Journal of Humanistic Mathematics

Volume 1 | Issue 2 July 2011

A Mathematician Weighs in on the Evolution Debate

Kris H. Green
St. John Fisher College, kgreen@sjfc.edu

Recommended Citation
Green, K. H. "A Mathematician Weighs in on the Evolution Debate," *Journal of Humanistic Mathematics*, Volume 1 Issue 2 (July 2011), pages 75-95. DOI: 10.5642/jhummath.201102.09 . Available at: http://scholarship.claremont.edu/jhm/vol1/iss2/9

A Mathematician Weighs in on the Evolution Debate

Kris H. Green

Department of Mathematical and Computing Sciences, St. John Fisher College
kgreen@sjfc.edu

Synopsis

There are a variety of reasons underlying the lack of public acceptance for the theory of evolution in the United States. An overlooked cause is related to problems with the mathematics curriculum in the K-12 setting. In this essay, we examine this relationship and propose changes to the mathematics curriculum that could improve mathematical thinking while also providing a basis for understanding theories, like evolution, that are poorly understood.

1. Introduction

Despite the large body of support for the modern theory of evolution, there remains substantial public debate in the United States regarding its accuracy and correctness. In addition, there are numerous misconceptions about the theory that have little to do with the underlying biology, but are, rather, about the behavior of complex systems. In the interest of full disclosure, the author is not, by any measure, an expert on evolutionary biology or biology of any sort. What is offered here is a perspective on the relationship between the mathematics curriculum and the teaching and understanding of evolutionary biology.

This essay begins with a survey of some of the reasons why the public has yet to embrace the theory wholly. While not a comprehensive analysis, it does cover a wide range of well-known causes for the low rate of acceptance, and then offers a new reason related to the current mathematics curriculum. Some specific problems with the mathematics curriculum are then discussed, and these are contrasted with components of the theory of evolution, the understanding of which is not supported by the current mathematics curriculum. As a way past this mismatch, the introduction of the study of complexity and its associated nonlinear dynamics, chaos, recurrence, and agent-based models, is proposed. Others have already demonstrated that these topics are well within the intellectual reach of middle and high school students and can result in measurable gains in student scientific and quantitative literacy. The essay concludes with thoughts about how this might impact science and mathematics education overall.

2. Some reasons evolution is not more widely accepted

Religious fundamentalism is clearly an important – probably the primary – reason why much of the public fails to accept the theory of evolution. In recent years, the debate between scientific and religious theories concerning life on planet Earth has intensified. We have seen new "scientific theories of creation" arise in the form of Intelligent Design (ID). We have seen school boards in Kansas and politicians in Tennessee make decisions about the placement of evolution and ID in the K-12 classroom [5]. We have witnessed the Southern Baptist Association attempt to convince its members to pursue home-schooling rather than subject their children to teachings about evolution in the public schools. Indeed, recent work has shown that acceptance of evolution is lower in the United States than almost any other country in the world [19, 21] and that this is strongly correlated to fundamentalist beliefs. Nor is this debate restricted to the United States [10]. However, even the traditionally conservative Catholic Church has accepted the merits of evolution for explaining life, so the religious arguments, while currently important, can be overcome [22].

The second reason is a matter of language, particularly the false dichotomy of "scientific theory" versus "scientific law". In fact, in modern usage, both are essentially the same thing. But in the eyes of the public we see a significant misconception regarding what a scientist means when she uses the word "theory". In grade school most of the readers of this article probably learned that science starts with a hypothesis (educated guess) which develops into a theory (that is a little better than a hypothesis) and finally becomes a law when it has been tested "enough". This jibes well with the everyday language use of the word "theory". A quick Google search (type "Define: theory") turns up a bewildering array of definitions. One of these, not surprisingly, is a link to a Wikipedia article on the different uses of the word and the importance of the context in which it is used. This in turn links to a Wiktionary page on the word. By the end of this journey, the interested reader will have been exposed to many more possible interpretations of the word than discussed above. Most notable, though, are the definitions labeled A and B below, which seem at odds:

A. An unproven conjecture.
B. A logical structure that enables one to deduce the possible results of every experiment that falls within its purview. [26]

Is it any doubt then that the average person is unaware of the contextual distinctions in the use of the word? A quick survey of its use in common language would, I suspect, turn up more instances of usage A than usage B, further reinforcing notions about science, particularly ideas that until something becomes a "law" it has not been proven.

