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Cover Page Footnote
Thanks to Eri Noguchi and Tomomi Lewis-Noguchi for discussions that helped form this article. Also, thanks to Eri Noguchi and Samuel Arbesman for reading an earlier draft of it.

Mathematics and *The Hunger Games*[1]

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Synopsis

The Hunger Games plot features a dystopian future in which twelve outer districts are oppressed by a centralized capital. The story focuses on the heroism of a sixteen-year-old girl named Katniss and how she tries to rise above the oppression that she experiences. It also features a special lottery and other twists that are sources of mathematical interest. This essay focuses on some of the mathematical issues raised by *The Hunger Games* in an effort to show that this story can be used to teach students (as well as other interested parties) some important concepts from mathematics.

Recently my wife, ten-year-old daughter, fourteen-year-old nephew, and I went to see the movie *The Hunger Games*. Even though my more literary friends and acquaintances have told me that *The Hunger Games*, both the movie and the book on which it is based, is a bit too sophisticated and, perhaps, violent for children my daughter's age, *The Hunger Games* happens to be "all the rage" among the students in my daughter's fourth grade class. I am sure this is a big part of the reason that she begged to read the book and see the movie. After consulting with her teacher, a well-known writer of children's books, we decided to let her read the book and take her to the movie. And I am very glad that we did.

The Hunger Games is a story in the dystopian science fiction tradition, along with *1984*, *The Handmaid's Tale*, *Brave New World*, etc. The basic

[1] A shorter version of this essay, entitled *Probability and Game Theory in the Hunger Games*, appeared in The Social Dimension series on Wired.com's website on April 10, 2012; see http://www.wired.com/wiredscience/2012/04/probability-and-game-theory-in-the-hunger-games/.

premise is that there is a society in what used to be North America. This society is made up of a centralized capital and twelve outer districts. Seventy-four years ago the districts staged an uprising against the capital which was violently put down. As punishment for this transgression, every year each of the districts must send one boy and one girl (it is not clear what would happen to transgendered persons in this world) to take part in the Hunger Games. This is a televised "contest" in which twenty-four children between the ages of 12-18 inclusive fight to the death until there is a sole survivor who is declared the winner. The story centers on Katniss, a smart, brave, and compassionate participant in the Hunger Games who is from District 12. *The Hunger Games* is a gripping and suspenseful tale that, I am told, does the equally compelling book (I have not read it) justice. It is masterful at depicting a decadent and oppressive regime in contrast to a desperate, hopeless, and oppressed people. One of the things I found most interesting about *The Hunger Games*, though, is the mathematical reflection that it inspires.[2]

The way districts choose which boy and girl to send to the capital for the Hunger Games is by lot. The movie does not provide a lot of details about how the lottery works. There are lines from a couple of characters which make it clear that the more times one's name appears in the lottery the more likely one is to be chosen for the game. Now it just so happens that I am a quantitative social worker/sociologist who teaches, among other things, statistics. Because of this sort of "bio," I suppose, I immediately started wondering how this lottery works. Luckily the details of the lottery can be found in Suzanne Collins' 2010 book *The Hunger Games* [4] on which the movie is based.

Once a child in a district turns 12, his or her name goes into the drawing for the Hunger Games. If the kid's name is drawn, her or his name does not appear in any future drawings either because the kid ends up dying in the Hunger Game or wins the game. That is, the names of dead kids and winners do not re-appear in future drawings. Ignoring, for the moment, certain complications, each previous year that a kid's name is not drawn her or his name appears one more time the next year. A twelve-year-old whose

[2] In this respect, it seems that I am not alone; Bush and Karp also appear to have been mathematically inspired by *The Hunger Games*, see [3].

name is not drawn will have their name appear two times when they are 13 (given that their name was not drawn at age 12), three times when they are 14 (given that their name was not drawn at age 13), etc. In other words, the equation that represents how the number of times kid's names appear in the lottery changes over time is the arithmetic progression:

$$names = a + (n - 1)d \tag{1}$$

where *names* is the number of times a kid's name appears in a given lottery, a = 1, n ranges from 1-7 inclusive (standing for ages 12-18 inclusive), and d is the common difference of the arithmetic progression, which in this case is 1 [2].

