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Q.954 and A.954, Quickie Problem and Solution

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Q954. *Proposed by Arthur Benjamin, Harvey Mudd College, Claremont, CA, and Michel Bataille, Rouen, France.*

Show that for positive integer n,

$$\sum_{k=0}^{n} \binom{n}{k}\binom{n+k}{k} = \sum_{k=0}^{n} 2^k \binom{n}{k}^2.$$

A954. Let $[n]$ denote the set $\{1, 2, \ldots, n\}$ and S denote the set of ordered pairs (A, B) where A is a subset of $[n]$ and B is an n-subset of $[2n]$ that is disjoint from A. We can select elements for S in two ways:

(1) For $0 \leq k \leq n$, let Z be a k-subset of $[n]$. The let $A = Z^c$, the complement of Z, which is an $(n-k)$-subset of $[n]$, and let B be an n-subset of $\{n+1, \ldots, 2n\} \cup Z$. This yields

$$|S| = \sum_{k=0}^{n} \binom{n}{k}\binom{n+k}{k}.$$

(2) For $0 \leq k \leq n$, choose a k-subset B_1 from $\{n+1, \ldots, 2n\}$ and a k-subset B_2 of $[n]$. Then form $B = B_1 \cup B_2^c$, and choose A from among the 2^k subsets of B_2. This leads to

$$|S| = \sum_{k=0}^{n} 2^k \binom{n}{k}^2.$$

This completes the proof.

Note. Another proof, using lattice paths, can be found in Robert A. Sulanke's article, Objects Counted by the Central Delannoy Numbers, *The Journal of Integer Sequences*, Vol 6, 2003. A proof by polynomials is in Michel Bataille's paper Some Identities about an Old Combinatorial Sum, *The Mathematical Gazette*, March 2003, pp. 144-8.

A slight change in the above proof leads to

$$\sum_{k=0}^{n} \binom{n}{k}\binom{m+k}{n} = \sum_{k=0}^{n} 2^k \binom{n}{k}\binom{m}{k},$$

for $m \geq n$, a generalization proved by Li Zhou using lattice paths in *The Mathematical Gazette.*