In point of fact, however, science has not made use of the word "law" to describe a new well-documented and supported theory for quite some time [20]. While old explanations, such as Newton's Law of Universal Gravitation, are still on the books with this moniker, we know that later evidence has resulted in many, this one in particular, being supplanted as better evidence becomes available. In this case, Einstein's Theory of Relativity provides more accurate predictions for the motions of objects under the influence of gravity than Newton's "Law". Relativity has been tested and experimentally verified to an even greater precision than most theories in other areas, so we generally accept it as true, even though we understand that (a) new evidence (such as that generated from the search for dark matter) may result in further modifications to the theory and (b) it reduces to the Newton's version of gravity under the conditions which Newton had available at the time he developed his theory. This means, simply, that should a new theory of gravity be proposed – and many have been suggested – it must reduce to relativity under the conditions which led Einstein to his glorious achievement and must, in turn, reduce to Newton's laws under appropriate conditions as well. But none of this means that we should call it "the Law of Relativity" even though this revision of current nomenclature might help; after all, in the eyes of many the "Law of Evolution and Natural Selection" sounds more impressive than the "Theory of Evolution".

This misconception about the word "theory" is exacerbated by poor science journalism. When two climatologists propose different predictions about world climate, the urge to reduce the information to sound bites results in very different pictures of the scientific community and our understanding. The sound-bite version might easily be interpreted as "Two competing theories: which is right?" while a fuller picture would illustrate that these researchers probably used different methods and data, applied to different aspects of climate, and that the two models are complementary or supplementary rather than conflicting. Further, the real version would probably also clarify that these are both modifications to an existing theory, rather than competitors to overthrow commonly accepted theories of climate change.

Even worse, the person-on-the-street uses the word "theory" incorrectly as a substitute for the word "hypothesis". When the strange man next to you at the checkout counter points to a tabloid picture of a baby with wings and says "I have a theory about this ...", what he really means is that he has a hypothesis about this. Without testing to confirm it, a hypothesis is just a guess that is based on some type of evidence, be it anecdotal, observational, or imagined (in the case of the baby-with-wings). However, the word "evolution" is already so overloaded with connotation that the confusion about what a theory is may not be the most significant barrier to the theory's acceptance.

Another major problem in this debate is caused by the school science curriculum. It is typically presented as either experimental or as a set of facts to be learned from books. Admittedly, this is a somewhat false dichotomy of science teaching; many science teachers at all levels recognize the need for both aspects and use them appropriately, but the dichotomy still exists in many U.S. classrooms. For example, some falsely claim that since no one witnessed the past 4 billion years of earth's history, we have no observations on which to base the theory of evolution, and so all we can teach is "facts of evolution". This misconception grows from a further simplification of what science is about. Even when experimentally "collecting data" a scientist is not an unbiased machine. Indeed, the very decisions as to what data to collect, how to collect it, how to analyze it and what it means all grow from some past concept of what the experiment is likely to show. When we encounter evidence, it must fit within that framework to support our theories, or we must revise our theories to account for the discrepancies. It is this, ultimately, that leads different people to develop different theories about the world – they start from a theoretical framework, perceive the evidence, determine which evidence supports the theory and then determine what to do with the rest: Discard it as "bad data" or a random event, consider it as further evidence that "the supernatural works in mysterious ways", or use it to lead the way to a new theoretical understanding that encompasses both the confirming and disparate data. Science is self correcting; the latter one is what scientists actually do. Much of the current science education reform movement is addressing just such matters, giving hope that this particular barrier can be circumvented.

And, assuming that one can get past these issues, there are numerous other misconceptions about evolution that persist in textbooks and popular media representations [16]. These range from oversimplification of the theory to misattribution of purpose and intent to assuming that there is no way to see evolution in action. Some of these details are clearly the purview of biology educators; but others are far removed from the science classroom.

In the end, none of these explanations is wholly satisfactory to explain the vast gulf separating the public from science. Evidence and logic have, in the past, won against non-scientific explanations in many cases. The disciplines of science and science education face many competing pressures, and school science, especially at the K-6 level, has come a long way since my childhood. Indeed, scientists and science educators are to be commended for their work, even though there is still a long way to go. And the blame cannot be entirely on the popular misuses of words or misunderstanding the intricate details of the theory.

There must be another, more subtle reason why evolution is so misunderstood and misperceived. In the interest of being provocative, mathematicians and mathematics education are to blame. Unlike evolutionary biology, mathematics has an established place in the school curriculum; it is not fighting for ground from which to make arguments and influence the public. And this complacency has led to the current situation. For, as Lynn Arthur Steen has pointed out (see the Preface and Part I of [25] for details) mathematicians are teaching, in many ways, a curriculum that is a century old while students are growing up in a world that has passed it by. Understanding that world requires a new understanding of mathematics.

3. Problems in secondary mathematics

It is no secret that many consider the U.S. secondary mathematics curriculum and the way it is implemented to be flawed. Any complex system involving the competing interests of so many different groups is likely to have flaws. Others have raised their voices on this matter, accusing it of being "a mile wide and an inch deep". Here we do not seek to enumerate all the issues that may or may not exist; the curriculum itself is the focus. And while the move to a national common core curriculum in mathematics is in process, a cursory survey shows that most existing K-12 mathematics curricula share these common characteristics.