For example, a twelve-year-old would have names = 1 + (1 - 1)1 = 1. If that twelve-year-old's name is not picked, then he or she is eligible for the drawing at age 13 and their name would appear 1 + (2 -1)1 = 2 times. I suspect that many of the eleven-year-olds reading and watching *The Hunger Games* have no idea what an arithmetic progression is but, perhaps, those who teach them can use their interest in this story to introduce them to it.

As I said earlier, the movie makes it clear that as kid's names appear in given drawings a higher number of times they have a higher chance of being chosen for the game. Making some unrealistic assumptions, even in the dystopian world of *The Hunger Games*, it is easy to provide an example where this would be the case.

Suppose the parents in a given district gave birth to only ten children, five boys and five girls, and that all of these kids were born at the same time. This would mean that they would all turn 12 at the same time and that all their names would go into the lottery at the same time. Since the boys and girls drawings are done separately, each boy and each girl would have a 1/5 or 20% chance of being chosen for the game. Now in any given year, one girl and one boy will be chosen for the game and either because of victory or death, their names will not appear the next year. Thus, in the next year all the kids that are eligible for the drawing would be thirteen years old and all of their names would appear in the drawing two times. There would now be eight boys' names in the pool for boys ($2 \times 4 = 8$ names), eight girls' names in the pool for girls, and each boy and girl would have a 2/8 or 25% of being chosen for the game. That is, the number of times that each person's name appears in the lottery will have increased and the chance of being chosen will have as well. It should not be too difficult to see that each boy and girl will have a 3/9 or 33% chance of being chosen when they are 14, a 4/8 or 50% chance when they are 15, and at age 16 each would have a 5/5 or 100% chance of being chosen for the game. Figure 1 shows how the chance of being chosen increases with age:

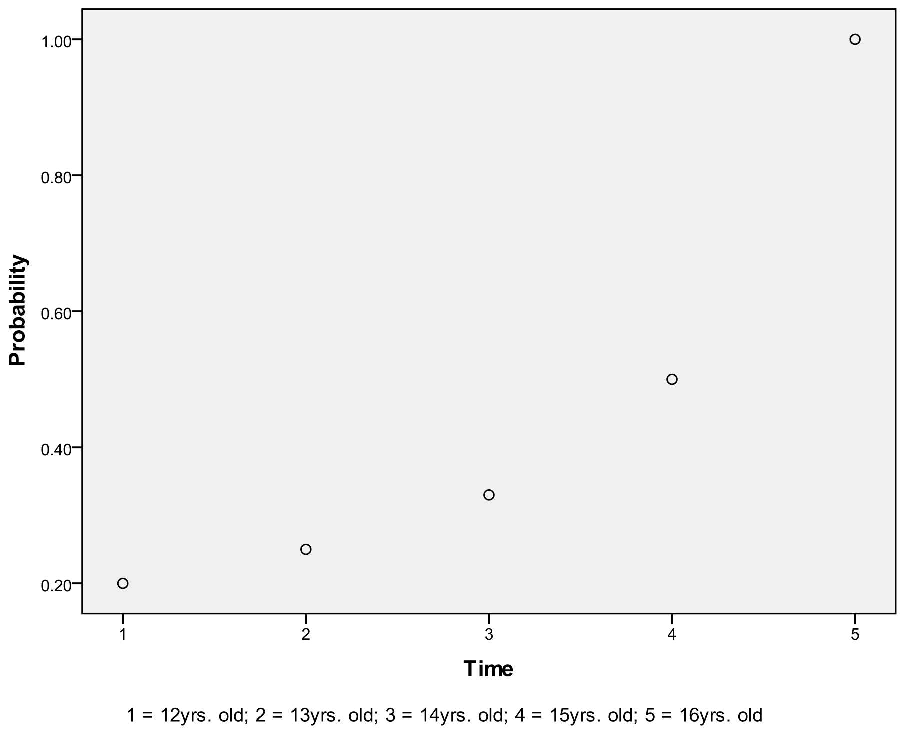

Figure 1: Probability of being chosen for the Hunger Games at certain ages, given not having been chosen at an earlier age.

