1-1-1973

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Summary

The construction of sorting networks has been a topic of much recent discussion [1] - [5]. In view of the apparent difficulty of verifying whether a reasonably large proposed sorting network actually does sort, the most useful approach for constructing large networks seems to be to devise a recursive scheme which constructs a network which is guaranteed to sort, obviating the verification phase. Examples of this approach are presented in [1],[5]. In this note, another such approach is presented.

The most economical 16-line sorter known has been constructed by Green [3], [4]. His approach is to successively sort lines whose indices differ in one component of the binary expansion. This yields a partial ordering of the lines which is isomorphic to a Boolean "n-cube" configuration. This configuration is then further sorted to yield a linear order. The network for accomplishing this is constructed in a clever, but *ad hoc* manner, and no techniques for extending this approach to larger numbers of lines have appeared.

In this note such a technique is presented. However, it suffers from the fact that it produces networks which are no more economical than the odd-even merge networks of Batcher [1]. Nevertheless, some insight may result from a knowledge of this technique.

The approach is to reduce an n-cube configuration to an n-m cube in which the vertices represent linear orders of m components. A recursive rule is given which applies this technique to obtain a complete sorting network and the correctness of the rule is proved. It is then shown that the number of comparisons for an n-line network are the same as Batcher's construction, although the networks are definitely not isomorphic to Batcher's. For certain numbers of lines, this method yields networks which are related to Batcher's by a kind of "flipping" operation described in [2]. Precisely what relation holds between these two constructions has not yet been discovered.

A complete presentation of these results appears in [6]. The construction is derived for the more general k-ary n-cube, but upper bounds are only shown for k = 2 (the "Boolean" case). Whether other values of k yield better results has not been thoroughly investigated. Proofs of correctness are done in terms of partial orders, using a useful and general lemma about "cross products" of partial orders and the technique of Liu [7].

This work was sponsored by the National Science Foundation through grant GJ-3012, and by the Bell Laboratories, Murray Hill, N.J.

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