3.1. Narrow focus

It may seem odd to quote the "mile wide and inch deep" criticism and then turn around and accuse the curriculum of narrowness, but the current U.S. mathematics curriculum is designed to provide students with a comprehensive set of skills primarily in one branch of mathematics – the branch that leads to calculus [25, page 4]. Unfortunately, this is but one field of the discipline, and most of the students who are shoved into this pipeline do not reach the end of it [25]. As a result it serves more as a barrier to learning further mathematics than as a pathway to learning more mathematics. Certainly there are compelling reasons for including calculus in the curriculum as a cornerstone of mathematics. It has been well-studied and well-developed to help explain certain types of phenomena: The motion of a falling raindrop (in the absence of air resistance and wind currents and Coriolis forces and . . .), the growth of a cell (based on its size and some physical characteristics, but not accounting for the distribution of food in the environment or competition among nearby cells or . . .), or computing the mass distribution or moment of inertia of a physical object (whose shape is not too complicated . . .), among many others. And while the branch of mathematics which includes calculus has many important open questions, students rarely find this out. In many cases, it is somewhat difficult to even get mathematics majors in college to understand that we do not know everything about mathematics. To be certain, many high schools now offer courses related to some other branches of mathematics – statistics is particularly popular, made so largely by the AP exams, and most curricula provide a brief diversion into planar geometry – but this hardly provides students a chance to see the breadth of mathematical ideas. Such a narrow focus may be appropriate and may be necessary in order to implement standards for high school students, but it does ignore branches of mathematics that illustrate more complexity, give rise to more obvious open questions, and are still largely accessible to high school students.

3.2. Lack of context and depth

Most U.S. mathematics curricula also lack context. True enough, all involve some sort of "word" or "story" problems, but part of the real beauty of mathematics is lost in such problems because the process of mathematizing a real-world situation has been completed for the students. Thus, the problems are abstracted, and assumptions have been made that, to many students, divorce the problems from the real world so much as to make the two unrelated. This issue is not limited to the K-12 mathematics curriculum. It extends to almost any mathematics course in which the real world is only seen through word problems (or not at all). Green and Emerson in [14] contrast two problems to illustrate this issue. One shows what a real application of mathematics might look like in a situation involving mathematical tools to help quantify ill-defined concepts and make judgments in the face of uncertainty. The other problem has been abstracted to the point where the students are not required to engage the real world past the point of writing the answer in a full sentence with the proper units.

Wigner claimed, over a half a century ago, that mathematics is "unreasonably effective" when it comes to explaining and predicting natural phenomena [28]. How sad then, that students do not see the truth of this, simply due to the abstracted and far-removed nature of the curriculum in which mathematics is offered to them. The curriculum is driven not by natural questions about the world which mathematics can then help explore and understand, but rather by a list of topics, much like hurdles on a track, which must be overcome.

3.3. Over-reliance on linear and one-variable thinking

The narrowness of the curriculum leads to an additional set of problems. By building from "simpler" to more "complex" mathematics strictly in service to the calculus, students intuit that everything is linear (or approximately linear) and that all situations can be reduced to finding the one variable, X, that is unknown. Yet many real problems are not linear (if a 12-pound turkey needs 8 hours to cook, should we cook a 24-pound bird for 16 hours?) and very few are dependent on only a single variable (is fuel economy a factor only of speed, or should we account for driving style, road conditions, tire pressure, and car body types?)

One ignored branch of mathematical study includes linear algebra; this course is often used in college as a bridge between computation mathematics courses and more abstract courses emphasizing proofs. However, the computations in linear algebra are largely related to the addition and multiplication of numbers (something most students could use more practice at anyway) and solving systems of linear equations (already a major part of the high school curriculum). Further, these operations are easily implemented on most graphing calculators, a tool available to many of our high school students already. What is significant conceptually, though, is that linear algebra – the study of the properties and applications of matrices – is inherently designed to model more complex phenomena through the interaction of several variables at once. If you have several individuals competing for resources, there are many matrix-based models for this. If you want to study the possible paths a rat may take in navigating a maze in order to test whether the rat is improving its navigation ability or memory, a Markov chain and its associated transition matrix (an example of a matrix utilizing probability) is easily constructed and studied. If you would like to explore the accuracy and tendencies of a quarterback in throwing to different areas of the field, this can be represented by a matrix.

The emphasis on linearity has resulted in oversimplified thinking on the part of many students. An important example related to public policy and personal decision making is the chronic misunderstanding associated with savings in fuel economy from trading in a vehicle [15]. Some claim that the curriculum must be simplified in order to function. In many application problems in the high school curriculum we make the broad simplification of assuming that the problem is linear or we limit ourselves to only considering such cases. Certainly, students at the upper end of the path to calculus explore quadratics, trigonometric functions, logarithms and exponential functions, but even these do not exhibit real complexity in the modern sense of the word. This emphasis on linearity has the benefit of being easier to teach and helping set up skills that are somewhat transferable to other models. But it has disadvantages outweighing these. Students focus so much on linear situations that they attempt to apply this reasoning to many nonlinear situations, leading to significant errors in thought [4, 11, 27].