It should not be too difficult to tell from the graph that the chance of being chosen not only increases with time but does so at an increasing rate. This can also be shown through the use of difference quotients. The general notation for such quotients is:

$$\text{difference}_{i-(i-1)} = (Y_i - Y_{i-1})/(X_i - X_{i-1}) \tag{2}$$

where Y_i refers to the value of the Y variable at a given point, Y_{i-1} refers to the value of that variable one point earlier, X_i refers to the value of the X

variable at point i, and X_{i-1} refers to its value at point $i-1$. Using the data from the previous paragraph we get the following difference quotients:

$$(25-20)/1=5, \qquad (33-25)/1=8, \qquad (50-33)/1=17,$$

and

$$(100-50)/1=50.$$

That is, we go through the differences 5, 8, 17, and 50, indicating clearly the increasing rate of change in the probabilities of being selected over time. In this highly simplified example, it is inevitable that parents will see their kids chosen for the game at some point. But highly simplified or not, this example could be an excellent way to introduce students interested in *The Hunger Games* to important mathematical topics such as graphs (both their construction and interpretation), difference quotients, and rates of change. These are all important steps toward the differential calculus.

Now let us consider some of the complications. The simple arithmetic progression discussed earlier is not a good model of how the number of times children's names appear in the lottery would change as they age. This is because *The Hunger Games* makes it clear that there is another way for children's names to appear more often in given drawings than merely getting older. The world of *The Hunger Games* is one of near starvation for many of those residing in the districts. One way to get more food is for a family to volunteer to have a child's name entered into the lottery a higher number of times. That is, a family with a thirteen-year-old whose name would ordinarily appear in the drawing twice could enter their child's name more than two times in return for a higher portion of food. Also, presumably, parents in the *The Hunger Games* world did not all have their kids at the same time and then have no more children. They would continue to have kids at different times. So some kids would be aging out of the Hunger Game drawings and others would be aging in. The math gets more complicated as these contingencies do. Changes in the numbers of times names appear in drawings and in probabilities of being selected could not really be figured out unless a lot of detail were known about demographics and the "choices" people made regarding risking a greater chance of peril for their children in the Hunger Games in return for eating a little better. I will not go into these complications but will briefly discuss another area of mathematics that relates to families choices about whether to enter their children's names more times in exchange for food—the mathematics of decision theory.

Decision theory is a branch of mathematics that models how people actually do or, if they were rational, should make decisions, especially in the face of uncertainty. By "uncertainty" decision theorists mean to refer to the ubiquitous fact that we often do not know what will happen in the world and, therefore, what the consequences of our actions will be. These theorists consider decision makers' preferences and possible states of the world as they model people's choices. From an abstract point of view, a typical decision maker faced with the necessity of making a choice under uncertainty is faced with the following question: given my preferences, given that I know at least some of the possible states of the world but do not know which will obtain, what should I do if I want to maximize the gain to me (or my whole family, etc.)? The decisions that are finally made in such situations depend crucially on people's assessments of the probability of certain states of the world obtaining.

How does decision theory apply to *The Hunger Games*? Families in the districts prefer more food but also prefer not to have their kids or themselves (if they are kids) chosen for the game. If they choose to enter their kids' names into the game more times, in exchange for more food, they may be increasing their kids' chances of being chosen for the games. But if they do not enter their kids' names more times, they may be increasing their chances of starving to death. They are faced with uncertainty because they do not know if their kids' names will be chosen (a possible state of the world), even if the chances of this occurring increases, and they do not know if they will actually starve to death (another possible state of the world), even if they refuse to enter their kids' names more times. The decision a family would end up making would depend crucially on their judgments about the probabilities of various states of the world obtaining (will they starve to death if they make such and such a choice, will their child get chosen for the game if they do such and such a thing, etc.). The minute one connects judgments and probabilities one raises the contentious issue among mathematicians/statisticians regarding how we should interpret probabilities.

