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The Nature and Experience of Mathematical Beauty

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This special issue grew out of a wish to bring fresh voices to bear on the age-old question of what is the nature of beauty in mathematics. This question has a long history, rooted in philosophical discussions about aesthetics going back to Hutchenson, Kant, and even the ancient Pythagoreans. The goal was not to recapitulate this history, but rather to connect those discussions to contemporary work in aesthetics and philosophy of mathematics.

We, that is Raman-Sundström and Öhman, began our exploration with the question of whether beauty and explanation might be related. Explanatory proofs are somehow privileged over non-explanatory proofs. Could this difference be aesthetic? Could there be some quality inherent to explanation that brings about the feeling that a proof is beautiful, or elegant, or cool, or pleasurable in some other positive way? The question is a natural one, but difficult to answer, particularly since aesthetic judgments are likely to be contextual and subjective, as Wells [13] and Burton [1] have suggested. Moreover, to make the question tractable we would need a model of mathematical explanation, and at present there is no such agreed upon model[1].

[1]See Mancosu [7] as a starting point for the debate.

Our first attempt to answer these questions took place in the context of a seminar run at Umeå University, attended mostly by local mathematicians[2], but occasionally also included philosophers, computer scientists, and mathematics educators. Together we read literature on mathematical aesthetics and mathematical explanation. Every other week we presented and discussed examples that group members considered to be beautiful, often contrasting two or more different proofs of the same theorem.

However, it did not take long to realize that, especially as it concerns philosophical questions about the nature of explanation, we were quickly out of our depth. To make some progress we brought together researchers from a variety of related fields, primalirily philosophy, education, and mathematics. Several of the members of that meeting, namely Marc Lange, Marcus Giaquinto, Lars-Daniel Öhman, and Boris Koichu, contributed papers to this special issue[3], and Nathalie Sinclair joined us as an editor. We also initiated an open call for this issue to bring in additional voices. We were happily surprised by the interest in this topic. With the help of a well-organized staff at the *Journal of Humanistic Mathematics*, we selected a number of these submissions which we thought most effectively brought new life and grounding into this subject.

One of our goals in selecting papers, to mirror our seminar, was to focus on rich, new examples (new in the context of discussions about mathematical aesthetics, not necessarily new in mathematics) that could shake, or at least nudge, our common conceptions. The standard examples, such as of the irrationality of the square root of two[4], felt overtrodden. We wanted new examples, both to test our prejudices about the nature of beauty and to deepen our understanding of its connection to other properties, such as explanatoriness.

[2]Regular members, who contributed with both their intellect and enthusiasm, included Lars Hellström, Olow Sande, Tord Sjödin, Jonas Hägglund, Victor Falgas-Ravry, Per-Anders Boo, and Håkan Persson.

[3]Other members, in addition to our beauty seminar members, included Hendrik Lenstra, Gila Hanna, Mark Steiner, Juliette Kennedy, Logan Fletcher, Fenner Tanswell, Kim-Erik Berts, Josephine Salverda, Daniel Molinini. For the full program see: https://mathbeauty.wordpress.com/wbem-schedule/, accessed on January 25, 2016.

[4]Discussed famously by Hardy [3], and also by contemporary authors such as Jullien [5] and Montano [8] and even, we must admit, two of the guest editors [9].

Most authors in this issue, we feel, have met this challenge. Lange offers a set of examples which, he suggests, might possibly bridge discussions on mathematical explanation with mathematical beauty. Giaquinto also addresses the relationship between beauty and explantion, via a separate set of examples from geometry, number theory, and graph theory. A nice proof by induction is offered by Öhman who argues it is genuinely beautiful, despite our prejudices against proofs by induction in general. Borwein's example of short random walks provides an account of changing aesthetic experiences in the course of a research undertaking. And Brown's Kaleidoscope chessboards provide a non-standard example of visual representation, raising the question about the role of symmetry in our perception of beauty.

Another goal of this special issue was to try to capture the *experience* of mathematical beauty, in contrast to studies of aesthetic judgements[5]. While analyses of judgements are informative, they quickly lead to difficult questions, such as how reliably people report their own experiences. Our emphasis on experience tries to shift away from what people say to what people do. Towards this goal, we offer three accounts by mathematicians, mentioned above (Öhman, Brown, Borwein), who describe striking proofs or solutions that arose out of their practice. There is still a good deal of reporting in these accounts, but there is hopefully sufficient detail to allow readers to draw their own conclusions, and enough mathematical content to make that reflection rewarding.

In addition to mathematicians' accounts of their practice, we have put an emphasis on the aesthetic experience of school students, joining what we hope to be a growing trend within mathematics education to study the aesthetic potential of school mathematics (e.g. [11], [12]). Do we do enough in school mathematics to bring about the kind of joy and satisfaction that mathematicians often experience in their practice? Two papers in this issue (Koichu, Dietiker) address this question. Koichu focuses on the element of surprise and discusses design choices to help bring about a sense of surprise for mathematics students. Dietiker likens a mathematics lesson to a story, showing how the sequencing of a mathematical activity can affect students aesthetic experience.

[5]For example Dewey [2], Rota [10], and Inglis & Aberdein [4].

Finally, we wanted to include at least one paper that dealt with the underbelly of aesthetics, that is to say what might be ugly or unsettling or even unappealing in mathematics. This is a theme that was considered, for instance, by the French mathematician François Le Lionnais, who contrasted what he called the *classical* form of beauty, marked by equilibrium, harmony, and order with *romantic* form of beauty, marked by lack of balance, pathology and form obliteration [6]. Maheaux's paper on wabi-sabi mathematics, which describes a Japanese aesthetic similar to Le Lionnais's romantic form of beauty, includes imperfection and crudeness as some elements worthy of our attention.

The result of our desire to include different voices means that this volume is a bit uneven in terms of tone, content, and perspective. Perhaps that is an advantage–there are many entry points into the subject, for people with different backgrounds and dispositions. Some papers are a bit more weighty and some more whimsical. Some are written by mathematicians, some by philosophers, and some by educators. But all the papers are meant to move you in some way, to find a new example in an unfamiliar place, to provide a fresh way of thinking about the subject, to bridge across ideas that you might have thought were not related at all.

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