Furthermore, the skills of solving linear equations, while useful and theoretically applicable to higher order polynomial equations, are often impractical to apply. Consider that the *sine qua non* in the study of second degree polynomial equations – the next level of difficulty up from linear equations – is their solution by the quadratic formula, something that obscures the relationship between solving linear equations and solving quadratic equations. Finally, the study of linear equations in high school is confined to applications with a single explanatory (independent) variable. This is certainly not true in most real situations. For example, my decision to sell a stock will rarely depend on a single input. It is more realistic to think that my decision will be based not only on its performance yesterday, but on my comfort with risk, my understanding of the Federal Reserve and interest rates, and some knowledge of what the company does and how this is likely to fare in the near future. A single linear variable cannot hope to capture this level of complexity, but other models can account for many of these variables and provide a more reliable decision-making process. Even more important is the fact that although we rarely talk about the secret that the real world is multivariable, students are all-too-often aware of this. Their awareness further distances them from learning the mathematics we seek to teach, since they see this oversimplification as a lack of consideration for reality.

This reliance on linear thinking extends into the interpretation of most mathematical models. Consider the approach followed in calculus: We take a derivative, evaluate it at a point, and discuss the rate of change at that point. Yet the derivative is an inherently linear concept. Only straight lines emerge from differentiation with a simple rate-of-change rule; by forcing this interpretation of change on all functions, we lose the ability to deal with other situations more naturally. The rate of change of exponential functions can be more naturally expressed as an exact "percent change in the dependent variable per unit change in the independent variable" rather than an approximate "fixed change in the dependent variable per unit change in the independent variable" (a linear interpretation.) Power functions and logarithmic functions have similar interpretations that are more natural and more easily applied [13].

3.4. Unintended Lessons

Another problem in the structure of the US mathematics curriculum is more subtle. Consider a typical data-based approach to learning algebra. One can start with simple data that show a clear upward or downward trend and fit a least-squares line to these data using a graphing calculator or spreadsheet. Thus, we need to explore how linear functions behave to understand constantly increasing or decreasing data. But suppose the data curve upward, or change direction? In these cases, we use a quadratic or exponential function to fit the data. What if the data change direction many times? Then a more complicated polynomial with more and more terms might be warranted.

This development is in many ways appropriate. It extends a basic idea, fitting a function to a set of data, to successively more complicated data. But it sends an underlying message that is at odds with complexity theory. This unintended lesson is that complicated behavior in one's data requires more and more complicated equations. Combined with some of the earlier issues, the following corollary emerges: Real situations are never as simple as they are in word problems, and must require such complicated formulas that no one can claim to understand them or really predict anything for sure. Students then reach an obvious conclusion: Only complicated equations are realistic, so studying these simple equations is a pointless school-type activity that will not be needed in the real world.

4. Evolution is a complex system

The previous list of issues with the mathematics curriculum would generate plenty of concern even if mathematics were an isolated field of study. But it is not. Mathematics has an important role to play in all of the sciences, whether natural, technical, or social. And while this relationship was not always highlighted in biology studies, it can no longer be ignored. The modern study of biology requires a deeper understanding of mathematical ideas, albeit ideas different from those in the focus of the current curriculum.

One of the driving forces of this connection is the evolution of the high school biology curriculum itself, which has certainly changed a great deal in the past decades, shifting from a taxonomic approach to the study of life to a more systems-oriented, first-principle study driven by genetics. An important part of this curriculum is the study of evolution. After studying the variety of life on earth and how it all has developed into balanced ecosystems over time, one would certainly not be wrong in thinking that evolution represents a very complex system. The theory describes the way that simple molecules combined into larger molecules, which formed the basis of single-celled life forms. These, in turn, interacted over time to lead to more and more complex organisms which competed for resources. All along, mutations – slight changes in the organisms – gave some a survival advantage that could be passed down to their offspring. As the number of distinct organisms and the variety of species increased, the niches formed were filled and vacated, competition and cooperation came and went, and life got more and more complicated. This all leads to the vast biodiversity seen today, ranging from bacteria, archeae, and viruses to felines, flounders, and falcons.

Clearly, if this theory is remotely correct, the evolution of life involves repeated interaction of components ranging in scale from single atoms and molecules to single-celled organisms to animals to entire species. At each of these levels, various rules dictate the way the components interact. Eventually, these interactions result in stable dynamics among the organisms, dead ends, oscillations, or any of a myriad of other possibilities. Based on these ideas, the field of artificial life [a-life] has been working from two directions to understand evolution. On the one hand is the "wet" side, seeking to build life from the ground up in the lab. On the other hand is the more theoretical side, seeking to develop robots that mimic living organisms (hard a-life) and software simulations (soft a-life) to reproduce life-like situations [2]. Developing and understanding these models requires the science of complexity, using agent-based models, nonlinear dynamics, recursion, and emergence to gain insights into the deepest secrets of life.

