, mainly under the influence of the philosophy of science, some new phenomena appeared in the field of philosophy of mathematics, which in turn represented a transition of the basic positions. The new position was that mathematics should be mainly regarded as creative activities of human beings rather than a specific body of fixed mathematical knowledge. Thus, in comparison with the traditional view of mathematics, the new conception—which may be called 'the human view of mathematics'—contains the following changes:

First, the new view emphasizes the development of mathematics: as creative activities of human beings, mathematics is not something static and fossilized but has been changing all the time and will keep on changing in the future. Particularly, as daily mathematical activities are concerned, they are necessarily complicated processes including conjectures, errors and tests.

Second, the development of mathematics is not only a process of accumulation but also includes qualitative changes. That is, there are also revolutions in mathematics.

Third, the human view of mathematics also confirms that mathematics consists of meaningful activities, so that it should not be identified as the mechanical manipulation of meaningless symbols.

As the human view of mathematics represents a big transition of basic ideas, it has also opened new directions for the study of the philosophy of mathematics.

For example, there is firstly the social-cultural approach to mathematics. To be concrete, mathematicians in modern society are all working in some social environment, and therefore are, in fact, members of 'mathematical communities'. In fact, the working aim for most mathematicians is to get mathematical statements which are representable by the language uniformly accepted by the community, and are resolutions to those problems uniformly regarded as important or significant by the community, and are based on arguments or methods uniformly accepted by the community. (cf. P. Kitcher, 1984) In fact, such a prescriptive role of the mathematical community based on individual mathematicians is just the concrete manifestation of what might be called 'mathematical culture' (in the level of graduate school and mathematics research).

Furthermore, as mathematical researches are social activities, we can therefore study the impetus and laws for the development of mathematics from a higher level. This is to say, we can transcend an individual's work and take the whole human society as a background for the study of the historical development of mathematics. (cf. R. Wilder, 1981) Obviously, such studies denote that philosophy of mathematics has extended from daily mathematical activities to macroscopic studies.

Also, from the microscopic view, mathematical activities are all mental processes. In particular, the creation of all mathematical concepts is a process of construction. To be concrete, mathematical entities are not objects existing in the empirical world but creations of abstraction. Furthermore, in strict research, no matter whether the entities concerned have or do not have empirical backgrounds, we cannot rely on intuition but on deduction from the corresponding definitions. Therefore, the process of mathematical abstraction is, in fact, an activity of construction. That is, mathematical entities are constructed by the corresponding definitions (including explicit and implicit definitions), and only by those processes of "logical construction" can the corresponding mathematical entities be transferred from 'the inner creations of the mind' to 'the outer independent existence'. (cf Y. Zheng, 1991b.) Furthermore, because mathematical entities are not objects in the empirical world, the study of mathematical entities must include a

as creative activities of human beings, mathematics is not something static and fossilized but has been changing all the time and will keep on changing in the future.

process of 're-creation' (in comparison with 'the primary creation'). That is, people must actually construct the corresponding mathematical entities in the mind, so that what had been 'objectified' with the aid of language can be transferred back into 'inner elements of the mind'.

Putting together the above discussion of the modern developments of mathematics education and of the philosophy of mathematics, we can see clearly that it is modern research in the philosophy of mathematics which has offered the important ideological foundation for the new reform movement of mathematics education in the USA. For example, the emphasis on problem solving is obviously a necessary consequence of the human view of mathematics. In fact, an important starting point of the new reform movement of mathematics education is just the recognition that school mathematics under the old tradition is not 'real mathematics', and the idea of 'focusing on problem solving' in turn is to put students in the same situation as mathematicians. T. Romberg says on this point:

the distinct feature of the 'new math' was that little attention was paid to the actual cognitive processes, of how human beings think about mathematics.

'For over two thousand years, mathematics has been viewed as a body of infallible truth far removed from the affairs and values of humanity. These views are being challenged by a growing number of philosophers of mathematics....Such a dynamic view of mathematics has powerful educational consequences. The aims of teaching mathematics need to include the empowerment of learners to create their own mathematical knowledge; ....When mathematics is seen in this way, it needs to be studied in living contexts that are meaningful and relevant to the learners, including their languages, cultures, and everyday lives, as well as their schoolbased experiences.' (T. Romberg, 1992, p. 751)

Secondly, although 'constructivist' is a new terminology in the world of mathematics education, it is quite familiar to philosophers of mathematics. Therefore, the 'rise' of the constructivist view on mathematical learning and teaching can be regarded as an extension or transition from philosophy of mathematics to mathematics education. What should also be noted is that, mathematics educators have found important illuminations for instruction from modern studies of the philosophy of mathematics and philosophy of science in general. For example, based on the discussion about scientific revolutions, especially about the transition of 'paradigm' in philosophy of science, some mathematics educators suggest that forming 'conceptual conflict' is a requisite and efficient way for promoting students' mathematical thoughts, especially for the correction of their wrong ideas.