Mathematicians, statisticians especially, have been engaged in a long debate about whether or not probabilities are "objective" or "subjective." The graph above can arguably be viewed as a graph of objective probabilities. To calculate these probabilities all I did was divide the number of times individual kids' names would appear in the pool by the total number of names in the pool—given that we agree on what it means to divide something by something else, what could be more objective than this procedure? The subjective, sometimes called Bayesian, view of probability, however, is different. Probabilities, according to this school, are simply numerical expressions of people's degree of belief in the occurrence of some uncertain outcome that they arrive at based on the information available to them. If they obtain more information, the probability of an outcome may change with this being perfectly reasonable. Assessments of probabilities, according to the Subjectivists (if you will), can be rational or irrational but they cannot be wrong in the sense of not corresponding to some probabilistic reality "out there" in the world somewhere [7]. The distinction between these two ways of interpreting the concept of probability can, perhaps, be seen more clearly by considering the decision about whether to enter kids' names into the lottery a higher number of times in return for more food.

A family's decision about whether to enter kids' names a higher number of times in exchange for more food will depend, as stated earlier, on their assessment of the probability of having their kid chosen for the game if they do so. Suppose a family's only child, Susan, decides to enter her name a higher number of times to get more food for herself and her family. She has concluded that by doing this she has increased her chance of being chosen for the game to 10%. The question is whether there is an objective probability "out there" that this 10% can be said to either correspond to or not, or if the 10% is just a numerical way of expressing Susan's belief regarding whether she will be chosen for the game or not. The debate about what probabilities are has implications for what we mean when we attach probabilities to plane crashes, dying from diseases, getting them in the first place, earthquakes in certain regions, and a host of other things that interest us, some of them life and death matters. *The Hunger Games* is an excellent way of introducing this important debate to students and others interested in mathematics.[3]

Another place where mathematics meets *The Hunger Games* can be seen by considering the actual game. Twenty-four kids engage in a contest of survival where the last one alive wins the game. If we thought of these twenty-four kids as approximated by twenty-four names in an urn where the names are well mixed, we might say that the probability (is it subjective or

[3]Freedman [6] and Hacking [8] provide nice overviews of the debate between objectivists and subjectivists on the foundations of statistics. Hacking is for more of a general audience while Freedman is a bit more technical.

objective?) of any given kid winning the game is 1/24, and the probability of the kid losing it is 23/24. Characters in *The Hunger Games* frequently wish children competing in the games "well" be expressing their desire that the odds always be in the children's favor. Now since odds are the probability of some outcome occurring divided by the probability of that outcome not occurring, using the urn model we can see that the odds of losing the game is $(23/24)/(1/24) = (23/24) \times (24/1) = 23/1$. That is, a kid is twenty-three times more likely to lose the game than to win it, certainly not favorable odds, assuming the kid would prefer not die in a Hunger Game. This example indicates how *The Hunger Games* can be used to explicate the notion of odds to students and other interested parties as well as how they are related to probabilities.

I suspect the urn model is not a good one for the Hunger Games. This is largely because of the fact that kids come to the Games with different skill sets. Katniss is an excellent archer, Cato (another main character) has trained all his life for the games, and others are faster, smarter, etc. than their competitors. So figuring out the probability of a given kid winning will depend on that kid's skill set and how their skill set compares to others. What we have here is a complicated exercise in calculating conditional probabilities, not a game that can be approximated by names being drawn from an urn. Simply having students think through what I have discussed so far, regarding probabilities and odds of winning, would be a great way of exploring conditional probabilities with them.

Yet another area of mathematics that is related to *The Hunger Games* is, rather appropriately, game theory. Game theory is a branch of mathematics that models *interdependent* decision making. By *interdependent* decision making, I mean situations (arguably most, if not all, those we face in life) where the outcomes of one's decision depends on decisions made by others. One of the most frequently discussed models in game theory is the Prisoner's Dilemma [5, 10].