Those developing and studying these models must make sense of the different "levels" that play a role in evolution [30]. Basically, rules must be developed to govern the micro level (genes and molecules) so that the expected behavior emerges at the macro level (species). In the words of Epstein [12] "If you didn't grow it, then you didn't explain its emergence" so that a theory about the micro level that does not produce the expected behavior at the macro level would be ruled out. And while life-like results at the macro-level would then be suggestive that one had discovered the appropriate rules for the micro level, all that would really be known is that one has a set of rules governing the micro level that are consistent with results and behaviors at the macro level.

Thus, it seems that in order to understand evolution, let alone to contribute to its scientific understanding, one clearly needs to have a way of thinking that is not narrow, linear, single variable, or de-contextualized. Wilensky and Resnick in [30] have discussed the need for us to rethink the content of the mathematics and science disciplines and how they are connected using the concept of "levels of organization." They demonstrate through three case studies how individuals, without some exploration and understanding of complexity, attribute "intentionality" and "leadership" to some elements of the system when it may in fact be completely absent. This is often the case in informal (and some formal) discussions of evolutionary biology when there is a discussion about "evolution leading to more advanced organisms" or "organisms are trying to adapt."

5. Teaching about complexity

So now we have established that the real world – especially the theory of evolution – involves complexity in a fundamental way and that the existing mathematics curriculum does not provide an adequate coverage of the concepts necessary for understanding and interacting meaningfully with this complexity. Thus, it seems that some changes to the curriculum are warranted. Fortunately there exist a number of excellent mathematical concepts that are appropriate for students to explore. And through these explorations, students can also develop many of the procedural skills that are the focal point of the current curriculum. Some examples which could easily be incorporated in the curriculum include the $3n + 1$ problem, the Game of Life, logistic growth models, networks, and agent-based models.

The $3n + 1$ problem and the associated Collatz Conjecture [8] are simple to understand. Pick any whole number. If it is even, divide it by two; if it is odd, multiply it by three and add one. Then iterate this process many times, each time dividing by two or multiplying by 3 and adding one as appropriate. If a class of students does this with the numbers between 11 and 30, each student working with their own number, it will not be long before they start to notice patterns. For example, all of these initial values eventually lead to a cycle repeating the numbers $4 - 2 - 1$. It is an open question, the Collatz Conjecture, whether this is true for all whole numbers. Some of the initial numbers result in very short sequences before settling into the cycle; others, which can be initially "close" to those with short paths, can lead to incredibly long paths. Students can easily generate and explore a variety of patterns using this simple starting point. The key idea is that even a simple arithmetic process like this can generate complex patterns if it is repeated many times. Engaging the problem requires only knowledge of even/odd numbers and the arithmetic of whole numbers. It offers students a chance to look for patterns, generate conjectures, and test hypotheses. The investigations can be enhanced through technology using spreadsheets, any of a multitude of applets, or even writing short programs.

The logistic map [18] provides a way to show how a simple computation – in this case a quadratic – can lead to really different dynamics as just a single number is changed. The mapping is commonly used to describe how a population changes over time in an environment with limited resources. As the intrinsic growth rate of the population changes, the behavior transitions from a smooth growth toward a maximum carrying capacity, to oscillations around that carrying capacity, to oscillations with additional "frequencies" to seemingly random behavior. Studying the logistic map requires some notation – especially of discrete models – and basic algebra skills for evaluating an expression by plugging in values. It offers an opportunity to introduce graphing skills and recurrence relations, leading to the study of deterministic chaos and more general discrete dynamical systems.

The mathematical subject of network theory is an outgrowth of the older subject of graph theory which was introduced by Leonard Euler as a way to study problems involving connections among different objects, see for instance [7]. These could be the way different land areas are connected by bridges, the way rooms are connected by doors, the way cities are connected by roads, buildings by power distribution systems, people by relationships, or molecules by metabolic functions. Over the past few decades, these models have been applied to almost everything in the natural and man-made worlds, with the result being a discovery that all networks have similar mathematical properties and structures. Biehl in [3] offers a series of simple activities using graphing calculators for exploring the properties of a network that grows over time – one similar to the World Wide Web. It provides an opportunity to explore a system with many variables in a systematic way that demonstrates a variety of patterns for study. The activities Biehl provides require little from students other than the basic idea of a graph (which takes only a few minutes to grasp) and the basic terminology of graph theory related to vertices, edges, and degrees. After these activities, though, students develop insight into some fundamental concepts, such as "the rich get richer" and gain experience in thinking about complex systems through a systematic process of investigation. The graphing calculator program even offers a chance to explore the interplay between mathematics and programming.