Finally, the social cultural approach to mathematics education obviously corresponds directly to the social cultural studies of mathematics. For example, in correspondence with the concept of 'mathematical community', mathematics educators have introduced the concept of 'mathematics education community', which consists of mathematics teachers, mathematics education researchers, directors for mathematics teacher's training, supervisors for mathematics curriculum, makers of policies for mathematics education, designers of mathematics examinations and so on, and the main feature of a mathematics education community is also that all its members share (consciously or unconsciously) somewhat the same conception of mathematics education.

In addition to the above discussion, what should be noted is that we can analyze the relationship between the philosophy of mathematics and mathematics education in a more general sense. For example, the traditional conception of mathematics education reflects to a great extent the 'absolute view of mathematics' (we should also see the influence of mechanism here). Besides, it is the foundational study mentioned above that offers the necessary ideological foundation for the 'new math movement', which was seen throughout all the western countries during the sixties. In fact, the distinct feature of the 'new math' was the emphasis on the logical structures of mathematical knowledge and little attention was paid to the actual cognitive processes, of how human beings think about mathematics. We can see here very clearly the influence of the 'foundationists'. The French mathematician R. Thom, while commenting on 'new math', clearly raised the following question:

'"Modern" Mathematics: an Educational and Philosophical Error?' And as an answer, he says, for example,

'Set theory....is the essential litany intoned by those who advocate the so-called modern mathematics. Some affirm that the use of set theory permits the entire renovation of mathematics teaching and that, thanks to this change, the average student will be able to achieve mastery of the curriculum. Needless to say, this is pure illusion ...Everything considered, the excessive optimism bred by the use of set theory symbol has its roots in a philosophical error.' (Thom, 1971, p. 75)

To summarize, we should definitely confirm that there is an important relationship between the philosophy of mathematics and mathematics education; and then, via mathematics education, philosophy of mathematics will exert great influence on the future of mathematics.

3. Towards a philosophy of mathematics education

The above discussion shows clearly the important relationship between the philosophy of mathematics and mathematics education; however, at the same time we should not identify the philosophy of mathematics with the theoretical foundation of mathematics education. In other words, mathematics education should have its own relatively independent theoretical foundation.

In fact, every subject has its own history during which it has formed its special field, problems and theories. With this view, we can see clearly the differences between the philosophy of mathematics and the theoretical foundation of mathematics education:

On the one hand, the philosophy of mathematics, as philosophical analysis of mathematics, has its special problems. In fact, in comparison with those problems mentioned above, the ontology and epistemology of mathematics are of a more basic nature. The ontology of mathematics can be described as: do mathematical entities have an independent existence? If the answer is 'yes', then what kind of existence is it; if the answer is 'no', what is the meaning of mathematics? On the other hand, the epistemology of mathematics is focusing on whether mathematical statements are a priori or empirical. The fact that the ontology and epistemology of mathematics have occupied important positions in the philosophy of mathematics is a natural result of the speciality of mathematics, especially its abstractness (the speciality of mathematics lies not only in the contents of mathematical abstraction, but also in its degree and method. cf. Y. Zheng, 1991). And just for this reason, although there have been some new directions in the field of the philosophy of mathematics since the sixties, any systematic theory in the philosophy of mathematics still has to give definite answers (or detailed analysis) to the ontology and epistemology of mathematics. In fact, just as P. Benacerraf points out in his paper Mathematical Truth, the difficulty in the study of philosophy of mathematics just lies in 'the dilemma of the ontology and epistemology of mathematics'. This is to say, those theories which are satisfactory in the ontology always have serious deficits in the epistemology; while the others which are satisfactory in epistemology always have deficits in the ontology. However, all these discussions do not seem to have important implications for mathematics education.(P. Benacerraf, 1983)

On the other hand, the theoretical foundation of mathematics education obviously should include the following contents:

(1) **The View of Mathematics.** This is the answer to the question 'what is mathematics'. It should include not only analysis about the relationship between objective mathematical knowledge and the creative activities of the human, but also an explication of the subject (and nature) of mathematics. For example, according to the modern view, mathematics should be defined as 'the science of patterns' (cf. L. Steen, 1988 and Y. Zheng, 1991), and this definition seems to be a confirmation of the duality of mathematics, i.e., it is both descriptive and prescriptive.

(2) **The Analysis of the Nature of Mathematical Learning.** Differing from the study of epistemology in the philosophy of mathematics, the final aim of the analysis of mathematical learning is not to get a definite conclusion about the a priori and empirical nature of mathematical statements but rather to study the actual information process of the mind and explicate its implications for mathematics education. Therefore, the key question here is

whether mathematics learning is a process of 'transmission of information' centering on teachers or an activity of discovery (re-creation) by students. Besides, from the social-cultural view, there is also the question whether mathematics education is an isolated activity or an organic part of the whole cultural system of the human.