Here is the story of the Prisoner's Dilemma. Two people who are suspected to have been involved in a serious crime are being interrogated separately by the police. The police inform each man that they know that they were involved in a serious crime but do not have enough evidence to convict them. They also inform the suspects that they know that they were involved in a minor crime and that they could easily convict them of this one. They offer each suspect the following deal. If one of them confesses but the other one does not, the one who confessed will be freed and the one who did not will do fifteen years in prison. If neither of them confesses, they will be easily convicted of the minor crime and both will do one year in prison. If both of them confess to the more serious crime, they will each do five years, instead of the full fifteen years, as a reward for their cooperation. Assuming the suspects would rather do less time in prison than more time, they would both be better off if they both kept quiet. But some simple tools of game theory can show that each prisoner is under compelling pressure to confess.

To see this consider the following table:

	Confess$_2$	Do Not Confess$_2$
Confess$_1$	5,5	0,15
Do Not Confess$_1$	15,0	1,1

Let us consider the matrix from Suspect 1's point of view (subscript "1" stands for Suspect 1 and subscript "2" is interpreted similarly). Suppose Suspect 2 confesses. If Suspect 1 confesses ze (I will use this instead of "he" and "she" out of respect for transgendered persons) will end up in prison for five years. If ze does not confess, ze will end up doing fifteen years. Now suppose Suspect 2 does not confess. In this case, if Suspect 1 confesses ze ends up a free person, and if ze does not confess ze ends up in prison for a year. Thus, regardless of what Suspect 2 does, Suspect 1 is better off confessing, and it can be shown that the same is true from Suspect 2's point of view. So there is great pressure on them to both confess. But, as can be seen from the table, if both suspects confess they will do worse (five years in prison) than they could have done if they had each kept quiet (only one year in prison). Thus, it looks like the individually rational thing to do is collectively irrational. Now let us apply this to *The Hunger Games*.

In both the movie and the book we see a coalition of some of the players develop where they attack other players as a group. As I considered this, being aware of game theory, I wondered how such an alliance could be stable, given the powerful incentive all members of the coalition have to kill each other in order to better position themselves to win the game. In fact, I wondered how members of the coalition would even get any sleep, especially given that they slept near each other. This may seem like a strange question, but the Prisoner's Dilemma game can show that it is not so strange.

Consider the following table:

	Do Not Sleep$_{all}$	Sleep$_{all}$
Do Not Sleep$_1$	Tired, Tired	Kill, Killed
Sleep$_1$	Killed, Kill	Rested, Rested

Here subscript "1" refers to any member of the coalition and subscript "all" refers to all other members of the coalition. Let us consider matters from any given member's perspective (the subscript 1 player). What if the other players do not sleep? If ze does not either, then ze will be tired and more vulnerable to better rested contestants. But if ze sleeps while others are awake, any one of them can kill zir (I will use this instead of him and her, again, out of respect for transgendered persons) in zir sleep. Presumably, it is better to be tired than dead, so ze is under tremendous pressure to stay awake. Now consider what is best from zir point of view if the others sleep. If ze does not sleep, ze can kill them all and be much better off in the sense that ze has fewer people to deal with to win the game. If ze does sleep, ze can be better rested. But assuming that being able to kill off a good number of potential opponents is better than being rested, ze is under tremendous pressure not to sleep. However all players are faced with this same dilemma. If they all choose not to sleep then all of them will end up being tired and more vulnerable to better rested contestants. So how do members of Cato's coalition in *The Hunger Games* get any sleep at all?

There is an extensive literature in mathematics and economics addressing the issue of why what would seem to be the most compelling outcome does not necessarily obtain in all situations resembling the Prisoner's Dilemma, see for instance [1, 9]. Relating this to *The Hunger Games*, this literature addresses the question of why coalition members would actually sleep when it would seem that there is a powerful incentive for them not to do so. This issue as well as others in game theory could be introduced to students by relating them to *The Hunger Games*.

As I write these lines, *The Hunger Games* is a blockbuster movie and top selling book. This, I suspect, is due to the fact that it is an interesting thriller. But what I have tried to show here is that it is also a great source of mathematical inspiration, perhaps the best such source we have seen from popular culture in some time. It could, I believe, easily be used to turn students and others on to some of the different, but no less compelling, thrills that can be found in mathematics.

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