John Conway introduced the Game of Life [9, 23], as a way to explore how simple rules can generate complex behavior. Traditionally it is played on a two-dimensional rectangular grid. Each square in the grid is either empty or contains a cell. The game of life progresses through turns, called generations, by updating the arrangement of the empty squares and cells according to three simple rules:

1. Death. If an occupied square is surrounded by 0, 1, 4, 5, 6, 7, or 8 occupied neighbors, the cell dies and the square becomes empty. If it has 0 or 1 neighbors, it dies of loneliness; if it has 4 or more, it dies from overcrowding.
2. Survival. If an occupied square has 2 or 3 neighbors, it survives to the next generation.
3. Birth. If an empty square has exactly three occupied neighbors, a new cell occupies that square at the next generation.

While there are now thousands of websites devoted to the game and numerous downloadable simulations that let the user change everything from the initial configuration of occupied and empty squares to the specific rules of the game, what is important is that this is all there is to the game. However, under many general conditions, a variety of behaviors can result. It is possible for every cell to die and the entire grid to empty out (extinction). It is possible to enter a stable state, where the pattern of occupied and empty cells is either unchanging from one generation to the next, or cycles through a set of patterns. Most interestingly, it is relatively easy to develop patterns which grow forever, with more occupied cells at each generation. In some of these growing patterns, one can create groups of cells which "reproduce," making copies of themselves.

In *The Recursive Universe* [24], Poundstone uses the game of life to make analogies to the physical world we inhabit. If a few simple rules can generate arbitrarily complex behavior in a game like this, why can't a similar set of rules, possibly a little more complex, like the rules for how electrons are shared in a chemical bond, lead to complicated organisms when applied to large groups of molecules? After all, an organism is simply a collection of cells that are working together under some rules, and these cells are collections of molecules. This study of cellular automata has developed considerably in the last quarter century, culminating, in some sense, with the publication of *A New Kind of Science* [32] which claims that all of mathematics and physics can be represented by computational systems ("programs") that are based on the mechanics of such cellular automata (pages 7–11, expanded in chapters 8–12). Regardless of the potential depth of the applications of this area, studying the game of life requires relatively little, past the ability to count and apply the logic built into the rules. After initial explorations, which can be conducted with pencil-and-paper or simple physical manipulatives, students can easily see a variety of behaviors, explore how changes in the rules affect the outcomes, and think about how changes to the system itself (such as switching to a hexagonal grid) influence the patterns that develop. Students can learn about programming and extend their understanding of logic and proof to contexts outside geometry.

The relatively simple cellular automata that began with Life have now developed into more complex, parallel-processed agent-based models [6]. These models take advantage of parallel-processing algorithms to allow the user to track the interactions of thousands of individual objects. Each object possesses its own rules for behavior, as does the underlying environment. Such models are relatively easy to explore by modifying existing examples. But more importantly, students can learn to create their own simulations using software like StarLogo[1] to explore whatever they are interested in, be it predator-prey dynamics, traffic jams, bird flocking, voter preferences, or hypothetical zombie attacks [1], Most of the software options provide the user with a multitude of reporting options, offering the chance to study the simulation results and search for patterns and connections. Agent-based modeling requires little up front other than some background about the basic ideas, which can be developed through explorations and examples [29]. After spending time to engage these types of models, students can interpret the connections linking parameters and initial values to the outcome of a simulation. They can begin to understand what is meant by "emergent behavior" at a higher level of a system. They can develop some skills in programming and logic, if they create their own agent-based models. By making use of the various outputs such software typically provides, they will be practicing quantitative skills and finding ways to meaningfully represent and interpret data.

Many of the outcomes that have been associated with the study of complexity above are touted as desired outcomes of the current curriculum. The way they are achieved is vastly different, offering many additional experiences that enrich each other and the understanding of the students. Most offer "back door" opportunities to approach some of the more traditional content of the curriculum, providing a motivation and connection to the real

[1]Available at `http://education.mit.edu/starlogo/`, accessed May 5, 2011.

world that might give students something they want to learn, rather than rules to memorize in order to play the teacher's game and survive to the next level.

6. Conclusion

One can certainly argue that making room for such examples in the K-12 mathematics curriculum would force out certain other topics that are important. However, we can build on the topics already in the curriculum quite easily, and we can use examples from complexity to explore and introduce some of the more traditional mathematics. Rather than over-emphasize the use of algebra for solving equations, we can incorporate the rule of four so that students explore such examples algebraically (for setting up the equations and spreadsheets), numerically (tables of data), graphically, and verbally (by describing the phenomena they are seeing and constructing some meaning for themselves). This would shift mathematics from being a tightly sequenced, finite subject that students must learn facts about to becoming more like an experimental science. The role of the mathematics teacher would then be to design learning experiences that build from real world questions about interesting phenomena to provide opportunities for students to explore the situations, construct meaning, and share their common experiences. They could then compare their understandings to the ways in which experts in the field have described the situation and connect it to other mathematics. Relatively few essential skills would be required up front, primarily the rule of four as a way of exploring the situation, and there is plenty of room for students to explore situations from the real world or ones that are more abstract.