(3) The Aim of Mathematics Education. As a conscious activity of humans, mathematics education has its definite aim which should reflect the features of the time, i.e., it should meet the needs of the time and reflect the advance of science and technology. Particularly, we should analyze carefully the great influence on mathematics education exerted by the transition from the industrial age to the information age and the rapid development of computer technology. For example, as the information age is in some sense 'the age of mathematizing', the development of the society has made a higher standard for every student an historical necessity for mathematics education (cf. NRC, 1990, 1991). In addition, the rapid development of computer technology has not only offered efficient tools but also opened a new prospect for mathematics education. For example, with the help of computers, people can really be freed from the influence of the traditional conception of mathematics education that emphasizes very routine skills, and then concentrate on the promotion of the students' ability in problem-solving.

By the above discussion, we can now see clearly that there are both some important relationship and differences between the philosophy of mathematics and the theoretical foundation of mathematics education. What is more, it is obvious that we should also differentiate naive conceptions of mathematics education from systematic theories. Therefore, there is a deep need to introduce the concept 'philosophy of mathematics education'. To be explicit, philosophy of mathematics education consists mainly of the following contents: the view of mathematics, the analysis of the nature of mathematics learning (and teaching), and the aim of mathematics education; and as a whole it forms the theoretical foundation for mathematics education.

To make things clearer, we are going to make a brief introduction and comment on the most popular view of mathematics education in China. According to this view, the theory of mathematics education mainly consists of the following three parts: the theory of mathematics curriculum, the theory of mathematics teaching, and the theory of mathematics learning. 'The theoretical foundations of the theory of mathematics education include the following subjects: dialectical materialism (philosophy), mathematics, education, psychology, logic, and computer science.' (Cao Cai-han, 1989, p. 9) So, the basic framework of this theory of mathematics education is as follows:

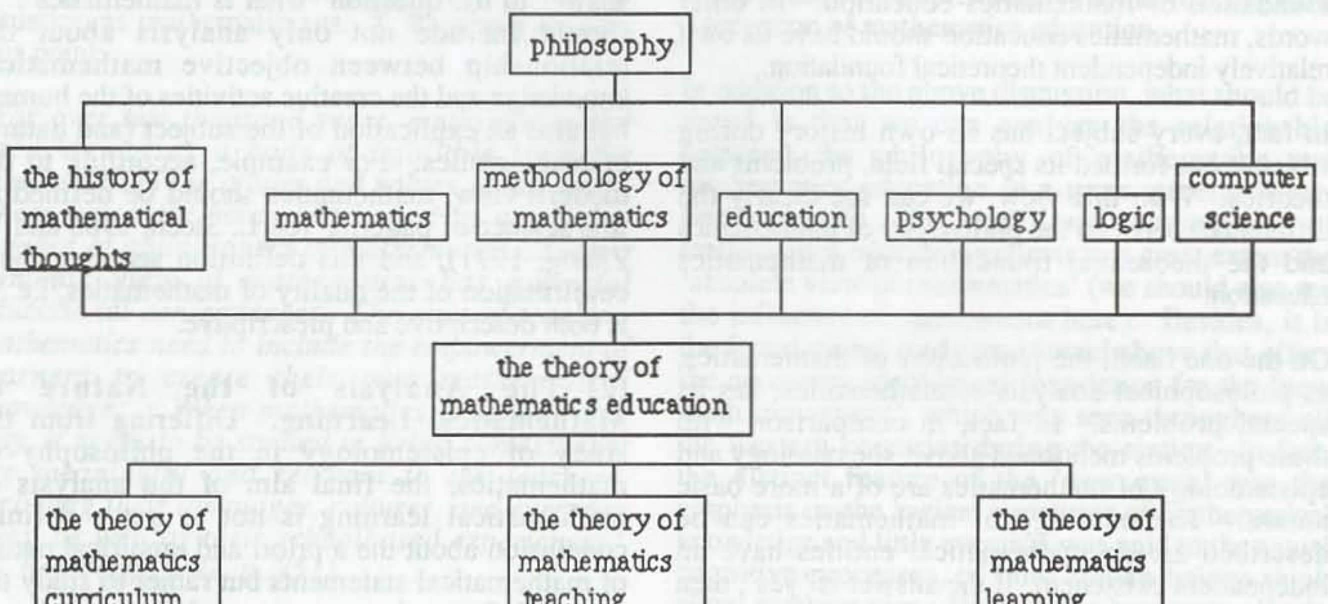

Figure 1

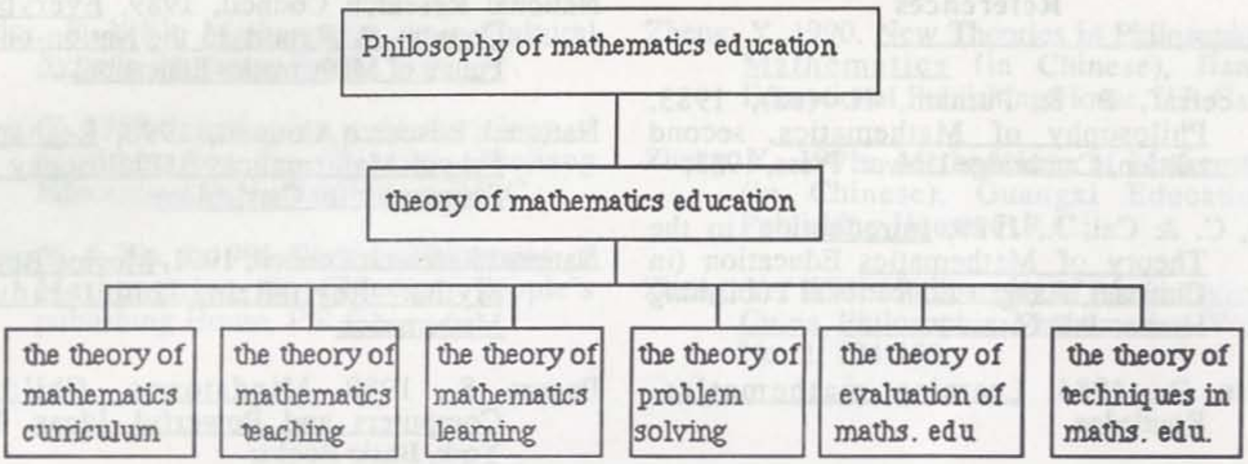

Figure 2

However, we now know clearly that the analysis of the theoretical foundations of mathematics education should not be limited to listing all the relevant subjects; instead, we should set up its own theoretical foundation, i.e., the philosophy of mathematics education. Therefore, we are, in fact, introducing the following new theoretical framework for mathematics education (in which we have also made some improvement and extension of the contents of the theory of mathematics education; however, it goes beyond the topic of this paper), as shown above.

Finally, what should be emphasized is that, although there is already some preliminary work in this direction (c.f., P. Ernest, 1990), philosophy of mathematics education is still a new field waiting for further studies. We can see by the above discussion that the founding of a systematic theory of philosophy of mathematics education needs cooperation between philosophers and educators. Actually, the most important thing is to introspect one's own concept of mathematics education, so as to transfer from the old, backward conception to the advanced and scientific conception of mathematics education. In fact, just as Everybody Counts, which is another important document for the new reform of mathematics education in the USA, points out, the following transitions 'will dominate the process of change during the remainder of this century':

Transition 1: The focus of school mathematics is shifting from a dualism mission—minimal mathematics for the majority, advanced mathematics for a few—to a singular focus on a significant common core of mathematics for all students.

Transition 2: The teaching of mathematics is shifting from an authoritarian model based on "transmission of knowledge" to a student centered practice featuring "stimulation of learning."

Transition 4: The teaching of Mathematics is shifting from preoccupation with inculcating routine skills to developing broad based mathematical power. (p. 81-82)

These transitions, of course, can not be carried out spontaneously in practice; just the opposite, 'naiveness' in philosophy (one frequent form of 'naiveness' is the ignoring of philosophy) always leads people to become slaves of some 'modern', but at the same time 'bad' philosophy. For example, what is called 'the radical constructivist view', which seems to be a 'modern fashion' in the world of mathematics education in the USA, is, in fact, a revision of intuitionism in the philosophy of mathematics. And as intuitionism necessarily leads to mathematical mysticism' and 'mathematical solipsism' by its denial of the representability and objectivity of mathematics, this philosophical view has already been widely criticized. Obviously, it shows more clearly the importance of the study of philosophy of mathematics education; and it in turn can also be regarded as an impetus for mathematics education to the further development of the philosophy of mathematics and philosophy in general.

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Poems by Lee Goldstein

Plighted Symbolism

Through the credential nonverbality
Are theorems of that that is of the not benamed abstraction,
And where the verbality, thence the symbolism seems may be to willing,
While the masters of the symbolism
Know that unwillingness and yet can act them;
For the locus of what is not benamed
Bodes mathematical,
And such constructivism is believed
In this very alterity.

1993

The Imagination

The infiniteth inbeing of desire expressed objectively,
For instance, 'the set of all sets which do not include themselves',
Implies an ineluctable phenomenon
That precludes mental escape,
Unless there is admitted the glamourous search
Of the not at the object,
But of a living, instead, past the paradoxes implicit in desired (or undesired) objects
Where truthful objectedness arises, identically,
Only upon a nonce imagination of the "things ideal."

1993