Another potential objection to this redesign relates to whether students are ready for and capable of investigating such examples in a meaningful way. Penner in [23] shows that even middle school students can develop an understanding of complexity and emergent behavior, so it is clear that students are capable of engaging with such concepts and materials. To do so, they may need help. But it is a different kind of help than simply repeating thousands of identical, template-driven, practice problems. Instead, they need help to develop a level of comprehension that bridges the gap between the micro- and macro- levels of the models being explored [17]. Once they make this transition, though, students are highly capable of using these techniques to participate as scientists by making observations and generating and testing hypotheses [29].

The examples above show that teaching about complex systems could improve mathematical learning by offering students more opportunities to create and explore their own conjectures. Students would have opportunities to focus on developing a deeper understanding while connecting more ideas across disciplines and across levels of organization. The examples would offer motivation for studying mathematics through their direct connection to the real world without too much abstraction. Together these would lead to students developing more of a sense that mathematics is a living, dynamic field. And by addressing some of the misconceptions that the curriculum currently fosters, teaching complex systems might improve the understanding of science in general and topics like evolution in particular, since students would have concrete examples of how a simple set of logical or arithmetical rules can lead to very complex emergent behavior that has no central director making decisions.

Will including more of the study of complexity in K-12 mathematics resolve all the issues in the mathematics curriculum or guarantee broader acceptance and understanding of evolution? Maybe not. But just possibly, if this is the "broken window" that, when repaired, puts education on a stronger path toward scientific and quantitative literacy, then we are obligated to consider it. (The Broken Window Theory is based on an essay by Wilson and Kelling [31] that claims there are seemingly innocuous signs of decay – broken windows – that lead to pervasive changes in attitude and behavior; by correcting these, the larger issues can be resolved indirectly. The theory has been used as a basis for several public safety plans.) As with any complex system like education, a small change in unlikely places can lead to dramatic differences in the final state of the system.

It is important to note, before closing, that the author's interest in complexity came not by way of classroom experiences in high school or college. Nor did they come about from seeing mathematics-specific materials in the popular press. They occurred almost by accident, as a sideline to an interest in computers and programming. Imagine, then, the number of students who never see this in school and never encounter it as part of their other interests, or who encounter it but perceive it as irrelevant. Changing the curriculum to provide students exposure to these ideas will certainly challenge many preconceived notions about mathematics. Most importantly, it will force us to re-examine what we think of as being fundamental to a good mathematical education. Is it specific mathematical facts? Is it specific computational procedures? Is it a lens through which to view phenomena? Is it the ability to create models of the world? Is it a process for systematic exploration of a phenomenon? These questions must be carefully considered if we are to achieve quantitatively literate citizens capable of using these concepts to inform their decision making and understand the world.

Obviously, this brief distillation of the large, historically rooted debates on evolution and the mathematics curriculum is guilty of hypocrisy. These are clearly not the only factors at work and emotional investments that individuals have in these issues are paramount. Many of these emotions are driven by elements of fear and uncertainty. As science has discovered more ways that we – once believing ourselves to be ordained by God as the pinnacle of life in the universe – are not an intended outcome or even an inevitable outcome of natural phenomena, we are relegated further from the center of the universe and our role of specialty. As a species, we have been slipping further from center stage since Galileo and Copernicus showed us our relationship to the sun in clearer light. The further we slip from the center of the room, the closer we get to the dark corners where anything could lurk. But as we approach these dark places, we are armed with a powerful torch to light the way: Mathematics.

Acknowledgments

The author would like to thank Dr. Barney Ricca for comments on an earlier draft of this manuscript and the editors for suggestions that have greatly improved the logic and organization of this essay.

References

[1] Badham, J. and Osborn, J. (2010). "Zombies in the city: a NetLogo model". Accessed May 7, 2011 from `http://maths.anu.edu.au/~osborn/publications/pdfs/Zombies.pdf`

[2] Bedau, M. (2003). "Artificial life: organization, adaptation, and complexity from the bottom up". *TRENDS in Cognitive Sciences*, **7**(11):505–512.

[3] Biehl, L. (2003). "Massive graphs, power laws, and the World Wide Web". *Mathematics Teacher*, **96**(6):434–446.

[4] Bonotto, C. (2003). "Suspension of sense-making in mathematical word problem solving: A possible remedy". Retrieved September 30, 2010, from http://cimm.ucr.ac.cr/resoluciondeproblemas/PDFs/Bonotto%20Cinzia.pdf

[5] Britt, R. (2011, April 13). "Attacks mount on teaching of evolution". Livescience. Retrieved May 3, 2011 at http://news.yahoo.com/s/livescience/20110413/sc_livescience/attacksmountonteachingofevolution

[6] Castiglione, F. (2006) "Agent based modeling". *Scholarpedia*, **1**(10):1562.

[7] Chartrand, G. (1985). *Introductory graph theory*. Dover, New York.

[8] *Collatz conjecture*. (2011, May 2). In Wikipedia, The Free Encyclopedia. Retrieved 14:09, May 3, 2011, from http://en.wikipedia.org/w/index.php?title=Collatz_conjecture&oldid=427012204

[9] *Conway's Game of Life*. (2007, January 12). In Wikipedia, The Free Encyclopedia. Retrieved 18:28, January 12, 2007, from http://en.wikipedia.org/w/index.php?title=Conway%27s_Game_of_Life&oldid=100238490

[10] Curry, A. (2009, February 27). "Creationist Beliefs Persist in Europe". *Science*, **323**:1159.

[11] De Bock, D., Van Dooren, W., Janssens, D., and Verschaffel, L. (2002). "Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors". *Educational Studies in Mathematics*, **50**:311–334.

[12] Epstein, J. (2006). *Generative social science: Studies in agent-based computational modeling*. Princeton University Press, Princeton.

[13] Green, K. (2008). "Using spreadsheets to discover meaning for parameters in nonlinear models". *Journal of Computers in Mathematics and Science Teaching*, **27**(4):423–441.

[14] Green, K. and Emerson, A. (2010). "Mathematical Reasoning in Service Courses: Why Students Need Mathematical Modeling Problems". *Montana Mathematics Enthusiast*, **7**(1):113–140.

[15] Larrick, R. and Soll, J. (2008). "The MPG illusion". *Science*, **320**:1593–1594.

[16] Le Page, M. (2008, April 16). "Evolution: 24 myths and misconceptions". Accessed May 3, 2011 from `http://www.newscientist.com/article/dn13620-evolution-24-myths-and-misconceptions.html`

[17] Levy, S. and Wilensky, U. (2008). "Inventing a 'mid level' to make ends meet: reasoning between the levels of complexity". *Cognition and Instruction*, **26**:1–47.

[18] *Logistic map.* (2007, January 10). In Wikipedia, The Free Encyclopedia. Retrieved 18:29, January 12, 2007, from `http://en.wikipedia.org/w/index.php?title=Logistic_map&oldid=99711208`

[19] Mazur, A. (2007, January 12). "Disbelievers in evolution [response to Miller, Scott and Okamoto]". Letter in *Science*, **315**:187.

[20] McLaughlin, J. (2006). "A gentle reminder that a hypothesis is never proven correct, nor is a theory ever proven to be true". *Journal of College Science Teaching*, **36**(1):60–61.

[21] Miller, J., Scott, E., and Okamoto, S. (2006, August 11). "Public acceptance of evolution". *Science*, **313**:765–766.

[22] Owen, R. (Feb 11, 2009). "Vatican buries the hatchet with Charles Darwin". *The Times*. Accessed May 6, 2011 from `http://www.timesonline.co.uk/tol/comment/faith/article5705331.ece`

[23] Penner, D. (2000). "Explaining systems: Investigating middles school students' understanding of emergent phenomena". *Journal of Research in Science Teaching*, **37**(8):784–806.

[24] Poundstone, W. (1985). *The Recursive Universe*. William Morrow and Company, Inc., New York.

[25] Steen, L. A. (2004). *Achieving Quantitative Literacy: An Urgent Challenge for Higher Education*. MAA Notes #62. Mathematical Association of America, Washington, D.C.

[26] *Theory*. (2007, February 9). In Wiktionary, the Free Dictionary. Retrieved 05:33, February 9, 2007, from `http://en.wiktionary.org/w/index.php?title=theory&oldid=1951713`

[27] Van Dooren, W., De Bock, D., Hessels, A., Janssens, D. and Verschaffel, L. (2005). "Not Everything is Proportional: Effects of Age and Problem Type on Propensities for Overgeneralization". *Cognition and Instruction*, **23**(1):57–86.

[28] Wigner, E. (1960). "The unreasonable effectiveness of mathematics in the natural sciences". *Communications in Pure and Applied Mathematics*, **13**:1–14.

[29] Wilensky, U. and Reisman, K. (2006). "Thinking like a wolf, a sheep, or a firefly: Learning biology through constructing and testing computational theories – an embodied modeling approach". *Cognition and Instruction*, **24**(2):171–209.

[30] Wilensky, U. and Resnick, M. (1999). "Thinking in levels: A dynamic systems approach to making sense of the world". *Journal of Science Education and Technology*, **8**(1):3–19.

[31] Wilson, J. Q. and G. L. Kelling. (1982). "Broken Windows". Downloaded 26 April 2011 from `http://www.codinghorror.com/blog/files/Atlantic%20Monthly%20-%20Broken%20Windows.htm`

[32] Wolfram, S. (2002). *A New Kind of Science*. Wolfram Media, Inc., Champaign, IL.